Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.5-Secant/119-4.5.1.3-d-sin-^n-a+b-sec-^m

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [306]. This is test number [119].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (306)	0.00 (0)
Rubi	99.67 (305)	0.33 (1)
Maple	87.25 (267)	12.75 (39)
Fricas	79.41 (243)	20.59 (63)
Mupad	63.07 (193)	36.93 (113)
Giac	62.42 (191)	37.58 (115)
Maxima	57.19 (175)	42.81 (131)
Sympy	2.61 (8)	97.39 (298)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

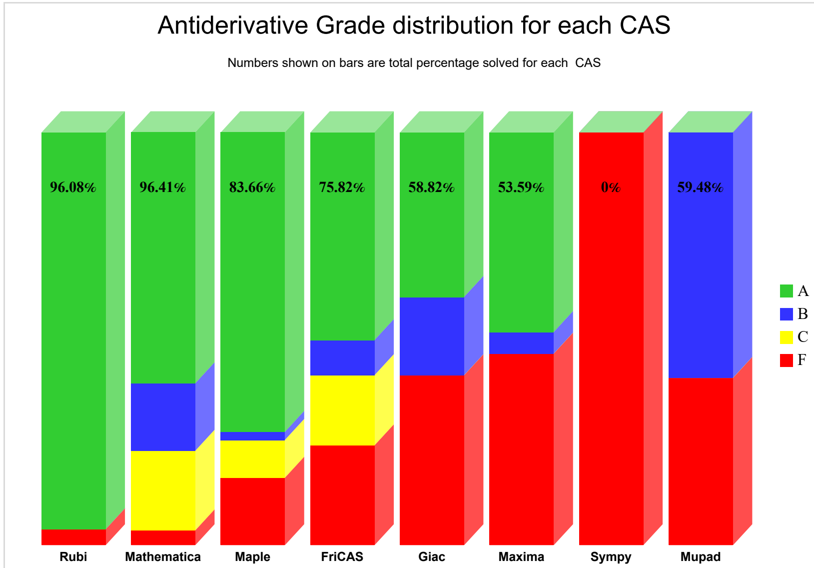
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

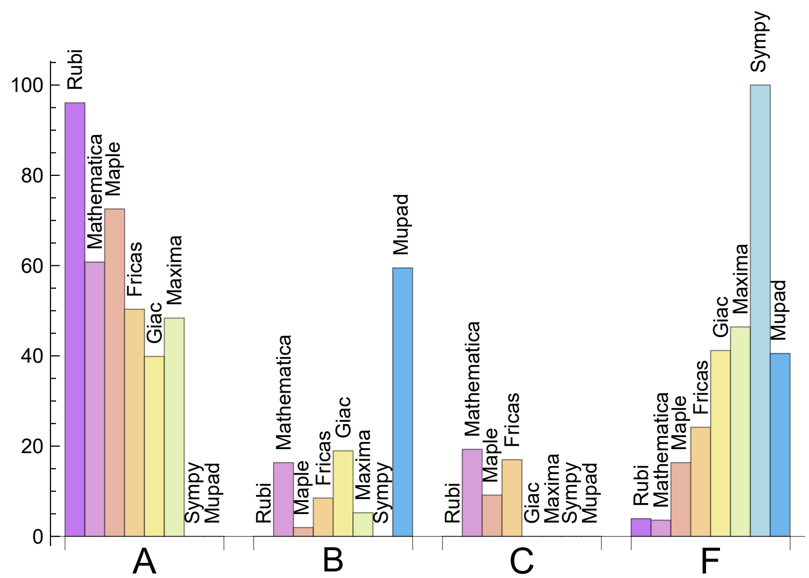
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.078	0.000	0.000	3.922
Maple	72.549	1.961	9.150	16.340
Mathematica	60.784	16.340	19.281	3.595
Fricas	50.327	8.497	16.993	24.183
Maxima	48.366	5.229	0.000	46.405
Giac	39.869	18.954	0.000	41.176
Mupad	0.000	59.477	0.000	40.523
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	1	100.00	0.00	0.00
Maple	39	100.00	0.00	0.00
Fricas	63	74.60	25.40	0.00
Mupad	113	0.00	100.00	0.00
Giac	115	97.39	0.00	2.61
Maxima	131	70.99	16.79	12.21
Sympy	298	62.75	37.25	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.27
Maxima	0.30
Rubi	0.44
Giac	1.14
Mathematica	4.09
Maple	4.09
Mupad	11.26
Sympy	46.23

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	22.50	0.96	22.00	0.96
Maxima	128.71	1.15	106.00	1.00
Rubi	169.94	1.00	127.00	1.00
Maple	240.84	1.27	128.00	1.01
Giac	249.48	1.76	149.00	1.42
Fricas	250.22	1.67	140.00	1.27
Mupad	250.50	1.49	107.00	1.05
Mathematica	324.61	1.82	136.50	1.09

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

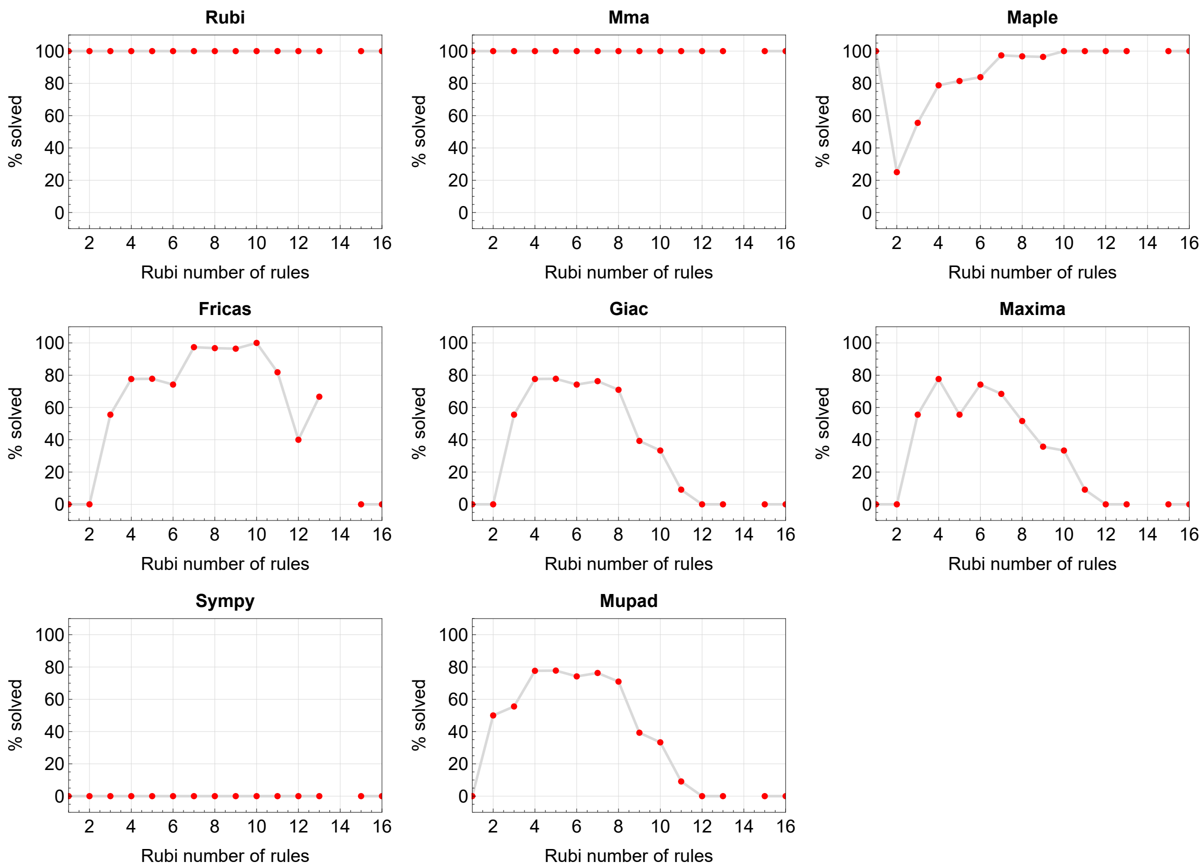


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

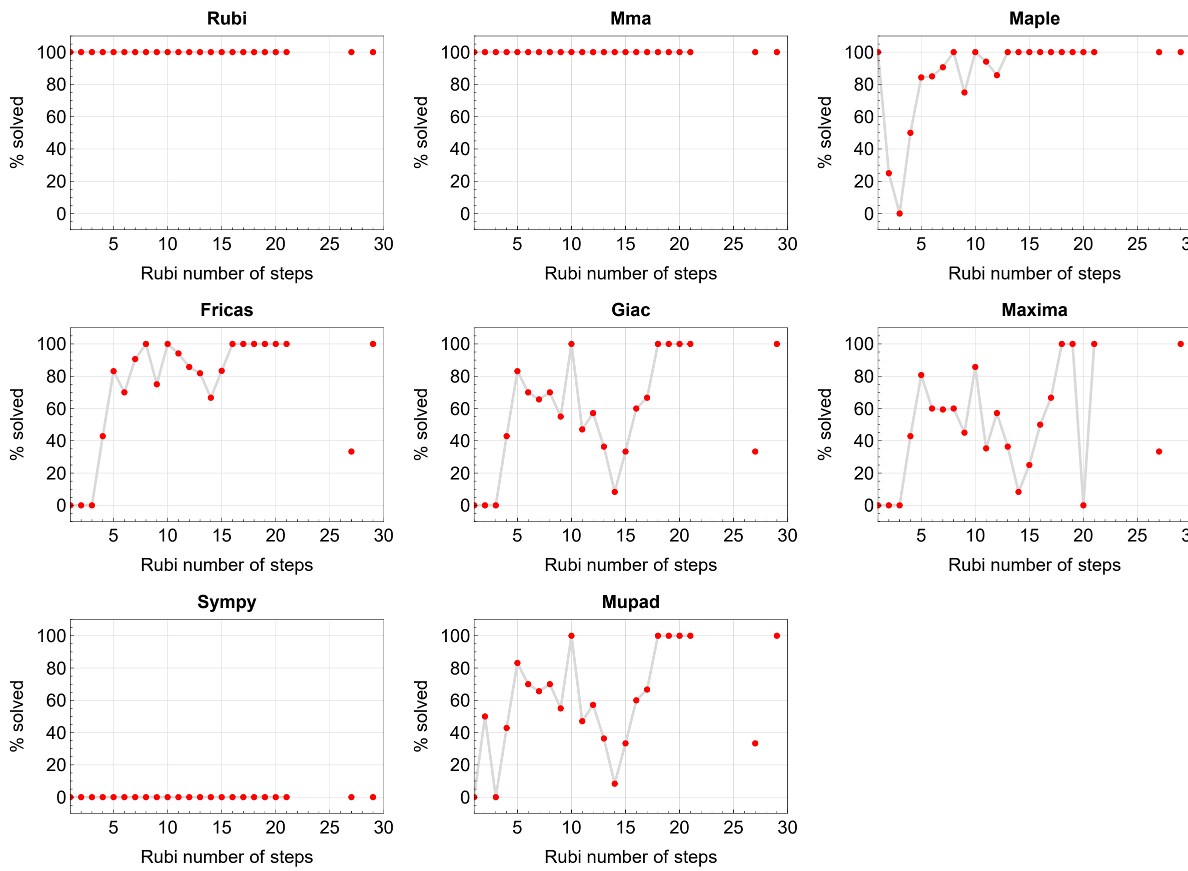


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

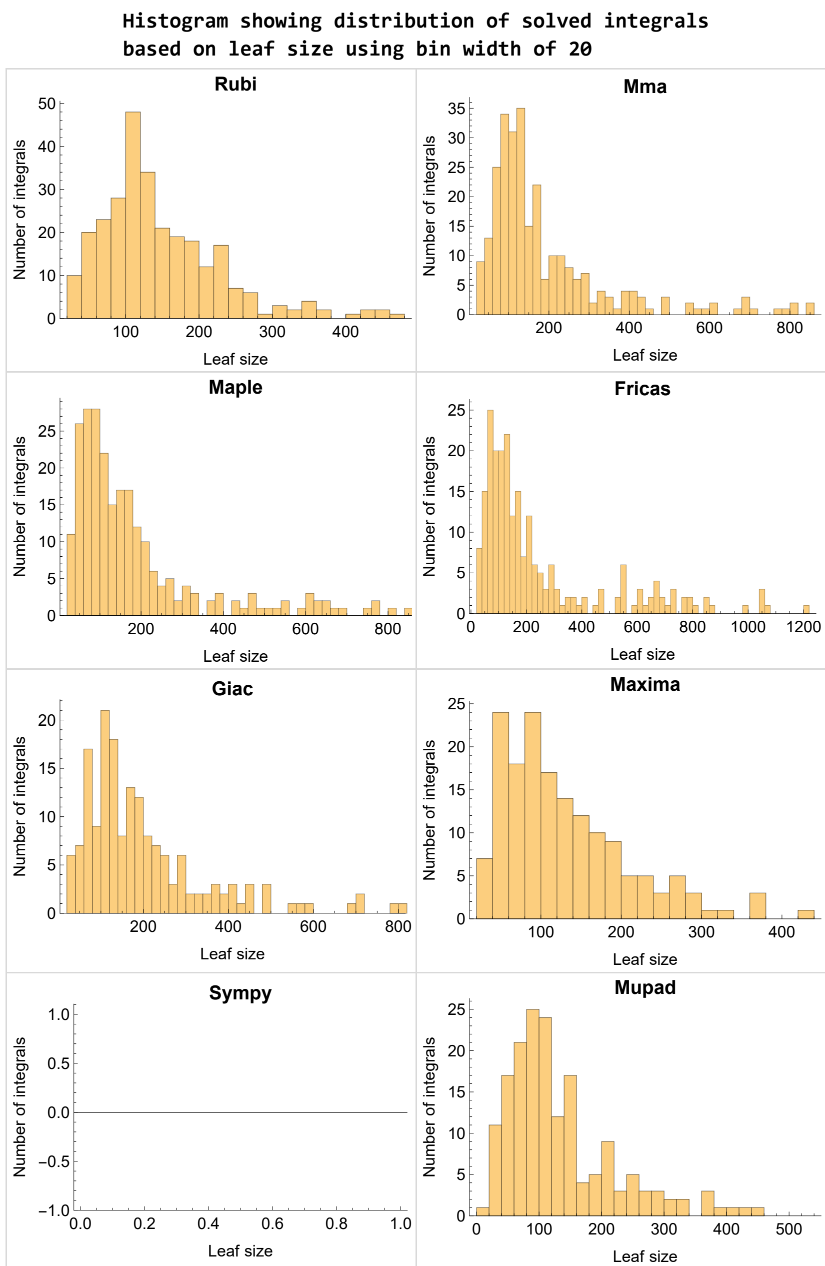


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

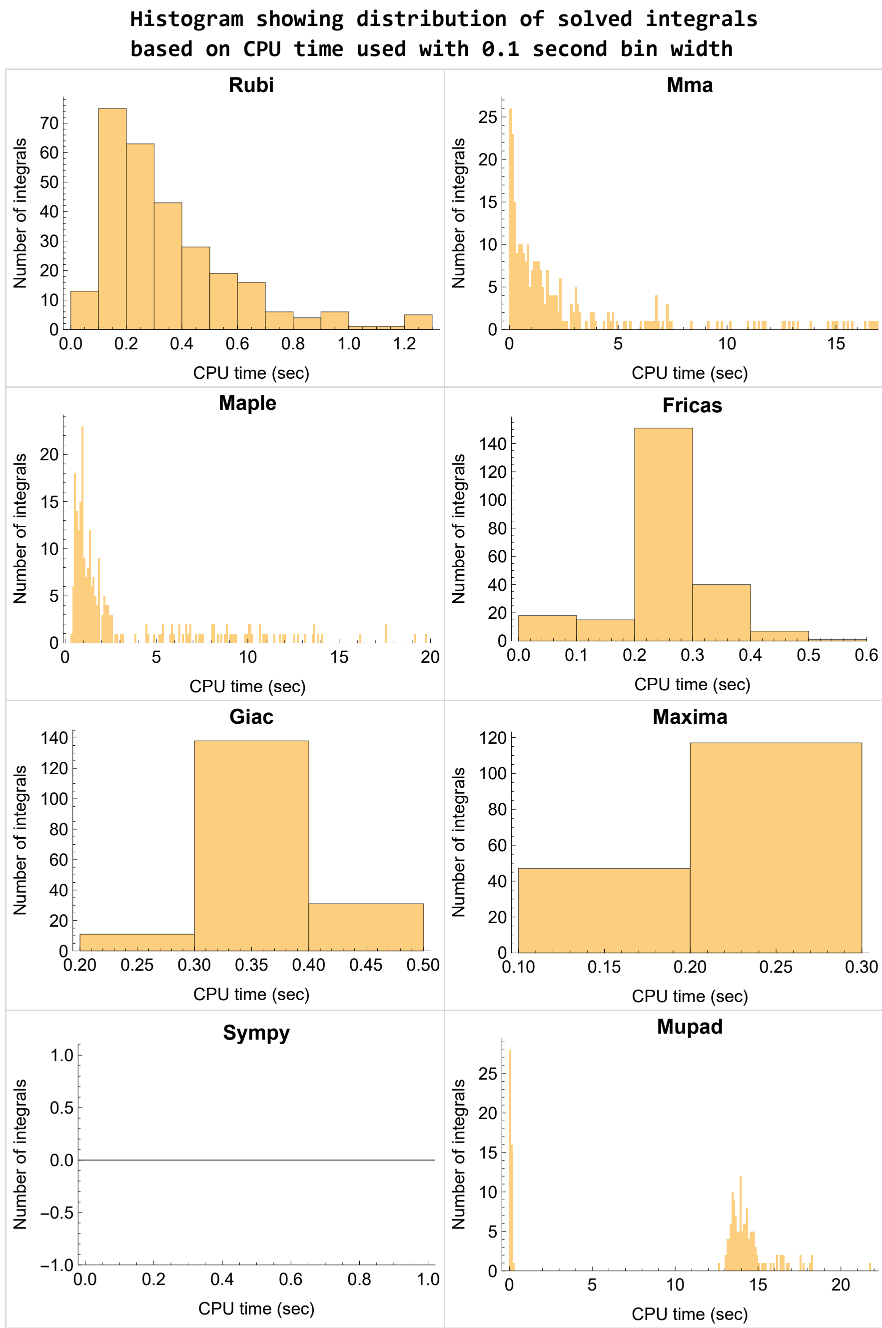


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

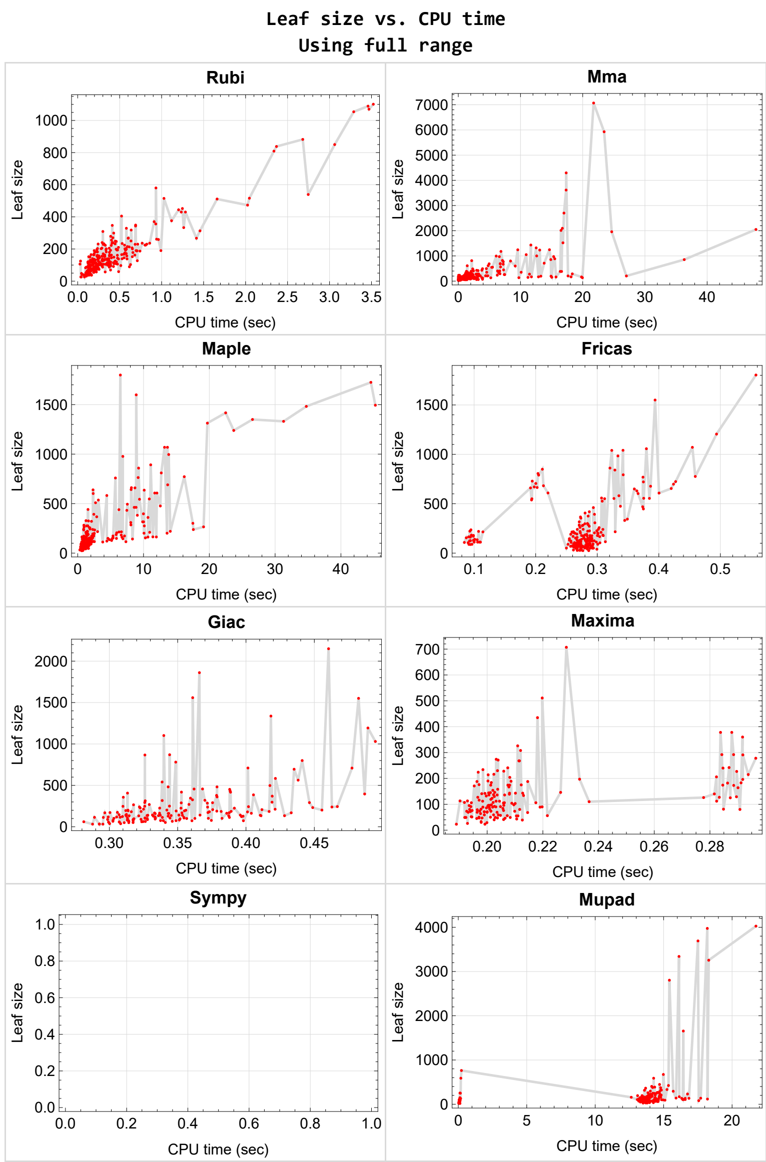


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{111, 114, 115, 116, 117, 118, 119, 139, 140, 141, 142, 143, 144, 152, 153, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 254, 260, 261, 262, 272, 275, 276, 287, 288, 289, 290, 291, 292\}$

Maple $\{241, 242, 243, 244, 245, 246, 247, 248, 255, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 300, 301\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

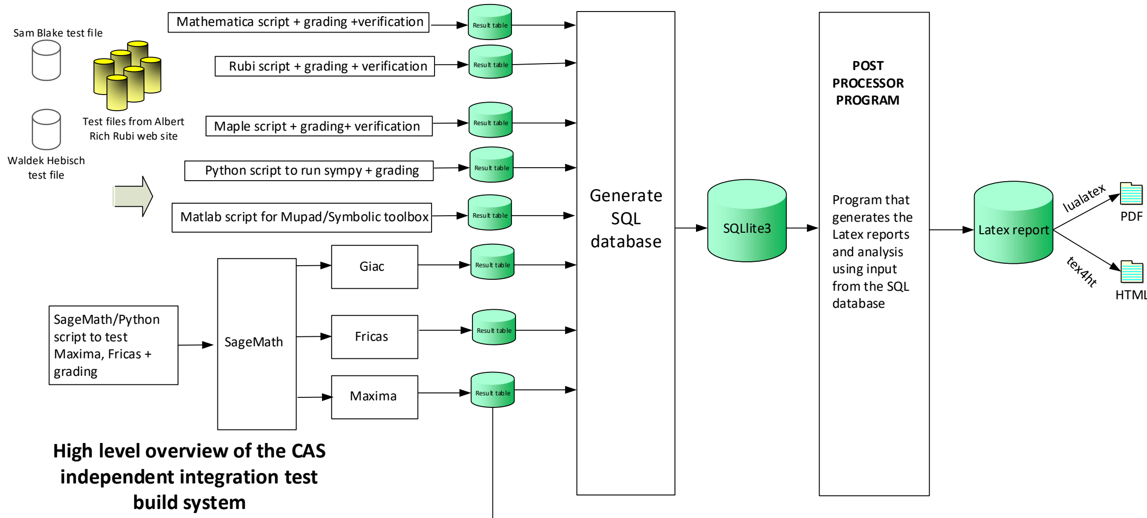
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

B grade { }

C grade { }

F normal fail { 276 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 120, 122, 124, 126, 127, 129, 131, 133, 134, 135, 136, 145, 146, 147, 148, 150, 154, 155, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 179, 180, 181, 185, 186, 187, 188, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 230, 231, 232, 249, 250, 252, 253, 254, 256, 257, 258, 259, 269, 270, 271, 281, 283, 285, 293, 295, 297, 299, 300, 302, 304, 306 }

B grade { 6, 32, 33, 34, 35, 36, 37, 51, 52, 53, 54, 55, 56, 70, 71, 72, 73, 140, 141, 142, 143, 144, 149, 151, 156, 157, 158, 159, 164, 178, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 229, 251, 255, 260, 261, 262, 268, 272, 275, 276 }

C grade { 14, 15, 16, 17, 18, 111, 114, 115, 116, 117, 118, 119, 121, 123, 125, 128, 130, 132, 137, 138, 139, 152, 153, 171, 172, 173, 226, 227, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 282, 284, 286, 287, 288, 289, 290, 291, 292, 294, 296, 298, 301, 303, 305 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253 }

B grade { 203, 251, 252, 254, 255, 256 }

C grade { 44, 56, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F normal fail { 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232 }

B grade { 9, 16, 17, 18, 28, 37, 46, 47, 64, 71, 72, 73, 82, 97, 98, 99, 165, 166, 167, 202, 208, 214, 215, 225, 226, 227 }

C grade { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F normal fail { 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276 }

F(-1) timedout fail { 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 209, 210, 211, 212, 213, 214, 221, 222, 223, 224, 225, 226 }

B grade { 65, 66, 67, 68, 71, 72, 73, 83, 84, 85, 86, 100, 101, 102, 215, 227 }

C grade { }

F normal fail { 108, 109, 110, 111, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 127, 128, 129, 130, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 242, 243, 244, 245, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276, 281, 283, 284, 285, 286, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 303, 304, 305, 306 }

F(-1) timedout fail { 112, 118, 119, 125, 126, 131, 132, 133, 238, 239, 240, 241, 246, 247, 248, 282, 287, 288, 294, 300, 301, 302 }

F(-2) exception fail { 203, 204, 205, 206, 207, 208, 216, 217, 218, 219, 220, 228, 229, 230, 231, 232 }

Giac

A grade { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 162, 163, 168, 175, 176, 177, 180, 186, 187, 191, 193, 194, 195, 198, 199, 200, 201, 206, 211, 212, 216, 217, 218, 219, 220, 223, 224, 229, 231, 232 }

B grade { 1, 2, 3, 19, 20, 21, 24, 38, 40, 43, 44, 76, 77, 81, 93, 160, 161, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 178, 179, 181, 182, 183, 184, 185, 188, 189, 190, 192, 196, 197, 202, 203, 204, 205, 207, 208, 209, 210, 213, 214, 215, 221, 222, 225, 226, 227, 228, 230 }

C grade { }

F normal fail { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 270, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F(-1) timedout fail { }

F(-2) exception fail { 143, 147, 269 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 148, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 270 }

C grade { }

F normal fail { }

F(-1) timedout fail { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 41, 42, 43, 44, 50, 51, 52, 53, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 110, 111, 112, 116, 117, 118, 122, 123, 124, 125, 130, 131, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 148, 149, 150, 153, 154, 157, 158, 159, 161, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 186, 187, 188, 189, 191, 192, 193, 194, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 212, 213, 214, 215, 216, 217, 218, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 245, 246, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 270, 271, 275, 282, 283, 284, 285, 286, 289, 290, 291, 294, 295, 296, 297, 301, 302, 303, 304 }

F(-1) timedout fail { 1, 2, 9, 10, 17, 18, 19, 20, 27, 28, 29, 30, 35, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 54, 55, 56, 57, 58, 59, 60, 73, 74, 75, 76, 77, 78, 83, 90, 91, 92, 93, 94, 95, 100, 107, 108, 109, 113, 114, 115, 119, 120, 121, 126, 127, 128, 129, 132, 133, 140, 145, 146, 147, 151, 152, 155, 156, 160, 167, 179, 184, 185, 190, 195, 196, 197, 198, 209, 210, 211, 221, 222, 223, 233, 234, 239, 240, 241, 242, 243, 244, 247, 248, 252, 263, 268, 269, 272, 273, 276, 277, 281, 287, 288, 292, 293, 298, 299, 300, 305, 306 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	106	107	113	115	0	293	111
N.S.	1	1.00	0.70	0.70	0.74	0.76	0.00	1.93	0.73
time (sec)	N/A	0.131	0.484	1.585	0.190	0.313	0.000	0.326	0.138

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	113	87	91	93	0	247	89
N.S.	1	1.00	0.95	0.73	0.76	0.78	0.00	2.08	0.75
time (sec)	N/A	0.121	0.183	1.462	0.194	0.289	0.000	0.310	0.082

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	83	67	69	71	0	201	67
N.S.	1	1.00	0.95	0.77	0.79	0.82	0.00	2.31	0.77
time (sec)	N/A	0.102	0.076	1.358	0.193	0.290	0.000	0.314	0.057

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	47	47	49	0	66	45
N.S.	1	1.00	0.98	0.81	0.81	0.84	0.00	1.14	0.78
time (sec)	N/A	0.107	0.050	1.308	0.197	0.292	0.000	0.303	0.053

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	37	24	23	25	0	32	20
N.S.	1	1.00	1.42	0.92	0.88	0.96	0.00	1.23	0.77
time (sec)	N/A	0.044	0.015	0.618	0.199	0.286	0.000	0.293	0.042

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	65	33	26	31	0	58	17
N.S.	1	1.00	2.17	1.10	0.87	1.03	0.00	1.93	0.57
time (sec)	N/A	0.082	0.043	0.423	0.193	0.261	0.000	0.307	0.122

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	120	59	52	93	0	102	53
N.S.	1	1.00	1.64	0.81	0.71	1.27	0.00	1.40	0.73
time (sec)	N/A	0.133	0.045	1.060	0.194	0.273	0.000	0.303	0.074

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	176	83	95	193	0	149	99
N.S.	1	1.00	1.49	0.70	0.81	1.64	0.00	1.26	0.84
time (sec)	N/A	0.166	0.036	0.821	0.194	0.267	0.000	0.313	0.103

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	232	103	136	307	0	196	142
N.S.	1	1.00	1.42	0.63	0.83	1.88	0.00	1.20	0.87
time (sec)	N/A	0.191	0.199	1.125	0.204	0.296	0.000	0.312	13.578

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	106	116	127	123	0	174	150
N.S.	1	1.00	0.64	0.70	0.77	0.75	0.00	1.05	0.91
time (sec)	N/A	0.196	1.095	1.855	0.197	0.289	0.000	0.306	13.774

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	86	96	106	102	0	146	120
N.S.	1	1.00	0.68	0.76	0.83	0.80	0.00	1.15	0.94
time (sec)	N/A	0.176	0.674	1.553	0.194	0.287	0.000	0.307	13.664

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	86	76	81	79	0	118	90
N.S.	1	1.00	0.97	0.85	0.91	0.89	0.00	1.33	1.01
time (sec)	N/A	0.147	0.293	1.471	0.198	0.281	0.000	0.305	13.455

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	55	59	55	0	88	80
N.S.	1	1.00	1.06	1.08	1.16	1.08	0.00	1.73	1.57
time (sec)	N/A	0.102	0.228	0.923	0.204	0.292	0.000	0.299	13.291

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	41	42	50	63	0	50	29
N.S.	1	1.00	1.11	1.14	1.35	1.70	0.00	1.35	0.78
time (sec)	N/A	0.135	0.156	0.749	0.201	0.278	0.000	0.311	13.688

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	63	76	108	0	79	65
N.S.	1	1.00	1.00	0.91	1.10	1.57	0.00	1.14	0.94
time (sec)	N/A	0.139	0.100	0.958	0.195	0.270	0.000	0.313	14.297

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	83	96	190	0	107	97
N.S.	1	1.00	0.90	0.82	0.95	1.88	0.00	1.06	0.96
time (sec)	N/A	0.138	0.081	0.889	0.201	0.283	0.000	0.326	14.525

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	113	103	116	281	0	136	128
N.S.	1	1.00	0.86	0.79	0.89	2.15	0.00	1.04	0.98
time (sec)	N/A	0.151	0.248	1.005	0.197	0.271	0.000	0.328	14.439

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	135	123	136	366	0	164	159
N.S.	1	1.00	0.82	0.75	0.82	2.22	0.00	0.99	0.96
time (sec)	N/A	0.160	0.302	1.028	0.206	0.273	0.000	0.337	14.790

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	127	181	146	167	0	370	146
N.S.	1	1.00	0.69	0.99	0.80	0.91	0.00	2.02	0.80
time (sec)	N/A	0.227	1.209	2.364	0.226	0.304	0.000	0.419	14.383

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	107	151	107	128	0	320	105
N.S.	1	1.00	0.82	1.15	0.82	0.98	0.00	2.44	0.80
time (sec)	N/A	0.196	0.389	2.054	0.192	0.283	0.000	0.370	14.311

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	121	94	115	0	270	91
N.S.	1	1.00	0.78	1.08	0.84	1.03	0.00	2.41	0.81
time (sec)	N/A	0.198	0.207	1.894	0.200	0.273	0.000	0.358	14.264

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	65	91	56	76	0	74	54
N.S.	1	1.00	1.05	1.47	0.90	1.23	0.00	1.19	0.87
time (sec)	N/A	0.155	0.210	2.018	0.222	0.269	0.000	0.349	0.060

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	34	41	51	0	51	41
N.S.	1	1.00	0.72	0.79	0.95	1.19	0.00	1.19	0.95
time (sec)	N/A	0.097	0.296	0.826	0.206	0.273	0.000	0.317	0.059

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	36	67	43	61	0	115	35
N.S.	1	1.00	0.75	1.40	0.90	1.27	0.00	2.40	0.73
time (sec)	N/A	0.139	0.131	0.448	0.211	0.262	0.000	0.308	14.252

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	75	95	68	112	0	135	61
N.S.	1	1.00	1.09	1.38	0.99	1.62	0.00	1.96	0.88
time (sec)	N/A	0.176	0.450	0.815	0.215	0.298	0.000	0.336	13.300

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	103	118	104	209	0	191	109
N.S.	1	1.00	0.90	1.03	0.90	1.82	0.00	1.66	0.95
time (sec)	N/A	0.216	1.214	0.967	0.209	0.261	0.000	0.349	0.109

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	136	172	143	289	0	238	147
N.S.	1	1.00	0.85	1.08	0.89	1.81	0.00	1.49	0.92
time (sec)	N/A	0.236	1.130	1.352	0.213	0.285	0.000	0.372	0.118

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	164	210	197	461	0	291	203
N.S.	1	1.00	0.80	1.02	0.96	2.25	0.00	1.42	0.99
time (sec)	N/A	0.289	2.620	1.267	0.233	0.294	0.000	0.376	14.143

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	144	164	215	185	0	225	293
N.S.	1	1.00	0.72	0.82	1.08	0.93	0.00	1.13	1.47
time (sec)	N/A	0.458	1.065	2.291	0.294	0.280	0.000	0.391	15.701

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	142	174	158	0	193	235
N.S.	1	1.00	0.79	0.90	1.11	1.01	0.00	1.23	1.50
time (sec)	N/A	0.377	0.469	2.128	0.285	0.288	0.000	0.355	14.710

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	94	116	126	133	0	161	177
N.S.	1	1.00	0.82	1.01	1.10	1.16	0.00	1.40	1.54
time (sec)	N/A	0.353	0.150	2.142	0.278	0.288	0.000	0.343	14.164

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	243	78	81	104	0	128	117
N.S.	1	1.00	3.33	1.07	1.11	1.42	0.00	1.75	1.60
time (sec)	N/A	0.181	1.224	1.273	0.285	0.304	0.000	0.343	13.581

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	401	77	74	101	0	90	70
N.S.	1	1.00	7.04	1.35	1.30	1.77	0.00	1.58	1.23
time (sec)	N/A	0.328	6.655	0.779	0.201	0.269	0.000	0.323	13.502

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	228	98	113	159	0	104	91
N.S.	1	1.00	2.62	1.13	1.30	1.83	0.00	1.20	1.05
time (sec)	N/A	0.376	1.971	1.335	0.193	0.267	0.000	0.334	15.097

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	317	154	144	206	0	136	124
N.S.	1	1.00	2.46	1.19	1.12	1.60	0.00	1.05	0.96
time (sec)	N/A	0.288	1.425	1.327	0.201	0.263	0.000	0.345	13.727

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	428	191	175	272	0	168	159
N.S.	1	1.00	2.63	1.17	1.07	1.67	0.00	1.03	0.98
time (sec)	N/A	0.323	2.955	1.326	0.212	0.287	0.000	0.353	13.467

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	1050	217	204	406	0	200	194
N.S.	1	1.00	5.22	1.08	1.01	2.02	0.00	1.00	0.97
time (sec)	N/A	0.340	10.916	1.484	0.208	0.280	0.000	0.356	13.097

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	148	217	158	182	0	396	157
N.S.	1	1.00	0.73	1.07	0.78	0.90	0.00	1.95	0.77
time (sec)	N/A	0.236	1.932	3.018	0.204	0.309	0.000	0.487	12.635

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	106	118	107	121	0	239	107
N.S.	1	1.00	0.81	0.90	0.82	0.92	0.00	1.82	0.82
time (sec)	N/A	0.210	0.587	2.361	0.202	0.269	0.000	0.463	13.503

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	108	172	106	130	0	297	107
N.S.	1	1.00	0.81	1.28	0.79	0.97	0.00	2.22	0.80
time (sec)	N/A	0.201	0.299	2.108	0.206	0.272	0.000	0.419	13.356

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	86	114	80	104	0	102	80
N.S.	1	1.00	0.88	1.16	0.82	1.06	0.00	1.04	0.82
time (sec)	N/A	0.129	0.156	2.169	0.212	0.263	0.000	0.382	13.276

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	65	46	55	65	0	64	52
N.S.	1	1.00	1.05	0.74	0.89	1.05	0.00	1.03	0.84
time (sec)	N/A	0.114	0.176	0.980	0.210	0.276	0.000	0.326	0.057

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	81	90	56	76	0	142	49
N.S.	1	1.00	1.21	1.34	0.84	1.13	0.00	2.12	0.73
time (sec)	N/A	0.163	0.115	0.492	0.207	0.266	0.000	0.332	13.299

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	132	84	132	0	189	75
N.S.	1	1.00	1.00	1.50	0.95	1.50	0.00	2.15	0.85
time (sec)	N/A	0.223	0.733	0.787	0.206	0.267	0.000	0.379	0.095

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	100	153	103	177	0	186	96
N.S.	1	1.00	0.90	1.38	0.93	1.59	0.00	1.68	0.86
time (sec)	N/A	0.201	0.723	0.996	0.210	0.279	0.000	0.389	0.111

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	129	164	145	297	0	243	151
N.S.	1	1.00	0.82	1.04	0.92	1.89	0.00	1.55	0.96
time (sec)	N/A	0.237	0.781	1.122	0.201	0.280	0.000	0.402	13.293

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	159	210	189	419	0	292	195
N.S.	1	1.00	0.79	1.04	0.94	2.07	0.00	1.45	0.97
time (sec)	N/A	0.279	0.886	1.251	0.209	0.287	0.000	0.446	13.426

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	156	197	291	204	0	244	320
N.S.	1	1.00	0.74	0.94	1.39	0.97	0.00	1.16	1.52
time (sec)	N/A	0.486	1.794	2.484	0.292	0.301	0.000	0.467	14.836

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	136	199	240	177	0	212	261
N.S.	1	1.00	0.75	1.09	1.32	0.97	0.00	1.16	1.43
time (sec)	N/A	0.341	0.570	2.516	0.285	0.300	0.000	0.421	14.533

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	114	153	182	152	0	180	204
N.S.	1	1.00	0.83	1.11	1.32	1.10	0.00	1.30	1.48
time (sec)	N/A	0.295	0.234	2.265	0.286	0.265	0.000	0.401	14.384

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	300	126	127	125	0	102	90
N.S.	1	1.00	3.06	1.29	1.30	1.28	0.00	1.04	0.92
time (sec)	N/A	0.223	2.197	1.548	0.283	0.295	0.000	0.372	13.396

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	244	103	137	122	0	106	98
N.S.	1	1.00	3.05	1.29	1.71	1.52	0.00	1.32	1.22
time (sec)	N/A	0.248	1.521	0.805	0.202	0.257	0.000	0.355	14.622

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	290	125	188	178	0	123	116
N.S.	1	1.00	2.64	1.14	1.71	1.62	0.00	1.12	1.05
time (sec)	N/A	0.274	6.706	1.487	0.215	0.288	0.000	0.397	18.203

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	353	137	228	225	0	141	136
N.S.	1	1.00	2.14	0.83	1.38	1.36	0.00	0.85	0.82
time (sec)	N/A	0.524	2.013	1.399	0.206	0.287	0.000	0.396	17.708

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	430	192	268	278	0	169	169
N.S.	1	1.00	2.24	1.00	1.40	1.45	0.00	0.88	0.88
time (sec)	N/A	0.392	3.788	1.556	0.211	0.276	0.000	0.433	16.142

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	1000	259	308	375	0	202	204
N.S.	1	1.00	4.31	1.12	1.33	1.62	0.00	0.87	0.88
time (sec)	N/A	0.446	12.602	1.751	0.212	0.285	0.000	0.456	14.120

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	62	89	89	89	0	141	110
N.S.	1	1.00	0.68	0.98	0.98	0.98	0.00	1.55	1.21
time (sec)	N/A	0.219	3.217	0.830	0.198	0.268	0.000	0.312	0.090

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	52	69	69	69	0	119	84
N.S.	1	1.00	0.71	0.95	0.95	0.95	0.00	1.63	1.15
time (sec)	N/A	0.192	1.116	0.659	0.204	0.257	0.000	0.299	0.063

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	42	49	49	49	0	97	58
N.S.	1	1.00	0.76	0.89	0.89	0.89	0.00	1.76	1.05
time (sec)	N/A	0.193	0.172	0.533	0.203	0.274	0.000	0.318	0.072

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	29	29	29	0	32	26
N.S.	1	1.00	0.86	0.78	0.78	0.78	0.00	0.86	0.70
time (sec)	N/A	0.163	0.161	0.506	0.200	0.273	0.000	0.288	13.499

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	28	32	30	28	0	34	25
N.S.	1	1.00	0.90	1.03	0.97	0.90	0.00	1.10	0.81
time (sec)	N/A	0.103	0.057	0.325	0.198	0.271	0.000	0.293	0.055

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	35	47	60	0	56	33
N.S.	1	1.00	1.16	0.60	0.81	1.03	0.00	0.97	0.57
time (sec)	N/A	0.117	0.078	0.535	0.204	0.283	0.000	0.299	13.502

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	92	55	86	138	0	129	75
N.S.	1	1.00	1.12	0.67	1.05	1.68	0.00	1.57	0.91
time (sec)	N/A	0.218	0.377	0.557	0.200	0.281	0.000	0.313	0.100

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	122	79	130	217	0	182	115
N.S.	1	1.00	1.15	0.75	1.23	2.05	0.00	1.72	1.08
time (sec)	N/A	0.242	0.358	0.578	0.203	0.261	0.000	0.316	13.383

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	132	99	360	91	0	139	132
N.S.	1	1.00	1.06	0.79	2.88	0.73	0.00	1.11	1.06
time (sec)	N/A	0.297	1.613	0.666	0.292	0.261	0.000	0.337	16.326

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	112	77	278	70	0	113	106
N.S.	1	1.00	1.13	0.78	2.81	0.71	0.00	1.14	1.07
time (sec)	N/A	0.231	0.511	0.591	0.296	0.285	0.000	0.296	16.459

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	83	44	196	51	0	87	55
N.S.	1	1.00	1.14	0.60	2.68	0.70	0.00	1.19	0.75
time (sec)	N/A	0.202	0.417	0.605	0.292	0.250	0.000	0.313	13.995

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	68	33	112	27	0	58	30
N.S.	1	1.00	1.55	0.75	2.55	0.61	0.00	1.32	0.68
time (sec)	N/A	0.152	0.190	0.608	0.282	0.267	0.000	0.307	13.794

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	66	35	49	41	0	37	32
N.S.	1	1.00	1.78	0.95	1.32	1.11	0.00	1.00	0.86
time (sec)	N/A	0.165	0.823	0.511	0.208	0.284	0.000	0.308	13.813

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	116	60	96	89	0	74	60
N.S.	1	1.00	2.11	1.09	1.75	1.62	0.00	1.35	1.09
time (sec)	N/A	0.190	0.684	0.530	0.200	0.253	0.000	0.317	13.991

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	158	86	136	131	0	103	153
N.S.	1	1.00	2.16	1.18	1.86	1.79	0.00	1.41	2.10
time (sec)	N/A	0.205	0.828	0.556	0.201	0.255	0.000	0.313	14.038

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	200	112	176	177	0	132	201
N.S.	1	1.00	2.20	1.23	1.93	1.95	0.00	1.45	2.21
time (sec)	N/A	0.205	2.066	0.607	0.197	0.310	0.000	0.322	13.927

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	242	138	216	219	0	161	139
N.S.	1	1.00	2.22	1.27	1.98	2.01	0.00	1.48	1.28
time (sec)	N/A	0.215	3.185	0.767	0.202	0.272	0.000	0.344	15.907

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	72	89	89	89	0	185	109
N.S.	1	1.00	0.53	0.65	0.65	0.65	0.00	1.35	0.80
time (sec)	N/A	0.239	3.896	1.160	0.202	0.301	0.000	0.382	0.090

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	62	79	79	79	0	141	96
N.S.	1	1.00	0.54	0.69	0.69	0.69	0.00	1.24	0.84
time (sec)	N/A	0.217	2.930	0.959	0.196	0.272	0.000	0.366	0.072

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	53	49	49	49	0	141	58
N.S.	1	1.00	0.73	0.67	0.67	0.67	0.00	1.93	0.79
time (sec)	N/A	0.211	1.373	0.850	0.192	0.288	0.000	0.351	0.074

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	42	39	39	39	0	119	36
N.S.	1	1.00	0.76	0.71	0.71	0.71	0.00	2.16	0.65
time (sec)	N/A	0.193	0.411	0.831	0.213	0.299	0.000	0.333	13.970

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	51	48	51	48	0	75	56
N.S.	1	1.00	0.77	0.73	0.77	0.73	0.00	1.14	0.85
time (sec)	N/A	0.203	0.099	0.962	0.195	0.265	0.000	0.314	0.066

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	64	44	46	58	0	52	46
N.S.	1	1.00	1.23	0.85	0.88	1.12	0.00	1.00	0.88
time (sec)	N/A	0.124	0.196	0.434	0.206	0.280	0.000	0.341	0.086

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	83	46	74	106	0	87	60
N.S.	1	1.00	1.38	0.77	1.23	1.77	0.00	1.45	1.00
time (sec)	N/A	0.150	0.215	0.515	0.197	0.263	0.000	0.328	0.097

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	38	46	59	60	0	82	58
N.S.	1	1.00	0.90	1.10	1.40	1.43	0.00	1.95	1.38
time (sec)	N/A	0.158	0.206	0.527	0.202	0.264	0.000	0.351	13.814

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	152	100	167	283	0	207	152
N.S.	1	1.00	1.04	0.68	1.14	1.94	0.00	1.42	1.04
time (sec)	N/A	0.262	0.621	0.611	0.208	0.277	0.000	0.376	13.638

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	131	99	378	90	0	139	133
N.S.	1	1.00	0.78	0.59	2.26	0.54	0.00	0.83	0.80
time (sec)	N/A	0.547	3.045	1.134	0.288	0.273	0.000	0.354	16.849

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	111	77	292	71	0	113	107
N.S.	1	1.00	1.07	0.74	2.81	0.68	0.00	1.09	1.03
time (sec)	N/A	0.374	0.619	0.933	0.288	0.259	0.000	0.339	16.577

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	91	55	206	50	0	87	81
N.S.	1	1.00	1.05	0.63	2.37	0.57	0.00	1.00	0.93
time (sec)	N/A	0.297	0.583	0.805	0.282	0.289	0.000	0.317	17.564

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	121	45	140	61	0	75	91
N.S.	1	1.00	1.75	0.65	2.03	0.88	0.00	1.09	1.32
time (sec)	N/A	0.392	0.456	0.800	0.282	0.266	0.000	0.337	13.160

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	105	58	90	71	0	74	71
N.S.	1	1.00	1.44	0.79	1.23	0.97	0.00	1.01	0.97
time (sec)	N/A	0.255	0.733	0.538	0.203	0.263	0.000	0.340	13.180

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	149	84	134	108	0	105	121
N.S.	1	1.00	1.64	0.92	1.47	1.19	0.00	1.15	1.33
time (sec)	N/A	0.426	1.110	0.638	0.213	0.258	0.000	0.344	13.187

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	191	110	174	169	0	134	106
N.S.	1	1.00	1.75	1.01	1.60	1.55	0.00	1.23	0.97
time (sec)	N/A	0.434	1.316	0.720	0.208	0.261	0.000	0.411	14.385

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	233	112	174	204	0	134	201
N.S.	1	1.00	1.86	0.90	1.39	1.63	0.00	1.07	1.61
time (sec)	N/A	0.446	2.321	0.623	0.207	0.273	0.000	0.387	14.015

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	120	89	89	89	0	207	110
N.S.	1	1.00	0.86	0.64	0.64	0.64	0.00	1.49	0.79
time (sec)	N/A	0.246	3.902	1.035	0.219	0.284	0.000	0.412	13.475

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	100	69	69	69	0	185	84
N.S.	1	1.00	0.92	0.63	0.63	0.63	0.00	1.70	0.77
time (sec)	N/A	0.218	1.856	0.991	0.200	0.265	0.000	0.417	13.185

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	80	49	49	49	0	163	58
N.S.	1	1.00	1.10	0.67	0.67	0.67	0.00	2.23	0.79
time (sec)	N/A	0.225	0.847	0.969	0.192	0.268	0.000	0.386	13.419

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	73	68	73	70	0	172	82
N.S.	1	1.00	0.72	0.67	0.72	0.69	0.00	1.69	0.80
time (sec)	N/A	0.229	0.361	1.047	0.210	0.272	0.000	0.372	13.356

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	99	60	72	82	0	94	75
N.S.	1	1.00	1.11	0.67	0.81	0.92	0.00	1.06	0.84
time (sec)	N/A	0.238	0.209	0.965	0.194	0.269	0.000	0.334	0.073

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	103	59	71	96	0	63	59
N.S.	1	1.00	1.37	0.79	0.95	1.28	0.00	0.84	0.79
time (sec)	N/A	0.162	0.150	0.446	0.200	0.268	0.000	0.325	0.089

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	61	98	151	0	113	83
N.S.	1	1.00	1.18	0.74	1.20	1.84	0.00	1.38	1.01
time (sec)	N/A	0.202	0.243	0.607	0.210	0.278	0.000	0.355	0.128

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	138	87	146	240	0	182	130
N.S.	1	1.00	1.10	0.69	1.16	1.90	0.00	1.44	1.03
time (sec)	N/A	0.185	0.743	0.680	0.203	0.268	0.000	0.373	0.170

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	137	91	188	317	0	232	173
N.S.	1	1.00	1.07	0.71	1.47	2.48	0.00	1.81	1.35
time (sec)	N/A	0.245	3.777	0.989	0.196	0.271	0.000	0.449	13.757

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	131	99	378	91	0	139	132
N.S.	1	1.00	0.83	0.63	2.41	0.58	0.00	0.89	0.84
time (sec)	N/A	0.585	5.223	0.970	0.284	0.265	0.000	0.410	16.578

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	111	77	292	70	0	113	106
N.S.	1	1.00	0.86	0.60	2.26	0.54	0.00	0.88	0.82
time (sec)	N/A	0.373	1.728	0.955	0.284	0.261	0.000	0.388	16.396

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	173	65	227	83	0	101	98
N.S.	1	1.00	1.60	0.60	2.10	0.77	0.00	0.94	0.91
time (sec)	N/A	0.459	0.614	0.909	0.290	0.283	0.000	0.363	14.959

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	177	66	164	99	0	96	115
N.S.	1	1.00	1.82	0.68	1.69	1.02	0.00	0.99	1.19
time (sec)	N/A	0.514	0.491	1.064	0.290	0.271	0.000	0.352	13.317

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	137	58	90	95	0	73	84
N.S.	1	1.00	1.54	0.65	1.01	1.07	0.00	0.82	0.94
time (sec)	N/A	0.616	1.003	0.575	0.220	0.268	0.000	0.360	13.481

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	175	60	92	146	0	73	105
N.S.	1	1.00	1.70	0.58	0.89	1.42	0.00	0.71	1.02
time (sec)	N/A	0.630	1.354	0.806	0.203	0.255	0.000	0.398	13.077

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	223	110	174	191	0	134	201
N.S.	1	1.00	1.76	0.87	1.37	1.50	0.00	1.06	1.58
time (sec)	N/A	0.710	1.405	0.899	0.201	0.257	0.000	0.428	13.935

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	265	136	214	214	0	163	249
N.S.	1	1.00	1.83	0.94	1.48	1.48	0.00	1.12	1.72
time (sec)	N/A	0.553	3.100	1.111	0.200	0.259	0.000	0.404	14.481

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	106	210	0	649	0	0	0
N.S.	1	1.00	0.68	1.34	0.00	4.13	0.00	0.00	0.00
time (sec)	N/A	0.253	0.518	10.939	0.000	0.360	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	170	154	0	603	0	0	0
N.S.	1	1.00	1.10	1.00	0.00	3.92	0.00	0.00	0.00
time (sec)	N/A	0.231	0.622	10.279	0.000	0.366	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	69	142	0	555	0	0	0
N.S.	1	1.00	0.66	1.37	0.00	5.34	0.00	0.00	0.00
time (sec)	N/A	0.184	0.279	8.324	0.000	0.375	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	226	115	0	550	0	0	0
N.S.	1	1.00	2.19	1.12	0.00	5.34	0.00	0.00	0.00
time (sec)	N/A	0.176	0.951	7.324	0.000	0.194	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	143	201	0	670	0	0	0
N.S.	1	1.00	0.92	1.30	0.00	4.32	0.00	0.00	0.00
time (sec)	N/A	0.230	0.567	10.071	0.000	0.198	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	120	164	0	705	0	0	0
N.S.	1	1.00	0.75	1.02	0.00	4.41	0.00	0.00	0.00
time (sec)	N/A	0.232	0.569	11.421	0.000	0.200	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	205	265	0	769	0	0	0
N.S.	1	1.00	1.06	1.37	0.00	3.96	0.00	0.00	0.00
time (sec)	N/A	0.467	17.639	19.127	0.000	0.374	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	204	201	0	721	0	0	0
N.S.	1	1.00	1.06	1.05	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.570	27.023	13.577	0.000	0.375	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	168	220	0	677	0	0	0
N.S.	1	1.00	1.22	1.59	0.00	4.91	0.00	0.00	0.00
time (sec)	N/A	0.486	11.755	14.067	0.000	0.387	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	164	163	0	660	0	0	0
N.S.	1	1.00	1.18	1.17	0.00	4.75	0.00	0.00	0.00
time (sec)	N/A	0.529	19.808	11.951	0.000	0.192	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	135	238	0	790	0	0	0
N.S.	1	1.00	0.60	1.06	0.00	3.53	0.00	0.00	0.00
time (sec)	N/A	0.559	19.948	17.572	0.000	0.205	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	169	301	0	849	0	0	0
N.S.	1	1.00	0.72	1.29	0.00	3.63	0.00	0.00	0.00
time (sec)	N/A	0.530	18.127	17.500	0.000	0.211	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	122	128	0	126	0	0	0
N.S.	1	1.00	0.88	0.92	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.351	1.735	5.149	0.000	0.094	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	193	173	0	111	0	0	0
N.S.	1	1.00	1.86	1.66	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.287	3.189	5.362	0.000	0.100	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	69	112	0	88	0	0	0
N.S.	1	1.00	0.68	1.10	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.292	1.261	3.802	0.000	0.092	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	210	149	0	123	0	0	0
N.S.	1	1.00	2.21	1.57	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.290	4.750	4.590	0.000	0.093	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	121	0	108	0	0	0
N.S.	1	1.00	0.76	1.20	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.281	1.132	4.429	0.000	0.084	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	124	187	0	163	0	0	0
N.S.	1	1.00	0.92	1.39	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.328	1.518	5.841	0.000	0.093	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	91	136	0	215	0	0	0
N.S.	1	1.00	0.67	1.01	0.00	1.59	0.00	0.00	0.00
time (sec)	N/A	0.323	1.738	4.814	0.000	0.114	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	94	145	0	125	0	0	0
N.S.	1	1.00	0.58	0.90	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.681	2.539	6.867	0.000	0.094	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	446	173	0	171	0	0	0
N.S.	1	1.00	2.39	0.93	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.742	7.280	7.114	0.000	0.099	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	119	153	0	140	0	0	0
N.S.	1	1.00	0.63	0.81	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.707	2.836	5.378	0.000	0.101	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	234	205	0	179	0	0	0
N.S.	1	1.00	1.24	1.09	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.704	6.265	5.924	0.000	0.099	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	82	148	0	165	0	0	0
N.S.	1	1.00	0.43	0.78	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.696	1.967	5.886	0.000	0.091	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	163	213	0	222	0	0	0
N.S.	1	1.00	0.73	0.95	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.809	1.991	6.276	0.000	0.093	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	113	160	0	236	0	0	0
N.S.	1	1.00	0.50	0.71	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.802	1.648	6.726	0.000	0.095	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	247	160	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.460	0.687	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	127	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	0.423	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	97	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	383	0	0	0	0	0	0
N.S.	1	1.00	3.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	16.385	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	612	0	0	0	0	0	0
N.S.	1	1.00	2.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.649	4.799	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	236	236	995	0	0	0	0	0	0
N.S.	1	1.00	4.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.768	6.094	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	1243	0	0	0	0	0	0
N.S.	1	1.00	11.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	9.562	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	433	0	0	0	0	0	0
N.S.	1	1.00	4.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.392	2.373	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	277	0	0	0	0	0	0
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.427	2.849	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	484	0	0	0	0	0	0
N.S.	1	1.00	4.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	2.319	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	276	0	0	0	0	0	0
N.S.	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	2.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	180	180	116	0	0	0	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	0.842	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	84	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.378	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	67	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.094	0.119	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	0	0	64
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.52
time (sec)	N/A	0.051	0.039	0.000	0.000	0.000	0.000	0.000	14.428

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	92	0	0	0	0	0	0
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.506	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	179	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	1.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	240	492	0	0	0	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	3.891	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	230	230	7069	0	0	0	0	0	0
N.S.	1	1.00	30.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.822	21.752	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	4297	0	0	0	0	0	0
N.S.	1	1.00	45.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.394	17.343	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	142	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	1.509	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	349	350	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.690	4.574	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	382	0	0	0	0	0	0
N.S.	1	1.00	3.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	16.603	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	214	0	0	0	0	0	0
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	3.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	212	0	0	0	0	0	0
N.S.	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	3.532	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	212	0	0	0	0	0	0
N.S.	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	4.385	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	113	87	91	93	0	317	89
N.S.	1	1.00	0.95	0.73	0.76	0.78	0.00	2.66	0.75
time (sec)	N/A	0.132	0.146	1.855	0.205	0.293	0.000	0.339	14.350

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	83	67	69	71	0	248	67
N.S.	1	1.00	0.95	0.77	0.79	0.82	0.00	2.85	0.77
time (sec)	N/A	0.115	0.067	1.576	0.196	0.276	0.000	0.326	13.937

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	47	47	49	0	66	45
N.S.	1	1.00	0.98	0.81	0.81	0.84	0.00	1.14	0.78
time (sec)	N/A	0.107	0.037	1.639	0.198	0.273	0.000	0.314	0.054

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	37	26	23	25	0	32	23
N.S.	1	1.00	1.42	1.00	0.88	0.96	0.00	1.23	0.88
time (sec)	N/A	0.037	0.025	0.782	0.189	0.277	0.000	0.295	0.039

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	65	33	45	51	0	61	63
N.S.	1	1.00	2.50	1.27	1.73	1.96	0.00	2.35	2.42
time (sec)	N/A	0.081	0.019	0.567	0.196	0.280	0.000	0.281	0.112

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	120	61	71	123	0	169	76
N.S.	1	1.00	1.88	0.95	1.11	1.92	0.00	2.64	1.19
time (sec)	N/A	0.128	0.015	1.155	0.199	0.290	0.000	0.297	0.101

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	176	83	110	201	0	266	117
N.S.	1	1.00	1.76	0.83	1.10	2.01	0.00	2.66	1.17
time (sec)	N/A	0.150	0.020	1.296	0.237	0.264	0.000	0.318	13.909

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	232	103	143	284	0	357	148
N.S.	1	1.00	1.66	0.74	1.02	2.03	0.00	2.55	1.06
time (sec)	N/A	0.177	0.053	1.245	0.210	0.271	0.000	0.310	13.473

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	118	96	106	102	0	228	332
N.S.	1	1.00	0.93	0.76	0.83	0.80	0.00	1.80	2.61
time (sec)	N/A	0.152	0.465	1.887	0.217	0.283	0.000	0.323	15.246

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	86	76	81	79	0	172	267
N.S.	1	1.00	0.97	0.85	0.91	0.89	0.00	1.93	3.00
time (sec)	N/A	0.135	0.174	1.726	0.201	0.283	0.000	0.300	14.541

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	55	59	55	0	114	83
N.S.	1	1.00	1.06	1.08	1.16	1.08	0.00	2.24	1.63
time (sec)	N/A	0.097	0.122	1.217	0.199	0.284	0.000	0.289	13.933

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	41	42	50	63	0	77	60
N.S.	1	1.00	1.11	1.14	1.35	1.70	0.00	2.08	1.62
time (sec)	N/A	0.122	0.079	1.191	0.197	0.281	0.000	0.303	14.566

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	63	76	125	0	133	101
N.S.	1	1.00	1.00	0.91	1.10	1.81	0.00	1.93	1.46
time (sec)	N/A	0.127	0.038	0.967	0.206	0.285	0.000	0.315	14.849

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	83	96	174	0	194	142
N.S.	1	1.00	0.90	0.82	0.95	1.72	0.00	1.92	1.41
time (sec)	N/A	0.139	0.045	1.056	0.206	0.277	0.000	0.314	14.363

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	112	120	105	125	0	418	104
N.S.	1	1.00	0.90	0.97	0.85	1.01	0.00	3.37	0.84
time (sec)	N/A	0.247	2.221	1.802	0.206	0.292	0.000	0.389	13.948

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	72	90	71	92	0	100	69
N.S.	1	1.00	0.90	1.12	0.89	1.15	0.00	1.25	0.86
time (sec)	N/A	0.189	0.678	1.799	0.199	0.285	0.000	0.357	14.113

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	40	40	50	0	50	40
N.S.	1	1.00	0.88	0.95	0.95	1.19	0.00	1.19	0.95
time (sec)	N/A	0.102	0.107	0.538	0.203	0.294	0.000	0.314	0.055

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	91	66	73	97	0	124	62
N.S.	1	1.00	1.23	0.89	0.99	1.31	0.00	1.68	0.84
time (sec)	N/A	0.232	0.833	0.511	0.193	0.265	0.000	0.310	14.601

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	329	116	119	205	0	314	120
N.S.	1	1.00	2.89	1.02	1.04	1.80	0.00	2.75	1.05
time (sec)	N/A	0.373	1.385	0.942	0.201	0.295	0.000	0.326	0.126

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	193	163	173	176	0	379	231
N.S.	1	1.00	1.10	0.93	0.99	1.01	0.00	2.17	1.32
time (sec)	N/A	0.561	2.140	2.136	0.289	0.291	0.000	0.378	16.744

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	157	133	125	142	0	285	207
N.S.	1	1.00	0.88	0.75	0.70	0.80	0.00	1.60	1.16
time (sec)	N/A	0.658	1.457	1.801	0.287	0.289	0.000	0.372	14.822

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	121	77	80	108	0	159	143
N.S.	1	1.00	1.57	1.00	1.04	1.40	0.00	2.06	1.86
time (sec)	N/A	0.152	0.975	1.385	0.291	0.282	0.000	0.334	14.663

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	138	76	73	104	0	167	108
N.S.	1	1.00	2.34	1.29	1.24	1.76	0.00	2.83	1.83
time (sec)	N/A	0.488	1.143	0.995	0.201	0.267	0.000	0.333	14.577

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	259	116	112	178	0	226	182
N.S.	1	1.00	2.59	1.16	1.12	1.78	0.00	2.26	1.82
time (sec)	N/A	0.368	1.283	1.667	0.203	0.274	0.000	0.337	14.683

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	368	154	143	241	0	326	248
N.S.	1	1.00	2.57	1.08	1.00	1.69	0.00	2.28	1.73
time (sec)	N/A	0.475	1.439	1.763	0.208	0.277	0.000	0.361	14.213

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	174	142	175	0	695	143
N.S.	1	1.00	0.91	1.02	0.84	1.03	0.00	4.09	0.84
time (sec)	N/A	0.304	1.390	2.446	0.199	0.284	0.000	0.435	13.583

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	102	116	98	123	0	128	99
N.S.	1	1.00	0.88	1.00	0.84	1.06	0.00	1.10	0.85
time (sec)	N/A	0.157	0.860	2.534	0.202	0.292	0.000	0.398	0.075

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	57	57	67	0	66	57
N.S.	1	1.00	0.88	0.89	0.89	1.05	0.00	1.03	0.89
time (sec)	N/A	0.114	0.154	0.823	0.205	0.269	0.000	0.346	13.584

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	89	92	112	139	0	250	85
N.S.	1	1.00	0.87	0.90	1.10	1.36	0.00	2.45	0.83
time (sec)	N/A	0.259	0.957	0.651	0.198	0.274	0.000	0.344	0.131

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	669	162	171	290	0	482	159
N.S.	1	1.00	4.13	1.00	1.06	1.79	0.00	2.98	0.98
time (sec)	N/A	0.402	7.220	1.311	0.195	0.285	0.000	0.379	13.956

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	818	234	242	241	0	563	373
N.S.	1	1.00	2.74	0.78	0.81	0.81	0.00	1.88	1.25
time (sec)	N/A	0.429	7.091	2.570	0.287	0.302	0.000	0.438	14.780

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	696	194	183	196	0	431	281
N.S.	1	1.00	2.95	0.82	0.78	0.83	0.00	1.83	1.19
time (sec)	N/A	0.858	6.749	2.285	0.291	0.301	0.000	0.388	14.697

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	327	128	129	151	0	346	202
N.S.	1	1.00	2.37	0.93	0.93	1.09	0.00	2.51	1.46
time (sec)	N/A	0.588	1.203	1.856	0.290	0.284	0.000	0.359	13.879

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	406	129	139	151	0	225	181
N.S.	1	1.00	3.05	0.97	1.05	1.14	0.00	1.69	1.36
time (sec)	N/A	0.349	2.442	1.049	0.208	0.279	0.000	0.352	14.088

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	610	187	190	260	0	361	260
N.S.	1	1.00	2.98	0.91	0.93	1.27	0.00	1.76	1.27
time (sec)	N/A	0.375	1.438	1.600	0.200	0.272	0.000	0.379	13.855

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	812	243	230	354	0	498	363
N.S.	1	1.00	2.91	0.87	0.82	1.27	0.00	1.78	1.30
time (sec)	N/A	0.393	2.156	1.872	0.204	0.273	0.000	0.418	13.684

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	282	275	224	222	0	1559	249
N.S.	1	1.00	1.26	1.23	1.00	1.00	0.00	6.99	1.12
time (sec)	N/A	0.329	1.809	1.355	0.197	0.284	0.000	0.361	0.152

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	172	160	141	140	0	867	151
N.S.	1	1.00	1.13	1.05	0.93	0.92	0.00	5.70	0.99
time (sec)	N/A	0.238	0.311	0.834	0.198	0.280	0.000	0.326	13.419

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	82	80	78	0	102	79
N.S.	1	1.00	1.00	0.92	0.90	0.88	0.00	1.15	0.89
time (sec)	N/A	0.196	0.098	0.775	0.194	0.288	0.000	0.326	13.510

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	48	33	31	0	38	30
N.S.	1	1.00	0.88	1.41	0.97	0.91	0.00	1.12	0.88
time (sec)	N/A	0.098	0.018	0.471	0.198	0.274	0.000	0.300	0.057

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	63	57	64	64	0	100	68
N.S.	1	1.00	0.85	0.77	0.86	0.86	0.00	1.35	0.92
time (sec)	N/A	0.131	0.225	0.722	0.198	0.282	0.000	0.328	13.674

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	123	110	132	216	0	202	133
N.S.	1	1.00	1.06	0.95	1.14	1.86	0.00	1.74	1.15
time (sec)	N/A	0.264	0.889	0.720	0.200	0.329	0.000	0.342	13.695

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	207	172	268	469	0	419	297
N.S.	1	1.00	1.16	0.96	1.50	2.62	0.00	2.34	1.66
time (sec)	N/A	0.378	4.624	0.943	0.212	0.374	0.000	0.353	14.030

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	268	441	0	553	0	781	3341
N.S.	1	1.00	1.17	1.92	0.00	2.40	0.00	3.40	14.53
time (sec)	N/A	0.715	2.318	1.563	0.000	0.327	0.000	0.349	16.123

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	172	264	0	393	0	407	317
N.S.	1	1.00	1.07	1.64	0.00	2.44	0.00	2.53	1.97
time (sec)	N/A	0.473	0.752	0.970	0.000	0.295	0.000	0.313	14.780

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	96	142	0	258	0	185	147
N.S.	1	1.00	0.96	1.42	0.00	2.58	0.00	1.85	1.47
time (sec)	N/A	0.242	0.195	0.704	0.000	0.303	0.000	0.323	13.945

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	118	96	0	300	0	129	109
N.S.	1	1.00	1.40	1.14	0.00	3.57	0.00	1.54	1.30
time (sec)	N/A	0.189	0.477	0.545	0.000	0.284	0.000	0.332	13.813

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	162	165	0	558	0	269	219
N.S.	1	1.00	1.16	1.18	0.00	3.99	0.00	1.92	1.56
time (sec)	N/A	0.370	1.071	0.663	0.000	0.307	0.000	0.329	13.998

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	277	282	0	861	0	541	387
N.S.	1	1.00	1.38	1.40	0.00	4.28	0.00	2.69	1.93
time (sec)	N/A	0.652	1.397	0.938	0.000	0.321	0.000	0.339	14.349

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	417	321	271	344	0	1861	588
N.S.	1	1.00	1.56	1.20	1.01	1.29	0.00	6.97	2.20
time (sec)	N/A	0.601	3.185	2.094	0.204	0.349	0.000	0.366	0.187

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	280	198	184	240	0	1102	253
N.S.	1	1.00	1.44	1.02	0.95	1.24	0.00	5.68	1.30
time (sec)	N/A	0.476	1.123	1.387	0.201	0.309	0.000	0.340	0.121

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	167	112	112	150	0	139	113
N.S.	1	1.00	1.40	0.94	0.94	1.26	0.00	1.17	0.95
time (sec)	N/A	0.393	0.323	1.386	0.204	0.291	0.000	0.328	0.086

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	76	67	55	75	0	61	60
N.S.	1	1.00	1.33	1.18	0.96	1.32	0.00	1.07	1.05
time (sec)	N/A	0.222	0.072	0.515	0.211	0.292	0.000	0.324	0.076

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	165	98	123	210	0	213	103
N.S.	1	1.00	1.51	0.90	1.13	1.93	0.00	1.95	0.94
time (sec)	N/A	0.256	0.554	0.731	0.202	0.306	0.000	0.325	13.777

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	224	151	274	630	0	456	228
N.S.	1	1.00	1.33	0.90	1.63	3.75	0.00	2.71	1.36
time (sec)	N/A	0.556	1.803	0.973	0.203	0.364	0.000	0.362	14.042

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	320	221	511	1205	0	710	447
N.S.	1	1.00	1.24	0.85	1.97	4.65	0.00	2.74	1.73
time (sec)	N/A	0.962	1.621	1.470	0.220	0.494	0.000	0.401	14.702

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	402	509	0	793	0	870	3692
N.S.	1	1.00	0.85	1.08	0.00	1.68	0.00	1.84	7.81
time (sec)	N/A	2.024	6.316	2.711	0.000	0.342	0.000	0.344	17.510

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	282	325	0	581	0	482	2804
N.S.	1	1.00	1.08	1.25	0.00	2.23	0.00	1.85	10.74
time (sec)	N/A	0.940	3.056	1.635	0.000	0.335	0.000	0.343	15.420

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	178	199	0	551	0	240	1655
N.S.	1	1.00	1.17	1.31	0.00	3.62	0.00	1.58	10.89
time (sec)	N/A	0.651	0.732	1.295	0.000	0.312	0.000	0.322	16.439

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	128	162	0	526	0	289	245
N.S.	1	1.00	0.63	0.80	0.00	2.59	0.00	1.42	1.21
time (sec)	N/A	0.575	0.917	0.661	0.000	0.308	0.000	0.353	14.317

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	281	242	0	1040	0	457	403
N.S.	1	1.00	0.82	0.71	0.00	3.03	0.00	1.33	1.17
time (sec)	N/A	0.690	1.380	0.896	0.000	0.324	0.000	0.368	14.125

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	550	367	326	447	0	2150	762
N.S.	1	1.00	1.67	1.12	0.99	1.36	0.00	6.53	2.32
time (sec)	N/A	0.580	5.501	2.889	0.211	0.375	0.000	0.460	0.229

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	388	239	234	331	0	1337	315
N.S.	1	1.00	1.62	1.00	0.98	1.38	0.00	5.59	1.32
time (sec)	N/A	0.420	2.048	2.280	0.198	0.345	0.000	0.418	13.600

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	208	144	154	226	0	170	167
N.S.	1	1.00	1.32	0.91	0.97	1.43	0.00	1.08	1.06
time (sec)	N/A	0.319	0.880	1.658	0.202	0.299	0.000	0.388	13.549

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	111	85	87	126	0	77	93
N.S.	1	1.00	1.34	1.02	1.05	1.52	0.00	0.93	1.12
time (sec)	N/A	0.152	0.155	0.747	0.201	0.281	0.000	0.375	13.479

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	203	148	241	474	0	452	182
N.S.	1	1.00	1.25	0.91	1.48	2.91	0.00	2.77	1.12
time (sec)	N/A	0.379	0.862	1.070	0.207	0.338	0.000	0.388	14.161

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	332	196	435	1071	0	800	378
N.S.	1	1.00	1.45	0.86	1.90	4.68	0.00	3.49	1.65
time (sec)	N/A	0.684	6.493	1.465	0.218	0.455	0.000	0.441	14.201

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	496	255	707	1803	0	1551	673
N.S.	1	1.00	1.58	0.81	2.26	5.76	0.00	4.96	2.15
time (sec)	N/A	1.457	4.514	1.847	0.228	0.558	0.000	0.482	14.980

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	539	539	599	582	0	1057	0	1030	3975
N.S.	1	1.00	1.11	1.08	0.00	1.96	0.00	1.91	7.37
time (sec)	N/A	2.746	9.152	4.422	0.000	0.380	0.000	0.495	18.191

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	1178	392	0	1041	0	584	3255
N.S.	1	1.00	3.54	1.18	0.00	3.13	0.00	1.75	9.77
time (sec)	N/A	1.263	6.857	2.448	0.000	0.342	0.000	0.422	18.295

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	282	276	0	984	0	1193	4026
N.S.	1	1.00	1.06	1.03	0.00	3.69	0.00	4.47	15.08
time (sec)	N/A	1.415	3.291	1.525	0.000	0.334	0.000	0.489	21.743

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	231	234	0	841	0	386	423
N.S.	1	1.00	0.61	0.62	0.00	2.24	0.00	1.03	1.12
time (sec)	N/A	1.118	1.205	0.911	0.000	0.329	0.000	0.405	15.354

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	388	328	0	1550	0	709	588
N.S.	1	1.00	0.75	0.64	0.00	3.01	0.00	1.38	1.14
time (sec)	N/A	1.028	1.304	1.259	0.000	0.394	0.000	0.478	14.270

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	516	516	2049	772	0	0	0	0	0
N.S.	1	1.00	3.97	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.045	47.845	16.193	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	430	430	853	690	0	0	0	0	0
N.S.	1	1.00	1.98	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.283	36.321	13.672	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	1959	636	0	0	0	0	0
N.S.	1	1.00	4.41	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.202	24.660	10.104	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	351	439	0	0	0	0	0
N.S.	1	1.00	0.99	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.930	10.107	6.260	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	546	494	0	0	0	0	0
N.S.	1	1.00	1.48	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.911	5.373	7.544	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	430	430	711	661	0	0	0	0	0
N.S.	1	1.00	1.65	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.234	13.803	8.162	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	1233	607	0	0	0	0	0
N.S.	1	1.00	2.73	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.247	13.086	12.038	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	511	511	797	892	0	0	0	0	0
N.S.	1	1.00	1.56	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.661	8.397	11.093	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1070	1070	974	1725	0	0	0	0	0
N.S.	1	1.00	0.91	1.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.470	15.325	44.553	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1101	1101	2095	1494	0	0	0	0	0
N.S.	1	1.00	1.90	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.522	16.726	45.234	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	850	850	886	1482	0	0	0	0	0
N.S.	1	1.00	1.04	1.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.061	15.507	34.740	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	882	882	2012	1350	0	0	0	0	0
N.S.	1	1.00	2.28	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.681	16.510	26.564	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	809	809	854	1313	0	0	0	0	0
N.S.	1	1.00	1.06	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.338	14.872	19.706	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	838	838	1246	1239	0	0	0	0	0
N.S.	1	1.00	1.49	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.368	14.655	23.712	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1054	1054	772	1417	0	0	0	0	0
N.S.	1	1.00	0.73	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.288	7.229	22.484	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1089	1089	1320	1331	0	0	0	0	0
N.S.	1	1.00	1.21	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.459	12.517	31.287	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	151	182	0	0	0	0	0
N.S.	1	1.00	1.21	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.559	6.610	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	120	215	0	0	0	0	0
N.S.	1	1.00	0.99	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	1.744	6.640	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	684	1599	0	0	0	0	0
N.S.	1	1.00	2.21	5.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	6.733	8.893	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	276	763	0	0	0	0	0
N.S.	1	1.00	1.21	3.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	11.446	9.183	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	138	147	0	0	0	0	0
N.S.	1	1.00	1.30	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.392	5.247	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	259	759	0	0	0	0	0
N.S.	1	1.00	1.02	2.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.403	7.392	5.731	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	347	347	972	1799	0	0	0	0	0
N.S.	1	1.00	2.80	5.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.412	6.598	6.481	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	259	977	0	0	0	0	0
N.S.	1	1.00	0.81	3.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.638	7.468	6.858	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	249	182	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	0.280	0.000	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	190	190	134	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.992	0.195	0.000	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	98	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	232	232	687	0	0	0	0	0	0
N.S.	1	1.00	2.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	4.957	0.000	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	405	405	1433	0	0	0	0	0	0
N.S.	1	1.00	3.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.523	11.663	0.000	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	580	580	2700	0	0	0	0	0	0
N.S.	1	1.00	4.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.932	16.986	0.000	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	0	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.00	1.00	1.08
time (sec)	N/A	0.079	39.216	0.908	1.077	0.404	0.000	1.071	18.518

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.08
time (sec)	N/A	0.072	0.749	0.973	1.011	0.358	5.958	0.660	15.792

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.08
time (sec)	N/A	0.077	21.347	0.842	1.101	0.423	1.359	78.037	18.982

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	53	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	2.12	0.96	1.00	1.08
time (sec)	N/A	0.082	20.853	0.847	1.128	0.521	30.712	38.514	25.881

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	27
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.17
time (sec)	N/A	0.050	7.656	0.909	1.517	0.424	161.887	0.638	15.168

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	562	0	0	0	0	0	0
N.S.	1	1.00	3.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	6.795	0.000	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	155	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	1.763	0.000	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	72	0	0	0	0	0	73
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	1.52
time (sec)	N/A	0.045	0.672	0.000	0.000	0.000	0.000	0.000	14.483

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	132	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	1.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	231	231	1520	0	0	0	0	0	0
N.S.	1	1.00	6.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	16.825	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	35	0	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.67	0.00	1.10	1.19
time (sec)	N/A	0.046	7.952	2.449	3.220	0.343	0.000	0.488	18.334

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	26	20	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.24	0.95	1.10	1.19
time (sec)	N/A	0.046	6.252	1.209	1.953	0.283	37.698	0.485	15.724

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	3614	0	0	0	0	0	0
N.S.	1	1.00	26.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	17.330	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	424	0	5928	0	0	0	0	0	0
N.S.	1	0.00	13.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	23.443	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	0	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.00	1.00	1.09
time (sec)	N/A	0.047	8.893	0.822	1.595	0.357	0.000	22.551	16.293

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.09
time (sec)	N/A	0.049	11.612	0.859	1.606	0.350	16.753	0.509	15.532

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.09
time (sec)	N/A	0.048	10.182	0.819	1.598	0.365	3.621	0.540	16.219

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	36	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.57	0.96	1.00	1.09
time (sec)	N/A	0.046	10.084	0.790	1.656	0.465	111.854	10.804	16.773

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	190	190	135	536	0	682	0	0	0
N.S.	1	1.00	0.71	2.82	0.00	3.59	0.00	0.00	0.00
time (sec)	N/A	0.197	2.337	3.137	0.000	0.213	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	169	169	146	639	0	609	0	0	0
N.S.	1	1.00	0.86	3.78	0.00	3.60	0.00	0.00	0.00
time (sec)	N/A	0.194	11.201	2.329	0.000	0.220	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	111	162	0	538	0	0	0
N.S.	1	1.00	0.92	1.34	0.00	4.45	0.00	0.00	0.00
time (sec)	N/A	0.164	1.186	10.672	0.000	0.193	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	122	122	130	544	0	554	0	0	0
N.S.	1	1.00	1.07	4.46	0.00	4.54	0.00	0.00	0.00
time (sec)	N/A	0.184	0.756	9.390	0.000	0.385	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	182	182	135	547	0	608	0	0	0
N.S.	1	1.00	0.74	3.01	0.00	3.34	0.00	0.00	0.00
time (sec)	N/A	0.236	9.737	10.881	0.000	0.400	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	197	197	165	602	0	654	0	0	0
N.S.	1	1.00	0.84	3.06	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.229	1.266	2.351	0.000	0.420	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	270	270	195	607	0	806	0	0	0
N.S.	1	1.00	0.72	2.25	0.00	2.99	0.00	0.00	0.00
time (sec)	N/A	0.419	13.270	11.702	0.000	0.204	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	240	240	195	1069	0	729	0	0	0
N.S.	1	1.00	0.81	4.45	0.00	3.04	0.00	0.00	0.00
time (sec)	N/A	0.414	14.905	13.196	0.000	0.194	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	154	154	168	476	0	664	0	0	0
N.S.	1	1.00	1.09	3.09	0.00	4.31	0.00	0.00	0.00
time (sec)	N/A	0.315	12.886	12.555	0.000	0.202	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	153	153	287	996	0	698	0	0	0
N.S.	1	1.00	1.88	6.51	0.00	4.56	0.00	0.00	0.00
time (sec)	N/A	0.324	18.326	13.827	0.000	0.424	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	222	222	164	810	0	724	0	0	0
N.S.	1	1.00	0.74	3.65	0.00	3.26	0.00	0.00	0.00
time (sec)	N/A	0.400	15.706	12.700	0.000	0.427	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	236	236	152	1069	0	776	0	0	0
N.S.	1	1.00	0.64	4.53	0.00	3.29	0.00	0.00	0.00
time (sec)	N/A	0.409	15.032	13.642	0.000	0.459	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	155	131	360	0	163	0	0	0
N.S.	1	1.00	0.85	2.32	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.288	1.786	10.678	0.000	0.094	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	230	642	0	127	0	0	0
N.S.	1	1.00	1.59	4.43	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.402	1.812	8.114	0.000	0.100	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	60	285	0	105	0	0	0
N.S.	1	1.00	0.57	2.71	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.956	8.081	0.000	0.090	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	95	432	0	87	0	0	0
N.S.	1	1.00	0.96	4.36	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.357	1.148	7.465	0.000	0.094	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	70	275	0	89	0	0	0
N.S.	1	1.00	0.66	2.59	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.378	1.045	10.124	0.000	0.096	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	100	462	0	111	0	0	0
N.S.	1	1.00	0.83	3.85	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.370	1.537	8.586	0.000	0.106	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	91	316	0	111	0	0	0
N.S.	1	1.00	0.61	2.12	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.431	1.488	10.010	0.000	0.110	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	268	268	115	382	0	220	0	0	0
N.S.	1	1.00	0.43	1.43	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.604	2.106	9.063	0.000	0.107	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	250	250	247	859	0	182	0	0	0
N.S.	1	1.00	0.99	3.44	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.635	2.363	9.250	0.000	0.096	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	82	309	0	155	0	0	0
N.S.	1	1.00	0.41	1.54	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.563	1.437	8.061	0.000	0.087	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	252	659	0	129	0	0	0
N.S.	1	1.00	1.27	3.31	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.580	2.269	8.700	0.000	0.099	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	101	397	0	135	0	0	0
N.S.	1	1.00	0.47	1.86	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.614	1.585	9.988	0.000	0.105	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	125	462	0	111	0	0	0
N.S.	1	1.00	0.58	2.15	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.592	2.821	8.864	0.000	0.110	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	94	316	0	111	0	0	0
N.S.	1	1.00	0.55	1.84	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.579	3.013	9.865	0.000	0.109	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [241] had the largest ratio of [.640000000000000013]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	19	0.210
2	A	5	4	1.00	19	0.210
3	A	5	4	1.00	19	0.210
4	A	5	4	1.00	19	0.210
5	A	4	3	1.00	17	0.176
6	A	6	6	1.00	17	0.353
7	A	5	4	1.00	19	0.210
8	A	5	4	1.00	19	0.210
9	A	5	4	1.00	19	0.210
10	A	11	7	1.00	19	0.368
11	A	10	7	1.00	19	0.368
12	A	9	7	1.00	19	0.368
13	A	7	7	1.00	19	0.368
14	A	7	7	1.00	19	0.368
15	A	8	6	1.00	19	0.316
16	A	8	6	1.00	19	0.316
17	A	8	6	1.00	19	0.316
18	A	8	6	1.00	19	0.316
19	A	5	4	1.00	21	0.190
20	A	5	4	1.00	21	0.190
21	A	5	4	1.00	21	0.190
22	A	5	4	1.00	21	0.190
23	A	5	4	1.00	19	0.210
24	A	5	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	4	1.00	21	0.190
26	A	5	4	1.00	21	0.190
27	A	5	4	1.00	21	0.190
28	A	5	4	1.00	21	0.190
29	A	27	8	1.00	21	0.381
30	A	18	8	1.00	21	0.381
31	A	14	8	1.00	21	0.381
32	A	9	7	1.00	21	0.333
33	A	11	9	1.00	21	0.429
34	A	8	8	1.00	21	0.381
35	A	12	8	1.00	21	0.381
36	A	12	8	1.00	21	0.381
37	A	12	8	1.00	21	0.381
38	A	5	4	1.00	21	0.190
39	A	5	4	1.00	21	0.190
40	A	5	4	1.00	21	0.190
41	A	4	3	1.00	21	0.143
42	A	5	4	1.00	19	0.210
43	A	5	4	1.00	19	0.210
44	A	5	4	1.00	21	0.190
45	A	5	4	1.00	21	0.190
46	A	5	4	1.00	21	0.190
47	A	5	4	1.00	21	0.190
48	A	29	9	1.00	21	0.429
49	A	18	9	1.00	21	0.429
50	A	16	9	1.00	21	0.429
51	A	11	8	1.00	21	0.381
52	A	9	7	1.00	21	0.333
53	A	11	8	1.00	21	0.381
54	A	10	9	1.00	21	0.429
55	A	17	9	1.00	21	0.429
56	A	17	9	1.00	21	0.429
57	A	7	6	1.00	21	0.286
58	A	7	6	1.00	21	0.286
59	A	7	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	6	5	1.00	21	0.238
61	A	5	4	1.00	19	0.210
62	A	6	6	1.00	19	0.316
63	A	7	7	1.00	21	0.333
64	A	8	7	1.00	21	0.333
65	A	9	7	1.00	21	0.333
66	A	8	7	1.00	21	0.333
67	A	7	7	1.00	21	0.333
68	A	5	5	1.00	21	0.238
69	A	6	5	1.00	21	0.238
70	A	7	6	1.00	21	0.286
71	A	7	6	1.00	21	0.286
72	A	7	6	1.00	21	0.286
73	A	7	6	1.00	21	0.286
74	A	5	4	1.00	21	0.190
75	A	5	4	1.00	21	0.190
76	A	5	4	1.00	21	0.190
77	A	5	4	1.00	21	0.190
78	A	5	4	1.00	21	0.190
79	A	5	4	1.00	19	0.210
80	A	6	5	1.00	19	0.263
81	A	4	4	1.00	21	0.190
82	A	6	5	1.00	21	0.238
83	A	16	8	1.00	21	0.381
84	A	7	6	1.00	21	0.286
85	A	11	6	1.00	21	0.286
86	A	9	8	1.00	21	0.381
87	A	11	6	1.00	21	0.286
88	A	13	7	1.00	21	0.333
89	A	13	7	1.00	21	0.333
90	A	13	7	1.00	21	0.333
91	A	5	4	1.00	21	0.190
92	A	5	4	1.00	21	0.190
93	A	5	4	1.00	21	0.190
94	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	5	4	1.00	21	0.190
96	A	5	4	1.00	19	0.210
97	A	6	5	1.00	19	0.263
98	A	5	4	1.00	21	0.190
99	A	6	5	1.00	21	0.238
100	A	19	9	1.00	21	0.429
101	A	15	6	1.00	21	0.286
102	A	13	8	1.00	21	0.381
103	A	10	8	1.00	21	0.381
104	A	15	8	1.00	21	0.381
105	A	16	7	1.00	21	0.333
106	A	16	7	1.00	21	0.333
107	A	16	7	1.00	21	0.333
108	A	11	11	1.00	23	0.478
109	A	11	11	1.00	23	0.478
110	A	9	9	1.00	23	0.391
111	A	9	9	1.00	23	0.391
112	A	11	11	1.00	23	0.478
113	A	11	11	1.00	23	0.478
114	A	15	12	1.00	25	0.480
115	A	15	12	1.00	25	0.480
116	A	13	10	1.00	25	0.400
117	A	13	10	1.00	25	0.400
118	A	16	13	1.00	25	0.520
119	A	16	13	1.00	25	0.520
120	A	8	8	1.00	25	0.320
121	A	7	7	1.00	25	0.280
122	A	7	7	1.00	25	0.280
123	A	7	7	1.00	25	0.280
124	A	7	7	1.00	25	0.280
125	A	8	8	1.00	25	0.320
126	A	8	8	1.00	25	0.320
127	A	14	8	1.00	25	0.320
128	A	14	9	1.00	25	0.360
129	A	14	9	1.00	25	0.360

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	15	9	1.00	25	0.360
131	A	15	9	1.00	25	0.360
132	A	17	9	1.00	25	0.360
133	A	17	9	1.00	25	0.360
134	A	9	6	1.00	23	0.261
135	A	7	6	1.00	23	0.261
136	A	5	5	1.00	21	0.238
137	A	5	5	1.00	23	0.217
138	A	9	6	1.00	23	0.261
139	A	12	7	1.00	23	0.304
140	A	5	4	1.00	25	0.160
141	A	5	4	1.00	25	0.160
142	A	5	4	1.00	25	0.160
143	A	5	4	1.00	25	0.160
144	A	5	4	1.00	23	0.174
145	A	4	4	1.00	21	0.190
146	A	4	4	1.00	21	0.190
147	A	3	3	1.00	21	0.143
148	A	2	2	1.00	19	0.105
149	A	2	2	1.00	19	0.105
150	A	4	4	1.00	21	0.190
151	A	5	5	1.00	21	0.238
152	A	11	9	1.00	21	0.429
153	A	6	5	1.00	21	0.238
154	A	4	4	1.00	21	0.190
155	A	7	6	1.00	21	0.286
156	A	5	4	1.00	23	0.174
157	A	5	4	1.00	23	0.174
158	A	5	4	1.00	23	0.174
159	A	5	4	1.00	23	0.174
160	A	5	4	1.00	19	0.210
161	A	5	4	1.00	19	0.210
162	A	5	4	1.00	19	0.210
163	A	4	3	1.00	17	0.176
164	A	5	5	1.00	17	0.294

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	7	6	1.00	19	0.316
166	A	9	7	1.00	19	0.368
167	A	10	7	1.00	19	0.368
168	A	10	7	1.00	19	0.368
169	A	9	7	1.00	19	0.368
170	A	7	7	1.00	19	0.368
171	A	7	7	1.00	19	0.368
172	A	8	6	1.00	19	0.316
173	A	8	6	1.00	19	0.316
174	A	5	4	1.00	21	0.190
175	A	5	4	1.00	21	0.190
176	A	5	4	1.00	19	0.210
177	A	5	4	1.00	19	0.210
178	A	6	5	1.00	21	0.238
179	A	12	10	1.00	21	0.476
180	A	7	7	1.00	21	0.333
181	A	10	8	1.00	21	0.381
182	A	8	6	1.00	21	0.286
183	A	9	6	1.00	21	0.286
184	A	9	6	1.00	21	0.286
185	A	5	4	1.00	21	0.190
186	A	4	3	1.00	21	0.143
187	A	5	4	1.00	19	0.210
188	A	5	4	1.00	19	0.210
189	A	6	5	1.00	21	0.238
190	A	21	11	1.00	21	0.524
191	A	8	7	1.00	21	0.333
192	A	8	8	1.00	21	0.381
193	A	15	10	1.00	21	0.476
194	A	17	9	1.00	21	0.429
195	A	17	9	1.00	21	0.429
196	A	5	4	1.00	21	0.190
197	A	5	4	1.00	21	0.190
198	A	5	4	1.00	21	0.190
199	A	5	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	4	3	1.00	19	0.158
201	A	6	5	1.00	21	0.238
202	A	7	5	1.00	21	0.238
203	A	7	5	1.00	21	0.238
204	A	6	5	1.00	21	0.238
205	A	5	5	1.00	21	0.238
206	A	5	5	1.00	21	0.238
207	A	6	5	1.00	21	0.238
208	A	7	5	1.00	21	0.238
209	A	5	4	1.00	21	0.190
210	A	5	4	1.00	21	0.190
211	A	5	4	1.00	21	0.190
212	A	5	4	1.00	19	0.210
213	A	5	4	1.00	19	0.210
214	A	6	5	1.00	21	0.238
215	A	7	5	1.00	21	0.238
216	A	10	8	1.00	21	0.381
217	A	8	7	1.00	21	0.333
218	A	8	8	1.00	21	0.381
219	A	11	7	1.00	21	0.333
220	A	15	8	1.00	21	0.381
221	A	5	4	1.00	21	0.190
222	A	5	4	1.00	21	0.190
223	A	5	4	1.00	21	0.190
224	A	5	4	1.00	19	0.210
225	A	5	4	1.00	19	0.210
226	A	5	4	1.00	21	0.190
227	A	7	5	1.00	21	0.238
228	A	11	8	1.00	21	0.381
229	A	9	7	1.00	21	0.333
230	A	9	8	1.00	21	0.381
231	A	16	8	1.00	21	0.381
232	A	20	9	1.00	21	0.429
233	A	15	12	1.00	25	0.480
234	A	14	12	1.00	25	0.480

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	14	12	1.00	25	0.480
236	A	13	11	1.00	25	0.440
237	A	13	11	1.00	25	0.440
238	A	14	12	1.00	25	0.480
239	A	14	12	1.00	25	0.480
240	A	15	12	1.00	25	0.480
241	A	35	16	1.00	25	0.640
242	A	35	16	1.00	25	0.640
243	A	32	15	1.00	25	0.600
244	A	32	15	1.00	25	0.600
245	A	27	13	1.00	25	0.520
246	A	27	13	1.00	25	0.520
247	A	33	16	1.00	25	0.640
248	A	33	16	1.00	25	0.640
249	A	1	1	1.00	14	0.071
250	A	2	2	1.00	23	0.087
251	A	5	5	1.00	14	0.357
252	A	4	4	1.00	23	0.174
253	A	1	1	1.00	14	0.071
254	A	6	6	1.00	23	0.261
255	A	6	6	1.00	14	0.429
256	A	6	6	1.00	23	0.261
257	A	9	6	1.00	23	0.261
258	A	9	8	1.00	23	0.348
259	A	5	5	1.00	21	0.238
260	A	4	4	1.00	23	0.174
261	A	6	4	1.00	23	0.174
262	A	7	4	1.00	23	0.174
263	N/A	0	0	1.00	25	0.000
264	N/A	0	0	1.00	25	0.000
265	N/A	0	0	1.00	25	0.000
266	N/A	0	0	1.00	25	0.000
267	N/A	0	0	1.00	23	0.000
268	A	6	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
269	A	3	3	1.00	21	0.143
270	A	2	2	1.00	19	0.105
271	A	4	3	1.00	19	0.158
272	A	9	4	1.00	21	0.190
273	N/A	0	0	1.00	21	0.000
274	N/A	0	0	1.00	21	0.000
275	A	4	4	1.00	21	0.190
276	F	0	0	N/A	0.000	N/A
277	N/A	0	0	1.00	23	0.000
278	N/A	0	0	1.00	23	0.000
279	N/A	0	0	1.00	23	0.000
280	N/A	0	0	1.00	23	0.000
281	A	11	11	1.00	23	0.478
282	A	11	11	1.00	23	0.478
283	A	9	9	1.00	23	0.391
284	A	9	9	1.00	23	0.391
285	A	11	11	1.00	23	0.478
286	A	11	11	1.00	23	0.478
287	A	15	13	1.00	25	0.520
288	A	15	13	1.00	25	0.520
289	A	12	10	1.00	25	0.400
290	A	12	10	1.00	25	0.400
291	A	14	12	1.00	25	0.480
292	A	14	12	1.00	25	0.480
293	A	8	8	1.00	25	0.320
294	A	8	8	1.00	25	0.320
295	A	7	7	1.00	25	0.280
296	A	7	7	1.00	25	0.280
297	A	7	7	1.00	25	0.280
298	A	7	7	1.00	25	0.280
299	A	8	8	1.00	25	0.320
300	A	16	9	1.00	25	0.360
301	A	16	9	1.00	25	0.360
302	A	14	9	1.00	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
303	A	14	9	1.00	25	0.360
304	A	13	9	1.00	25	0.360
305	A	13	9	1.00	25	0.360
306	A	13	8	1.00	25	0.320

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+a\sec(c+dx))\sin^9(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 152

$$\int (a+a\sec(c+dx))\sin^9(c+dx)\,dx=-\frac{a\cos(c+dx)}{d}+\frac{2a\cos^2(c+dx)}{d}+\frac{4a\cos^3(c+dx)}{3d}-\frac{3a\cos^4(c+dx)}{2d}-\frac{6a\cos^5(c+dx)}{5d}+\frac{2a\cos^6(c+dx)}{3d}+\frac{4a\cos^7(c+dx)}{7d}-\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^9(c+dx)}{9d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+2*a*cos(d*x+c)^2/d+4/3*a*cos(d*x+c)^3/d-3/2*a*cos(d*x+c)^4/
d-6/5*a*cos(d*x+c)^5/d+2/3*a*cos(d*x+c)^6/d+4/7*a*cos(d*x+c)^7/d-1/8*a*cos(
d*x+c)^8/d-1/9*a*cos(d*x+c)^9/d-a*ln(cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2915, 12, 90}

$$\int (a+a\sec(c+dx))\sin^9(c+dx)\,dx=-\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^8(c+dx)}{8d}+\frac{4a\cos^7(c+dx)}{7d}+\frac{2a\cos^6(c+dx)}{3d}-\frac{6a\cos^5(c+dx)}{5d}-\frac{3a\cos^4(c+dx)}{2d}+\frac{4a\cos^3(c+dx)}{3d}+\frac{2a\cos^2(c+dx)}{d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^9,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (2*a*Cos[c + d*x]^2)/d + (4*a*Cos[c + d*x]^3)/(3*d)
 - (3*a*Cos[c + d*x]^4)/(2*d) - (6*a*Cos[c + d*x]^5)/(5*d) + (2*a*Cos[c + d
*x]^6)/(3*d) + (4*a*Cos[c + d*x]^7)/(7*d) - (a*Cos[c + d*x]^8)/(8*d) - (a*C
os[c + d*x]^9)/(9*d) - (a*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))\sin^8(c+dx)\tan(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a(-a-x)^4(-a+x)^5}{x}\,dx,x,-a\cos(c+dx)\right)}{a^9d}\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^5}{x}\,dx,x,-a\cos(c+dx)\right)}{a^8d}\\
&= \frac{\text{Subst}\left(\int \left(a^8-\frac{a^9}{x}+4a^7x-4a^6x^2-6a^5x^3+6a^4x^4+4a^3x^5-4a^2x^6-ax^7+x^8\right)\,dx,x,-a\cos(c
\end{aligned}
$$

$$a^8d$$

$$
\begin{aligned}
&= -\frac{a\cos(c+dx)}{d} + \frac{2a\cos^2(c+dx)}{d} + \frac{4a\cos^3(c+dx)}{3d} - \frac{3a\cos^4(c+dx)}{2d} - \frac{6a\cos^5(c+dx)}{5d} \\
&\quad + \frac{2a\cos^6(c+dx)}{3d} + \frac{4a\cos^7(c+dx)}{7d} - \frac{a\cos^8(c+dx)}{8d} - \frac{a\cos^9(c+dx)}{9d} - \frac{a\log(\cos(c+dx))}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.70

$$
\int (a+a\sec(c+dx))\sin^9(c+dx)\,dx = -\frac{a(39690\cos(c+dx) - 161280\cos^2(c+dx) + 120960\cos^4(c+dx) - 53760\cos^6(c+dx) + 10080\cos^8(c
$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^9,x]
```

```
[Out] -1/80640*(a*(39690*Cos[c + d*x] - 161280*Cos[c + d*x]^2 + 120960*Cos[c + d*
x]^4 - 53760*Cos[c + d*x]^6 + 10080*Cos[c + d*x]^8 - 8820*Cos[3*(c + d*x)]
+ 2268*Cos[5*(c + d*x)] - 405*Cos[7*(c + d*x)] + 35*Cos[9*(c + d*x)] + 8064
0*Log[Cos[c + d*x]]))/d
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^8}{8}-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\right.}{9}}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^8}{8}-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\right.}{9}}{d}$
parts	$-\frac{a\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\frac{64\sin(dx+c)^2}{35}\right)\cos(dx+c)}{9d}+\frac{a\left(-\frac{\sin(dx+c)^8}{8}-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c}{4}\right.}{d}$
parallelrisch	$-\frac{a\left(-322560\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+322560\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+322560\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+269777+28980\cos(4dx+4\right.}{}$
risch	$iax+\frac{2iac}{d}+\frac{65a\,e^{2i(dx+c)}}{256d}+\frac{65a\,e^{-2i(dx+c)}}{256d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{63a\cos(dx+c)}{128d}-\frac{a\cos(9dx+9c)}{2304d}-\frac{a\cos}{}$

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^8-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^
2-ln(cos(d*x+c)))-1/9*a*(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*sin(d*x
+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx =$$
$$-\frac{280\,a\cos\left(dx+c\right)^9 + 315\,a\cos\left(dx+c\right)^8 - 1440\,a\cos\left(dx+c\right)^7 - 1680\,a\cos\left(dx+c\right)^6 + 3024\,a\cos\left(dx\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="fricas")
[Out] -1/2520*(280*a*cos(d*x + c)^9 + 315*a*cos(d*x + c)^8 - 1440*a*cos(d*x + c)^
7 - 1680*a*cos(d*x + c)^6 + 3024*a*cos(d*x + c)^5 + 3780*a*cos(d*x + c)^4 -
 3360*a*cos(d*x + c)^3 - 5040*a*cos(d*x + c)^2 + 2520*a*cos(d*x + c) + 2520
*a*log(-cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**9,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.74

$$\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx =$$
$$-\frac{280\,a\cos\left(dx+c\right)^9 + 315\,a\cos\left(dx+c\right)^8 - 1440\,a\cos\left(dx+c\right)^7 - 1680\,a\cos\left(dx+c\right)^6 + 3024\,a\cos\left(dx\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="maxima")
[Out] -1/2520*(280*a*cos(d*x + c)^9 + 315*a*cos(d*x + c)^8 - 1440*a*cos(d*x + c)^
7 - 1680*a*cos(d*x + c)^6 + 3024*a*cos(d*x + c)^5 + 3780*a*cos(d*x + c)^4 -
 3360*a*cos(d*x + c)^3 - 5040*a*cos(d*x + c)^2 + 2520*a*cos(d*x + c) + 2520
*a*log(cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(138) = 276$.

Time = 0.33 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.93

$$\int (a + a\sec(c + dx))\sin^9(c + dx)\,dx = \frac{2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{9177\,a - \frac{87633\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{375732\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{}}{}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2520*
a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (9177*a - 87633*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 375732*a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 953988*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1594
782*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1336734*a*(cos(d*x + c) -
 1)^5/(cos(d*x + c) + 1)^5 + 781956*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 - 302004*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 69201*a*(cos(d*
x + c) - 1)^8/(cos(d*x + c) + 1)^8 - 7129*a*(cos(d*x + c) - 1)^9/(cos(d*x +
 c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.73

$$\int (a + a\sec(c + dx))\sin^9(c + dx)\,dx = -\frac{a\cos(c + d\,x) - 2\,a\cos(c + d\,x)^2 - \frac{4\,a\cos(c+d\,x)^3}{3} + \frac{3\,a\cos(c+d\,x)^4}{2} + \frac{6\,a\cos(c+d\,x)^5}{5} - \frac{2\,a\cos(c+d\,x)^6}{3} - \frac{4\,a\cos(c+d\,x}{7}}{d}$$

```
[In] int(sin(c + d*x)^9*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - 2*a*cos(c + d*x)^2 - (4*a*cos(c + d*x)^3)/3 + (3*a*cos(c
 + d*x)^4)/2 + (6*a*cos(c + d*x)^5)/5 - (2*a*cos(c + d*x)^6)/3 - (4*a*cos(c
 + d*x)^7)/7 + (a*cos(c + d*x)^8)/8 + (a*cos(c + d*x)^9)/9 + a*log(cos(c +
d*x)))/d
```

3.2 $\int (a+a\sec(c+dx))\sin^7(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 119

$$\int (a+a\sec(c+dx))\sin^7(c+dx)\,dx=-\frac{a\cos(c+dx)}{d}+\frac{3a\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{d}-\frac{3a\cos^4(c+dx)}{4d}-\frac{3a\cos^5(c+dx)}{5d}+\frac{a\cos^6(c+dx)}{6d}+\frac{a\cos^7(c+dx)}{7d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+3/2*a*cos(d*x+c)^2/d+a*cos(d*x+c)^3/d-3/4*a*cos(d*x+c)^4/d-
3/5*a*cos(d*x+c)^5/d+1/6*a*cos(d*x+c)^6/d+1/7*a*cos(d*x+c)^7/d-a*ln(cos(d*x
+c))/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2915, 12, 90}

$$\int (a+a\sec(c+dx))\sin^7(c+dx)\,dx=\frac{a\cos^7(c+dx)}{7d}+\frac{a\cos^6(c+dx)}{6d}-\frac{3a\cos^5(c+dx)}{5d}-\frac{3a\cos^4(c+dx)}{4d}+\frac{a\cos^3(c+dx)}{d}+\frac{3a\cos^2(c+dx)}{2d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (3*a*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/d -
 (3*a*Cos[c + d*x]^4)/(4*d) - (3*a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^
6)/(6*d) + (a*Cos[c + d*x]^7)/(7*d) - (a*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int(-a-a\cos(c+dx))\sin^6(c+dx)\tan(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int\frac{a(-a-x)^3(-a+x)^4}{x}\,dx,x,-a\cos(c+dx)\right)}{a^7d} \\
&= \frac{\text{Subst}\left(\int\frac{(-a-x)^3(-a+x)^4}{x}\,dx,x,-a\cos(c+dx)\right)}{a^6d} \\
&= \frac{\text{Subst}\left(\int\left(a^6-\frac{a^7}{x}+3a^5x-3a^4x^2-3a^3x^3+3a^2x^4+ax^5-x^6\right)\,dx,x,-a\cos(c+dx)\right)}{a^6d} \\
&= -\frac{a\cos(c+dx)}{d}+\frac{3a\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{d}-\frac{3a\cos^4(c+dx)}{4d} \\
&\quad -\frac{3a\cos^5(c+dx)}{5d}+\frac{a\cos^6(c+dx)}{6d}+\frac{a\cos^7(c+dx)}{7d}-\frac{a\log(\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\begin{aligned}&\int (a+a\sec(c+dx))\sin^7(c+dx)\,dx\\&=-\frac{35a\cos(c+dx)}{64d}+\frac{7a\cos(3(c+dx))}{64d}-\frac{7a\cos(5(c+dx))}{320d}+\frac{a\cos(7(c+dx))}{448d}\\&\quad-\frac{a\left(-\frac{3}{2}\cos^2(c+dx)+\frac{3}{4}\cos^4(c+dx)-\frac{1}{6}\cos^6(c+dx)+\log(\cos(c+dx))\right)}{d}\end{aligned}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] (-35*a*Cos[c + d*x])/(64*d) + (7*a*Cos[3*(c + d*x)])/(64*d) - (7*a*Cos[5*(c
 + d*x)])/(320*d) + (a*Cos[7*(c + d*x)])/(448*d) - (a*((-3*Cos[c + d*x]^2)/
2 + (3*Cos[c + d*x]^4)/4 - Cos[c + d*x]^6/6 + Log[Cos[c + d*x]]))/d
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}}{d}$
parts	$-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7d}+\frac{a\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+\right.}{d}$
parallelrisch	$\frac{\left(192\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-192\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-192\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{5732}{35}-12\cos(4dx+4c)+21\cos(3dx+3c)-\right.}{192d}$
risch	$iax+\frac{2iac}{d}+\frac{29a\,e^{2i(dx+c)}}{128d}+\frac{29a\,e^{-2i(dx+c)}}{128d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{35a\cos(dx+c)}{64d}+\frac{a\cos(7dx+7c)}{448d}+\frac{a\,\mathrm{c}}{}$
norman	$\frac{-\frac{32a}{35d}-\frac{128a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3d}-\frac{166a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{5d}-\frac{224a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-\frac{42a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{5d}-\frac{14a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^1}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7}$

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))
-1/7*a*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int (a+a\sec(c+dx))\sin^7(c+dx)\,dx$$
$$=\frac{60\,a\cos{(dx+c)}^7+70\,a\cos{(dx+c)}^6-252\,a\cos{(dx+c)}^5-315\,a\cos{(dx+c)}^4+420\,a\cos{(dx+c)}^3+63}{420\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*a*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*a*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*a*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*a*log(-cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))\sin^7(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int (a+a\sec(c+dx))\sin^7(c+dx)\,dx$$
$$=\frac{60\,a\cos{(dx+c)}^7+70\,a\cos{(dx+c)}^6-252\,a\cos{(dx+c)}^5-315\,a\cos{(dx+c)}^4+420\,a\cos{(dx+c)}^3+63}{420\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*a*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*a*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*a*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*a*log(cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(109) = 218$.

Time = 0.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.08

$$\int (a + a\sec(c+dx))\sin^7(c+dx)\,dx = \frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{1473\,a - \frac{11151\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{36813\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \cdots}{\cdots}}{420\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1473*a - 11151*a*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 36813*a*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 69475*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 28749*a*(cos(d*x + c) - 1)^5/(co
s(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089
*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int (a + a\sec(c+dx))\sin^7(c+dx)\,dx = -\frac{a\cos(c+d\,x) - \frac{3\,a\cos(c+d\,x)^2}{2} - a\cos(c+d\,x)^3 + \frac{3\,a\cos(c+d\,x)^4}{4} + \frac{3\,a\cos(c+d\,x)^5}{5} - \frac{a\cos(c+d\,x)^6}{6} - \frac{a\cos(c+d\,x)^7}{7} \cdots}{d}$$

```
[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - (3*a*cos(c + d*x)^2)/2 - a*cos(c + d*x)^3 + (3*a*cos(c +
 d*x)^4)/4 + (3*a*cos(c + d*x)^5)/5 - (a*cos(c + d*x)^6)/6 - (a*cos(c + d*x
)^7)/7 + a*log(cos(c + d*x)))/d
```

3.3 $\int (a+a\sec(c+dx))\sin^5(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int (a+a\sec(c+dx))\sin^5(c+dx)\,dx = -\frac{a\cos(c+dx)}{d}+\frac{a\cos^2(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+a*cos(d*x+c)^2/d+2/3*a*cos(d*x+c)^3/d-1/4*a*cos(d*x+c)^4/d-
1/5*a*cos(d*x+c)^5/d-a*ln(cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2915, 12, 90}

$$\int (a+a\sec(c+dx))\sin^5(c+dx)\,dx = -\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^4(c+dx)}{4d}+\frac{2a\cos^3(c+dx)}{3d}+\frac{a\cos^2(c+dx)}{d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (a*Cos[c + d*x]^2)/d + (2*a*Cos[c + d*x]^3)/(3*d) -
 (a*Cos[c + d*x]^4)/(4*d) - (a*Cos[c + d*x]^5)/(5*d) - (a*Log[Cos[c + d*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a - a\cos(c+dx))\sin^4(c+dx)\tan(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{a(-a-x)^2(-a+x)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^5 d} \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^4 d} \\
&= \frac{\text{Subst}\left(\int \left(a^4 - \frac{a^5}{x} + 2a^3x - 2a^2x^2 - ax^3 + x^4\right)dx, x, -a\cos(c+dx)\right)}{a^4 d} \\
&= -\frac{a\cos(c+dx)}{d} + \frac{a\cos^2(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} \\
&\quad - \frac{a\cos^4(c+dx)}{4d} - \frac{a\cos^5(c+dx)}{5d} - \frac{a\log(\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$
\begin{aligned}
&\int (a + a\sec(c+dx))\sin^5(c+dx)\,dx \\
&= -\frac{5a\cos(c+dx)}{8d} + \frac{5a\cos(3(c+dx))}{48d} - \frac{a\cos(5(c+dx))}{80d} \\
&\quad - \frac{a\left(-\cos^2(c+dx) + \frac{1}{4}\cos^4(c+dx) + \log(\cos(c+dx))\right)}{d}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] (-5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*x)])/(48*d) - (a*Cos[5*(c + d
*x)])/(80*d) - (a*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c + d*x]]))
/d
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}}{d}$
parts	$-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{a\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parallelrisch	$-\frac{a\left(-480\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+480\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+480\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+421+6\cos(5dx+5c)+15\cos(4dx+4c)\right.}{480d}$
risch	$iax+\frac{3a\,e^{2i(dx+c)}}{16d}+\frac{3a\,e^{-2i(dx+c)}}{16d}+\frac{2iac}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{5a\cos(dx+c)}{8d}-\frac{a\cos(5dx+5c)}{80d}-\frac{a\cos(4d}{32d}$
norman	$\frac{-\frac{16a}{15d}-\frac{22a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}-\frac{62a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{10a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}+\frac{a\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}-\frac{a\ln\left(\tan\right.}{}$

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/5*a*(8/3+sin(d
*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c + dx))\sin^5(c + dx)\,dx = -\frac{12\,a\cos{(dx + c)}^5 + 15\,a\cos{(dx + c)}^4 - 40\,a\cos{(dx + c)}^3 - 60\,a\cos{(dx + c)}^2 + 60\,a\cos{(dx + c)} + 60}{60\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="fricas")
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*a*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*a*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*a*log(-cos(d*x + c)))/d
```

Sympy [F]

$$\int (a+a\sec(c+dx))\sin^5(c+dx)\,dx = a\left(\int \sin^5{(c+dx)}\sec{(c+dx)}\,dx + \int \sin^5{(c+dx)}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**5,x)
[Out] a*(Integral(sin(c + d*x)**5*sec(c + d*x), x) + Integral(sin(c + d*x)**5, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int (a + a\sec(c + dx))\sin^5(c + dx)\,dx = -\frac{12\,a\cos{(dx + c)}^5 + 15\,a\cos{(dx + c)}^4 - 40\,a\cos{(dx + c)}^3 - 60\,a\cos{(dx + c)}^2 + 60\,a\cos{(dx + c)} + 60}{60\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="maxima")
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*a*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*a*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*a*log(cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(81) = 162$.

Time = 0.31 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.31

$$\int (a + a\sec(c + dx))\sin^5(c + dx)\, dx$$
$$= \frac{60\, a \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 60\, a \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{201\, a - \frac{1125\, a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{2610\, a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{1970\, a(\cos}{(\cos(dx}}{\left(\frac{\cos(dx+c)-}{\cos(dx+c)+}\right.}}{60\, d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (201*a - 1125*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2610*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 - 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*a*(cos(d*x + c) - 1)^5/(cos(d*x + c)
+ 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int (a + a\sec(c + dx))\sin^5(c + dx)\, dx =$$
$$-\frac{a\,\cos\left(c+d\,x\right)-a\,{\cos\left(c+d\,x\right)}^2-\frac{2\,a\,{\cos\left(c+d\,x\right)}^3}{3}+\frac{a\,{\cos\left(c+d\,x\right)}^4}{4}+\frac{a\,{\cos\left(c+d\,x\right)}^5}{5}+a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - a*cos(c + d*x)^2 - (2*a*cos(c + d*x)^3)/3 + (a*cos(c + d
*x)^4)/4 + (a*cos(c + d*x)^5)/5 + a*log(cos(c + d*x)))/d
```

3.4 $\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx = -\frac{a\cos(c+dx)}{d}+\frac{a\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{3d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+1/2*a*cos(d*x+c)^2/d+1/3*a*cos(d*x+c)^3/d-a*ln(cos(d*x+c))/
d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2915, 12, 76}

$$\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx = \frac{a\cos^3(c+dx)}{3d}+\frac{a\cos^2(c+dx)}{2d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (a*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/(3*d)
 - (a*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))\sin^2(c+dx)\tan(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a(-a-x)(-a+x)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^3 d}\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)(-a+x)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^2 d}\\
&= \frac{\text{Subst}\left(\int \left(a^2-\frac{a^3}{x}+ax-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{a^2 d}\\
&= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{3d}-\frac{a\log(\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx = -\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d} - \frac{a\left(-\frac{1}{2}\cos^2(c+dx)+\log(\cos(c+dx))\right)}{d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^3,x]
[Out] (-3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (a*(-1/2*Cos[c +
d*x]^2 + Log[Cos[c + d*x]]))/d
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}}{d}$
parts	$-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{a\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parallelrisch	$\frac{a\left(12\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-11+\cos(3dx+3c)+3\cos(2dx+2c)-9\cos(d\right.}{12d}$
risch	$iax+\frac{a\,e^{2i(dx+c)}}{8d}+\frac{a\,e^{-2i(dx+c)}}{8d}+\frac{2iac}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{3a\cos(dx+c)}{4d}+\frac{a\cos(3dx+3c)}{12d}$
norman	$\frac{-\frac{4a}{3d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{6a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{a\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{d}$

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/3*a*(2+sin(d*x+c)^2)*cos(d*x+c)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx = \frac{2\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)^2-6\,a\cos\left(dx+c\right)-6\,a\log\left(-\cos\left(dx+c\right)\right)}{6\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*a*log(-
cos(d*x + c)))/d
```

Sympy [F]

$$\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx = a\left(\int \sin^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**3,x)
```

```
[Out] a*(Integral(sin(c + d*x)**3*sec(c + d*x), x) + Integral(sin(c + d*x)**3, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx = \frac{2\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)^2-6\,a\cos\left(dx+c\right)-6\,a\log\left(\cos\left(dx+c\right)\right)}{6\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*a*log(c
os(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int (a + a\sec(c + dx))\sin^3(c + dx)\,dx = -\frac{a\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{2\,ad^2\cos(dx+c)^3 + 3\,ad^2\cos(dx+c)^2 - 6\,ad^2\cos(dx+c)}{6\,d^3}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -a*log(abs(cos(d*x + c))/abs(d))/d + 1/6*(2*a*d^2*cos(d*x + c)^3 + 3*a*d^2*
cos(d*x + c)^2 - 6*a*d^2*cos(d*x + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int (a + a\sec(c + dx))\sin^3(c + dx)\,dx = -\frac{a\,\cos(c+d\,x) - \frac{a\cos(c+d\,x)^2}{2} - \frac{a\cos(c+d\,x)^3}{3} + a\,\ln(\cos(c+d\,x))}{d}$$

```
[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - (a*cos(c + d*x)^2)/2 - (a*cos(c + d*x)^3)/3 + a*log(cos(
c + d*x)))/d
```

3.5 $\int (a+a\sec(c+dx))\sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 26

$$\int (a+a\sec(c+dx))\sin(c+dx)\,dx=-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d-a*ln(cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3957, 2786, 45}

$$\int (a+a\sec(c+dx))\sin(c+dx)\,dx=-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (a*Log[Cos[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
```

```
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int (-a - a\cos(c+dx))\tan(c+dx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{-a+x}{x}\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(1-\frac{a}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= -\frac{a\cos(c+dx)}{d} - \frac{a\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int (a+a\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(c)\cos(dx)}{d} - \frac{a\log(\cos(c+dx))}{d} + \frac{a\sin(c)\sin(dx)}{d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x],x]
[Out] -((a*Cos[c]*Cos[d*x])/d) - (a*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{\sec(dx+c)}+\ln(\sec(dx+c))\right)}{d}$	24
default	$\frac{a\left(-\frac{1}{\sec(dx+c)}+\ln(\sec(dx+c))\right)}{d}$	24
parts	$-\frac{a\cos(dx+c)}{d}+\frac{a\ln(\sec(dx+c))}{d}$	26
risch	$iax+\frac{2iac}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{a\cos(dx+c)}{d}$	45
parallelrisch	$-\frac{a\left(-\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+1+\cos(dx+c)\right)}{d}$	53
norman	$-\frac{2a}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}+\frac{a\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	78

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*a*(-1/sec(d*x+c)+ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int (a+a\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(dx+c)+a\log(-\cos(dx+c))}{d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="fricas")
[Out] -(a*cos(d*x + c) + a*log(-cos(d*x + c)))/d
```

Sympy [F]

$$\int (a+a\sec(c+dx))\sin(c+dx)\,dx = a\left(\int \sin(c+dx)\sec(c+dx)\,dx+\int \sin(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x)
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int (a+a\sec(c+dx))\sin(c+dx)\,dx=-\frac{a\cos(dx+c)+a\log(\cos(dx+c))}{d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="maxima")
[Out] -(a*cos(d*x + c) + a*log(cos(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (a+a\sec(c+dx))\sin(c+dx)\,dx=-\frac{a\cos(dx+c)}{d}-\frac{a\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="giac")
[Out] -a*cos(d*x + c)/d - a*log(abs(cos(d*x + c))/abs(d))/d
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int (a+a\sec(c+dx))\sin(c+dx)\,dx=-\frac{a(\cos(c+d\,x)+\ln(\cos(c+d\,x)))}{d}$$

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x)),x)
[Out] -(a*(cos(c + d*x) + log(cos(c + d*x))))/d
```

3.6 $\int \csc(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log(1-\cos(c+dx))}{d} - \frac{a\log(\cos(c+dx))}{d}$$

```
[Out] a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3957, 2915, 12, 36, 31, 29}

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log(1-\cos(c+dx))}{d} - \frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Log[1 - Cos[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 29</u>

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a - a\cos(c+dx))\csc(c+dx)\sec(c+dx)\,dx \\
&= \frac{a\text{Subst}\left(\int \frac{a}{(-a-x)x}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^2\text{Subst}\left(\int \frac{1}{(-a-x)x}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= -\frac{a\text{Subst}\left(\int \frac{1}{-a-x}\,dx, x, -a\cos(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int \frac{1}{x}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a\log(1-\cos(c+dx))}{d} - \frac{a\log(\cos(c+dx))}{d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a\log(\cos(c+dx))}{d} + \frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log(\sin(c+dx))}{d}$$

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x]),x]
[Out] -((a*Log[Cos[c/2 + (d*x)/2]])/d) - (a*Log[Cos[c + d*x]])/d + (a*Log[Sin[c/2
 + (d*x)/2]])/d + (a*Log[Sin[c + d*x]])/d
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{a\ln(\tan(dx+c))+a\ln(-\cot(dx+c)+\csc(dx+c))}{d}$	33
default	$\frac{a\ln(\tan(dx+c))+a\ln(-\cot(dx+c)+\csc(dx+c))}{d}$	33
risch	$\frac{2a\ln(\mathrm{e}^{i(dx+c)}-1)}{d}-\frac{a\ln(\mathrm{e}^{2i(dx+c)}+1)}{d}$	38
parallelrisch	$\frac{a\left(2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{d}$	47
norman	$\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	54

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*ln(tan(d*x+c))+a*ln(-cot(d*x+c)+csc(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\log\left(-\cos\left(dx+c\right)\right)-a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{d}$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -(a*log(-cos(d*x + c)) - a*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc(c+dx)\sec(c+dx)\,dx + \int \csc(c+dx)\,dx\right)$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(csc(c + d*x)*sec(c + d*x), x) + Integral(csc(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log(\cos(dx+c)-1) - a\log(\cos(dx+c))}{d}$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] (a*log(cos(d*x + c) - 1) - a*log(cos(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{d}$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] (a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a*log(abs(-(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}(1-2\,\cos(c+d\,x))}{d}$$

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x),x)
[Out] (2*a*atanh(1 - 2*cos(c + d*x)))/d
```

3.7 $\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a^2}{2d(a-a\cos(c+dx))}+\frac{3a\log(1-\cos(c+dx))}{4d} - \frac{a\log(\cos(c+dx))}{d}+\frac{a\log(1+\cos(c+dx))}{4d}$$

```
[Out] -1/2*a^2/d/(a-a*cos(d*x+c))+3/4*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+1/4
*a*ln(1+cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2915, 12, 84}

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a^2}{2d(a-a\cos(c+dx))}+\frac{3a\log(1-\cos(c+dx))}{4d} - \frac{a\log(\cos(c+dx))}{d}+\frac{a\log(\cos(c+dx)+1)}{4d}$$

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*a^2/(d*(a - a*Cos[c + d*x])) + (3*a*Log[1 - Cos[c + d*x]])/(4*d) - (a*
Log[Cos[c + d*x]])/d + (a*Log[1 + Cos[c + d*x]])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a - a\cos(c+dx))\csc^3(c+dx)\sec(c+dx)\,dx \\
&= \frac{a^3\text{Subst}\left(\int \frac{a}{(-a-x)^2x(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^4\text{Subst}\left(\int \frac{1}{(-a-x)^2x(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^4\text{Subst}\left(\int \left(-\frac{1}{4a^3(a-x)} - \frac{1}{a^3x} + \frac{1}{2a^2(a+x)^2} + \frac{3}{4a^3(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= -\frac{a^2}{2d(a-a\cos(c+dx))} + \frac{3a\log(1-\cos(c+dx))}{4d} \\
&\quad - \frac{a\log(\cos(c+dx))}{d} + \frac{a\log(1+\cos(c+dx))}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\csc^2(c+dx)}{2d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\log(\cos(c+dx))}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\log(\sin(c+dx))}{d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[Cos[c + d*x]])/d + (a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Log[Sin[c + d*x]])/d + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$-\frac{a\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{4d}$	59
derivativedivides	$\frac{a\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)}{d}$	61
default	$\frac{a\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)}{d}$	61
norman	$-\frac{a}{4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	71
risch	$\frac{a\,e^{i(dx+c)}}{d\left(e^{i(dx+c)}-1\right)^2}+\frac{3a\ln\left(e^{i(dx+c)}-1\right)}{2d}+\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	83

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a*(cot(1/2*d*x+1/2*c)^2-6*ln(tan(1/2*d*x+1/2*c))+4*ln(tan(1/2*d*x+1/2*c)-1)+4*ln(tan(1/2*d*x+1/2*c)+1))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{4\left(a\cos\left(dx+c\right)-a\right)\log\left(-\cos\left(dx+c\right)\right)-\left(a\cos\left(dx+c\right)-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(a\cos\left(dx\right.\right.}{4\left(d\cos\left(dx+c\right)-d\right)}$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a*cos(d*x + c) - a)*log(-cos(d*x + c)) - (a*cos(d*x + c) - a)*log(
1/2*cos(d*x + c) + 1/2) - 3*(a*cos(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/
2) - 2*a)/(d*cos(d*x + c) - d)
```

Sympy [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc^3\left(c+dx\right)\sec\left(c+dx\right)dx + \int \csc^3\left(c+dx\right)dx\right)$$

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)**3*sec(c + d*x), x) + Integral(csc(c + d*x)**3, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\cos\left(dx+c\right)+1\right)+3\,a\log\left(\cos\left(dx+c\right)-1\right)-4\,a\log\left(\cos\left(dx+c\right)\right)+\frac{2\,a}{\cos\left(dx+c\right)-1}}{4\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(cos(d*x + c) + 1) + 3*a*log(cos(d*x + c) - 1) - 4*a*log(cos(d*x
+ c)) + 2*a/(cos(d*x + c) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{3\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 4\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{\left(a - \frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{4\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(3*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a*log(abs(-(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a - 3*a*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{\frac{a}{2\,(\cos(c+d\,x)-1)} - a\,\ln\left(\cos\left(c+d\,x\right)\right) + \frac{3\,a\,\ln(\cos(c+d\,x)-1)}{4} + \frac{a\,\ln(\cos(c+d\,x)+1)}{4}}{d}$$

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^3,x)
```

```
[Out] (a/(2*(cos(c + d*x) - 1)) - a*log(cos(c + d*x)) + (3*a*log(cos(c + d*x) - 1
))/4 + (a*log(cos(c + d*x) + 1))/4)/d
```

3.8 $\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx=-\frac{a^3}{8d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^2}{8d(a+a\cos(c+dx))}+\frac{11a\log(1-\cos(c+dx))}{16d}-\frac{a\log(\cos(c+dx))}{d}+\frac{5a\log(1+\cos(c+dx))}{16d}$$

```
[Out] -1/8*a^3/d/(a-a*cos(d*x+c))^2-1/2*a^2/d/(a-a*cos(d*x+c))-1/8*a^2/d/(a+a*cos
(d*x+c))+11/16*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+5/16*a*ln(1+cos(d*x+
c))/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2915, 12, 90}

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx=-\frac{a^3}{8d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^2}{8d(a\cos(c+dx)+a)}+\frac{11a\log(1-\cos(c+dx))}{16d}-\frac{a\log(\cos(c+dx))}{d}+\frac{5a\log(\cos(c+dx)+1)}{16d}$$

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*a^3/(d*(a - a*Cos[c + d*x])^2) - a^2/(2*d*(a - a*Cos[c + d*x])) - a^2/
(8*d*(a + a*Cos[c + d*x])) + (11*a*Log[1 - Cos[c + d*x]])/(16*d) - (a*Log[C
os[c + d*x]])/d + (5*a*Log[1 + Cos[c + d*x]])/(16*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))\csc^5(c+dx)\sec(c+dx)\,dx \\
&= \frac{a^5\text{Subst}\left(\int \frac{a}{(-a-x)^3x(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^6\text{Subst}\left(\int \frac{1}{(-a-x)^3x(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^6\text{Subst}\left(\int \left(-\frac{1}{8a^4(a-x)^2}-\frac{5}{16a^5(a-x)}-\frac{1}{a^5x}+\frac{1}{4a^3(a+x)^3}+\frac{1}{2a^4(a+x)^2}+\frac{11}{16a^5(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d} \\
&= -\frac{a^3}{8d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^2}{8d(a+a\cos(c+dx))} \\
&\quad +\frac{11a\log(1-\cos(c+dx))}{16d}-\frac{a\log(\cos(c+dx))}{d}+\frac{5a\log(1+\cos(c+dx))}{16d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.49

$$
\begin{aligned}
\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx = &-\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} \\
&-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^4(c+dx)}{4d} \\
&-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{a\log(\cos(c+dx))}{d} \\
&+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{a\log(\sin(c+dx))}{d} \\
&+\frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}
\end{aligned}
$$

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Csc[c
 + d*x]^2)/(2*d) - (a*Csc[c + d*x]^4)/(4*d) - (3*a*Log[Cos[(c + d*x)/2]])/(
8*d) - (a*Log[Cos[c + d*x]])/d + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (a*Log
[Sin[c + d*x]])/d + (3*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4
)/(64*d)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{a\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{\csc(dx+c)^3}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(-\cot(dx+c)+\csc(dx+c}{8}\right.}{d}$
default	$\frac{a\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{\csc(dx+c)^3}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(-\cot(dx+c)+\csc(dx+c}{8}\right.}{d}$
parallelrisch	$-\frac{a\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+10\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2+32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-44\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+32\ln\left(\tan\left(\frac{dx}{2}+\right.\right.\right.}{32d}$
norman	$\frac{-\frac{a}{32d}-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{16d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{11a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$
risch	$\frac{a\left(3\,e^{5i(dx+c)}+2\,e^{4i(dx+c)}-18\,e^{3i(dx+c)}+2\,e^{2i(dx+c)}+3\,e^{i(dx+c)}\right)}{4d\left(e^{i(dx+c)}-1\right)^4\left(e^{i(dx+c)}+1\right)^2}+\frac{11a\ln\left(e^{i(dx+c)}-1\right)}{8d}+\frac{5a\ln\left(e^{i(dx+c)}+1\right)}{8d}-\frac{a\,l}{}$

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a*((-1/4*csc(d*x
+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(-cot(d*x+c)+csc(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.64

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{6\,a\cos\left(dx+c\right)^2 + 2\,a\cos\left(dx+c\right) - 16\left(a\cos\left(dx+c\right)^3 - a\cos\left(dx+c\right)^2 - a\cos\left(dx+c\right) + a\right)\log\left(-\cos\left(\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(6*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - 16*(a*cos(d*x + c)^3 - a*cos(
d*x + c)^2 - a*cos(d*x + c) + a)*log(-cos(d*x + c)) + 5*(a*cos(d*x + c)^3 -
 a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(1/2*cos(d*x + c) + 1/2) + 11*(a
*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-1/2*cos(d*x +
 c) + 1/2) - 12*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) +
d)
```

Sympy [F]

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc^5\left(c+dx\right)\sec\left(c+dx\right)dx + \int \csc^5\left(c+dx\right)dx\right)$$

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)**5*sec(c + d*x), x) + Integral(csc(c + d*x)**5, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{5\,a\log\left(\cos\left(dx+c\right)+1\right) + 11\,a\log\left(\cos\left(dx+c\right)-1\right) - 16\,a\log\left(\cos\left(dx+c\right)\right) + \frac{2\left(3\,a\cos(dx+c)^2 + a\cos(dx+c) - 6\right.}{\cos(dx+c)^3 - \cos(dx+c)^2 - \cos(dx+}}{16\,d}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(5*a*log(cos(d*x + c) + 1) + 11*a*log(cos(d*x + c) - 1) - 16*a*log(cos
(d*x + c)) + 2*(3*a*cos(d*x + c)^2 + a*cos(d*x + c) - 6*a)/(cos(d*x + c)^3
- cos(d*x + c)^2 - cos(d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{22\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 32\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{\left(a-\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{33\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2} +}{32\,d}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(22*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a - 10*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 33*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx = \frac{11\,a\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{16\,d} - \frac{a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d} + \frac{5\,a\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{16\,d} + \frac{\frac{3\,a\cos(c+d\,x)^2}{8}+\frac{a\,\cos(c+d\,x)}{8}-\frac{3\,a}{4}}{d\,\left(\cos\left(c+d\,x\right)^3-\cos\left(c+d\,x\right)+\sin\left(c+d\,x\right)^2\right)}$$

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^5,x)
```

```
[Out] (11*a*log(cos(c + d*x) - 1))/(16*d) - (a*log(cos(c + d*x)))/d + (5*a*log(co
s(c + d*x) + 1))/(16*d) + ((a*cos(c + d*x))/8 - (3*a)/4 + (3*a*cos(c + d*x)
^2)/8)/(d*(cos(c + d*x)^3 - cos(c + d*x) + sin(c + d*x)^2))
```

3.9 $\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 163

$$\begin{aligned}\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx = & -\frac{a^4}{24d(a-a\cos(c+dx))^3}-\frac{5a^3}{32d(a-a\cos(c+dx))^2} \\ & -\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^3}{32d(a+a\cos(c+dx))^2} \\ & -\frac{3a^2}{16d(a+a\cos(c+dx))}+\frac{21a\log(1-\cos(c+dx))}{32d} \\ & -\frac{a\log(\cos(c+dx))}{d}+\frac{11a\log(1+\cos(c+dx))}{32d}\end{aligned}$$

```
[Out] -1/24*a^4/d/(a-a*cos(d*x+c))^3-5/32*a^3/d/(a-a*cos(d*x+c))^2-1/2*a^2/d/(a-a
*cos(d*x+c))-1/32*a^3/d/(a+a*cos(d*x+c))^2-3/16*a^2/d/(a+a*cos(d*x+c))+21/3
2*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+11/32*a*ln(1+cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2915, 12, 90}

$$\begin{aligned}\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx = & -\frac{a^4}{24d(a-a\cos(c+dx))^3}-\frac{5a^3}{32d(a-a\cos(c+dx))^2} \\ & -\frac{a^3}{32d(a\cos(c+dx)+a)^2}-\frac{a^2}{2d(a-a\cos(c+dx))} \\ & -\frac{3a^2}{16d(a\cos(c+dx)+a)}+\frac{21a\log(1-\cos(c+dx))}{32d} \\ & -\frac{a\log(\cos(c+dx))}{d}+\frac{11a\log(\cos(c+dx)+1)}{32d}\end{aligned}$$

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/24*a^4/(d*(a - a*Cos[c + d*x])^3) - (5*a^3)/(32*d*(a - a*Cos[c + d*x])^2
) - a^2/(2*d*(a - a*Cos[c + d*x])) - a^3/(32*d*(a + a*Cos[c + d*x])^2) - (3
*a^2)/(16*d*(a + a*Cos[c + d*x])) + (21*a*Log[1 - Cos[c + d*x]])/(32*d) - (
a*Log[Cos[c + d*x]])/d + (11*a*Log[1 + Cos[c + d*x]])/(32*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int (-a - a\cos(c+dx))\csc^7(c+dx)\sec(c+dx)\,dx$$

$$= \frac{a^7\text{Subst}\left(\int \frac{a}{(-a-x)^4x(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^8\text{Subst}\left(\int \frac{1}{(-a-x)^4x(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^8\text{Subst}\left(\int \left(-\frac{1}{16a^5(a-x)^3} - \frac{3}{16a^6(a-x)^2} - \frac{11}{32a^7(a-x)} - \frac{1}{a^7x} + \frac{1}{8a^4(a+x)^4} + \frac{5}{16a^5(a+x)^3} + \frac{1}{2a^6(a+x)^2} + \frac{21}{32a^7(a\cdots}\right.\right.}{d}$$

$$= -\frac{a^4}{24d(a-a\cos(c+dx))^3} - \frac{5a^3}{32d(a-a\cos(c+dx))^2} - \frac{a^2}{2d(a-a\cos(c+dx))} - \frac{a^3}{32d(a+a\cos(c+dx))^2} - \frac{3a^2}{16d(a+a\cos(c+dx))} + \frac{21a\log(1-\cos(c+dx))}{32d} - \frac{a\log(\cos(c+dx))}{d} + \frac{11a\log(1+\cos(c+dx))}{32d}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.42

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx = -\frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\csc^2(c+dx)}{2d} - \frac{a\csc^4(c+dx)}{4d} - \frac{a\csc^6(c+dx)}{6d} - \frac{5a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{a\log(\cos(c+dx))}{d} + \frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{a\log(\sin(c+dx))}{d} + \frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-5*a*Csc[(c + d*x)/2]^2)/(64*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Csc[(
c + d*x)/2]^6)/(384*d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^4)/(4*d
) - (a*Csc[c + d*x]^6)/(6*d) - (5*a*Log[Cos[(c + d*x)/2]])/(16*d) - (a*Log[
Cos[c + d*x]])/d + (5*a*Log[Sin[(c + d*x)/2]])/(16*d) + (a*Log[Sin[c + d*x]
])/d + (5*a*Sec[(c + d*x)/2]^2)/(64*d) + (a*Sec[(c + d*x)/2]^4)/(64*d) + (a
*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{a\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{\csc(dx+c)^5}{6}-\frac{5\csc(dx+c)^3}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx+c\right.}{d}$
default	$\frac{a\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{\csc(dx+c)^5}{6}-\frac{5\csc(dx+c)^3}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx+c\right.}{d}$
parallelrisch	$-\frac{a\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\frac{21\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}+66\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2+21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-252\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+192\,\mathrm{l}\right.}{192d}$
norman	$\frac{-\frac{a}{192d}-\frac{7a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{128d}-\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{32d}-\frac{7a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{64d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{128d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}+\frac{21a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{a\ln\left(\tan\left(\right.\right.}{}$
risch	$\frac{a\left(15\,e^{9i(dx+c)}+18\,e^{8i(dx+c)}-136\,e^{7i(dx+c)}-34\,e^{6i(dx+c)}+402\,e^{5i(dx+c)}-34\,e^{4i(dx+c)}-136\,e^{3i(dx+c)}+18\,e^{2i(dx+c)}+15\right.}{24d\left(e^{i(dx+c)}-1\right)^6\left(e^{i(dx+c)}+1\right)^4}$

[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/6/sin(d*x+c)^6-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c))) +a*((-1/6*csc(d*x+c)^5-5/24*csc(d*x+c)^3-5/16*csc(d*x+c))*cot(d*x+c)+5/16*ln(-cot(d*x+c)+csc(d*x+c))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(152) = 304$.

Time = 0.30 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.88

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{30\,a\cos\left(dx+c\right)^4+18\,a\cos\left(dx+c\right)^3-98\,a\cos\left(dx+c\right)^2-22\,a\cos\left(dx+c\right)-96\,\left(a\cos\left(dx+c\right)^5-a\,\mathrm{co}\right.}{}$$

[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/96*(30*a*cos(d*x + c)^4 + 18*a*cos(d*x + c)^3 - 98*a*cos(d*x + c)^2 - 22*a*cos(d*x + c) - 96*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(-cos(d*x + c)) + 33*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 63*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/2) + 88*a)/(d*cos(d*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*cos(d*x + c) - d)

Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{33\,a\log(\cos(dx+c)+1)+63\,a\log(\cos(dx+c)-1)-96\,a\log(\cos(dx+c))+\frac{2\left(15\,a\cos(dx+c)^4+9\,a\cos(dx+c)\cdots\right.}{\cos(dx+c)^5-\cos(dx+c)^4-2\,\mathrm{co}\cdots}}{96\,d}$$

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/96*(33*a*log(cos(d*x + c) + 1) + 63*a*log(cos(d*x + c) - 1) - 96*a*log(co
s(d*x + c)) + 2*(15*a*cos(d*x + c)^4 + 9*a*cos(d*x + c)^3 - 49*a*cos(d*x +
c)^2 - 11*a*cos(d*x + c) + 44*a)/(cos(d*x + c)^5 - cos(d*x + c)^4 - 2*cos(d
*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{252\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(2\,a-\frac{21\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{462\,a(\cos(dx+c)-\cdots}{(\cos(dx+c)+1)^3}\cdots\right.}{(\cos(dx+c)-1)^3}}{384\,d}$$

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 1/384*(252*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 384*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a - 21*a*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 132*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 462*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(c
os(d*x + c) - 1)^3 + 42*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{\frac{5\,a\cos(c+d\,x)^4}{16}+\frac{3\,a\cos(c+d\,x)^3}{16}-\frac{49\,a\cos(c+d\,x)^2}{48}-\frac{11\,a\,\cos(c+d\,x)}{48}+\frac{11\,a}{12}}{d\,\left(\cos\left(c+d\,x\right)^5-\cos\left(c+d\,x\right)^4-2\cos\left(c+d\,x\right)^3+2\cos\left(c+d\,x\right)^2+\cos\left(c+d\,x\right)-1\right)}$$
$$-\frac{a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}+\frac{21\,a\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{32\,d}+\frac{11\,a\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{32\,d}$$

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^7,x)
```

```
[Out] ((11*a)/12 - (11*a*cos(c + d*x))/48 - (49*a*cos(c + d*x)^2)/48 + (3*a*cos(c
 + d*x)^3)/16 + (5*a*cos(c + d*x)^4)/16)/(d*(cos(c + d*x) + 2*cos(c + d*x)^
2 - 2*cos(c + d*x)^3 - cos(c + d*x)^4 + cos(c + d*x)^5 - 1)) - (a*log(cos(c
 + d*x)))/d + (21*a*log(cos(c + d*x) - 1))/(32*d) + (11*a*log(cos(c + d*x)
+ 1))/(32*d)
```

3.10 $\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 165

$$\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx = \frac{35ax}{128}+\frac{a\text{arctanh}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{35a\cos(c+dx)\sin(c+dx)}{128d}-\frac{a\sin^3(c+dx)}{3d}-\frac{35a\cos(c+dx)\sin^3(c+dx)}{192d}-\frac{a\sin^5(c+dx)}{5d}-\frac{7a\cos(c+dx)\sin^5(c+dx)}{48d}-\frac{a\sin^7(c+dx)}{7d}-\frac{a\cos(c+dx)\sin^7(c+dx)}{8d}$$

```
[Out] 35/128*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-35/128*a*cos(d*x+c)*sin(d
*x+c)/d-1/3*a*sin(d*x+c)^3/d-35/192*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*a*sin(d
*x+c)^5/d-7/48*a*cos(d*x+c)*sin(d*x+c)^5/d-1/7*a*sin(d*x+c)^7/d-1/8*a*cos(d
*x+c)*sin(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used

$= \{3957, 2917, 2672, 308, 212, 2715, 8\}$

$$\int (a + a\sec(c+dx))\sin^8(c+dx)\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\sin^7(c+dx)}{7d} - \frac{a\sin^5(c+dx)}{5d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\sin(c+dx)}{d} - \frac{a\sin^7(c+dx)\cos(c+dx)}{8d} - \frac{7a\sin^5(c+dx)\cos(c+dx)}{48d} - \frac{35a\sin^3(c+dx)\cos(c+dx)}{192d} - \frac{35a\sin(c+dx)\cos(c+dx)}{128d} + \frac{35ax}{128}$$

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^8,x]
```

```
[Out] (35*a*x)/128 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (35*a*Cos
[c + d*x]*Sin[c + d*x])/(128*d) - (a*Sin[c + d*x]^3)/(3*d) - (35*a*Cos[c +
d*x]*Sin[c + d*x]^3)/(192*d) - (a*Sin[c + d*x]^5)/(5*d) - (7*a*Cos[c + d*x]
*Sin[c + d*x]^5)/(48*d) - (a*Sin[c + d*x]^7)/(7*d) - (a*Cos[c + d*x]*Sin[c
+ d*x]^7)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))\sin^7(c+dx)\tan(c+dx)\,dx\\
&= a\int \sin^8(c+dx)\,dx + a\int \sin^7(c+dx)\tan(c+dx)\,dx\\
&= -\frac{a\cos(c+dx)\sin^7(c+dx)}{8d}+\frac{1}{8}(7a)\int \sin^6(c+dx)\,dx+\frac{a\text{Subst}\left(\int \frac{x^8}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\
&= -\frac{7a\cos(c+dx)\sin^5(c+dx)}{48d}-\frac{a\cos(c+dx)\sin^7(c+dx)}{8d}\\
&\quad+\frac{1}{48}(35a)\int \sin^4(c+dx)\,dx\\
&\quad+\frac{a\text{Subst}\left(\int \left(-1-x^2-x^4-x^6+\frac{1}{1-x^2}\right)\,dx,x,\sin(c+dx)\right)}{d}\\
&= -\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}-\frac{35a\cos(c+dx)\sin^3(c+dx)}{192d}-\frac{a\sin^5(c+dx)}{5d}\\
&\quad-\frac{7a\cos(c+dx)\sin^5(c+dx)}{48d}-\frac{a\sin^7(c+dx)}{7d}-\frac{a\cos(c+dx)\sin^7(c+dx)}{8d}\\
&\quad+\frac{1}{64}(35a)\int \sin^2(c+dx)\,dx+\frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{35a\cos(c+dx)\sin(c+dx)}{128d} \\
&\quad - \frac{a\sin^3(c+dx)}{3d} - \frac{35a\cos(c+dx)\sin^3(c+dx)}{192d} \\
&\quad - \frac{a\sin^5(c+dx)}{5d} - \frac{7a\cos(c+dx)\sin^5(c+dx)}{48d} - \frac{a\sin^7(c+dx)}{7d} \\
&\quad - \frac{a\cos(c+dx)\sin^7(c+dx)}{8d} + \frac{1}{128}(35a)\int 1\,dx \\
&= \frac{35ax}{128} + \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{35a\cos(c+dx)\sin(c+dx)}{128d} \\
&\quad - \frac{a\sin^3(c+dx)}{3d} - \frac{35a\cos(c+dx)\sin^3(c+dx)}{192d} - \frac{a\sin^5(c+dx)}{5d} \\
&\quad - \frac{7a\cos(c+dx)\sin^5(c+dx)}{48d} - \frac{a\sin^7(c+dx)}{7d} - \frac{a\cos(c+dx)\sin^7(c+dx)}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.64

$$
\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx
$$
$$
= \frac{a(107520\operatorname{arctanh}(\sin(c+dx)) - 107520\sin(c+dx) - 35840\sin^3(c+dx) - 21504\sin^5(c+dx) - 15360\,\mathrm{s}}{}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^8,x]
```

```
[Out] (a*(107520*ArcTanh[Sin[c + d*x]] - 107520*Sin[c + d*x] - 35840*Sin[c + d*x]
^3 - 21504*Sin[c + d*x]^5 - 15360*Sin[c + d*x]^7 + 35*(840*c + 840*d*x - 67
2*Sin[2*(c + d*x)] + 168*Sin[4*(c + d*x)] - 32*Sin[6*(c + d*x)] + 3*Sin[8*(
c + d*x)])))/(107520*d)
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^7}{7}-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin\cdots}{}\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^7}{7}-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin\cdots}{}\right.}{}\right.}{d}$
parts	$\frac{a\left(-\frac{\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}+\frac{a\left(-\frac{\sin(dx+c)^7}{7}-\frac{\sin(dx+c)^5}{5}-\cdots\right.}{}$
parallelrisch	$-\frac{a\left(-\frac{105dx}{4}+\sin(6dx+6c)-\frac{3\sin(7dx+7c)}{14}-\frac{3\sin(8dx+8c)}{32}+96\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-96\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{279\sin(dx+c)}{2}\cdots\right.}{96d}$
risch	$\frac{35ax}{128}-\frac{93ia\,e^{-i(dx+c)}}{128d}+\frac{93ia\,e^{i(dx+c)}}{128d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}+\frac{a\sin(8dx+8c)}{1024d}+\frac{a\sin(7dx+7c\cdots}{448d}$
norman	$\frac{\frac{35ax}{128}-\frac{163a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}-\frac{1335a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{64d}-\frac{24223a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{320d}-\frac{359453a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{2240d}-\frac{724649a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{6720d}-\frac{45859a\tan\cdots}{}}{}$

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)^7-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(s
ec(d*x+c)+tan(d*x+c)))+a*(-1/8*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x
+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/128*d*x+35/128*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx$$
$$=\frac{3675\,adx+6720\,a\log\left(\sin\left(dx+c\right)+1\right)-6720\,a\log\left(-\sin\left(dx+c\right)+1\right)+\left(1680\,a\cos\left(dx+c\right)^7+1920\,a\,\mathrm{c}\cdots\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/13440*(3675*a*d*x + 6720*a*log(sin(d*x + c) + 1) - 6720*a*log(-sin(d*x +
c) + 1) + (1680*a*cos(d*x + c)^7 + 1920*a*cos(d*x + c)^6 - 7000*a*cos(d*x +
 c)^5 - 8448*a*cos(d*x + c)^4 + 11410*a*cos(d*x + c)^3 + 15616*a*cos(d*x +
c)^2 - 9765*a*cos(d*x + c) - 22528*a)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**8,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

$$\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx =$$
$$-\frac{512\left(30\sin(dx+c)^7+42\sin(dx+c)^5+70\sin(dx+c)^3-105\log(\sin(dx+c)+1)+105\log(\sin(\cdots\right.}{\cdots}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="maxima")
[Out] -1/107520*(512*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3 -
 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))*
a - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(8*d*x + 8*c) + 168
*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.05

$$\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx$$
$$=\frac{3675\,(dx+c)a+13440\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-13440\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9765\,a\tan(\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="giac")
[Out] 1/13440*(3675*(d*x + c)*a + 13440*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13
440*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9765*a*tan(1/2*d*x + 1/2*c)^1
5 + 83825*a*tan(1/2*d*x + 1/2*c)^13 + 321013*a*tan(1/2*d*x + 1/2*c)^11 + 72
4649*a*tan(1/2*d*x + 1/2*c)^9 + 1078359*a*tan(1/2*d*x + 1/2*c)^7 + 508683*a
*tan(1/2*d*x + 1/2*c)^5 + 140175*a*tan(1/2*d*x + 1/2*c)^3 + 17115*a*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91

$$\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx = \frac{35\,a\,x}{128}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{7\,a\,\sin\left(2\,c+2\,d\,x\right)}{32\,d}+\frac{37\,a\,\sin\left(3\,c+3\,d\,x\right)}{192\,d} + \frac{7\,a\,\sin\left(4\,c+4\,d\,x\right)}{128\,d}-\frac{9\,a\,\sin\left(5\,c+5\,d\,x\right)}{320\,d} - \frac{a\,\sin\left(6\,c+6\,d\,x\right)}{96\,d}+\frac{a\,\sin\left(7\,c+7\,d\,x\right)}{448\,d} + \frac{a\,\sin\left(8\,c+8\,d\,x\right)}{1024\,d}-\frac{93\,a\,\sin\left(c+d\,x\right)}{64\,d}$$

```
[In] int(sin(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

```
[Out] (35*a*x)/128 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (7*a*
sin(2*c + 2*d*x))/(32*d) + (37*a*sin(3*c + 3*d*x))/(192*d) + (7*a*sin(4*c +
 4*d*x))/(128*d) - (9*a*sin(5*c + 5*d*x))/(320*d) - (a*sin(6*c + 6*d*x))/(9
6*d) + (a*sin(7*c + 7*d*x))/(448*d) + (a*sin(8*c + 8*d*x))/(1024*d) - (93*a
*sin(c + d*x))/(64*d)
```

3.11 $\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 127

$$\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx = \frac{5ax}{16}+\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{5a\cos(c+dx)\sin(c+dx)}{16d}-\frac{a\sin^3(c+dx)}{3d}-\frac{5a\cos(c+dx)\sin^3(c+dx)}{24d}-\frac{a\sin^5(c+dx)}{5d}-\frac{a\cos(c+dx)\sin^5(c+dx)}{6d}$$

```
[Out] 5/16*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-5/16*a*cos(d*x+c)*sin(d*x+c
)/d-1/3*a*sin(d*x+c)^3/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*a*sin(d*x+c)^
5/d-1/6*a*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3957, 2917, 2672, 308, 212, 2715, 8}

$$\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}-\frac{a\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}$$

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*x)/16 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (5*a*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) - (a*Sin[c + d*x]^3)/(3*d) - (5*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) - (a*Sin[c + d*x]^5)/(5*d) - (a*Cos[c + d*x]*Sin[c +
 d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a - a\cos(c+dx))\sin^5(c+dx)\tan(c+dx)\,dx \\
&= a\int \sin^6(c+dx)\,dx + a\int \sin^5(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \sin^4(c+dx)\,dx + \frac{a\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= -\frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d} \\
&\quad + \frac{1}{8}(5a)\int \sin^2(c+dx)\,dx + \frac{a\text{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)\,dx, x, \sin(c+dx)\right)}{d} \\
&= -\frac{a\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\sin^3(c+dx)}{3d} \\
&\quad - \frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a\sin^5(c+dx)}{5d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d} \\
&\quad + \frac{1}{16}(5a)\int 1\,dx + \frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= \frac{5ax}{16} + \frac{a\,\text{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad - \frac{a\sin^3(c+dx)}{3d} - \frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a\sin^5(c+dx)}{5d} \\
&\quad - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.68

$$
\int (a + a\sec(c+dx))\sin^6(c+dx)\,dx = \frac{a(960\,\text{arctanh}(\sin(c+dx)) - 960\sin(c+dx) - 320\sin^3(c+dx) - 192\sin^5(c+dx) + 5(60c + 60dx - 45
}{960d}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (a*(960*ArcTanh[Sin[c + d*x]] - 960*Sin[c + d*x] - 320*Sin[c + d*x]^3 - 192
*Sin[c + d*x]^5 + 5*(60*c + 60*d*x - 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x
)] - Sin[6*(c + d*x)])))/(960*d)
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(}{6}\right.}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(}{6}\right.}{d}$
parts	$\frac{a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{a\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(\right.}{d}$
parallelrisch	$-\frac{a\left(-60dx+\sin(6dx+6c)+192\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-192\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+264\sin(dx+c)+45\sin(2dx+2c)-28\sin(3\right.}{192d}$
risch	$\frac{5ax}{16}+\frac{11ia\,\mathrm{e}^{i(dx+c)}}{16d}-\frac{11ia\,\mathrm{e}^{-i(dx+c)}}{16d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a\sin(6dx+6c)}{192d}-\frac{a\sin(5dx+5c)}{80d}$
norman	$\frac{\frac{5ax}{16}-\frac{21a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{389a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24d}-\frac{853a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20d}-\frac{523a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{20d}-\frac{73a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{8d}-\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8d}+}{\left(1+\tan\left(\frac{dx}{2}\right.\right.}$

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x
+c)))+a*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/
16*d*x+5/16*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx$$
$$=\frac{75\,adx+120\,a\log(\sin(dx+c)+1)-120\,a\log(-\sin(dx+c)+1)-\left(40\,a\cos(dx+c)^5+48\,a\cos(dx+\right.}{240\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*a*d*x + 120*a*log(sin(d*x + c) + 1) - 120*a*log(-sin(d*x + c) + 1
) - (40*a*cos(d*x + c)^5 + 48*a*cos(d*x + c)^4 - 130*a*cos(d*x + c)^3 - 176
*a*cos(d*x + c)^2 + 165*a*cos(d*x + c) + 368*a)*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx = a\left(\int \sin^6(c+dx)\sec(c+dx)\,dx + \int \sin^6(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**6,x)
[Out] a*(Integral(sin(c + d*x)**6*sec(c + d*x), x) + Integral(sin(c + d*x)**6, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx = -\frac{32\left(6\sin(dx+c)^5+10\sin(dx+c)^3-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)+30\sin(dx+c)\right)a - 5\left(4\sin(2dx+2c)^3+60dx+60c+9\sin(4dx+4c)-48\sin(2dx+2c)\right)a}{960\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="maxima")
[Out] -1/960*(32*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a - 5*(4*sin(2*d*x + 2*c)^3
+ 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.15

$$\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx = \frac{75(dx+c)a+240\,a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-240\,a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(165\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+1095\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+3138\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+5118\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1945\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+315\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^6}}{240\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="giac")
[Out] 1/240*(75*(d*x + c)*a + 240*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*a*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(165*a*tan(1/2*d*x + 1/2*c)^11 + 1095*
a*tan(1/2*d*x + 1/2*c)^9 + 3138*a*tan(1/2*d*x + 1/2*c)^7 + 5118*a*tan(1/2*d
*x + 1/2*c)^5 + 1945*a*tan(1/2*d*x + 1/2*c)^3 + 315*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

$$\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx = \frac{5\,a\,x}{16}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{15\,a\,\sin\left(2\,c+2\,d\,x\right)}{64\,d}+\frac{7\,a\,\sin\left(3\,c+3\,d\,x\right)}{48\,d} + \frac{3\,a\,\sin\left(4\,c+4\,d\,x\right)}{64\,d}-\frac{a\,\sin\left(5\,c+5\,d\,x\right)}{80\,d} - \frac{a\,\sin\left(6\,c+6\,d\,x\right)}{192\,d}-\frac{11\,a\,\sin\left(c+d\,x\right)}{8\,d}$$

```
[In] int(sin(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

```
[Out] (5*a*x)/16 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (15*a*s
in(2*c + 2*d*x))/(64*d) + (7*a*sin(3*c + 3*d*x))/(48*d) + (3*a*sin(4*c + 4*
d*x))/(64*d) - (a*sin(5*c + 5*d*x))/(80*d) - (a*sin(6*c + 6*d*x))/(192*d) -
 (11*a*sin(c + d*x))/(8*d)
```

3.12 $\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx = \frac{3ax}{8}+\frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d}-\frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\sin^3(c+dx)}{3d}-\frac{a\cos(c+dx)\sin^3(c+dx)}{4d}$$

```
[Out] 3/8*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-3/8*a*cos(d*x+c)*sin(d*x+c)/
d-1/3*a*sin(d*x+c)^3/d-1/4*a*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3957, 2917, 2672, 308, 212, 2715, 8}

$$\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{a\sin^3(c+dx)}{3d} - \frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*x)/8 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (3*a*Cos[c +
 d*x]*Sin[c + d*x])/(8*d) - (a*Sin[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Sin[
c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int (-a - a\cos(c+dx))\sin^3(c+dx)\tan(c+dx)\,dx$$

$$= a\int \sin^4(c+dx)\,dx + a\int \sin^3(c+dx)\tan(c+dx)\,dx$$

$$= -\frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{4}(3a)\int \sin^2(c+dx)\,dx + \frac{a\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{8}(3a)\int 1\,dx + \frac{a\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{3ax}{8} - \frac{a\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{3ax}{8} + \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4d}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx = \frac{3a(c+dx)}{8d} + \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d -
(a*Sin[c + d*x]^3)/(3*d) - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)]
)/(32*d)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
parts	$\frac{a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{a\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
parallelrisch	$-\frac{a\left(-36dx+120\sin(dx+c)-96\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+96\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+24\sin(2dx+2c)-3\sin(4dx+4c)-8\sin(3dx\right.}{96d}$
risch	$\frac{3ax}{8}+\frac{5ia\,e^{i(dx+c)}}{8d}-\frac{5ia\,e^{-i(dx+c)}}{8d}+\frac{a\ln(e^{i(dx+c)}+i)}{d}-\frac{a\ln(e^{i(dx+c)}-i)}{d}+\frac{a\sin(4dx+4c)}{32d}+\frac{a\sin(3dx+3c)}{12d}-$
norman	$\frac{\frac{3ax}{8}-\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{137a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}-\frac{71a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{9ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{3ax\,\mathrm{t}}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-1/4*(si
n(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx$$
$$=\frac{9\,adx+12\,a\log\left(\sin\left(dx+c\right)+1\right)-12\,a\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,a\cos\left(dx+c\right)^3+8\,a\cos\left(dx+c\right)^2-\right.}{24\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + 12*a*log(sin(d*x + c) + 1) - 12*a*log(-sin(d*x + c) + 1) +
(6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 32*a)*sin(d*
x + c))/d
```

Sympy [F]

$$\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx = a\left(\int \sin^4(c+dx)\sec(c+dx)\,dx+\int \sin^4(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**4,x)
[Out] a*(Integral(sin(c + d*x)**4*sec(c + d*x), x) + Integral(sin(c + d*x)**4, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx = -\frac{16\,(2\,\sin(dx+c)^3-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)+6\,\sin(dx+c))a-3\,(12\,dx+}{96\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="maxima")
[Out] -1/96*(16*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*
x + 2*c))*a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.33

$$\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx = \frac{9\,(dx+c)a+24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+71\,a}{}}{24\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="giac")
[Out] 1/24*(9*(d*x + c)*a + 24*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*a*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a*tan(1/2*d*x + 1/2*c)^7 + 71*a*tan(1/
2*d*x + 1/2*c)^5 + 137*a*tan(1/2*d*x + 1/2*c)^3 + 33*a*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx = \frac{3\,a\,x}{8}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\,\sin\left(3\,c+3\,d\,x\right)}{12\,d} + \frac{a\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}-\frac{5\,a\,\sin\left(c+d\,x\right)}{4\,d}$$

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*sin(2
*c + 2*d*x))/(4*d) + (a*sin(3*c + 3*d*x))/(12*d) + (a*sin(4*c + 4*d*x))/(32
*d) - (5*a*sin(c + d*x))/(4*d)
```

3.13 $\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx=\frac{ax}{2}+\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-1/2*a*cos(d*x+c)*sin(d*x+c)/
d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3957, 2917, 2672, 327, 212, 2715, 8}

$$\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx=\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*x)/2 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a - a\cos(c+dx))\sin(c+dx)\tan(c+dx)\,dx \\
&= a\int \sin^2(c+dx)\,dx + a\int \sin(c+dx)\tan(c+dx)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx + \frac{a\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= \frac{ax}{2} - \frac{a\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= \frac{ax}{2} + \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$
\begin{aligned}
\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx = {} & \frac{a(c+dx)}{2d} + \frac{a\text{arctanh}(\sin(c+dx))}{d} \\
& - \frac{a\sin(c+dx)}{d} - \frac{a\sin(2(c+dx))}{4d}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*(c + d*x))/(2*d) + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a
*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parallelrisch	$-\frac{a\left(-2dx+4\sin(dx+c)+\sin(2dx+2c)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)}{4d}$
parts	$\frac{a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
risch	$\frac{ax}{2}+\frac{ia\,\mathrm{e}^{i(dx+c)}}{2d}-\frac{ia\,\mathrm{e}^{-i(dx+c)}}{2d}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}-\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{ax}{2}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{d}$

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-1/2*cos(d*x+c)*sin(d*x+c
)+1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx = \frac{adx+a\log(\sin(dx+c)+1)-a\log(-\sin(dx+c)+1)-(a\cos(dx+c)+2\,a)\sin(dx+c)}{2\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="fricas")
[Out] 1/2*(a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1) - (a*cos(d*
x + c) + 2*a)*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx = a\left(\int \sin^2(c+dx)\sec(c+dx)\,dx + \int \sin^2(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**2,x)
[Out] a*(Integral(sin(c + d*x)**2*sec(c + d*x), x) + Integral(sin(c + d*x)**2, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx = \frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a+2\,a(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))}{4\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="maxima")
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a + 2*a*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx$$
$$= \frac{(dx+c)a+2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.57

$$\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx = \frac{a\,x}{2}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} + \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

```
[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*x)/2 - (3*a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 +
 (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) + (2*a*atanh(tan(c/2 + (d*x)/2)))/
d
```

3.14 $\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-a*csc(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3957, 2917, 2701, 327, 213, 3852, 8}

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a - a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^2(c+dx)\,dx + a\int \csc^2(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{Subst}(\int 1\,dx, x, \cot(c+dx))}{d} - \frac{a\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)}{d} - \frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&= \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)}{d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(c+dx)\right)}{d}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x]),x]
[Out] -((a*Cot[c + d*x])/d) - (a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin
[c + d*x]^2])/d
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a\cot(dx+c)}{d}$	42
default	$\frac{a\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a\cot(dx+c)}{d}$	42
norman	$-\frac{a}{d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	54
risch	$-\frac{2ia}{d\left(e^{i(dx+c)}-1\right)}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}$	59
parallelrisch	$\frac{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-1\right)}{d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$	64

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-a*cot(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log(\sin(dx+c)+1)\sin(dx+c) - a\log(-\sin(dx+c)+1)\sin(dx+c) - 2\,a\cos(dx+c) - 2\,a}{2\,d\sin(dx+c)}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(a*log(sin(d*x + c) + 1)*sin(d*x + c) - a*log(-sin(d*x + c) + 1)*sin(d*
x + c) - 2*a*cos(d*x + c) - 2*a)/(d*sin(d*x + c))
```

Sympy [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc^2(c+dx)\sec(c+dx)\,dx + \int \csc^2(c+dx)\,dx\right)$$

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(csc(c + d*x)**2*sec(c + d*x), x) + Integral(csc(c + d*x)**2, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + \frac{2\,a}{\tan(dx+c)}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(a*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) +
2*a/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - a/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\left(2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^2,x)
```

```
[Out] (a*(2*atanh(tan(c/2 + (d*x)/2)) - cot(c/2 + (d*x)/2)))/d
```

3.15 $\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d-a*csc(d*x+c)/d-
1/3*a*csc(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3957, 2917, 2701, 308, 213, 3852}

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))\csc^4(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^4(c+dx)\,dx + a\int \csc^4(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int (1+x^2)\,dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{d}
\end{aligned}$$

$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}$$

$$= \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = -\frac{2a\cot(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d} - \frac{a\csc^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, \sin^2(c+dx)\right)}{3d}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x]),x]
[Out] (-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (a*Csc[
c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)}{d}$	63
default	$\frac{a\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)}{d}$	63
parallelrisch	$-\frac{a\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+12\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}$	69
norman	$\frac{-\frac{a}{12d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	90
risch	$-\frac{2ia\left(3\,e^{3i(dx+c)}-6\,e^{2i(dx+c)}-e^{i(dx+c)}+2\right)}{3d\left(e^{i(dx+c)}-1\right)^3\left(e^{i(dx+c)}+1\right)}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}$	107

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] 1/d*(a*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-2/3-1
/3*csc(d*x+c)^2)*cot(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.57

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = -\frac{4\,a\cos\left(dx+c\right)^2-3\left(a\cos\left(dx+c\right)-a\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\left(a\cos\left(dx+c\right)-a\right)\log\left(-\right.}{6\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/6*(4*a*cos(d*x + c)^2 - 3*(a*cos(d*x + c) - a)*log(sin(d*x + c) + 1)*sin
(d*x + c) + 3*(a*cos(d*x + c) - a)*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*
a*cos(d*x + c) - 8*a)/((d*cos(d*x + c) - d)*sin(d*x + c))

Sympy [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc^4\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int \csc^4\left(c+dx\right)\,dx\right)$$

[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c)),x)

[Out] a*(Integral(csc(c + d*x)**4*sec(c + d*x), x) + Integral(csc(c + d*x)**4, x)
)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(\frac{2\left(3\sin\left(dx+c\right)^2+1\right)}{\sin\left(dx+c\right)^3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\left(3\tan\left(dx+c\right)^2+1\right)a}{\tan\left(dx+c\right)^3}}{6\,d}$$

[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/6*(a*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*a/tan(d*x + c)^3)/d

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}}{12\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*a*log(abs(tan(1/2*d*x +
1/2*c) - 1)) - 3*a*tan(1/2*d*x + 1/2*c) - (12*a*tan(1/2*d*x + 1/2*c)^2 + a)
/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx=\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a}{12}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}$$

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^4,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (a/12 + a*tan(c/2 + (d*x)/2)^2)/(d*tan(
c/2 + (d*x)/2)^3) - (a*tan(c/2 + (d*x)/2))/(4*d)
```

3.16 $\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-2/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+c
)^5/d-a*csc(d*x+c)/d-1/3*a*csc(d*x+c)^3/d-1/5*a*csc(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3957, 2917, 2701, 308, 213, 3852}

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot^5(c+dx)}{5d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (2*a*Cot[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*
d) - (a*Csc[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a - a\cos(c+dx))\csc^6(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^6(c+dx)\,dx + a\int \csc^6(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int (1+2x^2+x^4)\,dx, x, \cot(c+dx)\right)}{d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} \\
&\quad - \frac{a\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right) dx, x, \csc(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\csc(c+dx)}{d} \\
&\quad - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \csc(c+dx)\right)}{d} \\
&= \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} \\
&\quad - \frac{a\cot^5(c+dx)}{5d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$
\begin{aligned}
&\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx \\
&= -\frac{8a\cot(c+dx)}{15d} - \frac{4a\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} \\
&\quad - \frac{a\csc^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2}, 1, -\frac{3}{2}, \sin^2(c+dx)\right)}{5d}
\end{aligned}
$$

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*
Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (a*Csc[c + d*x]^5*Hypergeometric2F1[-5
/2, 1, -3/2, Sin[c + d*x]^2])/(5*d)
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c}{d}$
default	$\frac{a\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c}{d}$
parallelrisch	$-\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5+10\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+80\cot\left(\frac{dx}{2}+\frac{c}{2}\right)+30\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+80\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-80\ln(\right.}{80d}$
norman	$\frac{-\frac{a}{80d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{48d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{d}$
risch	$-\frac{2ia\left(15\,e^{7i(dx+c)}-30\,e^{6i(dx+c)}-35\,e^{5i(dx+c)}+100\,e^{4i(dx+c)}+13\,e^{3i(dx+c)}-46\,e^{2i(dx+c)}-e^{i(dx+c)}+8\right)}{15d\left(e^{i(dx+c)}-1\right)^5\left(e^{i(dx+c)}+1\right)^3}-\frac{a\ln(e^{i(dx+}}{d}$

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d
*x+c)))+a*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(93) = 186$.

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.88

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{16\,a\cos(dx+c)^4+14\,a\cos(dx+c)^3-54\,a\cos(dx+c)^2-15\,\left(a\cos(dx+c)^3-a\cos(dx+c)^2-a\,\mathrm{co}\right.}{3}$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(16*a*cos(d*x + c)^4 + 14*a*cos(d*x + c)^3 - 54*a*cos(d*x + c)^2 - 15
*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(sin(d*x + c
) + 1)*sin(d*x + c) + 15*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x +
 c) + a)*log(-sin(d*x + c) + 1)*sin(d*x + c) - 16*a*cos(d*x + c) + 46*a)/((
d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F]

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \csc^6(c+dx)\sec(c+dx)\,dx + \int \csc^6(c+dx)\,dx\right)$$

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(csc(c + d*x)**6*sec(c + d*x), x) + Integral(csc(c + d*x)**6, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{a\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5} - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1)\right) + \frac{2\left(15\tan(dx+c)^4+\right.}{\tan(d}}{30d}$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/30*(a*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 2*(15*tan(d*x + c)^4 +
10*tan(d*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + 90\,a\tan\left(\right.}{240\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] -1/240*(5*a*tan(1/2*d*x + 1/2*c)^3 - 240*a*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 240*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 90*a*tan(1/2*d*x + 1/2*c) +
 3*(80*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2*d*x + 1/2*c)^2 + a)/tan(1/2*
d*x + 1/2*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{48\,d} - \frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a}{5}\right)}{16\,d}$$

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^6,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (3*a*tan(c/2 + (d*x)/2))/(8*d) - (a*tan
(c/2 + (d*x)/2)^3)/(48*d) - (cot(c/2 + (d*x)/2)^5*(a/5 + 2*a*tan(c/2 + (d*x
)/2)^2 + 16*a*tan(c/2 + (d*x)/2)^4))/(16*d)
```

3.17 $\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{d} - \frac{3a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-a*cot(d*x+c)^3/d-3/5*a*cot(d*x+c)^5/
d-1/7*a*cot(d*x+c)^7/d-a*csc(d*x+c)/d-1/3*a*csc(d*x+c)^3/d-1/5*a*csc(d*x+c)
^5/d-1/7*a*csc(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3957, 2917, 2701, 308, 213, 3852}

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot^7(c+dx)}{7d} - \frac{3a\cot^5(c+dx)}{5d} - \frac{a\cot^3(c+dx)}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc^7(c+dx)}{7d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/d - (
3*a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Csc[c + d*x])/d -
 (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(
7*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a - a\cos(c+dx))\csc^8(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^8(c+dx)\,dx + a\int \csc^8(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int (1+3x^2+3x^4+x^6)\,dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{d} - \frac{3a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} \\
&\quad - \frac{a\text{Subst}\left(\int \left(1+x^2+x^4+x^6+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{d} - \frac{3a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\csc(c+dx)}{d} \\
&\quad - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d} - \frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&= \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{d} - \frac{3a\cot^5(c+dx)}{5d} \\
&\quad - \frac{a\cot^7(c+dx)}{7d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$
\begin{aligned}
&\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx \\
&= -\frac{16a\cot(c+dx)}{35d} - \frac{8a\cot(c+dx)\csc^2(c+dx)}{35d} \\
&\quad - \frac{6a\cot(c+dx)\csc^4(c+dx)}{35d} - \frac{a\cot(c+dx)\csc^6(c+dx)}{7d} \\
&\quad - \frac{a\csc^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2}, 1, -\frac{5}{2}, \sin^2(c+dx)\right)}{7d}
\end{aligned}
$$

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-16*a*Cot[c + d*x])/(35*d) - (8*a*Cot[c + d*x]*Csc[c + d*x]^2)/(35*d) - (6
*a*Cot[c + d*x]*Csc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]*Csc[c + d*x]^6)/(7
*d) - (a*Csc[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, Sin[c + d*x]^2])/(
7*d)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{a\left(-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{16}{35}-\frac{\csc(dx+c)^6}{7}-\frac{6\csc(dx+c}{35}\right.}{d}$
default	$\frac{a\left(-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{16}{35}-\frac{\csc(dx+c)^6}{7}-\frac{6\csc(dx+c}{35}\right.}{d}$
parallelrisch	$-\frac{\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\frac{56\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{203\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+\frac{56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+448\cot\left(\frac{dx}{2}+\frac{c}{2}\right)+203\tan\left(\frac{dx}{2}\right.\right.}{448d}$
norman	$\frac{-\frac{a}{448d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{40d}-\frac{29a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{192d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{29a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{64d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{24d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{320d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}+\frac{a\,l}{}$
risch	$-\frac{2ia\left(105\,e^{11i(dx+c)}-210\,e^{10i(dx+c)}-455\,e^{9i(dx+c)}+1120\,e^{8i(dx+c)}+686\,e^{7i(dx+c)}-2492\,e^{6i(dx+c)}-274\,e^{5i(dx+c)}+13\right.}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)^5}$

[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/7/sin(d*x+c)^7-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-16/35-1/7*csc(d*x+c)^6-6/35*csc(d*x+c)^4-8/35*csc(d*x+c)^2)*cot(d*x+c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(121) = 242$.

Time = 0.27 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.15

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx = -\frac{96\,a\cos(dx+c)^6+114\,a\cos(dx+c)^5-450\,a\cos(dx+c)^4-250\,a\cos(dx+c)^3+670\,a\cos(dx+c)^2}{}$$

[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/210*(96*a*cos(d*x + c)^6 + 114*a*cos(d*x + c)^5 - 450*a*cos(d*x + c)^4 - 250*a*cos(d*x + c)^3 + 670*a*cos(d*x + c)^2 - 105*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(-sin(d*x + c) + 1)*sin(d*x + c) + 142*a*cos(d*x + c) - 352*a)/((d*cos(d*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*cos(d*x + c) - d)*sin(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.89

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{a\left(\frac{2\left(105\sin(dx+c)^6+35\sin(dx+c)^4+21\sin(dx+c)^2+15\right)}{\sin(dx+c)^7}-105\log\left(\sin\left(dx+c\right)+1\right)+105\log\left(\sin\left(dx+c\right)-1\right)\right)}{210\,d}$$

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/210*(a*(2*(105*sin(d*x + c)^6 + 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2 +
15)/sin(d*x + c)^7 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1))
 + 6*(35*tan(d*x + c)^6 + 35*tan(d*x + c)^4 + 21*tan(d*x + c)^2 + 5)*a/tan(
d*x + c)^7)/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6720\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+6720\,a\log\left(\left|\tan\left(\right.\right.\right.}{6720}$$

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] -1/6720*(21*a*tan(1/2*d*x + 1/2*c)^5 + 280*a*tan(1/2*d*x + 1/2*c)^3 - 6720*
a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6720*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 3045*a*tan(1/2*d*x + 1/2*c) + (6720*a*tan(1/2*d*x + 1/2*c)^6 + 1015
*a*tan(1/2*d*x + 1/2*c)^4 + 168*a*tan(1/2*d*x + 1/2*c)^2 + 15*a)/tan(1/2*d*
x + 1/2*c)^7)/d
```

Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$
\begin{aligned}
&\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx \\
&= \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{320\,d} \\
&\quad-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(64\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{a}{7}\right)}{64\,d}
\end{aligned}
$$

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^8,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (29*a*tan(c/2 + (d*x)/2))/(64*d) - (a*t
an(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^5)/(320*d) - (cot(c/2 +
 (d*x)/2)^7*(a/7 + (8*a*tan(c/2 + (d*x)/2)^2)/5 + (29*a*tan(c/2 + (d*x)/2)^
4)/3 + 64*a*tan(c/2 + (d*x)/2)^6))/(64*d)
```

3.18 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 165

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{6a\cot^5(c+dx)}{5d} - \frac{4a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d} - \frac{a\csc^9(c+dx)}{9d}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-4/3*a*cot(d*x+c)^3/d-6/5*a*cot(d*x+c
)^5/d-4/7*a*cot(d*x+c)^7/d-1/9*a*cot(d*x+c)^9/d-a*csc(d*x+c)/d-1/3*a*csc(d*
x+c)^3/d-1/5*a*csc(d*x+c)^5/d-1/7*a*csc(d*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

$= \{3957, 2917, 2701, 308, 213, 3852\}$

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot^9(c+dx)}{9d} - \frac{4a\cot^7(c+dx)}{7d} - \frac{6a\cot^5(c+dx)}{5d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc^9(c+dx)}{9d} - \frac{a\csc^7(c+dx)}{7d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (4*a*Cot[c + d*x]^3)/(3*
d) - (6*a*Cot[c + d*x]^5)/(5*d) - (4*a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c + d
*x]^9)/(9*d) - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d
*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))\csc^{10}(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^{10}(c+dx)\,dx + a\int \csc^{10}(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int \frac{x^{10}}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&\quad -\frac{a\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d}-\frac{4a\cot^3(c+dx)}{3d}-\frac{6a\cot^5(c+dx)}{5d}-\frac{4a\cot^7(c+dx)}{7d} \\
&\quad -\frac{a\cot^9(c+dx)}{9d}-\frac{a\text{Subst}\left(\int \left(1+x^2+x^4+x^6+x^8+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d}-\frac{4a\cot^3(c+dx)}{3d}-\frac{6a\cot^5(c+dx)}{5d}-\frac{4a\cot^7(c+dx)}{7d} \\
&\quad -\frac{a\cot^9(c+dx)}{9d}-\frac{a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^5(c+dx)}{5d} \\
&\quad -\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^9(c+dx)}{9d}-\frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&= \frac{a\text{arctanh}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{4a\cot^3(c+dx)}{3d} \\
&\quad -\frac{6a\cot^5(c+dx)}{5d}-\frac{4a\cot^7(c+dx)}{7d}-\frac{a\cot^9(c+dx)}{9d}-\frac{a\csc(c+dx)}{d} \\
&\quad -\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^9(c+dx)}{9d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.82

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= -\frac{128a\cot(c+dx)}{315d} - \frac{64a\cot(c+dx)\csc^2(c+dx)}{315d} - \frac{16a\cot(c+dx)\csc^4(c+dx)}{105d} - \frac{8a\cot(c+dx)\csc^6(c+dx)}{63d} - \frac{a\cot(c+dx)\csc^8(c+dx)}{9d} - \frac{a\csc^9(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},\sin^2(c+dx)\right)}{9d}$$

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-128*a*Cot[c + d*x])/(315*d) - (64*a*Cot[c + d*x]*Csc[c + d*x]^2)/(315*d)
- (16*a*Cot[c + d*x]*Csc[c + d*x]^4)/(105*d) - (8*a*Cot[c + d*x]*Csc[c + d*
x]^6)/(63*d) - (a*Cot[c + d*x]*Csc[c + d*x]^8)/(9*d) - (a*Csc[c + d*x]^9*Hy
pergeometric2F1[-9/2, 1, -7/2, Sin[c + d*x]^2])/(9*d)
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{a\left(-\frac{1}{9\sin(dx+c)^9}-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{128}{315}-\frac{\csc(dx+}{9}\right.}{d}$
default	$\frac{a\left(-\frac{1}{9\sin(dx+c)^9}-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{128}{315}-\frac{\csc(dx+}{9}\right.}{d}$
parallelrisch	$-\frac{a\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^9+\frac{90\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{414\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+390\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+138\tan\left(\right.\right.}{2304d}$
risch	$-\frac{2ia\left(315\,e^{15i(dx+c)}-630\,e^{14i(dx+c)}-1995\,e^{13i(dx+c)}+4620\,e^{12i(dx+c)}+5103\,e^{11i(dx+c)}-14826\,e^{10i(dx+c)}-6303\,e^{9i(d\right.}{315d(e}$

```
[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9-1/7/sin(d*x+c)^7-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^
3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-128/315-1/9*csc(d*x+c)^8-8/63
*csc(d*x+c)^6-16/105*csc(d*x+c)^4-64/315*csc(d*x+c)^2)*cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(149) = 298$.

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.22

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{256\,a\cos\left(dx+c\right)^{8}+374\,a\cos\left(dx+c\right)^{7}-1526\,a\cos\left(dx+c\right)^{6}-1204\,a\cos\left(dx+c\right)^{5}+3220\,a\cos\left(dx+\right.}{}$$

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/630*(256*a*cos(d*x + c)^8 + 374*a*cos(d*x + c)^7 - 1526*a*cos(d*x + c)^6
 - 1204*a*cos(d*x + c)^5 + 3220*a*cos(d*x + c)^4 + 1316*a*cos(d*x + c)^3 -
2996*a*cos(d*x + c)^2 - 315*(a*cos(d*x + c)^7 - a*cos(d*x + c)^6 - 3*a*cos(
d*x + c)^5 + 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^3 - 3*a*cos(d*x + c)^2 -
 a*cos(d*x + c) + a)*log(sin(d*x + c) + 1)*sin(d*x + c) + 315*(a*cos(d*x +
c)^7 - a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^5 + 3*a*cos(d*x + c)^4 + 3*a*cos
(d*x + c)^3 - 3*a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-sin(d*x + c) +
1)*sin(d*x + c) - 496*a*cos(d*x + c) + 1126*a)/((d*cos(d*x + c)^7 - d*cos(d
*x + c)^6 - 3*d*cos(d*x + c)^5 + 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 -
3*d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.82

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{a\left(\frac{2\left(315\,\sin(dx+c)^{8}+105\,\sin(dx+c)^{6}+63\,\sin(dx+c)^{4}+45\,\sin(dx+c)^{2}+35\right)}{\sin(dx+c)^{9}}-315\,\log\left(\sin\left(dx+c\right)+1\right)+315\,\log\left(\sin\left(dx\right.\right.\right.}{630\,d}$$

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/630*(a*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4 +
 45*sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315*l
og(sin(d*x + c) - 1)) + 2*(315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*ta
n(d*x + c)^4 + 180*tan(d*x + c)^2 + 35)*a/tan(d*x + c)^9)/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.99

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4830\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-80640\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(45*a*tan(1/2*d*x + 1/2*c)^7 + 630*a*tan(1/2*d*x + 1/2*c)^5 + 4830
*a*tan(1/2*d*x + 1/2*c)^3 - 80640*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 80
640*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 40950*a*tan(1/2*d*x + 1/2*c) + (
80640*a*tan(1/2*d*x + 1/2*c)^8 + 13650*a*tan(1/2*d*x + 1/2*c)^6 + 2898*a*ta
n(1/2*d*x + 1/2*c)^4 + 450*a*tan(1/2*d*x + 1/2*c)^2 + 35*a)/tan(1/2*d*x + 1
/2*c)^9)/d
```

Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{65\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d} - \frac{23\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{384\,d} - \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{128\,d} - \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{1792\,d} - \frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\left(256\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{130\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{46\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}+\frac{10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{7}+\frac{a}{9}\right)}{256\,d}$$

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^10,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (65*a*tan(c/2 + (d*x)/2))/(128*d) - (23
*a*tan(c/2 + (d*x)/2)^3)/(384*d) - (a*tan(c/2 + (d*x)/2)^5)/(128*d) - (a*ta
n(c/2 + (d*x)/2)^7)/(1792*d) - (cot(c/2 + (d*x)/2)^9*(a/9 + (10*a*tan(c/2 +
 (d*x)/2)^2)/7 + (46*a*tan(c/2 + (d*x)/2)^4)/5 + (130*a*tan(c/2 + (d*x)/2)^
6)/3 + 256*a*tan(c/2 + (d*x)/2)^8))/(256*d)
```

3.19 $\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 183

$$\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx=\frac{3a^2\cos(c+dx)}{d}+\frac{4a^2\cos^2(c+dx)}{d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{3a^2\cos^4(c+dx)}{d}-\frac{2a^2\cos^5(c+dx)}{5d}+\frac{4a^2\cos^6(c+dx)}{3d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{a^2\cos^8(c+dx)}{4d}-\frac{a^2\cos^9(c+dx)}{9d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] 3*a^2*cos(d*x+c)/d+4*a^2*cos(d*x+c)^2/d-2/3*a^2*cos(d*x+c)^3/d-3*a^2*cos(d*
x+c)^4/d-2/5*a^2*cos(d*x+c)^5/d+4/3*a^2*cos(d*x+c)^6/d+3/7*a^2*cos(d*x+c)^7
/d-1/4*a^2*cos(d*x+c)^8/d-1/9*a^2*cos(d*x+c)^9/d-2*a^2*ln(cos(d*x+c))/d+a^2
*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

= {3957, 2915, 12, 90}

$$\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx = -\frac{a^2\cos^9(c+dx)}{9d} - \frac{a^2\cos^8(c+dx)}{4d} + \frac{3a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^6(c+dx)}{3d} - \frac{2a^2\cos^5(c+dx)}{5d} - \frac{3a^2\cos^4(c+dx)}{d} - \frac{2a^2\cos^3(c+dx)}{3d} + \frac{4a^2\cos^2(c+dx)}{d} + \frac{3a^2\cos(c+dx)}{d} + \frac{a^2\sec(c+dx)}{d} - \frac{2a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^9,x]
```

```
[Out] (3*a^2*Cos[c + d*x])/d + (4*a^2*Cos[c + d*x]^2)/d - (2*a^2*Cos[c + d*x]^3)/
(3*d) - (3*a^2*Cos[c + d*x]^4)/d - (2*a^2*Cos[c + d*x]^5)/(5*d) + (4*a^2*Co
s[c + d*x]^6)/(3*d) + (3*a^2*Cos[c + d*x]^7)/(7*d) - (a^2*Cos[c + d*x]^8)/(
4*d) - (a^2*Cos[c + d*x]^9)/(9*d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[
c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\sin^7(c+dx)\tan^2(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)^4(-a+x)^6}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^9d}\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^6}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^7d}\\
&= \frac{\text{Subst}\left(\int \left(-3a^8+\frac{a^{10}}{x^2}-\frac{2a^9}{x}+8a^7x+2a^6x^2-12a^5x^3+2a^4x^4+8a^3x^5-3a^2x^6-2ax^7+x^8\right)\,dx,
}{a^7d}\\
&= \frac{3a^2\cos(c+dx)}{d}+\frac{4a^2\cos^2(c+dx)}{d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{3a^2\cos^4(c+dx)}{d}\\
&\quad -\frac{2a^2\cos^5(c+dx)}{5d}+\frac{4a^2\cos^6(c+dx)}{3d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{a^2\cos^8(c+dx)}{4d}\\
&\quad -\frac{a^2\cos^9(c+dx)}{9d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$
\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx = -\frac{a^2(-714420-361620\cos(2(c+dx))-134820\cos(3(c+dx))+29232\cos(4(c+dx))+24780\cos(5(c+
}{}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^9,x]
[Out] -1/322560*(a^2*(-714420 - 361620*Cos[2*(c + d*x)] - 134820*Cos[3*(c + d*x)]
 + 29232*Cos[4*(c + d*x)] + 24780*Cos[5*(c + d*x)] - 1458*Cos[6*(c + d*x)]
- 3885*Cos[7*(c + d*x)] - 380*Cos[8*(c + d*x)] + 315*Cos[9*(c + d*x)] + 70*
Cos[10*(c + d*x)] + 210*Cos[c + d*x]*(205 + 3072*Log[Cos[c + d*x]]))*Sec[c
+ d*x])/d
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^{10}}{\cos(dx+c)}+\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\frac{64\sin(dx+c)^2}{35}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^8}{8}-\sin(\right.}{}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^{10}}{\cos(dx+c)}+\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\frac{64\sin(dx+c)^2}{35}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^8}{8}-\sin(\right.}{}$
parallelrisch	$\frac{a^2\left(645120\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)-645120\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-645120\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx\right.}{}$
parts	$-\frac{a^2\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\frac{64\sin(dx+c)^2}{35}\right)\cos(dx+c)}{9d}+\frac{a^2\left(\frac{\sin(dx+c)^{10}}{\cos(dx+c)}+\left(\frac{128}{35}+\sin(dx+c)\right.\right.}{}$
risch	$\frac{2a^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}+\frac{65a^2e^{2i(dx+c)}}{128d}+2ia^2x+\frac{4ia^2c}{d}+\frac{65a^2e^{-2i(dx+c)}}{128d}+\frac{311a^2e^{-i(dx+c)}}{256d}+\frac{311a^2e^{i(dx+c)}}{256d}-$

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^10/cos(d*x+c)+(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48
/35*sin(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(-1/8*sin(d*x+c)^8-1
/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/9*a^2*(
128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*sin(d*x+c)^4+64/35*sin(d*x+c)^2)
*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx =$$
$$-\frac{17920\,a^2\cos(dx+c)^{10}+40320\,a^2\cos(dx+c)^9-69120\,a^2\cos(dx+c)^8-215040\,a^2\cos(dx+c)^7+64}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/161280*(17920*a^2*cos(d*x + c)^10 + 40320*a^2*cos(d*x + c)^9 - 69120*a^2
*cos(d*x + c)^8 - 215040*a^2*cos(d*x + c)^7 + 64512*a^2*cos(d*x + c)^6 + 48
3840*a^2*cos(d*x + c)^5 + 107520*a^2*cos(d*x + c)^4 - 645120*a^2*cos(d*x +
c)^3 - 483840*a^2*cos(d*x + c)^2 + 322560*a^2*cos(d*x + c)*log(-cos(d*x + c
)) + 197295*a^2*cos(d*x + c) - 161280*a^2)/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**9,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.80

$$\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx = -\frac{140\,a^2\cos(dx+c)^9+315\,a^2\cos(dx+c)^8-540\,a^2\cos(dx+c)^7-1680\,a^2\cos(dx+c)^6+504\,a^2\cos(dx}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/1260*(140*a^2*cos(d*x + c)^9 + 315*a^2*cos(d*x + c)^8 - 540*a^2*cos(d*x
+ c)^7 - 1680*a^2*cos(d*x + c)^6 + 504*a^2*cos(d*x + c)^5 + 3780*a^2*cos(d*
x + c)^4 + 840*a^2*cos(d*x + c)^3 - 5040*a^2*cos(d*x + c)^2 - 3780*a^2*cos(
d*x + c) + 2520*a^2*log(cos(d*x + c)) - 1260*a^2/cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(171) = 342$.

Time = 0.42 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.02

$$\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx = \frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2520\left(2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}+\frac{1457\,a^2-\frac{20673}{}}{}}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/1260*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 2520*(2*a^2 +
a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1) + (1457*a^2 - 20673*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
```

```
 123012*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 421428*a^2*(cos(d*x
 + c) - 1)^3/(cos(d*x + c) + 1)^3 + 949662*a^2*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 - 1009134*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 666
036*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 276804*a^2*(cos(d*x + c
) - 1)^7/(cos(d*x + c) + 1)^7 + 66681*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c
) + 1)^8 - 7129*a^2*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 1)^9)/d
```

Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.80

$$\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx = -\frac{\frac{2\,a^2\cos(c+d\,x)^3}{3}-\frac{a^2}{\cos(c+d\,x)}-4\,a^2\cos\left(c+d\,x\right)^2-3\,a^2\,\cos\left(c+d\,x\right)+3\,a^2\cos\left(c+d\,x\right)^4+\frac{2\,a^2\cos(c+d\,x)^5}{5}-}{d}$$

```
[In] int(sin(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

```
[Out] -((2*a^2*cos(c + d*x)^3)/3 - a^2/cos(c + d*x) - 4*a^2*cos(c + d*x)^2 - 3*a^
2*cos(c + d*x) + 3*a^2*cos(c + d*x)^4 + (2*a^2*cos(c + d*x)^5)/5 - (4*a^2*c
os(c + d*x)^6)/3 - (3*a^2*cos(c + d*x)^7)/7 + (a^2*cos(c + d*x)^8)/4 + (a^2
*cos(c + d*x)^9)/9 + 2*a^2*log(cos(c + d*x)))/d
```

3.20 $\int (a+a\sec(c+dx))^2 \sin^7(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 131

$$\int (a+a\sec(c+dx))^2\sin^7(c+dx)\,dx=\frac{2a^2\cos(c+dx)}{d}+\frac{3a^2\cos^2(c+dx)}{d}-\frac{3a^2\cos^4(c+dx)}{2d}-\frac{2a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^6(c+dx)}{3d}+\frac{a^2\cos^7(c+dx)}{7d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] 2*a^2*cos(d*x+c)/d+3*a^2*cos(d*x+c)^2/d-3/2*a^2*cos(d*x+c)^4/d-2/5*a^2*cos(
d*x+c)^5/d+1/3*a^2*cos(d*x+c)^6/d+1/7*a^2*cos(d*x+c)^7/d-2*a^2*ln(cos(d*x+c
))/d+a^2*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$\int (a+a\sec(c+dx))^2\sin^7(c+dx)\,dx=\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^6(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{3a^2\cos^4(c+dx)}{2d}+\frac{3a^2\cos^2(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^7,x]
```

```
[Out] (2*a^2*Cos[c + d*x])/d + (3*a^2*Cos[c + d*x]^2)/d - (3*a^2*Cos[c + d*x]^4)/
(2*d) - (2*a^2*Cos[c + d*x]^5)/(5*d) + (a^2*Cos[c + d*x]^6)/(3*d) + (a^2*Co
s[c + d*x]^7)/(7*d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\sin^5(c+dx)\tan^2(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)^3(-a+x)^5}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^7d}\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^3(-a+x)^5}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \left(-2a^6+\frac{a^8}{x^2}-\frac{2a^7}{x}+6a^5x-6a^3x^3+2a^2x^4+2ax^5-x^6\right)\,dx, x, -a\cos(c+dx)\right)}{a^5d}\\
&= \frac{2a^2\cos(c+dx)}{d}+\frac{3a^2\cos^2(c+dx)}{d}-\frac{3a^2\cos^4(c+dx)}{2d}-\frac{2a^2\cos^5(c+dx)}{5d}\\
&\quad+\frac{a^2\cos^6(c+dx)}{3d}+\frac{a^2\cos^7(c+dx)}{7d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c+dx))^2 \sin^7(c+dx)\,dx = \frac{a^2(25725 + 11760\cos(2(c+dx)) + 5250\cos(3(c+dx)) - 588\cos(4(c+dx)) - 770\cos(5(c+dx)) - 48\cos}{13}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^7,x]
```

```
[Out] (a^2*(25725 + 11760*Cos[2*(c + d*x)] + 5250*Cos[3*(c + d*x)] - 588*Cos[4*(c
 + d*x)] - 770*Cos[5*(c + d*x)] - 48*Cos[6*(c + d*x)] + 70*Cos[7*(c + d*x)]
 + 15*Cos[8*(c + d*x)] - 70*Cos[c + d*x]*(5 + 384*Log[Cos[c + d*x]]))*Sec[c
 + d*x])/(13440*d)
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^8}{\cos(dx+c)}+\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}\right.}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^8}{\cos(dx+c)}+\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}\right.}{d}$
parts	$-\frac{a^2\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7d}+\frac{a^2\left(\frac{\sin(dx+c)^8}{\cos(dx+c)}+\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin}{}\right.\right.}{d}$
parallelrisch	$\frac{a^2\left(26880\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)-26880\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-26880\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+3\right.}{}$
risch	$2ia^2x+\frac{29a^2e^{2i(dx+c)}}{64d}+\frac{117a^2e^{i(dx+c)}}{128d}+\frac{117a^2e^{-i(dx+c)}}{128d}+\frac{29a^2e^{-2i(dx+c)}}{64d}+\frac{4ia^2c}{d}+\frac{2a^2e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}-$
norman	$\frac{-\frac{192a^2}{35d}-\frac{64a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{d}-\frac{24a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-\frac{172a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{3d}-\frac{264a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{5d}-\frac{292a^2\tan}{1}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7}$

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*s
in(d*x+c)^2)*cos(d*x+c))+2*a^2*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(
d*x+c)^2-ln(cos(d*x+c)))-1/7*a^2*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*si
n(d*x+c)^2)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int (a+a\sec(c+dx))^2\sin^7(c+dx)\,dx$$
$$= \frac{120\,a^2\cos{(dx+c)}^8+280\,a^2\cos{(dx+c)}^7-336\,a^2\cos{(dx+c)}^6-1260\,a^2\cos{(dx+c)}^5+2520\,a^2\cos{(dx}{840\,d\cos{(}}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(120*a^2*cos(d*x + c)^8 + 280*a^2*cos(d*x + c)^7 - 336*a^2*cos(d*x +
c)^6 - 1260*a^2*cos(d*x + c)^5 + 2520*a^2*cos(d*x + c)^3 + 1680*a^2*cos(d*x
 + c)^2 - 1680*a^2*cos(d*x + c)*log(-cos(d*x + c)) - 875*a^2*cos(d*x + c) +
 840*a^2)/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^2\sin^7(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int (a+a\sec(c+dx))^2\sin^7(c+dx)\,dx$$
$$= \frac{30\,a^2\cos{(dx+c)}^7+70\,a^2\cos{(dx+c)}^6-84\,a^2\cos{(dx+c)}^5-315\,a^2\cos{(dx+c)}^4+630\,a^2\cos{(dx+c)}^2}{210\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/210*(30*a^2*cos(d*x + c)^7 + 70*a^2*cos(d*x + c)^6 - 84*a^2*cos(d*x + c)^
5 - 315*a^2*cos(d*x + c)^4 + 630*a^2*cos(d*x + c)^2 + 420*a^2*cos(d*x + c)
- 420*a^2*log(cos(d*x + c)) + 210*a^2/cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(123) = 246$.

Time = 0.37 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.44

$$\int (a + a\sec(c + dx))^2 \sin^7(c + dx)\, dx$$
$$= \frac{420\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 420\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{420\left(2\,a^2 + \frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1} + \frac{357\,a^2 - \frac{3759\,a^2(\cos(\dots}{\cos(dx+\dots}}{}}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/210*(420*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 420*(2*a^2 + a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1) + (357*a^2 - 3759*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16737
*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 42595*a^2*(cos(d*x + c) -
1)^3/(cos(d*x + c) + 1)^3 + 43855*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
1)^4 - 25389*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8043*a^2*(cos(
d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089*a^2*(cos(d*x + c) - 1)^7/(cos(d
*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c + dx))^2 \sin^7(c + dx)\, dx$$
$$= \frac{2\,a^2\,\cos(c+d\,x) + \frac{a^2}{\cos(c+d\,x)} + 3\,a^2\cos(c+d\,x)^2 - \frac{3\,a^2\cos(c+d\,x)^4}{2} - \frac{2\,a^2\cos(c+d\,x)^5}{5} + \frac{a^2\cos(c+d\,x)^6}{3} + \frac{a^2\cos(c+d\,x)^7}{7}\dots}{d}$$

```
[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*cos(c + d*x) + a^2/cos(c + d*x) + 3*a^2*cos(c + d*x)^2 - (3*a^2*cos(
c + d*x)^4)/2 - (2*a^2*cos(c + d*x)^5)/5 + (a^2*cos(c + d*x)^6)/3 + (a^2*co
s(c + d*x)^7)/7 - 2*a^2*log(cos(c + d*x)))/d
```

3.21 $\int (a+a\sec(c+dx))^2\sin^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int (a+a\sec(c+dx))^2\sin^5(c+dx)\,dx=\frac{a^2\cos(c+dx)}{d}+\frac{2a^2\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos^4(c+dx)}{2d}-\frac{a^2\cos^5(c+dx)}{5d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] a^2*cos(d*x+c)/d+2*a^2*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-1/2*a^2*cos(d*
x+c)^4/d-1/5*a^2*cos(d*x+c)^5/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$\int (a+a\sec(c+dx))^2\sin^5(c+dx)\,dx=-\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos^4(c+dx)}{2d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos^2(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] (a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x]^3)/(3*d
) - (a^2*Cos[c + d*x]^4)/(2*d) - (a^2*Cos[c + d*x]^5)/(5*d) - (2*a^2*Log[Co
s[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\sin^3(c+dx)\tan^2(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)^2(-a+x)^4}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^4}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(-a^4+\frac{a^6}{x^2}-\frac{2a^5}{x}+4a^3x-a^2x^2-2ax^3+x^4\right)dx, x, -a\cos(c+dx)\right)}{a^3d}\\
&= \frac{a^2\cos(c+dx)}{d}+\frac{2a^2\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos^4(c+dx)}{2d}\\
&\quad -\frac{a^2\cos^5(c+dx)}{5d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int (a+a\sec(c+dx))^2\sin^5(c+dx)\,dx = -\frac{a^2(-750-275\cos(2(c+dx))-165\cos(3(c+dx))-2\cos(4(c+dx))+15\cos(5(c+dx))+3\cos(6(c$$
$$480d$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] -1/480*(a^2*(-750 - 275*Cos[2*(c + d*x)] - 165*Cos[3*(c + d*x)] - 2*Cos[4*(
c + d*x)] + 15*Cos[5*(c + d*x)] + 3*Cos[6*(c + d*x)] + 30*Cos[c + d*x]*(-3
+ 32*Log[Cos[c + d*x]]))*Sec[c + d*x])/d
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(\frac{8}{3}\right.}{}}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(\frac{8}{3}\right.}{}}{d}$
parts	$-\frac{a^2\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{a^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)}{d}+$
parallelrisch	$-\frac{\left(-320\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)+320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\cos\right.}{160d\cos(dx+c)}$
risch	$2ia^2x+\frac{3a^2e^{2i(dx+c)}}{8d}+\frac{9a^2e^{i(dx+c)}}{16d}+\frac{9a^2e^{-i(dx+c)}}{16d}+\frac{3a^2e^{-2i(dx+c)}}{8d}+\frac{4ia^2c}{d}+\frac{2a^2e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}-\frac{2a^2\ln}{}$
norman	$\frac{-\frac{64a^2}{15d}-\frac{64a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{16a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{16a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}-\frac{196a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{15d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}-\frac{2a^2\ln(t}{}$

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d
*x+c))+2*a^2*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/5*a^2*(8
/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int (a+a\sec(c+dx))^2\sin^5(c+dx)\,dx =$$
$$-\frac{48\,a^2\cos\left(dx+c\right)^6+120\,a^2\cos\left(dx+c\right)^5-80\,a^2\cos\left(dx+c\right)^4-480\,a^2\cos\left(dx+c\right)^3-240\,a^2\cos\left(dx+c\right.}{240\,d\cos\left(dx+c\right)}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^6 + 120*a^2*cos(d*x + c)^5 - 80*a^2*cos(d*x + c
)^4 - 480*a^2*cos(d*x + c)^3 - 240*a^2*cos(d*x + c)^2 + 480*a^2*cos(d*x + c
)*log(-cos(d*x + c)) + 195*a^2*cos(d*x + c) - 240*a^2)/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+a\sec(c+dx))^2\sin^5(c+dx)\,dx = a^2\left(\int 2\sin^5\left(c+dx\right)\sec\left(c+dx\right)dx + \int \sin^5\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \sin^5\left(c+dx\right)dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**5,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)**5*sec(c + d*x), x) + Integral(sin(c + d*x)**
5*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**5, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int (a+a\sec(c+dx))^2\sin^5(c+dx)\,dx =$$
$$-\frac{6\,a^2\cos\left(dx+c\right)^5+15\,a^2\cos\left(dx+c\right)^4-10\,a^2\cos\left(dx+c\right)^3-60\,a^2\cos\left(dx+c\right)^2-30\,a^2\cos\left(dx+c\right)+6}{30\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/30*(6*a^2*cos(d*x + c)^5 + 15*a^2*cos(d*x + c)^4 - 10*a^2*cos(d*x + c)^3
 - 60*a^2*cos(d*x + c)^2 - 30*a^2*cos(d*x + c) + 60*a^2*log(cos(d*x + c)) -
 30*a^2/cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(106) = 212$.

Time = 0.36 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.41

$$\int (a + a\sec(c + dx))^2 \sin^5(c + dx)\, dx$$
$$= \frac{60\, a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 60\, a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{60\left(2\,a^2 + \frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1} + \frac{69\,a^2 - \frac{525\,a^2(\cos(dx+c}{\cos(dx+c)+1} \cdots}{\cdots}}{30\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/30*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 60*(2*a^2 + a^2*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1) + (69*a^2 - 525*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1650*a^2*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1610*a^2*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 745*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 137
*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int (a + a\sec(c + dx))^2 \sin^5(c + dx)\, dx$$
$$= \frac{a^2\, \cos(c + d\,x) + \frac{a^2}{\cos(c+d\,x)} + 2\,a^2 \cos(c + d\,x)^2 + \frac{a^2 \cos(c+d\,x)^3}{3} - \frac{a^2 \cos(c+d\,x)^4}{2} - \frac{a^2 \cos(c+d\,x)^5}{5} - 2\,a^2\, \ln(\cos(c \cdots}{d}$$

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*cos(c + d*x) + a^2/cos(c + d*x) + 2*a^2*cos(c + d*x)^2 + (a^2*cos(c +
d*x)^3)/3 - (a^2*cos(c + d*x)^4)/2 - (a^2*cos(c + d*x)^5)/5 - 2*a^2*log(cos
(c + d*x)))/d
```

3.22 $\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx=\frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] a^2*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*
x+c)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 76}

$$\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx=\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] (a^2*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x]^3)/(3*d) - (2*a^2*Log[Cos[c + d*
x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\sin(c+dx)\tan^2(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)(-a+x)^3}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d} \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)(-a+x)^3}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^4}{x^2}-\frac{2a^3}{x}+2ax-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int (a + a\sec(c + dx))^2 \sin^3(c + dx)\, dx$$
$$= \frac{a^2(27 + 4\cos(2(c + dx)) + 6\cos(3(c + dx)) + \cos(4(c + dx)) - 6\cos(c + dx)(1 + 8\log(\cos(c + dx))))\sec(c}{24d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] (a^2*(27 + 4*Cos[2*(c + d*x)] + 6*Cos[3*(c + d*x)] + Cos[4*(c + d*x)] - 6*C
os[c + d*x]*(1 + 8*Log[Cos[c + d*x]]))*Sec[c + d*x])/(24*d)
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+2a^2\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}}{d}$
parts	$-\frac{a^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{a^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)}{d}-\frac{a^2\sin(dx+c)^2}{d}-\frac{2a^2\ln(\cos(dx}{d}$
parallelrisch	$\frac{a^2\left(-48\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-48\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+48\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)+26\cos(dx}{24d\cos(dx+c)}$
risch	$2ia^2x+\frac{a^2e^{2i(dx+c)}}{4d}+\frac{a^2e^{i(dx+c)}}{8d}+\frac{a^2e^{-i(dx+c)}}{8d}+\frac{a^2e^{-2i(dx+c)}}{4d}+\frac{4ia^2c}{d}+\frac{2a^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2a^2\ln(e^{2i(dx}}{d}$
norman	$\frac{-\frac{8a^2}{3d}-\frac{8a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(-1/2*
sin(d*x+c)^2-ln(cos(d*x+c)))-1/3*a^2*(2+sin(d*x+c)^2)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx = \frac{2\,a^2\cos\left(dx+c\right)^4+6\,a^2\cos\left(dx+c\right)^3-12\,a^2\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-3\,a^2\cos\left(dx+c\right)+6\,a^2}{6\,d\cos\left(dx+c\right)}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="fricas")
[Out] 1/6*(2*a^2*cos(d*x + c)^4 + 6*a^2*cos(d*x + c)^3 - 12*a^2*cos(d*x + c)*log(
-cos(d*x + c)) - 3*a^2*cos(d*x + c) + 6*a^2)/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx = a^2\left(\int 2\sin^3\left(c+dx\right)\sec\left(c+dx\right)dx + \int \sin^3\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \sin^3\left(c+dx\right)dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**3,x)
[Out] a**2*(Integral(2*sin(c + d*x)**3*sec(c + d*x), x) + Integral(sin(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx = \frac{a^2\cos\left(dx+c\right)^3+3\,a^2\cos\left(dx+c\right)^2-6\,a^2\log\left(\cos\left(dx+c\right)\right)+\frac{3\,a^2}{\cos(dx+c)}}{3\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="maxima")
[Out] 1/3*(a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c)^2 - 6*a^2*log(cos(d*x + c)) +
3*a^2/cos(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int (a + a\sec(c+dx))^2 \sin^3(c+dx)\,dx = -\frac{2\,a^2\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{a^2}{d\cos(dx+c)} + \frac{a^2d^5\cos(dx+c)^3 + 3\,a^2d^5\cos(dx+c)^2}{3\,d^6}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2*a^2*log(abs(cos(d*x + c))/abs(d))/d + a^2/(d*cos(d*x + c)) + 1/3*(a^2*d^
5*cos(d*x + c)^3 + 3*a^2*d^5*cos(d*x + c)^2)/d^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int (a + a\sec(c+dx))^2 \sin^3(c+dx)\,dx = \frac{\frac{a^2}{\cos(c+d\,x)} + a^2\cos(c+d\,x)^2 + \frac{a^2\cos(c+d\,x)^3}{3} - 2\,a^2\,\ln(\cos(c+d\,x))}{d}$$

```
[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2/cos(c + d*x) + a^2*cos(c + d*x)^2 + (a^2*cos(c + d*x)^3)/3 - 2*a^2*log
(cos(c + d*x)))/d
```

3.23 $\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx=-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] -a^2*cos(d*x+c)/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2912, 12, 45}

$$\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx=-\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x],x]
```

```
[Out] -((a^2*Cos[c + d*x])/d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/
d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^2(-a+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}\\
&= \frac{a\text{Subst}\left(\int \frac{(-a+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}-\frac{2a}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= -\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx = \frac{a^2(1-2\log(\cos(c+dx))+\sin(c+dx)\tan(c+dx))}{d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x],x]
[Out] (a^2*(1 - 2*Log[Cos[c + d*x]] + Sin[c + d*x]*Tan[c + d*x]))/d
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{a^2\left(\sec(dx+c)-\frac{1}{\sec(dx+c)}+2\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^2\left(\sec(dx+c)-\frac{1}{\sec(dx+c)}+2\ln(\sec(dx+c))\right)}{d}$
parts	$-\frac{a^2\cos(dx+c)}{d}+\frac{a^2\sec(dx+c)}{d}+\frac{2a^2\ln(\sec(dx+c))}{d}$
parallelrisch	$-\frac{a^2\left(4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-4\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)+\cos(2dx+2\right.}{2d\cos(dx+c)}$
risch	$2ia^2x-\frac{a^2\mathrm{e}^{i(dx+c)}}{2d}-\frac{a^2\mathrm{e}^{-i(dx+c)}}{2d}+\frac{4ia^2c}{d}+\frac{2a^2\mathrm{e}^{i(dx+c)}}{d(\mathrm{e}^{2i(dx+c)}+1)}-\frac{2a^2\ln(\mathrm{e}^{2i(dx+c)}+1)}{d}$
norman	$-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{2a^2\ln\left(1+\right.}{}$

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*a^2*(sec(d*x+c)-1/sec(d*x+c)+2*ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx$$
$$=-\frac{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)\log(-\cos(dx+c))-a^2}{d\cos(dx+c)}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="fricas")
[Out] -(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c)*log(-cos(d*x + c)) - a^2)/(d*cos(
d*x + c))
```

Sympy [F]

$$\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx = a^2\left(\int 2\sin(c+dx)\sec(c+dx)\,dx + \int \sin(c+dx)\sec^2(c+dx)\,dx + \int \sin(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c),x)
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x)*sec(
c + d*x)**2, x) + Integral(sin(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx = -\frac{a^2\cos(dx+c) + 2\,a^2\log(\cos(dx+c)) - \frac{a^2}{\cos(dx+c)}}{d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="maxima")
[Out] -(a^2*cos(d*x + c) + 2*a^2*log(cos(d*x + c)) - a^2/cos(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx = -\frac{a^2\cos(dx+c)}{d} - \frac{2\,a^2\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{a^2}{d\cos(dx+c)}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="giac")
[Out] -a^2*cos(d*x + c)/d - 2*a^2*log(abs(cos(d*x + c))/abs(d))/d + a^2/(d*cos(d*
x + c))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int (a + a\sec(c + dx))^2 \sin(c + dx)\, dx$$
$$= -\frac{a^2 \left(2\, \cos\left(c + d\, x\right)\, \ln\left(\cos\left(c + d\, x\right)\right) + {\cos\left(c + d\, x\right)}^2 - 1\right)}{d\, \cos\left(c + d\, x\right)}$$

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x))^2,x)
[Out] -(a^2*(2*cos(c + d*x)*log(cos(c + d*x)) + cos(c + d*x)^2 - 1))/(d*cos(c + d
*x))
```

3.24 $\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\log(1-\cos(c+dx))}{d} - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{a^2\sec(c+dx)}{d}$$

```
[Out] 2*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2915, 12, 78}

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\sec(c+dx)}{d} + \frac{2a^2\log(1-\cos(c+dx))}{d} - \frac{2a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Log[1 - Cos[c + d*x]])/d - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c
+ d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a - a\cos(c+dx))^2 \csc(c+dx)\sec^2(c+dx)\,dx \\
&= \frac{a\text{Subst}\left(\int \frac{a^2(-a+x)}{(-a-x)x^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^3\text{Subst}\left(\int \frac{-a+x}{(-a-x)x^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{x^2} - \frac{2}{ax} + \frac{2}{a(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{2a^2\log(1-\cos(c+dx))}{d} - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{a^2\sec(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{a^2\left(-2\log(\cos(c+dx))+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec(c+dx)\right)}{d}$$

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^2,x]
[Out] (a^2*(-2*Log[Cos[c + d*x]] + 4*Log[Sin[(c + d*x)/2]] + Sec[c + d*x]))/d
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+2a^2\ln(\tan(dx+c))+a^2\ln(-\cot(dx+c)+\csc(dx+c))}{d}$
default	$\frac{a^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+2a^2\ln(\tan(dx+c))+a^2\ln(-\cot(dx+c)+\csc(dx+c))}{d}$
risch	$\frac{2a^2 e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}+\frac{4a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{2a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$
parallelrisch	$\frac{a^2\left(4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(dx+c)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\cos(dx+c)+1\right)}{d\cos(dx+c)}$
norman	$-\frac{2a^2}{d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{4a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(1/cos(d*x+c)+ln(-cot(d*x+c)+csc(d*x+c)))+2*a^2*ln(tan(d*x+c))+a^2
*ln(-cot(d*x+c)+csc(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=-\frac{2\,a^2\cos(dx+c)\log(-\cos(dx+c))-2\,a^2\cos(dx+c)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)-a^2}{d\cos(dx+c)}$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(2*a^2*cos(d*x + c)*log(-cos(d*x + c)) - 2*a^2*cos(d*x + c)*log(-1/2*cos(d
*x + c) + 1/2) - a^2)/(d*cos(d*x + c))
```

Sympy [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\csc(c+dx)\sec(c+dx)\,dx + \int \csc(c+dx)\sec^2(c+dx)\,dx + \int \csc(c+dx)\,dx\right)$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**2,x)
[Out] a**2*(Integral(2*csc(c + d*x)*sec(c + d*x), x) + Integral(csc(c + d*x)*sec(
c + d*x)**2, x) + Integral(csc(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\,a^2\log\left(\cos\left(dx+c\right)-1\right)-2\,a^2\log\left(\cos\left(dx+c\right)\right)+\frac{a^2}{\cos(dx+c)}}{d}$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] (2*a^2*log(cos(d*x + c) - 1) - 2*a^2*log(cos(d*x + c)) + a^2/cos(d*x + c))/
d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(48) = 96$.

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.40

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\left(a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}\right)}{d}$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^2*log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a^2 + a^2*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 14.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2}{d\,\cos\left(c+d\,x\right)}-\frac{4\,a^2\,\mathrm{atanh}\left(2\,\cos\left(c+d\,x\right)-1\right)}{d}$$

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x),x)
```

```
[Out] a^2/(d*cos(c + d*x)) - (4*a^2*atanh(2*cos(c + d*x) - 1))/d
```

3.25 $\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^3}{d(a-a\cos(c+dx))}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] -a^3/d/(a-a*cos(d*x+c))+2*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos(d*x+c))/d+a^2
*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 46}

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^3}{d(a-a\cos(c+dx))}+\frac{a^2\sec(c+dx)}{d}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^3/(d*(a - a*Cos[c + d*x]))) + (2*a^2*Log[1 - Cos[c + d*x]])/d - (2*a^2*
Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\csc^3(c+dx)\sec^2(c+dx)\,dx\\
&= \frac{a^3\text{Subst}\left(\int \frac{a^2}{(-a-x)^2x^2}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^5\text{Subst}\left(\int \frac{1}{(-a-x)^2x^2}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^5\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2}{a^3x}+\frac{1}{a^2(a+x)^2}+\frac{2}{a^3(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= -\frac{a^3}{d(a-a\cos(c+dx))}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\csc^2\left(\frac{1}{2}(c+dx)\right)+4\log(\cos(c+dx))-8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.}{8d}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/8*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(Csc[(c + d*x)/2]^2 + 4*L
og[Cos[c + d*x]] - 8*Log[Sin[(c + d*x)/2]] - 2*Sec[c + d*x]))/d
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

method	result
parallelrisch	$-\frac{2a^2\left(\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(dx+c)+\frac{3\left(\cos(dx+c)-\right.}{}\right.}{d\cos(dx+c)}$
risch	$\frac{4a^2\left(e^{3i(dx+c)}-e^{2i(dx+c)}+e^{i(dx+c)}\right)}{d\left(e^{i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}+\frac{4a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{2a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{a^2}{2d}-\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{4a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}{d}$
derivativedivides	$\frac{a^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+2a^2\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(-\frac{\cot}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+2a^2\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(-\frac{\cot}{}\right.}{d}$

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^2*(ln(tan(1/2*d*x+1/2*c)-1)*cos(d*x+c)+ln(tan(1/2*d*x+1/2*c)+1)*cos(d*
x+c)-2*ln(tan(1/2*d*x+1/2*c))*cos(d*x+c)+3/4*(cos(d*x+c)-2/3)*cot(1/2*d*x+1
/2*c)^2)/d/cos(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.62

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{2\,a^2\cos\left(dx+c\right)-a^2-2\left(a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)\right)\log\left(-\cos\left(dx+c\right)\right)+2\left(a^2\cos\left(dx+c\right)^2-a^2\right.}{d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)}$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*cos(d*x + c) - a^2 - 2*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-
cos(d*x + c)) + 2*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-1/2*cos(d*x
+ c) + 1/2))/(d*cos(d*x + c)^2 - d*cos(d*x + c))
```

Sympy [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\csc^3\left(c+dx\right)\sec\left(c+dx\right)dx + \int \csc^3\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \csc^3\left(c+dx\right)dx\right)$$

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)**3*sec(c + d*x), x) + Integral(csc(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{2\,a^2\log\left(\cos\left(dx+c\right)-1\right)-2\,a^2\log\left(\cos\left(dx+c\right)\right)+\frac{2\,a^2\cos(dx+c)-a^2}{\cos(dx+c)^2-\cos(dx+c)}}{d}$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*log(cos(d*x + c) - 1) - 2*a^2*log(cos(d*x + c)) + (2*a^2*cos(d*x + c
) - a^2)/(cos(d*x + c)^2 - cos(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.96

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{4\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-4\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a^2+\frac{5\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a^2*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a^2 + 5*a^2*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx=-\frac{2\,a^2\,\cos\left(c+d\,x\right)-a^2}{d\,\left(\cos\left(c+d\,x\right)-{\cos\left(c+d\,x\right)}^2\right)}-\frac{4\,a^2\,\mathrm{atanh}\left(2\,\cos\left(c+d\,x\right)-1\right)}{d}$$

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^3,x)
```

```
[Out] - (2*a^2*cos(c + d*x) - a^2)/(d*(cos(c + d*x) - cos(c + d*x)^2)) - (4*a^2*a
tanh(2*cos(c + d*x) - 1))/d
```

3.26 $\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^4}{4d(a-a\cos(c+dx))^2} - \frac{5a^3}{4d(a-a\cos(c+dx))} + \frac{17a^2\log(1-\cos(c+dx))}{8d} - \frac{2a^2\log(\cos(c+dx))}{d} - \frac{a^2\log(1+\cos(c+dx))}{8d} + \frac{a^2\sec(c+dx)}{d}$$

```
[Out] -1/4*a^4/d/(a-a*cos(d*x+c))^2-5/4*a^3/d/(a-a*cos(d*x+c))+17/8*a^2*ln(1-cos(
d*x+c))/d-2*a^2*ln(cos(d*x+c))/d-1/8*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c)/
d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^4}{4d(a-a\cos(c+dx))^2} - \frac{5a^3}{4d(a-a\cos(c+dx))} + \frac{a^2\sec(c+dx)}{d} + \frac{17a^2\log(1-\cos(c+dx))}{8d} - \frac{2a^2\log(\cos(c+dx))}{d} - \frac{a^2\log(\cos(c+dx)+1)}{8d}$$

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*a^4/(d*(a - a*Cos[c + d*x])^2) - (5*a^3)/(4*d*(a - a*Cos[c + d*x])) +
(17*a^2*Log[1 - Cos[c + d*x]])/(8*d) - (2*a^2*Log[Cos[c + d*x]])/d - (a^2*L
og[1 + Cos[c + d*x]])/(8*d) + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\csc^5(c+dx)\sec^2(c+dx)\,dx \\
&= \frac{a^5\text{Subst}\left(\int \frac{a^2}{(-a-x)^3x^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^7\text{Subst}\left(\int \frac{1}{(-a-x)^3x^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^7\text{Subst}\left(\int \left(\frac{1}{8a^5(a-x)}+\frac{1}{a^4x^2}-\frac{2}{a^5x}+\frac{1}{2a^3(a+x)^3}+\frac{5}{4a^4(a+x)^2}+\frac{17}{8a^5(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= -\frac{a^4}{4d(a-a\cos(c+dx))^2}-\frac{5a^3}{4d(a-a\cos(c+dx))}+\frac{17a^2\log(1-\cos(c+dx))}{8d} \\
&\quad -\frac{2a^2\log(\cos(c+dx))}{d}-\frac{a^2\log(1+\cos(c+dx))}{8d}+\frac{a^2\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(10\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)+4\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{64d}$$

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/64*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(10*Csc[(c + d*x)/2]^2 +
 Csc[(c + d*x)/2]^4 + 4*(Log[Cos[(c + d*x)/2]] + 8*Log[Cos[c + d*x]] - 17*L
og[Sin[(c + d*x)/2]] - 4*Sec[c + d*x])))/d
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

method	result
parallelrisch	$\frac{17\left(-\frac{8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)}{17}-\frac{8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)}{17}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(dx+c)-\frac{45\csc\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\cos(dx\right.}{}\right.}{4d\cos(dx+c)}$
norman	$\frac{\frac{a^2}{16d}+\frac{11a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{16d}-\frac{11a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{17a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{d}$
risch	$\frac{a^2\left(9\,e^{5i(dx+c)}-28\,e^{4i(dx+c)}+34\,e^{3i(dx+c)}-28\,e^{2i(dx+c)}+9\,e^{i(dx+c)}\right)}{2d\left(e^{i(dx+c)}-1\right)^4\left(e^{2i(dx+c)}+1\right)}-\frac{a^2\ln\left(e^{i(dx+c)}+1\right)}{4d}+\frac{17a^2\ln\left(e^{i(dx+c)}-1\right)}{4d}-$
derivativedivides	$\frac{a^2\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)}-\frac{5}{8\sin(dx+c)^2\cos(dx+c)}+\frac{15}{8\cos(dx+c)}+\frac{15\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+2a^2\left(-\frac{1}{4\sin(dx+c)^4}-\frac{}{2\sin}\right.}{d}$
default	$\frac{a^2\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)}-\frac{5}{8\sin(dx+c)^2\cos(dx+c)}+\frac{15}{8\cos(dx+c)}+\frac{15\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+2a^2\left(-\frac{1}{4\sin(dx+c)^4}-\frac{}{2\sin}\right.}{d}$

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 17/4*(-8/17*ln(tan(1/2*d*x+1/2*c)-1)*cos(d*x+c)-8/17*ln(tan(1/2*d*x+1/2*c)+
1)*cos(d*x+c)+ln(tan(1/2*d*x+1/2*c))*cos(d*x+c)-45/136*csc(1/2*d*x+1/2*c)^2
*(cos(d*x+c)-3/10*cos(2*d*x+2*c)-59/90)*cot(1/2*d*x+1/2*c)^2)*a^2/d/cos(d*x
+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.82

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{18\,a^2\cos(dx+c)^2 - 28\,a^2\cos(dx+c) + 8\,a^2 - 16\left(a^2\cos(dx+c)^3 - 2\,a^2\cos(dx+c)^2 + a^2\cos(dx+c)\right.}{}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(18*a^2*cos(d*x + c)^2 - 28*a^2*cos(d*x + c) + 8*a^2 - 16*(a^2*cos(d*x
+ c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-cos(d*x + c)) - (a^2
*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(1/2*cos(d*x
+ c) + 1/2) + 17*(a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 +
 d*cos(d*x + c))
```

Sympy [F]

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\csc^5(c+dx)\sec(c+dx)\,dx + \int \csc^5(c+dx)\sec^2(c+dx)\,dx + \int \csc^5(c+dx)\,dx\right)$$

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)**5*sec(c + d*x), x) + Integral(csc(c + d*x)**
5*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**5, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{a^2\log(\cos(dx+c)+1) - 17\,a^2\log(\cos(dx+c)-1) + 16\,a^2\log(\cos(dx+c)) - \frac{2\left(9\,a^2\cos(dx+c)^2 - 14\,a^2\cos\right.}{\cos(dx+c)^3 - 2\cos(dx+c)}}{8\,d}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^2*log(cos(d*x + c) + 1) - 17*a^2*log(cos(d*x + c) - 1) + 16*a^2*log
(cos(d*x + c)) - 2*(9*a^2*cos(d*x + c)^2 - 14*a^2*cos(d*x + c) + 4*a^2)/(co
s(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.66

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{34\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 32\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{\left(a^2-\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{51\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}}{16\,d}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(34*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 - 12*a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 51*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 + 32*(2*a^2 + a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/
d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{17\,a^2\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{8\,d}-\frac{a^2\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{8\,d} + \frac{\frac{9\,a^2\,{\cos\left(c+d\,x\right)}^2}{4}-\frac{7\,a^2\,\cos\left(c+d\,x\right)}{2}+a^2}{d\,\left({\cos\left(c+d\,x\right)}^3-2\,{\cos\left(c+d\,x\right)}^2+\cos\left(c+d\,x\right)\right)}-\frac{2\,a^2\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^5,x)
```

```
[Out] (17*a^2*log(cos(c + d*x) - 1))/(8*d) - (a^2*log(cos(c + d*x) + 1))/(8*d) +
(a^2 - (7*a^2*cos(c + d*x))/2 + (9*a^2*cos(c + d*x)^2)/4)/(d*(cos(c + d*x)
- 2*cos(c + d*x)^2 + cos(c + d*x)^3)) - (2*a^2*log(cos(c + d*x)))/d
```

3.27 $\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 160

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^5}{12d(a-a\cos(c+dx))^3}-\frac{3a^4}{8d(a-a\cos(c+dx))^2}-\frac{23a^3}{16d(a-a\cos(c+dx))}+\frac{a^3}{16d(a+a\cos(c+dx))}+\frac{9a^2\log(1-\cos(c+dx))}{4d}-\frac{2a^2\log(\cos(c+dx))}{d}-\frac{a^2\log(1+\cos(c+dx))}{4d}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] -1/12*a^5/d/(a-a*cos(d*x+c))^3-3/8*a^4/d/(a-a*cos(d*x+c))^2-23/16*a^3/d/(a-
a*cos(d*x+c))+1/16*a^3/d/(a+a*cos(d*x+c))+9/4*a^2*ln(1-cos(d*x+c))/d-2*a^2*
ln(cos(d*x+c))/d-1/4*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^5}{12d(a-a\cos(c+dx))^3}-\frac{3a^4}{8d(a-a\cos(c+dx))^2}-\frac{23a^3}{16d(a-a\cos(c+dx))}+\frac{a^3}{16d(a\cos(c+dx)+a)}+\frac{a^2\sec(c+dx)}{d}+\frac{9a^2\log(1-\cos(c+dx))}{4d}-\frac{2a^2\log(\cos(c+dx))}{d}-\frac{a^2\log(\cos(c+dx)+1)}{4d}$$

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/12*a^5/(d*(a - a*Cos[c + d*x])^3) - (3*a^4)/(8*d*(a - a*Cos[c + d*x])^2)
 - (23*a^3)/(16*d*(a - a*Cos[c + d*x])) + a^3/(16*d*(a + a*Cos[c + d*x])) +
 (9*a^2*Log[1 - Cos[c + d*x]])/(4*d) - (2*a^2*Log[Cos[c + d*x]])/d - (a^2*L
og[1 + Cos[c + d*x]])/(4*d) + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int (-a - a\cos(c+dx))^2 \csc^7(c+dx)\sec^2(c+dx)\,dx$$

$$= \frac{a^7 \text{Subst}\left(\int \frac{a^2}{(-a-x)^4 x^2 (-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^9 \text{Subst}\left(\int \frac{1}{(-a-x)^4 x^2 (-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^9 \text{Subst}\left(\int \left(\frac{1}{16a^6(a-x)^2} + \frac{1}{4a^7(a-x)} + \frac{1}{a^6x^2} - \frac{2}{a^7x} + \frac{1}{4a^4(a+x)^4} + \frac{3}{4a^5(a+x)^3} + \frac{23}{16a^6(a+x)^2} + \frac{9}{4a^7(a+x)}\right)dx, x,\right.}{d}$$

$$= -\frac{a^5}{12d(a - a\cos(c+dx))^3} - \frac{3a^4}{8d(a - a\cos(c+dx))^2} - \frac{23a^3}{16d(a - a\cos(c+dx))} + \frac{a^3}{16d(a + a\cos(c+dx))} + \frac{9a^2\log(1-\cos(c+dx))}{4d} - \frac{2a^2\log(\cos(c+dx))}{d} - \frac{a^2\log(1+\cos(c+dx))}{4d} + \frac{a^2\sec(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(120\csc^2\left(\frac{1}{2}(c+dx)\right)+36\csc^4\left(\frac{1}{2}(c+dx)\right)+48\left(\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(120*Csc[(c + d*x)/2]^2
 + 36*Csc[(c + d*x)/2]^4 + 48*(Log[Cos[(c + d*x)/2]] + 4*Log[Cos[c + d*x]]
- 9*Log[Sin[(c + d*x)/2]]) + Csc[(c + d*x)/2]^6*(16 - 3*Sec[(c + d*x)/2]^2*
(3 + 2*Sec[c + d*x]))))/d
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08

method	result
norman	$\frac{\frac{a^2}{96d}+\frac{11a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{96d}+\frac{13a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16d}+\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{32d}-\frac{95a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{32d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{9a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{2a^2\ln\left(\right.}{}$
parallelrisch	$\frac{a^2\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^6+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+11\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4+432\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-192\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\mathrm{ta}\right.}{}$
derivativedivides	$\frac{a^2\left(-\frac{1}{6\sin(dx+c)^6\cos(dx+c)}-\frac{7}{24\sin(dx+c)^4\cos(dx+c)}-\frac{35}{48\sin(dx+c)^2\cos(dx+c)}+\frac{35}{16\cos(dx+c)}+\frac{35\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right.}{}$
default	$\frac{a^2\left(-\frac{1}{6\sin(dx+c)^6\cos(dx+c)}-\frac{7}{24\sin(dx+c)^4\cos(dx+c)}-\frac{35}{48\sin(dx+c)^2\cos(dx+c)}+\frac{35}{16\cos(dx+c)}+\frac{35\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right.}{}$
risch	$\frac{a^2\left(15\,e^{9i(dx+c)}-48\,e^{8i(dx+c)}+32\,e^{7i(dx+c)}+40\,e^{6i(dx+c)}-62\,e^{5i(dx+c)}+40\,e^{4i(dx+c)}+32\,e^{3i(dx+c)}-48\,e^{2i(dx+c)}+15\,e^{i}\right.}{3d\left(e^{i(dx+c)}-1\right)^6\left(e^{i(dx+c)}+1\right)^2\left(e^{2i(dx+c)}+1\right)}$

```
[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/96/d*a^2+11/96*a^2/d*tan(1/2*d*x+1/2*c)^2+13/16*a^2/d*tan(1/2*d*x+1/2*c)
^4+1/32*a^2/d*tan(1/2*d*x+1/2*c)^10-95/32*a^2/d*tan(1/2*d*x+1/2*c)^6)/tan(1
/2*d*x+1/2*c)^6/(-1+tan(1/2*d*x+1/2*c)^2)+9/2/d*a^2*ln(tan(1/2*d*x+1/2*c))-
2*a^2/d*ln(tan(1/2*d*x+1/2*c)-1)-2*a^2/d*ln(tan(1/2*d*x+1/2*c)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.81

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{30\,a^2\cos\left(dx+c\right)^4 - 48\,a^2\cos\left(dx+c\right)^3 - 14\,a^2\cos\left(dx+c\right)^2 + 46\,a^2\cos\left(dx+c\right) - 12\,a^2 - 24\,\left(a^2\cos\left(dx+\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(30*a^2*cos(d*x + c)^4 - 48*a^2*cos(d*x + c)^3 - 14*a^2*cos(d*x + c)^2
 + 46*a^2*cos(d*x + c) - 12*a^2 - 24*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x +
c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-cos(d*x + c)) - 3*(a^2
*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x
 + c))*log(1/2*cos(d*x + c) + 1/2) + 27*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x
 + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-1/2*cos(d*x + c) +
1/2))/(d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2 - d*cos(d
*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{3\,a^2\log\left(\cos\left(dx+c\right)+1\right)-27\,a^2\log\left(\cos\left(dx+c\right)-1\right)+24\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{2\left(15\,a^2\cos(dx+c)^4-24\,a\right.}{\cos(dx+c)^5-}}{12\,d}$$

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^2*log(cos(d*x + c) + 1) - 27*a^2*log(cos(d*x + c) - 1) + 24*a^2*
log(cos(d*x + c)) - 2*(15*a^2*cos(d*x + c)^4 - 24*a^2*cos(d*x + c)^3 - 7*a^
2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) - 6*a^2)/(cos(d*x + c)^5 - 2*cos(d*x
 + c)^4 + 2*cos(d*x + c)^2 - cos(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.49

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$=\frac{216\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-192\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{3\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{\left(a^2-\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{90\,a^2(\cos}{(\cos(d}\right.}{}}{96\,d}$$

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(216*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 192*a^2*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 3*a^2*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + (a^2 - 12*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 90*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 396*a^2*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(cos(d*x + c) - 1)^3 +
192*(2*a^2 + a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{9\,a^2\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{4\,d}-\frac{a^2\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{4\,d}-\frac{2\,a^2\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$
$$+\frac{-\frac{5\,a^2\,\cos(c+d\,x)^4}{2}+4\,a^2\cos\left(c+d\,x\right)^3+\frac{7\,a^2\cos(c+d\,x)^2}{6}-\frac{23\,a^2\,\cos(c+d\,x)}{6}+a^2}{d\,\left(-\cos\left(c+d\,x\right)^5+2\cos\left(c+d\,x\right)^4-2\cos\left(c+d\,x\right)^2+\cos\left(c+d\,x\right)\right)}$$

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^7,x)
```

```
[Out] (9*a^2*log(cos(c + d*x) - 1))/(4*d) - (a^2*log(cos(c + d*x) + 1))/(4*d) - (
2*a^2*log(cos(c + d*x)))/d + (a^2 - (23*a^2*cos(c + d*x))/6 + (7*a^2*cos(c
+ d*x)^2)/6 + 4*a^2*cos(c + d*x)^3 - (5*a^2*cos(c + d*x)^4)/2)/(d*(cos(c +
d*x) - 2*cos(c + d*x)^2 + 2*cos(c + d*x)^4 - cos(c + d*x)^5))
```

3.28 $\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\begin{aligned}\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx = & -\frac{a^6}{32d(a-a\cos(c+dx))^4} \\ & -\frac{7a^5}{48d(a-a\cos(c+dx))^3} \\ & -\frac{15a^4}{32d(a-a\cos(c+dx))^2}-\frac{51a^3}{32d(a-a\cos(c+dx))} \\ & +\frac{a^4}{64d(a+a\cos(c+dx))^2}+\frac{9a^3}{64d(a+a\cos(c+dx))} \\ & +\frac{303a^2\log(1-\cos(c+dx))}{128d} \\ & -\frac{2a^2\log(\cos(c+dx))}{d}-\frac{47a^2\log(1+\cos(c+dx))}{128d} \\ & +\frac{a^2\sec(c+dx)}{d}\end{aligned}$$

```
[Out] -1/32*a^6/d/(a-a*cos(d*x+c))^4-7/48*a^5/d/(a-a*cos(d*x+c))^3-15/32*a^4/d/(a
-a*cos(d*x+c))^2-51/32*a^3/d/(a-a*cos(d*x+c))+1/64*a^4/d/(a+a*cos(d*x+c))^2
+9/64*a^3/d/(a+a*cos(d*x+c))+303/128*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos(d*
x+c))/d-47/128*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^6}{32d(a-a\cos(c+dx))^4}-\frac{7a^5}{48d(a-a\cos(c+dx))^3}-\frac{15a^4}{32d(a-a\cos(c+dx))^2}+\frac{a^4}{64d(a\cos(c+dx)+a)^2}-\frac{51a^3}{32d(a-a\cos(c+dx))}+\frac{9a^3}{64d(a\cos(c+dx)+a)}+\frac{a^2\sec(c+dx)}{d}+\frac{303a^2\log(1-\cos(c+dx))}{128d}-\frac{2a^2\log(\cos(c+dx))}{d}-\frac{47a^2\log(\cos(c+dx)+1)}{128d}$$

```
[In] Int[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/32*a^6/(d*(a - a*Cos[c + d*x])^4) - (7*a^5)/(48*d*(a - a*Cos[c + d*x])^3
) - (15*a^4)/(32*d*(a - a*Cos[c + d*x])^2) - (51*a^3)/(32*d*(a - a*Cos[c +
d*x])) + a^4/(64*d*(a + a*Cos[c + d*x])^2) + (9*a^3)/(64*d*(a + a*Cos[c + d
*x])) + (303*a^2*Log[1 - Cos[c + d*x]])/(128*d) - (2*a^2*Log[Cos[c + d*x]])
/d - (47*a^2*Log[1 + Cos[c + d*x]])/(128*d) + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-a - a\cos(c+dx))^2 \csc^9(c+dx)\sec^2(c+dx)\,dx \\
&= \frac{a^9 \text{Subst}\left(\int \frac{a^2}{(-a-x)^5 x^2 (-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^{11} \text{Subst}\left(\int \frac{1}{(-a-x)^5 x^2 (-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^{11} \text{Subst}\left(\int \left(\frac{1}{32a^7(a-x)^3} + \frac{9}{64a^8(a-x)^2} + \frac{47}{128a^9(a-x)} + \frac{1}{a^8x^2} - \frac{2}{a^9x} + \frac{1}{8a^5(a+x)^5} + \frac{7}{16a^6(a+x)^4} + \frac{15}{16a^7(a+x)^3}\right.\right.}{d} \\
&= -\frac{a^6}{32d(a-a\cos(c+dx))^4} - \frac{7a^5}{48d(a-a\cos(c+dx))^3} - \frac{15a^4}{32d(a-a\cos(c+dx))^2} \\
&\quad - \frac{51a^3}{32d(a-a\cos(c+dx))} + \frac{a^4}{64d(a+a\cos(c+dx))^2} \\
&\quad + \frac{9a^3}{64d(a+a\cos(c+dx))} + \frac{303a^2\log(1-\cos(c+dx))}{128d} \\
&\quad - \frac{2a^2\log(\cos(c+dx))}{d} - \frac{47a^2\log(1+\cos(c+dx))}{128d} + \frac{a^2\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(1224\csc^2\left(\frac{1}{2}(c+dx)\right)+180\csc^4\left(\frac{1}{2}(c+dx)\right)+28\csc^6\left(\frac{1}{2}(c+dx\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6144*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(1224*Csc[(c + d*x)/2]
^2 + 180*Csc[(c + d*x)/2]^4 + 28*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8
- 6*(18*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^4 + 4*(-47*Log[Cos[(c + d*x)/
2]] - 128*Log[Cos[c + d*x]] + 303*Log[Sin[(c + d*x)/2]] + 64*Sec[c + d*x]))
))/d
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.02

method	result
norman	$\frac{\frac{a^2}{512d}+\frac{37a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{1536d}+\frac{121a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{768d}+\frac{233a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{256d}+\frac{19a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{256d}+\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{256d}-\frac{203a^2\tan\left(\frac{dx}{2}+\right.}{64d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
parallelrisch	$a^2\left(3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^8+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+37\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^6+114\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+242\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4+7272\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\right.$
derivativedivides	$a^2\left(-\frac{1}{8\sin(dx+c)^8\cos(dx+c)}-\frac{3}{16\sin(dx+c)^6\cos(dx+c)}-\frac{21}{64\sin(dx+c)^4\cos(dx+c)}-\frac{105}{128\sin(dx+c)^2\cos(dx+c)}+\frac{315}{128\cos(dx+c)}+\frac{315}{}\right.$
default	$a^2\left(-\frac{1}{8\sin(dx+c)^8\cos(dx+c)}-\frac{3}{16\sin(dx+c)^6\cos(dx+c)}-\frac{21}{64\sin(dx+c)^4\cos(dx+c)}-\frac{105}{128\sin(dx+c)^2\cos(dx+c)}+\frac{315}{128\cos(dx+c)}+\frac{315}{}\right.$
risch	$\frac{a^2\left(525\,e^{13i(dx+c)}-1716\,e^{12i(dx+c)}+214\,e^{11i(dx+c)}+4652\,e^{10i(dx+c)}-4173\,e^{9i(dx+c)}-2552\,e^{8i(dx+c)}+4564\,e^{7i(dx+c)}-25\right.}{96d\left(e^{i(dx+c)}-1\right)^8\left(e^{i(dx+c)}+1\right)^4\left(e^{2i(d\right.}}$

```
[In] int(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/512/d*a^2+37/1536*a^2/d*tan(1/2*d*x+1/2*c)^2+121/768*a^2/d*tan(1/2*d*x+1
/2*c)^4+233/256*a^2/d*tan(1/2*d*x+1/2*c)^6+19/256*a^2/d*tan(1/2*d*x+1/2*c)^
12+1/256*a^2/d*tan(1/2*d*x+1/2*c)^14-203/64/d*a^2*tan(1/2*d*x+1/2*c)^8)/tan
(1/2*d*x+1/2*c)^8/(-1+tan(1/2*d*x+1/2*c)^2)+303/64/d*a^2*ln(tan(1/2*d*x+1/2
*c))-2*a^2/d*ln(tan(1/2*d*x+1/2*c)-1)-2*a^2/d*ln(tan(1/2*d*x+1/2*c)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(193) = 386$.

Time = 0.29 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.25

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{1050\,a^2\cos(dx+c)^6-1716\,a^2\cos(dx+c)^5-1468\,a^2\cos(dx+c)^4+3308\,a^2\cos(dx+c)^3-38\,a^2\cos(dx}{}$$

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/384*(1050*a^2*cos(d*x + c)^6 - 1716*a^2*cos(d*x + c)^5 - 1468*a^2*cos(d*x
 + c)^4 + 3308*a^2*cos(d*x + c)^3 - 38*a^2*cos(d*x + c)^2 - 1568*a^2*cos(d*
x + c) + 384*a^2 - 768*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos
(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c
)^2 + a^2*cos(d*x + c))*log(-cos(d*x + c)) - 141*(a^2*cos(d*x + c)^7 - 2*a^
2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x
```

```
+ c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/
2) + 909*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 +
4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(
d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^7 - 2*d*cos(d*x + c
)^6 - d*cos(d*x + c)^5 + 4*d*cos(d*x + c)^4 - d*cos(d*x + c)^3 - 2*d*cos(d*
x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**9*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.96

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{141\,a^2\log\left(\cos\left(dx+c\right)+1\right)-909\,a^2\log\left(\cos\left(dx+c\right)-1\right)+768\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{2\left(525\,a^2\cos(dx+c)\cdots\right.}{\cos\cdots}}{384\,d}$$

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/384*(141*a^2*log(cos(d*x + c) + 1) - 909*a^2*log(cos(d*x + c) - 1) + 768
*a^2*log(cos(d*x + c)) - 2*(525*a^2*cos(d*x + c)^6 - 858*a^2*cos(d*x + c)^5
 - 734*a^2*cos(d*x + c)^4 + 1654*a^2*cos(d*x + c)^3 - 19*a^2*cos(d*x + c)^2
 - 784*a^2*cos(d*x + c) + 192*a^2)/(cos(d*x + c)^7 - 2*cos(d*x + c)^6 - cos
(d*x + c)^5 + 4*cos(d*x + c)^4 - cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*
x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.42

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{3636\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 3072\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{120\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{6\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{\left(3\,a^2 \cdots\right.}{\cdots}}{1536\cdots}$$

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(3636*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 3072*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 120*a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 6*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 - (3*a^2 - 40*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 282*a^2*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1680*a^2*(cos(d*x + c) - 1)^3/(cos(d
*x + c) + 1)^3 + 7575*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)*(cos(d
*x + c) + 1)^4/(cos(d*x + c) - 1)^4 + 3072*(2*a^2 + a^2*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.99

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{-\frac{175\,a^2\cos(c+d\,x)^6}{64} + \frac{143\,a^2\cos(c+d\,x)^5}{32} + \frac{367\,a^2\cos(c+d\,x)^4}{96} - \frac{827\,a^2\cos(c+d\,x)^3}{96} + \frac{19\,a^2\cos(c+d\,x)^2}{192} + \frac{49\,a^2\cos(c+d\,x)}{12} \cdots}{d\left(-\cos(c+d\,x)^7 + 2\cos(c+d\,x)^6 + \cos(c+d\,x)^5 - 4\cos(c+d\,x)^4 + \cos(c+d\,x)^3 + 2\cos(c+d\,x)^2 - \cdots\right)}$$
$$+ \frac{303\,a^2\ln(\cos(c+d\,x)-1)}{128\,d} - \frac{47\,a^2\ln(\cos(c+d\,x)+1)}{128\,d} - \frac{2\,a^2\ln(\cos(c+d\,x))}{d}$$

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^9,x)
```

```
[Out] ((49*a^2*cos(c + d*x))/12 - a^2 + (19*a^2*cos(c + d*x)^2)/192 - (827*a^2*co
s(c + d*x)^3)/96 + (367*a^2*cos(c + d*x)^4)/96 + (143*a^2*cos(c + d*x)^5)/3
2 - (175*a^2*cos(c + d*x)^6)/64)/(d*(2*cos(c + d*x)^2 - cos(c + d*x) + cos(
c + d*x)^3 - 4*cos(c + d*x)^4 + cos(c + d*x)^5 + 2*cos(c + d*x)^6 - cos(c +
 d*x)^7)) + (303*a^2*log(cos(c + d*x) - 1))/(128*d) - (47*a^2*log(cos(c + d
*x) + 1))/(128*d) - (2*a^2*log(cos(c + d*x)))/d
```

3.29 $\int (a+a\sec(c+dx))^2 \sin^8(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 199

$$\begin{aligned}\int (a+a\sec(c+dx))^2 \sin^8(c+dx)\,dx = & -\frac{245a^2x}{128}+\frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} \\ & -\frac{2a^2\sin(c+dx)}{d}+\frac{139a^2\cos(c+dx)\sin(c+dx)}{128d} \\ & +\frac{11a^2\cos^3(c+dx)\sin(c+dx)}{192d} \\ & -\frac{17a^2\cos^5(c+dx)\sin(c+dx)}{48d} \\ & +\frac{a^2\cos^7(c+dx)\sin(c+dx)}{8d} \\ & -\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin^5(c+dx)}{5d} \\ & -\frac{2a^2\sin^7(c+dx)}{7d}+\frac{a^2\tan(c+dx)}{d}\end{aligned}$$

```
[Out] -245/128*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d+139/128*a^2*c
os(d*x+c)*sin(d*x+c)/d+11/192*a^2*cos(d*x+c)^3*sin(d*x+c)/d-17/48*a^2*cos(d
*x+c)^5*sin(d*x+c)/d+1/8*a^2*cos(d*x+c)^7*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3
/d-2/5*a^2*sin(d*x+c)^5/d-2/7*a^2*sin(d*x+c)^7/d+a^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2951, 2717, 2715, 8, 2713, 3855, 3852}

$$\int (a+a\sec(c+dx))^2\sin^8(c+dx)\,dx = \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin^7(c+dx)}{7d} - \frac{2a^2\sin^5(c+dx)}{5d} - \frac{2a^2\sin^3(c+dx)}{3d} - \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{a^2\sin(c+dx)\cos^7(c+dx)}{8d} - \frac{17a^2\sin(c+dx)\cos^5(c+dx)}{48d} + \frac{11a^2\sin(c+dx)\cos^3(c+dx)}{192d} + \frac{139a^2\sin(c+dx)\cos(c+dx)}{128d} - \frac{245a^2x}{128}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^8,x]
```

```
[Out] (-245*a^2*x)/128 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d
 + (139*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (11*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(192*d) - (17*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a^2*Cos
[c + d*x]^7*Sin[c + d*x])/(8*d) - (2*a^2*Sin[c + d*x]^3)/(3*d) - (2*a^2*Sin
[c + d*x]^5)/(5*d) - (2*a^2*Sin[c + d*x]^7)/(7*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int (-a - a\cos(c+dx))^2 \sin^6(c+dx)\tan^2(c+dx)\,dx$$

$$= \frac{\int \left(-3a^{10} - 8a^{10}\cos(c+dx) + 2a^{10}\cos^2(c+dx) + 12a^{10}\cos^3(c+dx) + 2a^{10}\cos^4(c+dx) - 8a^{10}\right.}{}$$

$$= -3a^2x + a^2\int \cos^8(c+dx)\,dx + a^2\int \sec^2(c+dx)\,dx + \left(2a^2\right)\int \cos^2(c+dx)\,dx$$
$$+ \left(2a^2\right)\int \cos^4(c+dx)\,dx + \left(2a^2\right)\int \cos^7(c+dx)\,dx$$
$$+ \left(2a^2\right)\int \sec(c+dx)\,dx - \left(3a^2\right)\int \cos^6(c+dx)\,dx - \left(8a^2\right)\int \cos(c+dx)\,dx$$
$$- \left(8a^2\right)\int \cos^5(c+dx)\,dx + \left(12a^2\right)\int \cos^3(c+dx)\,dx$$

$$
\begin{aligned}
&= -3a^2x + \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{8a^2\sin(c+dx)}{d} \\
&\quad + \frac{a^2\cos(c+dx)\sin(c+dx)}{d} + \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d} \\
&\quad - \frac{a^2\cos^5(c+dx)\sin(c+dx)}{2d} + \frac{a^2\cos^7(c+dx)\sin(c+dx)}{8d} \\
&\quad + \frac{1}{8}(7a^2)\int\cos^6(c+dx)\,dx + a^2\int 1\,dx + \frac{1}{2}(3a^2)\int\cos^2(c+dx)\,dx \\
&\quad - \frac{1}{2}(5a^2)\int\cos^4(c+dx)\,dx - \frac{a^2\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} \\
&\quad - \frac{(2a^2)\,\text{Subst}\left(\int(1-3x^2+3x^4-x^6)\,dx, x, -\sin(c+dx)\right)}{d} \\
&\quad + \frac{(8a^2)\,\text{Subst}\left(\int(1-2x^2+x^4)\,dx, x, -\sin(c+dx)\right)}{d} \\
&\quad - \frac{(12a^2)\,\text{Subst}\left(\int(1-x^2)\,dx, x, -\sin(c+dx)\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= -2a^2x + \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{4d} \\
&\quad - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{8d} - \frac{17a^2\cos^5(c+dx)\sin(c+dx)}{48d} \\
&\quad + \frac{a^2\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{2a^2\sin^3(c+dx)}{3d} - \frac{2a^2\sin^5(c+dx)}{5d} \\
&\quad - \frac{2a^2\sin^7(c+dx)}{7d} + \frac{a^2\tan(c+dx)}{d} + \frac{1}{48}(35a^2)\int\cos^4(c \\
&\qquad + dx)\,dx + \frac{1}{4}(3a^2)\int 1\,dx - \frac{1}{8}(15a^2)\int\cos^2(c+dx)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5a^2x}{4} + \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} \\
&\quad + \frac{13a^2\cos(c+dx)\sin(c+dx)}{16d} + \frac{11a^2\cos^3(c+dx)\sin(c+dx)}{192d} \\
&\quad - \frac{17a^2\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{a^2\cos^7(c+dx)\sin(c+dx)}{8d} \\
&\quad - \frac{2a^2\sin^3(c+dx)}{3d} - \frac{2a^2\sin^5(c+dx)}{5d} - \frac{2a^2\sin^7(c+dx)}{7d} \\
&\quad + \frac{a^2\tan(c+dx)}{d} + \frac{1}{64}(35a^2)\int\cos^2(c+dx)\,dx - \frac{1}{16}(15a^2)\int 1\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{35a^2x}{16} + \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} \\
&\quad + \frac{139a^2\cos(c+dx)\sin(c+dx)}{128d} + \frac{11a^2\cos^3(c+dx)\sin(c+dx)}{192d} \\
&\quad - \frac{17a^2\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{a^2\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{2a^2\sin^3(c+dx)}{3d} \\
&\quad - \frac{2a^2\sin^5(c+dx)}{5d} - \frac{2a^2\sin^7(c+dx)}{7d} + \frac{a^2\tan(c+dx)}{d} + \frac{1}{128}\left(35a^2\right)\int 1\,dx \\
&= -\frac{245a^2x}{128} + \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} \\
&\quad + \frac{139a^2\cos(c+dx)\sin(c+dx)}{128d} + \frac{11a^2\cos^3(c+dx)\sin(c+dx)}{192d} \\
&\quad - \frac{17a^2\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{a^2\cos^7(c+dx)\sin(c+dx)}{8d} \\
&\quad - \frac{2a^2\sin^3(c+dx)}{3d} - \frac{2a^2\sin^5(c+dx)}{5d} - \frac{2a^2\sin^7(c+dx)}{7d} + \frac{a^2\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.72

$$
\int (a+a\sec(c+dx))^2\sin^8(c+dx)\,dx = -\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(168000c+168000dx+37800\arctan(\tan(c+dx))-215040\text{arct}\right.}{}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^8,x]
```

```
[Out] -1/430080*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(168000*c + 168000*d
*x + 37800*ArcTan[Tan[c + d*x]] - 215040*ArcTanh[Sin[c + d*x]] + 215040*Sin
[c + d*x] + 71680*Sin[c + d*x]^3 + 43008*Sin[c + d*x]^5 + 30720*Sin[c + d*x
]^7 - 55440*Sin[2*(c + d*x)] + 2520*Sin[4*(c + d*x)] + 560*Sin[6*(c + d*x)]
 - 105*Sin[8*(c + d*x)] - 107520*Tan[c + d*x]))/d
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{a^2\left(-411600dx\cos(dx+c)-430080\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+430080\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+270480\sin(d\right.}{}$
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^9}{\cos(dx+c)}+\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)-\frac{35dx}{16}-\frac{35c}{16}\right)+2a^2\left(-\frac{\sin(dx+c)^7}{7}-\right.}{}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^9}{\cos(dx+c)}+\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)-\frac{35dx}{16}-\frac{35c}{16}\right)+2a^2\left(-\frac{\sin(dx+c)^7}{7}-\right.}{}$
parts	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}+\frac{a^2\left(\frac{\sin(dx+c)^9}{\cos(dx+c)}+\left(\sin(dx+c)^7+\right.\right.}{}$
risch	$-\frac{245a^2x}{128}+\frac{33ia^2\mathrm{e}^{-2i(dx+c)}}{128d}-\frac{93ia^2\mathrm{e}^{-i(dx+c)}}{64d}+\frac{93ia^2\mathrm{e}^{i(dx+c)}}{64d}-\frac{33ia^2\mathrm{e}^{2i(dx+c)}}{128d}+\frac{2ia^2}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{2a^2\ln(}{}$

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/215040*a^2*(-411600*d*x*cos(d*x+c)-430080*ln(tan(1/2*d*x+1/2*c)-1)*cos(d*
x+c)+430080*ln(tan(1/2*d*x+1/2*c)+1)*cos(d*x+c)+270480*sin(d*x+c)+35392*sin
(4*d*x+4*c)+105*sin(9*d*x+9*c)-271040*sin(2*d*x+2*c)+480*sin(8*d*x+8*c)-556
8*sin(6*d*x+6*c)-455*sin(7*d*x+7*c)-3080*sin(5*d*x+5*c)+52920*sin(3*d*x+3*c
))/d/cos(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93

$$\int (a+a\sec(c+dx))^2\sin^8(c+dx)\,dx =$$
$$-\frac{25725\,a^2dx\cos(dx+c)-13440\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+13440\,a^2\cos(dx+c)\log(-\sin(}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/13440*(25725*a^2*d*x*cos(d*x + c) - 13440*a^2*cos(d*x + c)*log(sin(d*x +
 c) + 1) + 13440*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) - (1680*a^2*cos(d*
x + c)^8 + 3840*a^2*cos(d*x + c)^7 - 4760*a^2*cos(d*x + c)^6 - 16896*a^2*co
s(d*x + c)^5 + 770*a^2*cos(d*x + c)^4 + 31232*a^2*cos(d*x + c)^3 + 14595*a^
2*cos(d*x + c)^2 - 45056*a^2*cos(d*x + c) + 13440*a^2)*sin(d*x + c))/(d*cos
(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c+dx))^2 \sin^8(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**8,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08

$$\int (a + a\sec(c+dx))^2 \sin^8(c+dx)\,dx =$$

$$-\frac{1024\left(30\,\sin\left(dx+c\right)^7 + 42\,\sin\left(dx+c\right)^5 + 70\,\sin\left(dx+c\right)^3 - 105\,\log\left(\sin\left(dx+c\right)+1\right) + 105\,\log\left(\sin\right.\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/107520*(1024*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3
- 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))
*a^2 - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(8*d*x + 8*c) +
168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^2 + 2240*(105*d*x + 105*c -
(87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x + c))/(tan(d*x + c)^6
+ 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*x + c))*a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.13

$$\int (a + a\sec(c+dx))^2 \sin^8(c+dx)\,dx =$$

$$-\frac{25725\,(dx+c)a^2 - 26880\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 26880\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{26880}{\tan}}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/13440*(25725*(d*x + c)*a^2 - 26880*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) + 26880*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 26880*a^2*tan(1/2*d*x +
```

```
1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(39165*a^2*tan(1/2*d*x + 1/2*c)^15
+ 300265*a^2*tan(1/2*d*x + 1/2*c)^13 + 989261*a^2*tan(1/2*d*x + 1/2*c)^11 +
 1791073*a^2*tan(1/2*d*x + 1/2*c)^9 + 1814943*a^2*tan(1/2*d*x + 1/2*c)^7 +
670131*a^2*tan(1/2*d*x + 1/2*c)^5 + 147735*a^2*tan(1/2*d*x + 1/2*c)^3 + 145
95*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 15.70 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.47

$$\int (a+a\sec(c+dx))^2\sin^8(c+dx)\,dx=\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{245\,a^2\,x}{128}+\frac{\frac{501\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64}+\frac{2633\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{48}+\frac{38047\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{240}+\frac{388613\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{1680}+\frac{13781\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{96}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+1\right.}$$

```
[In] int(sin(c + d*x)^8*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (245*a^2*x)/128 + ((13781*a^2*tan(c/2
 + (d*x)/2)^9)/96 - (1739*a^2*tan(c/2 + (d*x)/2)^5)/80 - (32681*a^2*tan(c/2
 + (d*x)/2)^7)/560 - (61*a^2*tan(c/2 + (d*x)/2)^3)/16 + (388613*a^2*tan(c/2
 + (d*x)/2)^11)/1680 + (38047*a^2*tan(c/2 + (d*x)/2)^13)/240 + (2633*a^2*ta
n(c/2 + (d*x)/2)^15)/48 + (501*a^2*tan(c/2 + (d*x)/2)^17)/64 - (11*a^2*tan(
c/2 + (d*x)/2))/64)/(d*(7*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2 + (d*x)/2)^4 +
28*tan(c/2 + (d*x)/2)^6 + 14*tan(c/2 + (d*x)/2)^8 - 14*tan(c/2 + (d*x)/2)^1
0 - 28*tan(c/2 + (d*x)/2)^12 - 20*tan(c/2 + (d*x)/2)^14 - 7*tan(c/2 + (d*x)
/2)^16 - tan(c/2 + (d*x)/2)^18 + 1))
```

3.30 $\int (a+a\sec(c+dx))^2 \sin^6(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int (a+a\sec(c+dx))^2\sin^6(c+dx)\,dx = -\frac{25a^2x}{16}+\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{7a^2\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin^5(c+dx)}{5d}+\frac{a^2\tan(c+dx)}{d}$$

```
[Out] -25/16*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d+7/16*a^2*cos(d*
x+c)*sin(d*x+c)/d+7/24*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a^2*cos(d*x+c)^5*s
in(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d-2/5*a^2*sin(d*x+c)^5/d+a^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used

$= \{3957, 2951, 2717, 2713, 2715, 8, 3855, 3852\}$

$$\int (a+a\sec(c+dx))^2\sin^6(c+dx)\,dx = \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin^5(c+dx)}{5d} - \frac{2a^2\sin^3(c+dx)}{3d} - \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\tan(c+dx)}{d} - \frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d} + \frac{7a^2\sin(c+dx)\cos^3(c+dx)}{24d} + \frac{7a^2\sin(c+dx)\cos(c+dx)}{16d} - \frac{25a^2x}{16}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] (-25*a^2*x)/16 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d +
 (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (7*a^2*Cos[c + d*x]^3*Sin[c + d
*x])/(24*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^2*Sin[c + d*x]
^3)/(3*d) - (2*a^2*Sin[c + d*x]^5)/(5*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
```

```
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\sin^4(c+dx)\tan^2(c+dx)\,dx \\
&= \frac{\int \left(-2a^8-6a^8\cos(c+dx)+6a^8\cos^3(c+dx)+2a^8\cos^4(c+dx)-2a^8\cos^5(c+dx)-a^8\cos^6(c+\right.}{a^6} \\
&= -2a^2x-a^2\int\cos^6(c+dx)\,dx+a^2\int\sec^2(c+dx)\,dx \\
&\quad+\left(2a^2\right)\int\cos^4(c+dx)\,dx-\left(2a^2\right)\int\cos^5(c+dx)\,dx \\
&\quad+\left(2a^2\right)\int\sec(c+dx)\,dx-\left(6a^2\right)\int\cos(c+dx)\,dx+\left(6a^2\right)\int\cos^3(c+dx)\,dx \\
&= -2a^2x+\frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{6a^2\sin(c+dx)}{d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d} \\
&\quad-\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{1}{6}\left(5a^2\right)\int\cos^4(c+dx)\,dx \\
&\quad+\frac{1}{2}\left(3a^2\right)\int\cos^2(c+dx)\,dx-\frac{a^2\operatorname{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d} \\
&\quad+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{d} \\
&\quad-\frac{\left(6a^2\right)\operatorname{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}
\end{aligned}$$

$$
\begin{aligned}
&= -2a^2x + \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d} \\
&\quad + \frac{7a^2\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{2a^2\sin^3(c+dx)}{3d} \\
&\quad - \frac{2a^2\sin^5(c+dx)}{5d} + \frac{a^2\tan(c+dx)}{d} - \frac{1}{8}(5a^2)\int\cos^2(c+dx)\,dx + \frac{1}{4}(3a^2)\int 1\,dx \\
&= -\frac{5a^2x}{4} + \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad + \frac{7a^2\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} \\
&\quad - \frac{2a^2\sin^3(c+dx)}{3d} - \frac{2a^2\sin^5(c+dx)}{5d} + \frac{a^2\tan(c+dx)}{d} - \frac{1}{16}(5a^2)\int 1\,dx \\
&= -\frac{25a^2x}{16} + \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad + \frac{7a^2\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} \\
&\quad - \frac{2a^2\sin^3(c+dx)}{3d} - \frac{2a^2\sin^5(c+dx)}{5d} + \frac{a^2\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$
\int (a+a\sec(c+dx))^2\sin^6(c+dx)\,dx = -\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)(1080c+1080dx+420\arctan(\tan(c+dx))-1920\operatorname{arctanh}(\sin(c-
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] -1/3840*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(1080*c + 1080*d*x + 4
20*ArcTan[Tan[c + d*x]] - 1920*ArcTanh[Sin[c + d*x]] + 1920*Sin[c + d*x] +
640*Sin[c + d*x]^3 + 384*Sin[c + d*x]^5 - 255*Sin[2*(c + d*x)] - 15*Sin[4*(
c + d*x)] + 5*Sin[6*(c + d*x)] - 960*Tan[c + d*x]))/d
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{a^2\left(-3000dx\cos(dx+c)-3840\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+3840\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+2175\sin(dx+c)-\right.}{1920d\cos(dx+c}$
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^7}{\cos(dx+c)}+\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2a^2\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\mathrm{si}\right.}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^7}{\cos(dx+c)}+\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2a^2\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\mathrm{si}\right.}{d}$
parts	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{a^2\left(\frac{\sin(dx+c)^7}{\cos(dx+c)}+\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\right.\right.}{d}$
risch	$-\frac{25a^2x}{16}+\frac{17ia^2e^{-2i(dx+c)}}{128d}-\frac{17ia^2e^{2i(dx+c)}}{128d}-\frac{11ia^2e^{-i(dx+c)}}{8d}+\frac{11ia^2e^{i(dx+c)}}{8d}+\frac{2ia^2}{d(e^{2i(dx+c)}+1)}-\frac{2a^2\ln}{}$
norman	$\frac{25a^2x}{16}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{27a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4d}+\frac{797a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{40d}-\frac{91a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{2d}-\frac{8041a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{120d}-\frac{431a^2\tan\left(\frac{d}{2}\right.}{12d}$

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1920*a^2*(-3000*d*x*cos(d*x+c)-3840*ln(tan(1/2*d*x+1/2*c)-1)*cos(d*x+c)+3
840*ln(tan(1/2*d*x+1/2*c)+1)*cos(d*x+c)+2175*sin(d*x+c)-2360*sin(2*d*x+2*c)
-24*sin(6*d*x+6*c)+256*sin(4*d*x+4*c)-5*sin(7*d*x+7*c)+10*sin(5*d*x+5*c)+27
0*sin(3*d*x+3*c))/d/cos(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\int (a+a\sec(c+dx))^2\sin^6(c+dx)\,dx =$$
$$-\frac{375\,a^2dx\cos(dx+c)-240\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+240\,a^2\cos(dx+c)\log(-\sin(dx+c$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/240*(375*a^2*d*x*cos(d*x + c) - 240*a^2*cos(d*x + c)*log(sin(d*x + c) +
1) + 240*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (40*a^2*cos(d*x + c)^6 +
 96*a^2*cos(d*x + c)^5 - 70*a^2*cos(d*x + c)^4 - 352*a^2*cos(d*x + c)^3 - 1
```

```
05*a^2*cos(d*x + c)^2 + 736*a^2*cos(d*x + c) - 240*a^2)*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \sec(c + dx))^2 \sin^6(c + dx)\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**6,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.11

$$\int (a + a \sec(c + dx))^2 \sin^6(c + dx)\, dx =$$
$$-\frac{64\left(6\sin(dx+c)^5 + 10\sin(dx+c)^3 - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1) + 30\sin(dx\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="maxima")
[Out] -1/960*(64*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^2 - 5*(4*sin(2*d*x + 2*c)^
3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 + 120*(15
*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d
*x + c)^2 + 1) - 8*tan(d*x + c))*a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.23

$$\int (a + a \sec(c + dx))^2 \sin^6(c + dx)\, dx =$$
$$-\frac{375\,(dx+c)a^2 - 480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{480\,a^2\tan\left(\frac{1}{2}\,dx+\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(375*(d*x + c)*a^2 - 480*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 48
0*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 480*a^2*tan(1/2*d*x + 1/2*c)/(ta
n(1/2*d*x + 1/2*c)^2 - 1) + 2*(615*a^2*tan(1/2*d*x + 1/2*c)^11 + 3485*a^2*t
an(1/2*d*x + 1/2*c)^9 + 7926*a^2*tan(1/2*d*x + 1/2*c)^7 + 8586*a^2*tan(1/2*
d*x + 1/2*c)^5 + 2595*a^2*tan(1/2*d*x + 1/2*c)^3 + 345*a^2*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 14.71 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.50

$$\int (a + a\sec(c+dx))^2 \sin^6(c+dx)\,dx$$
$$= \frac{\frac{57\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8} + \frac{431\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{12} + \frac{8041\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{120} + \frac{91\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2} - \frac{797\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{40} - \frac{27\,a^2\,\mathrm{t}}{}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} - 5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} + 5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + 9\tan\left(\right.\right.}$$
$$- \frac{25\,a^2\,x}{16} + \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

```
[In] int(sin(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((91*a^2*tan(c/2 + (d*x)/2)^7)/2 - (797*a^2*tan(c/2 + (d*x)/2)^5)/40 - (27*
a^2*tan(c/2 + (d*x)/2)^3)/4 + (8041*a^2*tan(c/2 + (d*x)/2)^9)/120 + (431*a^
2*tan(c/2 + (d*x)/2)^11)/12 + (57*a^2*tan(c/2 + (d*x)/2)^13)/8 - (7*a^2*tan
(c/2 + (d*x)/2))/8)/(d*(5*tan(c/2 + (d*x)/2)^2 + 9*tan(c/2 + (d*x)/2)^4 + 5
*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 - 9*tan(c/2 + (d*x)/2)^10 -
5*tan(c/2 + (d*x)/2)^12 - tan(c/2 + (d*x)/2)^14 + 1)) - (25*a^2*x)/16 + (4*
a^2*atanh(tan(c/2 + (d*x)/2)))/d
```

3.31 $\int (a + a\sec(c + dx))^2 \sin^4(c + dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx = -\frac{9a^2x}{8}+\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}$$

```
[Out] -9/8*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/8*a^2*cos(d*x+c
)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d+a^2
*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3957, 2951, 2717, 2715, 8, 2713, 3855, 3852}

$$\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx = \frac{2a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}-\frac{9a^2x}{8}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] (-9*a^2*x)/8 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (
a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4
*d) - (2*a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\sin^2(c+dx)\tan^2(c+dx)\,dx \\
&= \frac{\int \left(-a^6-4a^6\cos(c+dx)-a^6\cos^2(c+dx)+2a^6\cos^3(c+dx)+a^6\cos^4(c+dx)+2a^6\sec(c+dx)\right.}{a^4} \\
&= -a^2x-a^2\int\cos^2(c+dx)\,dx+a^2\int\cos^4(c+dx)\,dx+a^2\int\sec^2(c+dx)\,dx \\
&\quad+\left(2a^2\right)\int\cos^3(c+dx)\,dx+\left(2a^2\right)\int\sec(c+dx)\,dx-\left(4a^2\right)\int\cos(c+dx)\,dx \\
&= -a^2x+\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d} \\
&\quad+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{1}{2}a^2\int 1\,dx+\frac{1}{4}\left(3a^2\right)\int\cos^2(c+dx)\,dx \\
&\quad-\frac{a^2\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}-\frac{\left(2a^2\right)\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d} \\
&= -\frac{3a^2x}{2}+\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{8d} \\
&\quad+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{1}{8}\left(3a^2\right)\int 1\,dx \\
&= -\frac{9a^2x}{8}+\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{8d} \\
&\quad+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx = -\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(48c+48dx+60\arctan(\tan(c+dx))-192\text{arctanh}(\sin(c+dx))\right.}{384d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] -1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(48*c + 48*d*x + 60*Arc
Tan[Tan[c + d*x]] - 192*ArcTanh[Sin[c + d*x]] + 192*Sin[c + d*x] + 64*Sin[c
 + d*x]^3 - 3*Sin[4*(c + d*x)] - 96*Tan[c + d*x]))/d
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

method	result
parallelrisch	$\frac{a^2\left(-72dx\cos(dx+c)-128\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+128\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\sin(5dx+5c)+64\sin(d\right.}{64d\cos(dx+c)}$
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(\right.}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(\right.}{d}$
parts	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{a^2\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}\right.}{d}$
risch	$-\frac{9a^2x}{8}+\frac{5ia^2e^{i(dx+c)}}{4d}-\frac{5ia^2e^{-i(dx+c)}}{4d}+\frac{2ia^2}{d\left(e^{2i(dx+c)}+1\right)}+\frac{2a^2\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{2a^2\ln\left(e^{i(dx+c)}-i\right)}{d}+a^2$
norman	$\frac{\frac{9a^2x}{8}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{22a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{31a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2d}-\frac{58a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}-\frac{25a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}+\frac{27a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{8}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*a^2*(-72*d*x*cos(d*x+c)-128*ln(tan(1/2*d*x+1/2*c)-1)*cos(d*x+c)+128*ln
(tan(1/2*d*x+1/2*c)+1)*cos(d*x+c)+sin(5*d*x+5*c)+64*sin(d*x+c)-224/3*sin(2*
d*x+2*c)+sin(3*d*x+3*c)+16/3*sin(4*d*x+4*c))/d/cos(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16

$$\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx =$$
$$-\frac{27\,a^2dx\cos(dx+c)-24\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+24\,a^2\cos(dx+c)\log(-\sin(dx+c)+}{24\,d\,\mathrm{co}}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(27*a^2*d*x*cos(d*x + c) - 24*a^2*cos(d*x + c)*log(sin(d*x + c) + 1)
+ 24*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) - (6*a^2*cos(d*x + c)^4 + 16*a
^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c)^2 - 64*a^2*cos(d*x + c) + 24*a^2)*si
n(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx = a^2\left(\int 2\sin^4{(c+dx)}\sec{(c+dx)}\,dx + \int \sin^4{(c+dx)}\sec^2{(c+dx)}\,dx + \int \sin^4{(c+dx)}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**4,x)
[Out] a**2*(Integral(2*sin(c + d*x)**4*sec(c + d*x), x) + Integral(sin(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10

$$\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx = -\frac{32\left(2\sin{(dx+c)}^3-3\log{(\sin{(dx+c)}+1)}+3\log{(\sin{(dx+c)}-1)}+6\sin{(dx+c)}\right)a^2-3\,(12\,dx+}{96\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="maxima")
[Out] -1/96*(32*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*
d*x + 2*c))*a^2 + 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*t
an(d*x + c))*a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.40

$$\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx = -\frac{27\,(dx+c)a^2-48\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+48\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{48\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{24\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/24*(27*(d*x + c)*a^2 - 48*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 48*a^
2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 48*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1) + 2*(51*a^2*tan(1/2*d*x + 1/2*c)^7 + 187*a^2*tan(1/2*d
*x + 1/2*c)^5 + 229*a^2*tan(1/2*d*x + 1/2*c)^3 + 45*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 14.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.54

$$\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx=\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{9\,a^2\,x}{8}+\frac{\frac{25\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{58\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{31\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}-\frac{22\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-\frac{7\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (9*a^2*x)/8 + ((31*a^2*tan(c/2 + (d*x
)/2)^5)/2 - (22*a^2*tan(c/2 + (d*x)/2)^3)/3 + (58*a^2*tan(c/2 + (d*x)/2)^7)
/3 + (25*a^2*tan(c/2 + (d*x)/2)^9)/4 - (7*a^2*tan(c/2 + (d*x)/2))/4)/(d*(3*
tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6 - 3*
tan(c/2 + (d*x)/2)^8 - tan(c/2 + (d*x)/2)^10 + 1))
```

3.32 $\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx = -\frac{a^2x}{2}+\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^2\tan(c+dx)}{d}$$

```
[Out] -1/2*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/2*a^2*cos(d*x+c
)*sin(d*x+c)/d+a^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2788, 2717, 2715, 8, 3855, 3852}

$$\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx = \frac{2a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^2x}{2}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] -1/2*(a^2*x) + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (
a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\tan^2(c+dx)\,dx\\
&= \frac{\int \left(-2a^4\cos(c+dx)-a^4\cos^2(c+dx)+2a^4\sec(c+dx)+a^4\sec^2(c+dx)\right)\,dx}{a^2}\\
&= -\left(a^2\int \cos^2(c+dx)\,dx\right)+a^2\int \sec^2(c+dx)\,dx\\
&\quad -\left(2a^2\right)\int \cos(c+dx)\,dx+\left(2a^2\right)\int \sec(c+dx)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} \\
&\quad - \frac{1}{2}a^2\int 1\,dx - \frac{a^2\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} \\
&= -\frac{a^2x}{2} + \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} \\
&\quad - \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^2\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 243 vs. $2(73) = 146$.

Time = 1.22 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.33

$$
\begin{aligned}
&\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx \\
&= \frac{1}{16}a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-2x\right. \\
&\quad - \frac{8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} \\
&\quad - \frac{8\cos(dx)\sin(c)}{d} - \frac{\cos(2dx)\sin(2c)}{d} - \frac{8\cos(c)\sin(dx)}{d} - \frac{\cos(2c)\sin(2dx)}{d} \\
&\quad + \frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\
&\quad \left.+ \frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)
\end{aligned}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-2*x - (8*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]])/d + (8*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d -
 (8*Cos[d*x]*Sin[c])/d - (Cos[2*d*x]*Sin[2*c])/d - (8*Cos[c]*Sin[d*x])/d -
(Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/16
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a^2(\tan(dx+c)-dx-c)}{d}+\frac{2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
parallelrisch	$-\frac{a^2\left(4dx\cos(dx+c)+16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-7\sin(dx+c)+\sin(3dx+3c\right.}{8d\cos(dx+c)}$
risch	$-\frac{a^2x}{2}+\frac{ia^2e^{2i(dx+c)}}{8d}+\frac{ia^2e^{i(dx+c)}}{d}-\frac{ia^2e^{-i(dx+c)}}{d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{2ia^2}{d\left(e^{2i(dx+c)}+1\right)}+\frac{2a^2\ln(e^{i(dx+c)}+i}{d}$
norman	$\frac{\frac{a^2x}{2}+\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{6a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}-\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}-\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-$

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a
^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.42

$$\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx =$$
$$-\frac{a^2dx\cos(dx+c)-2a^2\cos(dx+c)\log(\sin(dx+c)+1)+2a^2\cos(dx+c)\log(-\sin(dx+c)+1)+}{2d\cos(dx+c)}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*d*x*cos(d*x + c) - 2*a^2*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*a
^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^2*cos(d*x + c)^2 + 4*a^2*cos(d*
x + c) - 2*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx = a^2\left(\int 2\sin^2{(c+dx)}\sec{(c+dx)}\,dx + \int \sin^2{(c+dx)}\sec^2{(c+dx)}\,dx + \int \sin^2{(c+dx)}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)**2*sec(c + d*x), x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx = \frac{(2\,dx+2\,c-\sin{(2\,dx+2\,c)})a^2-4\,(dx+c-\tan{(dx+c)})a^2+4\,a^2(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+}}{4\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^2 - 4*(d*x + c - tan(d*x + c))*a^2
+ 4*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.75

$$\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx = -\frac{(dx+c)a^2-4\,a^2\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+1\right|\right)}+4\,a^2\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-1\right|\right)}+\frac{4\,a^2\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}^2-1}+\frac{2\,(3\,a}{}}{2\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 4*a^2*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x +
1/2*c)^2 - 1) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx$$
$$=\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\,x}{2}$$
$$+\frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (a^2*x)/2 + (6*a^2*tan(c/2 + (d*x)/2)
^3 + 5*a^2*tan(c/2 + (d*x)/2)^5 - 3*a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (
d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1))
```

3.33 $\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d} + \frac{a^2\tan(c+dx)}{d}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-2*a^2*cot(d*x+c)/d-2*a^2*csc(d*x+c)/d+a^2*tan(d
*x+c)/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3957, 2952, 3852, 8, 2701, 327, 213, 2700, 14}

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} - \frac{2a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c + d
*x])/d + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\csc^2(c+dx)\sec^2(c+dx)\,dx \\
&= \int \left(a^2\csc^2(c+dx)+2a^2\csc^2(c+dx)\sec(c+dx)+a^2\csc^2(c+dx)\sec^2(c+dx)\right)\,dx \\
&= a^2\int \csc^2(c+dx)\,dx+a^2\int \csc^2(c+dx)\sec^2(c+dx)\,dx+\left(2a^2\right)\int \csc^2(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a^2\text{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{d}+\frac{a^2\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\tan(c+dx)\right)}{d} \\
&\quad -\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d} \\
&= -\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx,x,\tan(c+dx)\right)}{d} \\
&\quad -\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d} \\
&= \frac{2a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 401 vs. $2(57) = 114$.

Time = 6.65 (sec) , antiderivative size = 401, normalized size of antiderivative = 7.04

$$\begin{aligned}
&\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx \\
&= -\frac{\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{2d} \\
&\quad +\frac{\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{2d} \\
&\quad +\frac{\cos^2(c+dx)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{2d} \\
&\quad +\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{4d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} \\
&\quad +\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{4d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
\end{aligned}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*(Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 +
 (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/d + (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(2*d
) + (Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^4*(a + a
*Sec[c + d*x])^2*Sin[(d*x)/2])/(2*d) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4
*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
 (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a
+ a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{a^{2}\left(\frac{1}{\sin\left(dx+c\right)\cos\left(dx+c\right)}-2\cot\left(dx+c\right)\right)+2a^{2}\left(-\frac{1}{\sin\left(dx+c\right)}+\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)\right)-a^{2}\cot\left(dx+c\right)}{d}$
default	$\frac{a^{2}\left(\frac{1}{\sin\left(dx+c\right)\cos\left(dx+c\right)}-2\cot\left(dx+c\right)\right)+2a^{2}\left(-\frac{1}{\sin\left(dx+c\right)}+\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)\right)-a^{2}\cot\left(dx+c\right)}{d}$
parallelrisch	$-\frac{a^{2}\left(3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)\cos\left(dx+c\right)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos\left(dx+c\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos\left(dx+c\right)-\cot\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\cos\left(dx+c\right)}$
norman	$\frac{\frac{2a^{2}}{d}-\frac{4a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}-\frac{2a^{2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2a^{2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$
risch	$-\frac{2ia^{2}\left(2\,{\mathrm e}^{2i\left(dx+c\right)}-{\mathrm e}^{i\left(dx+c\right)}+3\right)}{d\left({\mathrm e}^{i\left(dx+c\right)}-1\right)\left({\mathrm e}^{2i\left(dx+c\right)}+1\right)}-\frac{2a^{2}\ln\left({\mathrm e}^{i\left(dx+c\right)}-i\right)}{d}+\frac{2a^{2}\ln\left({\mathrm e}^{i\left(dx+c\right)}+i\right)}{d}$

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+2*a^2*(-1/sin(d*x+c)+ln(sec
(d*x+c)+tan(d*x+c)))-a^2*cot(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.77

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{a^{2}\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-a^{2}\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-3}{d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*cos(d*x + c)*log(sin(d*x + c) + 1)*sin(d*x + c) - a^2*cos(d*x + c)*log
(-sin(d*x + c) + 1)*sin(d*x + c) - 3*a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c
) + a^2)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\csc^2(c+dx)\sec(c+dx)\,dx + \int \csc^2(c+dx)\sec^2(c+dx)\,dx + \int \csc^2(c+dx)\,dx\right)$$

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
[Out] a**2*(Integral(2*csc(c + d*x)**2*sec(c + d*x), x) + Integral(csc(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + a^2\left(\frac{1}{\tan(dx+c)} - \tan(dx+c)\right) + \frac{a^2}{\tan(dx+c)}}{d}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -(a^2*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + a^
2*(1/tan(d*x + c) - tan(d*x + c)) + a^2/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\left(a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{d}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) - (2*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2)/(tan(1/2*d*x + 1/2*c)^3 - ta
n(1/2*d*x + 1/2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx=\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^2,x)
```

```
[Out] (4*a^2*tan(c/2 + (d*x)/2)^2 - 2*a^2)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*
x)/2)^3)) + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d
```

3.34 $\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{10a^2\tan(c+dx)}{3d}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d+10/3*a^2*tan(d*x+c)/d-2*a^2*tan(d*x+c)/d/(1-cos
(d*x+c))-1/3*a^4*tan(d*x+c)/d/(a-a*cos(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3957, 2948, 2845, 3057, 2827, 3852, 8, 3855}

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}+\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{10a^2\tan(c+dx)}{3d}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}$$

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d + (10*a^2*Tan[c + d*x])/(3*d) - (2*a^2*Tan[
c + d*x])/(d*(1 - Cos[c + d*x])) - (a^4*Tan[c + d*x])/(3*d*(a - a*Cos[c + d
*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\csc^4(c+dx)\sec^2(c+dx)\,dx \\
&= a^4\int \frac{\sec^2(c+dx)}{(-a+a\cos(c+dx))^2}\,dx \\
&= -\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}+\frac{1}{3}a^2\int \frac{(-4a-2a\cos(c+dx))\sec^2(c+dx)}{-a+a\cos(c+dx)}\,dx \\
&= -\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}+\frac{1}{3}\int \left(10a^2+6a^2\cos(c+dx)\right)\sec^2(c+dx)\,dx \\
&= -\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2} \\
&\quad +\left(2a^2\right)\int \sec(c+dx)\,dx+\frac{1}{3}\left(10a^2\right)\int \sec^2(c+dx)\,dx \\
&= \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))} \\
&\quad -\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}-\frac{\left(10a^2\right)\operatorname{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{3d} \\
&= \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{10a^2\tan(c+dx)}{3d}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 228 vs. $2(87) = 174$.

Time = 1.97 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.62

$$
\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx
$$

$$
= \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-\cot\left(\frac{c}{2}\right)\csc^2\left(\frac{1}{2}(c+dx)\right)-(-8+7\cos(c+dx))\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{1}{2}(c\right.\right.}{}
$$

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-(Cot[c/2]*Csc[(c + d*x)/2]^2
) - (-8 + 7*Cos[c + d*x])*Csc[c/2]*Csc[(c + d*x)/2]^3*Sin[(d*x)/2] + 6*(-2*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(24
*d)
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

method	result
parallelrisch	$-\frac{2 (\ln (\tan (\frac{dx}{2}+\frac{c}{2})-1) \cos (dx+c)-\ln (\tan (\frac{dx}{2}+\frac{c}{2})+1) \cos (dx+c)+\frac{7 \csc (\frac{dx}{2}+\frac{c}{2})^{2} (\cos (dx+c)-\frac{5 \cos (2dx+2c)}{14}-\frac{4}{7}) \cot}{6}}{d \cos (dx+c)}$
norman	$\frac{\frac{a^{2}}{6d}+\frac{7a^{2} \tan (\frac{dx}{2}+\frac{c}{2})^{2}}{3d}-\frac{9a^{2} \tan (\frac{dx}{2}+\frac{c}{2})^{4}}{2d}}{\tan (\frac{dx}{2}+\frac{c}{2})^{3} (-1+\tan (\frac{dx}{2}+\frac{c}{2})^{2})}-\frac{2a^{2} \ln (\tan (\frac{dx}{2}+\frac{c}{2})-1)}{d}+\frac{2a^{2} \ln (\tan (\frac{dx}{2}+\frac{c}{2})+1)}{d}$
derivativedivides	$\frac{a^{2} (-\frac{1}{3 \sin (dx+c)^{3} \cos (dx+c)}+\frac{4}{3 \sin (dx+c) \cos (dx+c)}-\frac{8 \cot (dx+c)}{3})+2a^{2} (-\frac{1}{3 \sin (dx+c)^{3}}-\frac{1}{\sin (dx+c)}+\ln (\sec (dx+c)+\tan (dx}{d}$
default	$\frac{a^{2} (-\frac{1}{3 \sin (dx+c)^{3} \cos (dx+c)}+\frac{4}{3 \sin (dx+c) \cos (dx+c)}-\frac{8 \cot (dx+c)}{3})+2a^{2} (-\frac{1}{3 \sin (dx+c)^{3}}-\frac{1}{\sin (dx+c)}+\ln (\sec (dx+c)+\tan (dx}{d}$
risch	$-\frac{4ia^{2} (3 e^{4i(dx+c)}-9 e^{3i(dx+c)}+11 e^{2i(dx+c)}-12 e^{i(dx+c)}+5)}{3d (e^{i(dx+c)}-1)^{3} (e^{2i(dx+c)}+1)}+\frac{2a^{2} \ln (e^{i(dx+c)}+i)}{d}-\frac{2a^{2} \ln (e^{i(dx+c)}-i)}{d}$

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(ln(tan(1/2*d*x+1/2*c)-1)*cos(d*x+c)-ln(tan(1/2*d*x+1/2*c)+1)*cos(d*x+c)
+7/6*csc(1/2*d*x+1/2*c)^2*(cos(d*x+c)-5/14*cos(2*d*x+2*c)-4/7)*cot(1/2*d*x+
1/2*c))*a^2/d/cos(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.83

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{10\,a^{2} \cos (dx+c)^{3}-4\,a^{2} \cos (dx+c)^{2}-11\,a^{2} \cos (dx+c)-3\,(a^{2} \cos (dx+c)^{2}-a^{2} \cos (dx+c)) \log (}{3\,(d \cos (dx+c)^{2}-}$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(10*a^2*cos(d*x + c)^3 - 4*a^2*cos(d*x + c)^2 - 11*a^2*cos(d*x + c) -
3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c
) + 3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*
x + c) + 3*a^2)/((d*cos(d*x + c)^2 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\csc^4(c+dx)\sec(c+dx)\,dx + \int \csc^4(c+dx)\sec^2(c+dx)\,dx + \int \csc^4(c+dx)\,dx\right)$$

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)**4*sec(c + d*x), x) + Integral(csc(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.30

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right) + a^2\left(\frac{6\tan(dx+c)^2+1}{\tan(dx+c)^3} - 3\tan(dx}{3d}$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) + a^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 -
 3*tan(d*x + c)) + (3*tan(d*x + c)^2 + 1)*a^2/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{6\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(12*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*a^2*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 12*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1)
- (15*a^2*tan(1/2*d*x + 1/2*c)^2 + a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx=\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{14\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{a^2}{3}}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^4,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((14*a^2*tan(c/2 + (d*x)/2)^2)/3 - 9*
a^2*tan(c/2 + (d*x)/2)^4 + a^2/3)/(d*(2*tan(c/2 + (d*x)/2)^3 - 2*tan(c/2 +
(d*x)/2)^5))
```

3.35 $\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{4a^2\cot(c+dx)}{d} - \frac{5a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d} + \frac{a^2\tan(c+dx)}{d}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-4*a^2*cot(d*x+c)/d-5/3*a^2*cot(d*x+c)^3/d-2/5*a
^2*cot(d*x+c)^5/d-2*a^2*csc(d*x+c)/d-2/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d*x
+c)^5/d+a^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2952, 3852, 2701, 308, 213, 2700, 276}

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{5a^2\cot^3(c+dx)}{3d} - \frac{4a^2\cot(c+dx)}{d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (4*a^2*Cot[c + d*x])/d - (5*a^2*Cot[c + d
*x]^3)/(3*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d - (2*a
^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) + (a^2*Tan[c + d*x]
)/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\csc^6(c+dx)\sec^2(c+dx)\,dx \\
&= \int \left(a^2\csc^6(c+dx)+2a^2\csc^6(c+dx)\sec(c+dx)+a^2\csc^6(c+dx)\sec^2(c+dx)\right)\,dx \\
&= a^2\int \csc^6(c+dx)\,dx+a^2\int \csc^6(c+dx)\sec^2(c+dx)\,dx+\left(2a^2\right)\int \csc^6(c+dx)\sec(c+dx)\,dx \\
&= \frac{a^2\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6}\,dx,x,\tan(c+dx)\right)}{d}-\frac{a^2\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,\cot(c+dx)\right)}{d}-\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d} \\
&= -\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\text{Subst}\left(\int \left(1+\frac{1}{x^6}+\frac{3}{x^4}+\frac{3}{x^2}\right)\,dx,x,\tan(c+dx)\right)}{d}-\frac{(2a^2)\,\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)\,dx,x,\csc(c+dx)\right)}{d} \\
&= -\frac{4a^2\cot(c+dx)}{d}-\frac{5a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\tan(c+dx)}{d}-\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d} \\
&= \frac{2a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^2\cot(c+dx)}{d}-\frac{5a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 317 vs. $2(129) = 258$.

Time = 1.42 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.46

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-3840\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(-3840*Cos[c + d*
x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3840*Cos[c + d*x]*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + Csc[2*c]*Csc[(c + d*x)/2]^4*Csc[c + d*x]*(3
20*Sin[2*c] - 596*Sin[d*x] + 864*Sin[2*d*x] + 216*Sin[c - d*x] - 416*Sin[c
+ d*x] + 624*Sin[2*(c + d*x)] - 416*Sin[3*(c + d*x)] + 104*Sin[4*(c + d*x)]
 - 596*Sin[2*c + d*x] - 680*Sin[3*c + d*x] + 894*Sin[c + 2*d*x] + 224*Sin[2
*(c + 2*d*x)] + 894*Sin[3*c + 2*d*x] + 480*Sin[4*c + 2*d*x] - 776*Sin[c + 3
*d*x] - 596*Sin[2*c + 3*d*x] - 596*Sin[4*c + 3*d*x] - 120*Sin[5*c + 3*d*x]
+ 149*Sin[3*c + 4*d*x] + 149*Sin[5*c + 4*d*x])))/(7680*d)
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

method	result
norman	$\frac{\frac{a^2}{40d}+\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{15d}+\frac{31a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{12d}-\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}+\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2a^2\ln(}{}$
derivativedivides	$\frac{a^2\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)}-\frac{2}{5\sin(dx+c)^3\cos(dx+c)}+\frac{8}{5\sin(dx+c)\cos(dx+c)}-\frac{16\cot(dx+c)}{5}\right)+2a^2\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+}\right.}{d}$
default	$\frac{a^2\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)}-\frac{2}{5\sin(dx+c)^3\cos(dx+c)}+\frac{8}{5\sin(dx+c)\cos(dx+c)}-\frac{16\cot(dx+c)}{5}\right)+2a^2\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+}\right.}{d}$
parallelrisch	$\frac{a^2\left(3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5+240\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-240\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\right.}{120d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$
risch	$-\frac{4ia^2\left(15\,e^{7i(dx+c)}-60\,e^{6i(dx+c)}+85\,e^{5i(dx+c)}-40\,e^{4i(dx+c)}-27\,e^{3i(dx+c)}+108\,e^{2i(dx+c)}-97\,e^{i(dx+c)}+28\right)}{15d\left(e^{i(dx+c)}-1\right)^5\left(e^{2i(dx+c)}+1\right)\left(e^{i(dx+c)}+1\right)}-\frac{2a^2\ln}{}$

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/40/d*a^2+4/15*a^2/d*tan(1/2*d*x+1/2*c)^2+31/12*a^2/d*tan(1/2*d*x+1/2*c)^
4-5*a^2/d*tan(1/2*d*x+1/2*c)^6+1/8/d*a^2*tan(1/2*d*x+1/2*c)^8)/tan(1/2*d*x+
```

```
1/2*c)^5/(-1+tan(1/2*d*x+1/2*c)^2)-2*a^2/d*ln(tan(1/2*d*x+1/2*c)-1)+2*a^2/d
*ln(tan(1/2*d*x+1/2*c)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.60

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{56\,a^2\cos(dx+c)^4 - 82\,a^2\cos(dx+c)^3 - 32\,a^2\cos(dx+c)^2 + 76\,a^2\cos(dx+c) - 15\left(a^2\cos(dx+c)^3 -\right.}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -1/15*(56*a^2*cos(d*x + c)^4 - 82*a^2*cos(d*x + c)^3 - 32*a^2*cos(d*x + c)^
2 + 76*a^2*cos(d*x + c) - 15*(a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a
^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*(a^2*cos(d*x + c)^
3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x
 + c) - 15*a^2)/((d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x + c))*s
in(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.12

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5} - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1)\right) + 3\,a^2\left(\frac{15\tan(dx+}{t}\right.}{15\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(a^2*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3*a^2*((15*tan(d*x +
c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x + c)) + (15*tan(d*x
 + c)^4 + 10*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{240\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-240\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{240\,a^2\tan}{\tan\left(\frac{1}{2}\,d\right.}}{120\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/120*(240*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*a^2*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) + 15*a^2*tan(1/2*d*x + 1/2*c) - 240*a^2*tan(1/2*d*x + 1
/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (345*a^2*tan(1/2*d*x + 1/2*c)^4 + 35*a
^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx &= \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}\\&\quad-\frac{-39\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{62\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{a^2}{5}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}\end{aligned}$$

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^6,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((32*a^2*tan(c/2 + (d*x)/2)^2)/15 + (
62*a^2*tan(c/2 + (d*x)/2)^4)/3 - 39*a^2*tan(c/2 + (d*x)/2)^6 + a^2/5)/(d*(8
*tan(c/2 + (d*x)/2)^5 - 8*tan(c/2 + (d*x)/2)^7)) + (a^2*tan(c/2 + (d*x)/2))
/(8*d)
```

3.36 $\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{5a^2\cot(c+dx)}{d}-\frac{3a^2\cot^3(c+dx)}{d}-\frac{7a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^7(c+dx)}{7d}+\frac{a^2\tan(c+dx)}{d}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-5*a^2*cot(d*x+c)/d-3*a^2*cot(d*x+c)^3/d-7/5*a^2
*cot(d*x+c)^5/d-2/7*a^2*cot(d*x+c)^7/d-2*a^2*csc(d*x+c)/d-2/3*a^2*csc(d*x+c
)^3/d-2/5*a^2*csc(d*x+c)^5/d-2/7*a^2*csc(d*x+c)^7/d+a^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

= {3957, 2952, 3852, 2701, 308, 213, 2700, 276}

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d} - \frac{2a^2\cot^7(c+dx)}{7d}-\frac{7a^2\cot^5(c+dx)}{5d} - \frac{3a^2\cot^3(c+dx)}{d}-\frac{5a^2\cot(c+dx)}{d} - \frac{2a^2\csc^7(c+dx)}{7d}-\frac{2a^2\csc^5(c+dx)}{5d} - \frac{2a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (5*a^2*Cot[c + d*x])/d - (3*a^2*Cot[c + d
*x]^3)/d - (7*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (2
*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5
)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d) + (a^2*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\csc^8(c+dx)\sec^2(c+dx)\,dx\\
&= \int \left(a^2\csc^8(c+dx)+2a^2\csc^8(c+dx)\sec(c+dx)+a^2\csc^8(c+dx)\sec^2(c+dx)\right)\,dx\\
&= a^2\int \csc^8(c+dx)\,dx+a^2\int \csc^8(c+dx)\sec^2(c+dx)\,dx+\left(2a^2\right)\int \csc^8(c+dx)\sec(c+dx)\,dx\\
&= \frac{a^2\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^8}\,dx,x,\tan(c+dx)\right)}{d}\\
&\quad-\frac{a^2\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx,x,\cot(c+dx)\right)}{d}\\
&\quad-\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\
&= -\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{d}-\frac{3a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}\\
&\quad+\frac{a^2\text{Subst}\left(\int \left(1+\frac{1}{x^8}+\frac{4}{x^6}+\frac{6}{x^4}+\frac{4}{x^2}\right)\,dx,x,\tan(c+dx)\right)}{d}\\
&\quad-\frac{\left(2a^2\right)\text{Subst}\left(\int \left(1+x^2+x^4+x^6+\frac{1}{-1+x^2}\right)\,dx,x,\csc(c+dx)\right)}{d}
\end{aligned}$$

$$= -\frac{5a^2\cot(c+dx)}{d} - \frac{3a^2\cot^3(c+dx)}{d} - \frac{7a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{2a^2\csc^7(c+dx)}{7d} + \frac{a^2\tan(c+dx)}{d} - \frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}$$

$$= \frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{5a^2\cot(c+dx)}{d} - \frac{3a^2\cot^3(c+dx)}{d} - \frac{7a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{2a^2\csc^7(c+dx)}{7d} + \frac{a^2\tan(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 428 vs. $2(163) = 326$.

Time = 2.96 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.63

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-6881280\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(-6881280*Cos[c +
 d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6881280*Cos[c + d*x]*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] - 32*Csc[2*c]*Csc[(c + d*x)/2]^4*Csc[c
+ d*x]^3*(-9856*Sin[2*c] + 17288*Sin[d*x] - 29056*Sin[2*d*x] - 7264*Sin[c -
 d*x] + 14208*Sin[c + d*x] - 19536*Sin[2*(c + d*x)] + 7104*Sin[3*(c + d*x)]
 + 7104*Sin[4*(c + d*x)] - 7104*Sin[5*(c + d*x)] + 1776*Sin[6*(c + d*x)] +
17288*Sin[2*c + d*x] + 20384*Sin[3*c + d*x] - 23771*Sin[c + 2*d*x] + 7104*S
in[2*(c + 2*d*x)] - 23771*Sin[3*c + 2*d*x] - 8960*Sin[4*c + 2*d*x] + 19984*
Sin[c + 3*d*x] + 8644*Sin[2*c + 3*d*x] + 8644*Sin[4*c + 3*d*x] - 6160*Sin[5
*c + 3*d*x] + 8644*Sin[3*c + 4*d*x] + 8644*Sin[5*c + 4*d*x] + 6720*Sin[6*c
+ 4*d*x] - 12144*Sin[3*c + 5*d*x] - 8644*Sin[4*c + 5*d*x] - 8644*Sin[6*c +
5*d*x] - 1680*Sin[7*c + 5*d*x] + 3456*Sin[4*c + 6*d*x] + 2161*Sin[5*c + 6*d
*x] + 2161*Sin[7*c + 6*d*x])))/(13762560*d)
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.17

method	result
parallelrisch	$\frac{a^2\left(15\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+174\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5+910\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-6720\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\right.}{}$
norman	$\frac{\frac{a^2}{224d}+\frac{29a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{560d}+\frac{163a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{480d}+\frac{67a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{24d}+\frac{13a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{48d}+\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{96d}-\frac{175a^2\tan\left(\frac{dx}{2}+\right.}{32d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
derivativedivides	$\frac{a^2\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)}-\frac{8}{35\sin(dx+c)^5\cos(dx+c)}-\frac{16}{35\sin(dx+c)^3\cos(dx+c)}+\frac{64}{35\sin(dx+c)\cos(dx+c)}-\frac{128\cot(dx+c)}{35}\right)+2a^2}{}$
default	$\frac{a^2\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)}-\frac{8}{35\sin(dx+c)^5\cos(dx+c)}-\frac{16}{35\sin(dx+c)^3\cos(dx+c)}+\frac{64}{35\sin(dx+c)\cos(dx+c)}-\frac{128\cot(dx+c)}{35}\right)+2a^2}{}$
risch	$-\frac{4ia^2\left(105\,e^{11i(dx+c)}-420\,e^{10i(dx+c)}+385\,e^{9i(dx+c)}+560\,e^{8i(dx+c)}-1274\,e^{7i(dx+c)}+616\,e^{6i(dx+c)}+454\,e^{5i(dx+c)}-1816\right.}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}+1\right)}$

```
[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(15*cot(1/2*d*x+1/2*c)^7+35*tan(1/2*d*x+1/2*c)^5+174*cot(1/2*d*x+1/2*c)
^5+910*tan(1/2*d*x+1/2*c)^3-6720*ln(tan(1/2*d*x+1/2*c)-1)*tan(1/2*d*x+1/2*c
)^2+6720*ln(tan(1/2*d*x+1/2*c)+1)*tan(1/2*d*x+1/2*c)^2+1141*cot(1/2*d*x+1/2
*c)^3-18375*tan(1/2*d*x+1/2*c)+6720*ln(tan(1/2*d*x+1/2*c)-1)-6720*ln(tan(1/
2*d*x+1/2*c)+1)+9380*cot(1/2*d*x+1/2*c))/(3360*d*tan(1/2*d*x+1/2*c)^2-3360*
d)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.67

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{432\,a^2\cos(dx+c)^6-654\,a^2\cos(dx+c)^5-636\,a^2\cos(dx+c)^4+1226\,a^2\cos(dx+c)^3+74\,a^2\cos(dx-}{}$$

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(432*a^2*cos(d*x + c)^6 - 654*a^2*cos(d*x + c)^5 - 636*a^2*cos(d*x +
 c)^4 + 1226*a^2*cos(d*x + c)^3 + 74*a^2*cos(d*x + c)^2 - 562*a^2*cos(d*x +
 c) - 105*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2
 - a^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a^2*cos(d*x
+ c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*lo
g(-sin(d*x + c) + 1)*sin(d*x + c) + 105*a^2)/((d*cos(d*x + c)^5 - 2*d*cos(d
*x + c)^4 + 2*d*cos(d*x + c)^2 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\left(\frac{2\left(105\sin(dx+c)^6+35\sin(dx+c)^4+21\sin(dx+c)^2+15\right)}{\sin(dx+c)^7} - 105\log\left(\sin\left(dx+c\right)+1\right) + 105\log\left(\sin\left(dx+c\right)-1\right)\right.}{}$$

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/105*(a^2*(2*(105*sin(d*x + c)^6 + 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2
+ 15)/sin(d*x + c)^7 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1
)) + 3*a^2*((140*tan(d*x + c)^6 + 70*tan(d*x + c)^4 + 28*tan(d*x + c)^2 + 5
)/tan(d*x + c)^7 - 35*tan(d*x + c)) + 3*(35*tan(d*x + c)^6 + 35*tan(d*x + c
)^4 + 21*tan(d*x + c)^2 + 5)*a^2/tan(d*x + c)^7)/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 6720\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 6720\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + 945}{3360}$$

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] 1/3360*(35*a^2*tan(1/2*d*x + 1/2*c)^3 + 6720*a^2*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - 6720*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 945*a^2*tan(1/2*d*
x + 1/2*c) - 6720*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (
10710*a^2*tan(1/2*d*x + 1/2*c)^6 + 1330*a^2*tan(1/2*d*x + 1/2*c)^4 + 189*a^
2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{96\,d}+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32\,d}$$
$$-\frac{-166\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\frac{268\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{163\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{15}+\frac{58\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{35}+\frac{a^2}{7}}{d\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\right)}$$

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^8,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(96*d) + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d + (
9*a^2*tan(c/2 + (d*x)/2))/(32*d) - ((58*a^2*tan(c/2 + (d*x)/2)^2)/35 + (163
*a^2*tan(c/2 + (d*x)/2)^4)/15 + (268*a^2*tan(c/2 + (d*x)/2)^6)/3 - 166*a^2*
tan(c/2 + (d*x)/2)^8 + a^2/7)/(d*(32*tan(c/2 + (d*x)/2)^7 - 32*tan(c/2 + (d
*x)/2)^9))
```

3.37 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 201

$$\begin{aligned}\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx &= \frac{2a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{6a^2\cot(c+dx)}{d}\\&\quad-\frac{14a^2\cot^3(c+dx)}{3d}-\frac{16a^2\cot^5(c+dx)}{5d}\\&\quad-\frac{9a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^9(c+dx)}{9d}\\&\quad-\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}\\&\quad-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^7(c+dx)}{7d}\\&\quad-\frac{2a^2\csc^9(c+dx)}{9d}+\frac{a^2\tan(c+dx)}{d}\end{aligned}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-6*a^2*cot(d*x+c)/d-14/3*a^2*cot(d*x+c)^3/d-16/5
*a^2*cot(d*x+c)^5/d-9/7*a^2*cot(d*x+c)^7/d-2/9*a^2*cot(d*x+c)^9/d-2*a^2*csc
(d*x+c)/d-2/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d*x+c)^5/d-2/7*a^2*csc(d*x+c)^
7/d-2/9*a^2*csc(d*x+c)^9/d+a^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used

$= \{3957, 2952, 3852, 2701, 308, 213, 2700, 276\}$

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{9a^2\cot^7(c+dx)}{7d} - \frac{16a^2\cot^5(c+dx)}{5d} - \frac{14a^2\cot^3(c+dx)}{3d} - \frac{6a^2\cot(c+dx)}{d} - \frac{2a^2\csc^9(c+dx)}{9d} - \frac{2a^2\csc^7(c+dx)}{7d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (6*a^2*Cot[c + d*x])/d - (14*a^2*Cot[c +
d*x]^3)/(3*d) - (16*a^2*Cot[c + d*x]^5)/(5*d) - (9*a^2*Cot[c + d*x]^7)/(7*d
) - (2*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c +
d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d)
 - (2*a^2*Csc[c + d*x]^9)/(9*d) + (a^2*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\csc^{10}(c+dx)\sec^2(c+dx)\,dx \\
&= \int \left(a^2\csc^{10}(c+dx)+2a^2\csc^{10}(c+dx)\sec(c+dx)+a^2\csc^{10}(c+dx)\sec^2(c+dx)\right)\,dx \\
&= a^2\int \csc^{10}(c+dx)\,dx+a^2\int \csc^{10}(c+dx)\sec^2(c+dx)\,dx \\
&\quad +\left(2a^2\right)\int \csc^{10}(c+dx)\sec(c+dx)\,dx \\
&= \frac{a^2\text{Subst}\left(\int \frac{(1+x^2)^5}{x^{10}}\,dx,x,\tan(c+dx)\right)}{d} \\
&\quad -\frac{a^2\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx,x,\cot(c+dx)\right)}{d} \\
&\quad -\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^{10}}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}
\end{aligned}
$$

$$= -\frac{a^2\cot(c+dx)}{d} - \frac{4a^2\cot^3(c+dx)}{3d} - \frac{6a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} + \frac{a^2\text{Subst}\left(\int \left(1+\frac{1}{x^{10}}+\frac{5}{x^8}+\frac{10}{x^6}+\frac{10}{x^4}+\frac{5}{x^2}\right)\,dx,x,\tan(c+dx)\right)}{d} - \frac{(2a^2)\,\text{Subst}\left(\int \left(1+x^2+x^4+x^6+x^8+\frac{1}{-1+x^2}\right)\,dx,x,\csc(c+dx)\right)}{d}$$

$$= -\frac{6a^2\cot(c+dx)}{d} - \frac{14a^2\cot^3(c+dx)}{3d} - \frac{16a^2\cot^5(c+dx)}{5d} - \frac{9a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{2a^2\csc^7(c+dx)}{7d} - \frac{2a^2\csc^9(c+dx)}{9d} + \frac{a^2\tan(c+dx)}{d} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}$$

$$= \frac{2a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{6a^2\cot(c+dx)}{d} - \frac{14a^2\cot^3(c+dx)}{3d} - \frac{16a^2\cot^5(c+dx)}{5d} - \frac{9a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{2a^2\csc^7(c+dx)}{7d} - \frac{2a^2\csc^9(c+dx)}{9d} + \frac{a^2\tan(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1050 vs. $2(201) = 402$.

Time = 10.92 (sec) , antiderivative size = 1050, normalized size of antiderivative = 5.22

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= -\frac{6899\cos^2(c+dx)\cot\left(\frac{c}{2}\right)\csc^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{80640d}$$
$$-\frac{193\cos^2(c+dx)\cot\left(\frac{c}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{13440d}$$
$$-\frac{71\cos^2(c+dx)\cot\left(\frac{c}{2}\right)\csc^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{32256d}$$
$$-\frac{\cos^2(c+dx)\cot\left(\frac{c}{2}\right)\csc^8\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{4608d}$$
$$-\frac{\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{2d}$$
$$+\frac{\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{2d}$$
$$+\frac{123041\cos^2(c+dx)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{161280d}$$
$$+\frac{6899\cos^2(c+dx)\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{80640d}$$
$$+\frac{193\cos^2(c+dx)\csc\left(\frac{c}{2}\right)\csc^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{13440d}$$
$$+\frac{71\cos^2(c+dx)\csc\left(\frac{c}{2}\right)\csc^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{32256d}$$
$$+\frac{\cos^2(c+dx)\csc\left(\frac{c}{2}\right)\csc^9\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{4608d}$$
$$+\frac{803\cos^2(c+dx)\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{7680d}$$
$$+\frac{49\cos^2(c+dx)\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{7680d}$$
$$+\frac{\cos^2(c+dx)\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{2560d}$$
$$+\frac{\cos(c+dx)\sec(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin(dx)}{4d}$$
$$+\frac{49\cos^2(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\tan\left(\frac{c}{2}\right)}{7680d}$$
$$+\frac{\cos^2(c+dx)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\tan\left(\frac{c}{2}\right)}{2560d}$$

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-6899*Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^4*(a
 + a*Sec[c + d*x])^2)/(80640*d) - (193*Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d
```

```
*x)/2]^4*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(13440*d) - (71*Cos[c
 + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^6*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c +
 d*x])^2)/(32256*d) - (Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^8*Sec[c/2
 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(4608*d) - (Cos[c + d*x]^2*Log[Cos[c/
2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x]
)^2)/(2*d) + (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*S
ec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(2*d) + (123041*Cos[c + d*x]^2*
Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin
[(d*x)/2])/(161280*d) + (6899*Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^3*
Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(80640*d) + (193*
Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^5*Sec[c/2 + (d*x)/2]^4*(a + a*Se
c[c + d*x])^2*Sin[(d*x)/2])/(13440*d) + (71*Cos[c + d*x]^2*Csc[c/2]*Csc[c/2
 + (d*x)/2]^7*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(32
256*d) + (Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^9*Sec[c/2 + (d*x)/2]^4
*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4608*d) + (803*Cos[c + d*x]^2*Sec[c/
2]*Sec[c/2 + (d*x)/2]^5*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(7680*d) + (49
*Cos[c + d*x]^2*Sec[c/2]*Sec[c/2 + (d*x)/2]^7*(a + a*Sec[c + d*x])^2*Sin[(d
*x)/2])/(7680*d) + (Cos[c + d*x]^2*Sec[c/2]*Sec[c/2 + (d*x)/2]^9*(a + a*Sec
[c + d*x])^2*Sin[(d*x)/2])/(2560*d) + (Cos[c + d*x]*Sec[c]*Sec[c/2 + (d*x)/
2]^4*(a + a*Sec[c + d*x])^2*Sin[d*x])/(4*d) + (49*Cos[c + d*x]^2*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^2*Tan[c/2])/(7680*d) + (Cos[c + d*x]^2*Sec[
c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^2*Tan[c/2])/(2560*d)
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.08

method	result
parallelrisch	$\frac{a^2\left(35\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^9+63\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+460\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7+1092\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+3096\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5+16800\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\right.}{}$
derivativedivides	$\frac{a^2\left(-\frac{1}{9\sin(dx+c)^9\cos(dx+c)}-\frac{10}{63\sin(dx+c)^7\cos(dx+c)}-\frac{16}{63\sin(dx+c)^5\cos(dx+c)}-\frac{32}{63\sin(dx+c)^3\cos(dx+c)}+\frac{128}{63\sin(dx+c)\cos(dx+}\right.}{}$
default	$\frac{a^2\left(-\frac{1}{9\sin(dx+c)^9\cos(dx+c)}-\frac{10}{63\sin(dx+c)^7\cos(dx+c)}-\frac{16}{63\sin(dx+c)^5\cos(dx+c)}-\frac{32}{63\sin(dx+c)^3\cos(dx+c)}+\frac{128}{63\sin(dx+c)\cos(dx+}\right.}{}$
risch	$-\frac{4ia^2\left(315\,\mathrm{e}^{15i(dx+c)}-1260\,\mathrm{e}^{14i(dx+c)}+525\,\mathrm{e}^{13i(dx+c)}+4200\,\mathrm{e}^{12i(dx+c)}-5817\,\mathrm{e}^{11i(dx+c)}-2772\,\mathrm{e}^{10i(dx+c)}+10161\,\mathrm{e}^{9i(dx\right.}{315d\left(\mathrm{e}^{i(dx+c)}\right.}$

```
[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(35*cot(1/2*d*x+1/2*c)^9+63*tan(1/2*d*x+1/2*c)^7+460*cot(1/2*d*x+1/2*c)
^7+1092*tan(1/2*d*x+1/2*c)^5+3096*cot(1/2*d*x+1/2*c)^5+16800*tan(1/2*d*x+1/
2*c)^3-80640*ln(tan(1/2*d*x+1/2*c)-1)*tan(1/2*d*x+1/2*c)^2+80640*ln(tan(1/2
*d*x+1/2*c)+1)*tan(1/2*d*x+1/2*c)^2+16044*cot(1/2*d*x+1/2*c)^3-238140*tan(1
/2*d*x+1/2*c)+80640*ln(tan(1/2*d*x+1/2*c)-1)-80640*ln(tan(1/2*d*x+1/2*c)+1)
```

```
+119910*cot(1/2*d*x+1/2*c))/(40320*d*tan(1/2*d*x+1/2*c)^2-40320*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(185) = 370$.

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.02

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{1408\,a^2\cos(dx+c)^8 - 2186\,a^2\cos(dx+c)^7 - 3372\,a^2\cos(dx+c)^6 + 6200\,a^2\cos(dx+c)^5 + 2060\,a^2\,c}{}$$

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -1/315*(1408*a^2*cos(d*x + c)^8 - 2186*a^2*cos(d*x + c)^7 - 3372*a^2*cos(d*
x + c)^6 + 6200*a^2*cos(d*x + c)^5 + 2060*a^2*cos(d*x + c)^4 - 5784*a^2*cos
(d*x + c)^3 + 268*a^2*cos(d*x + c)^2 + 1756*a^2*cos(d*x + c) - 315*(a^2*cos
(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c
)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(sin
(d*x + c) + 1)*sin(d*x + c) + 315*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^
6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*
cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) - 31
5*a^2)/((d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^6 - d*cos(d*x + c)^5 + 4*d*cos
(d*x + c)^4 - d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x + c))*sin(d
*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{a^2\left(\frac{2\left(315\sin(dx+c)^8+105\sin(dx+c)^6+63\sin(dx+c)^4+45\sin(dx+c)^2+35\right)}{\sin(dx+c)^9}-315\log\left(\sin\left(dx+c\right)+1\right)+315\log\left(\sin\left(d\right.\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/315*(a^2*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4
 + 45*sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315
*log(sin(d*x + c) - 1)) + 5*a^2*((315*tan(d*x + c)^8 + 210*tan(d*x + c)^6 +
 126*tan(d*x + c)^4 + 45*tan(d*x + c)^2 + 7)/tan(d*x + c)^9 - 63*tan(d*x +
c)) + (315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*tan(d*x + c)^4 + 180*t
an(d*x + c)^2 + 35)*a^2/tan(d*x + c)^9)/d
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$=\frac{63\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80640\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-80640\,a^2\log\left(\right|}{}$$

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40320*(63*a^2*tan(1/2*d*x + 1/2*c)^5 + 1155*a^2*tan(1/2*d*x + 1/2*c)^3 +
80640*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 80640*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) + 17955*a^2*tan(1/2*d*x + 1/2*c) - 80640*a^2*tan(1/2*d*x +
 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (139545*a^2*tan(1/2*d*x + 1/2*c)^8 +
 19635*a^2*tan(1/2*d*x + 1/2*c)^6 + 3591*a^2*tan(1/2*d*x + 1/2*c)^4 + 495*a
^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{384\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{-699\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{1142\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+\frac{764\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}+\frac{344\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{35}+\frac{92\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{63}+\frac{a^2}{9}}{d\,\left(128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\right)}$$
$$+\frac{57\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}$$

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^10,x)
```

```
[Out] (11*a^2*tan(c/2 + (d*x)/2)^3)/(384*d) + (a^2*tan(c/2 + (d*x)/2)^5)/(640*d)
+ (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((92*a^2*tan(c/2 + (d*x)/2)^2)/63 +
 (344*a^2*tan(c/2 + (d*x)/2)^4)/35 + (764*a^2*tan(c/2 + (d*x)/2)^6)/15 + (1
142*a^2*tan(c/2 + (d*x)/2)^8)/3 - 699*a^2*tan(c/2 + (d*x)/2)^10 + a^2/9)/(d
*(128*tan(c/2 + (d*x)/2)^9 - 128*tan(c/2 + (d*x)/2)^11)) + (57*a^2*tan(c/2
+ (d*x)/2))/(128*d)
```

3.38 $\int (a+a\sec(c+dx))^3 \sin^9(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 203

$$\begin{aligned}\int (a+a\sec(c+dx))^3\sin^9(c+dx)\,dx = {} & \frac{11a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d} \\ & -\frac{14a^3\cos^3(c+dx)}{3d}-\frac{7a^3\cos^4(c+dx)}{2d} \\ & +\frac{6a^3\cos^5(c+dx)}{5d}+\frac{11a^3\cos^6(c+dx)}{6d} \\ & +\frac{a^3\cos^7(c+dx)}{7d}-\frac{3a^3\cos^8(c+dx)}{8d} \\ & -\frac{a^3\cos^9(c+dx)}{9d}+\frac{a^3\log(\cos(c+dx))}{d} \\ & +\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}\end{aligned}$$

```
[Out] 11*a^3*cos(d*x+c)/d+3*a^3*cos(d*x+c)^2/d-14/3*a^3*cos(d*x+c)^3/d-7/2*a^3*co
s(d*x+c)^4/d+6/5*a^3*cos(d*x+c)^5/d+11/6*a^3*cos(d*x+c)^6/d+1/7*a^3*cos(d*x
+c)^7/d-3/8*a^3*cos(d*x+c)^8/d-1/9*a^3*cos(d*x+c)^9/d+a^3*ln(cos(d*x+c))/d+
3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

= {3957, 2915, 12, 90}

$$\int (a+a\sec(c+dx))^3\sin^9(c+dx)\,dx = -\frac{a^3\cos^9(c+dx)}{9d}-\frac{3a^3\cos^8(c+dx)}{8d}+\frac{a^3\cos^7(c+dx)}{7d}+\frac{11a^3\cos^6(c+dx)}{6d}+\frac{6a^3\cos^5(c+dx)}{5d}-\frac{7a^3\cos^4(c+dx)}{2d}-\frac{14a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos^2(c+dx)}{d}+\frac{11a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^9,x]
```

```
[Out] (11*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/d - (14*a^3*Cos[c + d*x]^3
)/(3*d) - (7*a^3*Cos[c + d*x]^4)/(2*d) + (6*a^3*Cos[c + d*x]^5)/(5*d) + (11
*a^3*Cos[c + d*x]^6)/(6*d) + (a^3*Cos[c + d*x]^7)/(7*d) - (3*a^3*Cos[c + d*
x]^8)/(8*d) - (a^3*Cos[c + d*x]^9)/(9*d) + (a^3*Log[Cos[c + d*x]])/d + (3*a
^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3\sin^6(c+dx)\tan^3(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^4(-a+x)^7}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^9d}\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^7}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^6d}\\
&= \frac{\text{Subst}\left(\int \left(-11a^8-\frac{a^{11}}{x^3}+\frac{3a^{10}}{x^2}+\frac{a^9}{x}+6a^7x+14a^6x^2-14a^5x^3-6a^4x^4+11a^3x^5-a^2x^6-3ax^7+\right.\right.}{a^6d}\\
&= \frac{11a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}-\frac{14a^3\cos^3(c+dx)}{3d}-\frac{7a^3\cos^4(c+dx)}{2d}\\
&\quad+\frac{6a^3\cos^5(c+dx)}{5d}+\frac{11a^3\cos^6(c+dx)}{6d}+\frac{a^3\cos^7(c+dx)}{7d}-\frac{3a^3\cos^8(c+dx)}{8d}\\
&\quad-\frac{a^3\cos^9(c+dx)}{9d}+\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.73

$$\int (a+a\sec(c+dx))^3\sin^9(c+dx)\,dx = \frac{a^3(471450+11624760\cos(c+dx)+2188872\cos(3(c+dx))+41160\cos(4(c+dx))-204156\cos(5(c+d}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^9,x]
```

```
[Out] (a^3*(471450 + 11624760*Cos[c + d*x] + 2188872*Cos[3*(c + d*x)] + 41160*Cos
[4*(c + d*x)] - 204156*Cos[5*(c + d*x)] - 35805*Cos[6*(c + d*x)] + 22972*Co
s[7*(c + d*x)] + 9030*Cos[8*(c + d*x)] - 820*Cos[9*(c + d*x)] - 945*Cos[10*
(c + d*x)] - 140*Cos[11*(c + d*x)] + 645120*Log[Cos[c + d*x]] + 210*Cos[2*(
c + d*x)]*(-413 + 3072*Log[Cos[c + d*x]]))*Sec[c + d*x]^2)/(1290240*d)
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{a^3\left(-645120(1+\cos(2dx+2c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+645120(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+645120(1+\cos(2dx+\right.}{}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^{10}}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^8}{2}+\frac{2\sin(dx+c)^6}{3}+\sin(dx+c)^4+2\sin(dx+c)^2+4\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^{10}}{\cos(dx+c)}+\left(\frac{128}{35}+\mathrm{si}\right.\right.}{}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^{10}}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^8}{2}+\frac{2\sin(dx+c)^6}{3}+\sin(dx+c)^4+2\sin(dx+c)^2+4\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^{10}}{\cos(dx+c)}+\left(\frac{128}{35}+\mathrm{si}\right.\right.}{}$
parts	$-\frac{a^3\left(\frac{128}{35}+\sin(dx+c)^8+\frac{8\sin(dx+c)^6}{7}+\frac{48\sin(dx+c)^4}{35}+\frac{64\sin(dx+c)^2}{35}\right)\cos(dx+c)}{9d}+\frac{a^3\left(\frac{\sin(dx+c)^{10}}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^8}{2}+\frac{2\sin\right.}{}$
risch	$-\frac{2ia^3c}{d}-ia^3x-\frac{a^3\cos(9dx+9c)}{2304d}+\frac{a^3\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}-\frac{25a^3\mathrm{e}^{-3i(dx+c)}}{64d}+\frac{57a^3\mathrm{e}^{-2i(dx+c)}}{256d}+\frac{1059a^3\mathrm{e}^{-i(}}{256d}$

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/645120*a^3*(-645120*(1+cos(2*d*x+2*c))*ln(sec(1/2*d*x+1/2*c)^2)+645120*(1
+cos(2*d*x+2*c))*ln(tan(1/2*d*x+1/2*c)-1)+645120*(1+cos(2*d*x+2*c))*ln(tan(
1/2*d*x+1/2*c)+1)-35805*cos(6*d*x+6*c)+22972*cos(7*d*x+7*c)+9030*cos(8*d*x+
8*c)-820*cos(9*d*x+9*c)-945*cos(10*d*x+10*c)-140*cos(11*d*x+11*c)+11624760*
cos(d*x+c)+6529934*cos(2*d*x+2*c)+2188872*cos(3*d*x+3*c)+41160*cos(4*d*x+4*
c)-204156*cos(5*d*x+5*c)+7088114)/d/(1+cos(2*d*x+2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.90

$$\int (a+a\sec(c+dx))^3\sin^9(c+dx)\,dx =$$
$$-\frac{35840\,a^3\cos(dx+c)^{11}+120960\,a^3\cos(dx+c)^{10}-46080\,a^3\cos(dx+c)^9-591360\,a^3\cos(dx+c)^8-}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/322560*(35840*a^3*cos(d*x + c)^11 + 120960*a^3*cos(d*x + c)^10 - 46080*a
^3*cos(d*x + c)^9 - 591360*a^3*cos(d*x + c)^8 - 387072*a^3*cos(d*x + c)^7 +
 1128960*a^3*cos(d*x + c)^6 + 1505280*a^3*cos(d*x + c)^5 - 967680*a^3*cos(d
*x + c)^4 - 3548160*a^3*cos(d*x + c)^3 - 322560*a^3*cos(d*x + c)^2*log(-cos
(d*x + c)) + 212205*a^3*cos(d*x + c)^2 - 967680*a^3*cos(d*x + c) - 161280*a
^3)/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^3 \sin^9(c + dx)\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**9,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.78

$$\int (a + a\sec(c + dx))^3 \sin^9(c + dx)\, dx =$$
$$-\frac{280\, a^3 \cos(dx + c)^9 + 945\, a^3 \cos(dx + c)^8 - 360\, a^3 \cos(dx + c)^7 - 4620\, a^3 \cos(dx + c)^6 - 3024\, a^3 \cos(d}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="maxima")
[Out] -1/2520*(280*a^3*cos(d*x + c)^9 + 945*a^3*cos(d*x + c)^8 - 360*a^3*cos(d*x
+ c)^7 - 4620*a^3*cos(d*x + c)^6 - 3024*a^3*cos(d*x + c)^5 + 8820*a^3*cos(d
*x + c)^4 + 11760*a^3*cos(d*x + c)^3 - 7560*a^3*cos(d*x + c)^2 - 27720*a^3*
cos(d*x + c) - 2520*a^3*log(cos(d*x + c)) - 1260*(6*a^3*cos(d*x + c) + a^3)
/cos(d*x + c)^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(187) = 374$.

Time = 0.49 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.95

$$\int (a + a\sec(c + dx))^3 \sin^9(c + dx)\, dx =$$
$$-\frac{2520\, a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 2520\, a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) - \frac{1260\left(9\, a^3 + \frac{2\, a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{3\, a^3(\cos(dx+c)-}{(\cos(dx+c)+1)}\right.}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="giac")
[Out] -1/2520*(2520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 25
20*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 1260*(9*a^3 +
 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^3*(cos(d*x + c) - 1)^2/(
```

```
cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 + (45257
*a^3 - 392193*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1467972*a^3*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3001908*a^3*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 3232782*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 -
 2359854*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1190196*a^3*(cos(d
*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 397764*a^3*(cos(d*x + c) - 1)^7/(cos(
d*x + c) + 1)^7 + 79281*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 - 712
9*a^3*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^9)/d
```

Mupad [B] (verification not implemented)

Time = 12.63 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.77

$$\int (a + a\sec(c + dx))^3 \sin^9(c + dx)\, dx$$
$$= \frac{\frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2} + 11\,a^3\,\cos\left(c+d\,x\right) + 3\,a^3\cos\left(c+d\,x\right)^2 - \frac{14\,a^3\cos(c+d\,x)^3}{3} - \frac{7\,a^3\cos(c+d\,x)^4}{2} + \frac{6\,a^3\cos(c+d\,x)^5}{5} + \cdots}{d}$$

```
[In] int(sin(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 11*a^3*cos(c + d*x) + 3*a^3*
cos(c + d*x)^2 - (14*a^3*cos(c + d*x)^3)/3 - (7*a^3*cos(c + d*x)^4)/2 + (6*
a^3*cos(c + d*x)^5)/5 + (11*a^3*cos(c + d*x)^6)/6 + (a^3*cos(c + d*x)^7)/7
- (3*a^3*cos(c + d*x)^8)/8 - (a^3*cos(c + d*x)^9)/9 + a^3*log(cos(c + d*x))
)/d
```

3.39 $\int (a+a\sec(c+dx))^3\sin^7(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 131

$$\int (a+a\sec(c+dx))^3\sin^7(c+dx)\,dx=\frac{8a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}-\frac{2a^3\cos^3(c+dx)}{d}-\frac{2a^3\cos^4(c+dx)}{d}+\frac{a^3\cos^6(c+dx)}{2d}+\frac{a^3\cos^7(c+dx)}{7d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}$$

```
[Out] 8*a^3*cos(d*x+c)/d+3*a^3*cos(d*x+c)^2/d-2*a^3*cos(d*x+c)^3/d-2*a^3*cos(d*x+
c)^4/d+1/2*a^3*cos(d*x+c)^6/d+1/7*a^3*cos(d*x+c)^7/d+3*a^3*sec(d*x+c)/d+1/2
*a^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$\int (a+a\sec(c+dx))^3\sin^7(c+dx)\,dx=\frac{a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^6(c+dx)}{2d}-\frac{2a^3\cos^4(c+dx)}{d}-\frac{2a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}+\frac{8a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^7,x]
```

```
[Out] (8*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/d - (2*a^3*Cos[c + d*x]^3)/
d - (2*a^3*Cos[c + d*x]^4)/d + (a^3*Cos[c + d*x]^6)/(2*d) + (a^3*Cos[c + d*
x]^7)/(7*d) + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3\sin^4(c+dx)\tan^3(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^3(-a+x)^6}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^7d}\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^3(-a+x)^6}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^4d}\\
&= \frac{\text{Subst}\left(\int \left(-8a^6-\frac{a^9}{x^3}+\frac{3a^8}{x^2}+6a^5x+6a^4x^2-8a^3x^3+3ax^5-x^6\right)\,dx, x, -a\cos(c+dx)\right)}{a^4d}\\
&= \frac{8a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}-\frac{2a^3\cos^3(c+dx)}{d}-\frac{2a^3\cos^4(c+dx)}{d}\\
&\quad+\frac{a^3\cos^6(c+dx)}{2d}+\frac{a^3\cos^7(c+dx)}{7d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int (a+a\sec(c+dx))^3\sin^7(c+dx)\,dx$$
$$=\frac{a^3(427+14014\cos(c+dx)-210\cos(2(c+dx))+2548\cos(3(c+dx))+196\cos(4(c+dx))-188\cos(5(c}{1792d}$$

[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^7,x]

[Out] (a^3*(427 + 14014*Cos[c + d*x] - 210*Cos[2*(c + d*x)] + 2548*Cos[3*(c + d*x
)] + 196*Cos[4*(c + d*x)] - 188*Cos[5*(c + d*x)] - 56*Cos[6*(c + d*x)] + 9*
Cos[7*(c + d*x)] + 7*Cos[8*(c + d*x)] + Cos[9*(c + d*x)])*Sec[c + d*x]^2)/(
1792*d)

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{a^3(196\cos(4dx+4c)+2548\cos(3dx+3c)+14014\cos(dx+c)+7\cos(8dx+8c)+9\cos(7dx+7c)-56\cos(6dx+6c)-188\cos(5dx}{896d(1+\cos(2dx+2c))}$
derivativedivides	$a^3\left(\frac{\sin(dx+c)^8}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^6}{2}+\frac{3\sin(dx+c)^4}{4}+\frac{3\sin(dx+c)^2}{2}+3\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^8}{\cos(dx+c)}+\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(d}{5}\right.\right.$
default	$a^3\left(\frac{\sin(dx+c)^8}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^6}{2}+\frac{3\sin(dx+c)^4}{4}+\frac{3\sin(dx+c)^2}{2}+3\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^8}{\cos(dx+c)}+\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(d}{5}\right.\right.$
parts	$-\frac{a^3\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7d}+\frac{a^3\left(\frac{\sin(dx+c)^8}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^6}{2}+\frac{3\sin(dx+c)^4}{4}+\frac{3\sin(dx+c}{2}\right.}{d}$
risch	$-\frac{29a^3e^{3i(dx+c)}}{128d}+\frac{47a^3e^{2i(dx+c)}}{128d}+\frac{421a^3e^{i(dx+c)}}{128d}+\frac{421a^3e^{-i(dx+c)}}{128d}+\frac{47a^3e^{-2i(dx+c)}}{128d}-\frac{29a^3e^{-3i(dx+c)}}{128d}+2$

[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x,method=_RETURNVERBOSE)

[Out] 1/896/d*a^3*(196*cos(4*d*x+4*c)+2548*cos(3*d*x+3*c)+14014*cos(d*x+c)+7*cos(
8*d*x+8*c)+9*cos(7*d*x+7*c)-56*cos(6*d*x+6*c)-188*cos(5*d*x+5*c)+7800*cos(2
*d*x+2*c)+cos(9*d*x+9*c)+8437)/(1+cos(2*d*x+2*c))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int (a + a\sec(c + dx))^3 \sin^7(c + dx)\, dx$$
$$= \frac{32\, a^3 \cos\left(dx + c\right)^9 + 112\, a^3 \cos\left(dx + c\right)^8 - 448\, a^3 \cos\left(dx + c\right)^6 - 448\, a^3 \cos\left(dx + c\right)^5 + 672\, a^3 \cos\left(dx + c\right.}{224\, d \cos\left(dx + c\right)^2}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/224*(32*a^3*cos(d*x + c)^9 + 112*a^3*cos(d*x + c)^8 - 448*a^3*cos(d*x + c
)^6 - 448*a^3*cos(d*x + c)^5 + 672*a^3*cos(d*x + c)^4 + 1792*a^3*cos(d*x +
c)^3 - 203*a^3*cos(d*x + c)^2 + 672*a^3*cos(d*x + c) + 112*a^3)/(d*cos(d*x
+ c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^3 \sin^7(c + dx)\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c + dx))^3 \sin^7(c + dx)\, dx$$
$$= \frac{2\, a^3 \cos\left(dx + c\right)^7 + 7\, a^3 \cos\left(dx + c\right)^6 - 28\, a^3 \cos\left(dx + c\right)^4 - 28\, a^3 \cos\left(dx + c\right)^3 + 42\, a^3 \cos\left(dx + c\right)^2 + 11}{14\, d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/14*(2*a^3*cos(d*x + c)^7 + 7*a^3*cos(d*x + c)^6 - 28*a^3*cos(d*x + c)^4 -
 28*a^3*cos(d*x + c)^3 + 42*a^3*cos(d*x + c)^2 + 112*a^3*cos(d*x + c) + 7*(
6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.82

$$\int (a+a\sec(c+dx))^3\sin^7(c+dx)\,dx$$
$$= \frac{2\left(\frac{7\left(3a^3+\frac{2a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}-\frac{43a^3-\frac{273a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{672a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{630a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{343a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{105a^3(\cos(dx+c)}{(\cos(dx+c)}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}}{7d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 2/7*(7*(3*a^3 + 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 1)^2 - (43*a^3 - 273*a^3*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 672*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 630*a^
3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 343*a^3*(cos(d*x + c) - 1)^4/
(cos(d*x + c) + 1)^4 - 105*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 +
14*a^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int (a+a\sec(c+dx))^3\sin^7(c+dx)\,dx$$
$$= \frac{\frac{3a^3\cos(c+dx)+\frac{a^3}{2}}{\cos(c+dx)^2}+8a^3\cos(c+dx)+3a^3\cos(c+dx)^2-2a^3\cos(c+dx)^3-2a^3\cos(c+dx)^4+\frac{a^3\cos(c+}{2}}{d}$$

```
[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 8*a^3*cos(c + d*x) + 3*a^3*c
os(c + d*x)^2 - 2*a^3*cos(c + d*x)^3 - 2*a^3*cos(c + d*x)^4 + (a^3*cos(c +
d*x)^6)/2 + (a^3*cos(c + d*x)^7)/7)/d
```

3.40 $\int (a+a\sec(c+dx))^3\sin^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 134

$$\int (a+a\sec(c+dx))^3\sin^5(c+dx)\,dx = \frac{5a^3\cos(c+dx)}{d}+\frac{5a^3\cos^2(c+dx)}{2d}-\frac{a^3\cos^3(c+dx)}{3d}-\frac{3a^3\cos^4(c+dx)}{4d}-\frac{a^3\cos^5(c+dx)}{5d}-\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}$$

```
[Out] 5*a^3*cos(d*x+c)/d+5/2*a^3*cos(d*x+c)^2/d-1/3*a^3*cos(d*x+c)^3/d-3/4*a^3*co
s(d*x+c)^4/d-1/5*a^3*cos(d*x+c)^5/d-a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d
+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$\int (a+a\sec(c+dx))^3\sin^5(c+dx)\,dx = -\frac{a^3\cos^5(c+dx)}{5d}-\frac{3a^3\cos^4(c+dx)}{4d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos^2(c+dx)}{2d}+\frac{5a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] (5*a^3*Cos[c + d*x])/d + (5*a^3*Cos[c + d*x]^2)/(2*d) - (a^3*Cos[c + d*x]^3
)/(3*d) - (3*a^3*Cos[c + d*x]^4)/(4*d) - (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*
Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3\sin^2(c+dx)\tan^3(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^2(-a+x)^5}{x^3}\,dx,x,-a\cos(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^5}{x^3}\,dx,x,-a\cos(c+dx)\right)}{a^2d}\\
&= \frac{\text{Subst}\left(\int \left(-5a^4-\frac{a^7}{x^3}+\frac{3a^6}{x^2}-\frac{a^5}{x}+5a^3x+a^2x^2-3ax^3+x^4\right)\,dx,x,-a\cos(c+dx)\right)}{a^2d}\\
&= \frac{5a^3\cos(c+dx)}{d}+\frac{5a^3\cos^2(c+dx)}{2d}-\frac{a^3\cos^3(c+dx)}{3d}-\frac{3a^3\cos^4(c+dx)}{4d}\\
&\quad-\frac{a^3\cos^5(c+dx)}{5d}-\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int (a+a\sec(c+dx))^3\sin^5(c+dx)\,dx = -\frac{a^3(-120-12350\cos(c+dx)-2074\cos(3(c+dx))-330\cos(4(c+dx))+82\cos(5(c+dx))+45\cos(\ldots}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] -1/1920*(a^3*(-120 - 12350*Cos[c + d*x] - 2074*Cos[3*(c + d*x)] - 330*Cos[4
*(c + d*x)] + 82*Cos[5*(c + d*x)] + 45*Cos[6*(c + d*x)] + 6*Cos[7*(c + d*x)
] + 960*Log[Cos[c + d*x]] + 15*Cos[2*(c + d*x)]*(31 + 64*Log[Cos[c + d*x]])
)*Sec[c + d*x]^2)/d
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^6}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^4}{2}+\sin(dx+c)^2+2\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(d\ldots}{d}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^6}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^4}{2}+\sin(dx+c)^2+2\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(d\ldots}{d}$
parallelrisch	$\frac{a^3\left(960(1+\cos(2dx+2c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-960(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-960(1+\cos(2dx+2c))\ln\left(\tan\left(\ldots\right.\right.\right.}{}$
parts	$-\frac{a^3\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{a^3\left(\frac{\sin(dx+c)^6}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^4}{2}+\sin(dx+c)^2+2\ln(\cos(dx+c))\right)}{d}+\ldots$
norman	$\frac{\frac{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}+\frac{224a^3}{15d}-\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-\frac{6a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3d}+\frac{134a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{15d}+\frac{214a^3\tan\left(\frac{dx}{2}\ldots\right.}{5d}\ldots}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$
risch	$ia^3x-\frac{7a^3e^{3i(dx+c)}}{96d}+\frac{7a^3e^{2i(dx+c)}}{16d}+\frac{37a^3e^{i(dx+c)}}{16d}+\frac{37a^3e^{-i(dx+c)}}{16d}+\frac{7a^3e^{-2i(dx+c)}}{16d}-\frac{7a^3e^{-3i(dx+c)}}{96d}+\ldots$

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(
cos(d*x+c)))+3*a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c
)^2)*cos(d*x+c))+3*a^3*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-
1/5*a^3*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97

$$\int (a+a\sec(c+dx))^3\sin^5(c+dx)\,dx =$$
$$-\frac{96\,a^3\cos\left(dx+c\right)^7+360\,a^3\cos\left(dx+c\right)^6+160\,a^3\cos\left(dx+c\right)^5-1200\,a^3\cos\left(dx+c\right)^4-2400\,a^3\cos\left(dx\right.}{480\,d\cos\left(\right.}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^3*cos(d*x + c)^7 + 360*a^3*cos(d*x + c)^6 + 160*a^3*cos(d*x +
c)^5 - 1200*a^3*cos(d*x + c)^4 - 2400*a^3*cos(d*x + c)^3 + 480*a^3*cos(d*x
+ c)^2*log(-cos(d*x + c)) + 465*a^3*cos(d*x + c)^2 - 1440*a^3*cos(d*x + c)
- 240*a^3)/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^3\sin^5(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int (a+a\sec(c+dx))^3\sin^5(c+dx)\,dx =$$
$$-\frac{12\,a^3\cos\left(dx+c\right)^5+45\,a^3\cos\left(dx+c\right)^4+20\,a^3\cos\left(dx+c\right)^3-150\,a^3\cos\left(dx+c\right)^2-300\,a^3\cos\left(dx+c\right)}{60\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a^3*cos(d*x + c)^5 + 45*a^3*cos(d*x + c)^4 + 20*a^3*cos(d*x + c)^
3 - 150*a^3*cos(d*x + c)^2 - 300*a^3*cos(d*x + c) + 60*a^3*log(cos(d*x + c)
) - 30*(6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(124) = 248$.

Time = 0.42 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.22

$$\int (a + a\sec(c + dx))^3 \sin^5(c + dx)\,dx$$
$$= \frac{60\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 60\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{30\left(15\,a^3 + \frac{14\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{3\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2} -}{60\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^3*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 30*(15*a^3 + 14*a^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^3*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 - (399*a^3 - 13
95*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 390*a^3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 650*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 -
 565*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^3*(cos(d*x + c)
- 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)
/d
```

Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c + dx))^3 \sin^5(c + dx)\,dx =$$
$$-\frac{\frac{a^3\cos(c+d\,x)^3}{3} - 5\,a^3\,\cos\left(c+d\,x\right) - \frac{5\,a^3\cos(c+d\,x)^2}{2} - \frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2} + \frac{3\,a^3\cos(c+d\,x)^4}{4} + \frac{a^3\cos(c+d\,x)^5}{5} + a^3\,\ln(}{d}$$

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -((a^3*cos(c + d*x)^3)/3 - 5*a^3*cos(c + d*x) - (5*a^3*cos(c + d*x)^2)/2 -
(3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + (3*a^3*cos(c + d*x)^4)/4 + (a
^3*cos(c + d*x)^5)/5 + a^3*log(cos(c + d*x)))/d
```

3.41 $\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx=\frac{2a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}$$

```
[Out] 2*a^3*cos(d*x+c)/d+3/2*a^3*cos(d*x+c)^2/d+1/3*a^3*cos(d*x+c)^3/d-2*a^3*ln(c
os(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3957, 2786, 76}

$$\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx=\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{2a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{2a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] (2*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/(2*d) + (a^3*Cos[c + d*x]^3
)/(3*d) - (2*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c
 + d*x]^2)/(2*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3\tan^3(c+dx)\,dx\\
&= \frac{\operatorname{Subst}\left(\int \frac{(-a-x)(-a+x)^4}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(-2a^2-\frac{a^5}{x^3}+\frac{3a^4}{x^2}-\frac{2a^3}{x}+3ax-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{2a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}\\
&\quad -\frac{2a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$
\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx
= \frac{a^3(-41+226\cos(c+dx)+29\cos(3(c+dx))+9\cos(4(c+dx))+\cos(5(c+dx))-48\log(\cos(c+dx))}{48d}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

[Out] (a^3*(-41 + 226*Cos[c + d*x] + 29*Cos[3*(c + d*x)] + 9*Cos[4*(c + d*x)] + C
os[5*(c + d*x)] - 48*Log[Cos[c + d*x]] - 8*Cos[2*(c + d*x)]*(7 + 6*Log[Cos[
c + d*x]]))*Sec[c + d*x]^2)/(48*d)

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{a^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+3a^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-}{d}$
default	$\frac{a^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+3a^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)-}{d}$
parts	$-\frac{a^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{a^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)}{d}+\frac{3a^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}+\frac{3a^3\left(\right.}{}$
parallelrisch	$\frac{a^3\left(48(1+\cos(2dx+2c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-48\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)-48\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)-\right.}{24}$
norman	$\frac{\frac{32a^3}{3d}-\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}+\frac{20a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}+\frac{20a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{2a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-2a^3\ln$
risch	$2ia^3x+\frac{a^3e^{3i(dx+c)}}{24d}+\frac{3a^3e^{2i(dx+c)}}{8d}+\frac{9a^3e^{i(dx+c)}}{8d}+\frac{9a^3e^{-i(dx+c)}}{8d}+\frac{3a^3e^{-2i(dx+c)}}{8d}+\frac{a^3e^{-3i(dx+c)}}{24d}+\frac{4ia^3}{d}$

[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+(
2+sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/3*a^
3*(2+sin(d*x+c)^2)*cos(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

$$\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx$$
$$=\frac{4\,a^3\cos(dx+c)^5+18\,a^3\cos(dx+c)^4+24\,a^3\cos(dx+c)^3-24\,a^3\cos(dx+c)^2\log(-\cos(dx+c))-9\,a^3}{12\,d\cos(dx+c)^2}$$

[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="fricas")

[Out] 1/12*(4*a^3*cos(d*x + c)^5 + 18*a^3*cos(d*x + c)^4 + 24*a^3*cos(d*x + c)^3
- 24*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 9*a^3*cos(d*x + c)^2 + 36*a^3*
cos(d*x + c) + 6*a^3)/(d*cos(d*x + c)^2)

Sympy [F]

$$\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx = a^3\left(\int 3\sin^3(c+dx)\sec(c+dx)\,dx + \int 3\sin^3(c+dx)\sec^2(c+dx)\,dx + \int \sin^3(c+dx)\sec^3(c+dx)\,dx + \int \sin^3(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**3,x)
[Out] a**3*(Integral(3*sin(c + d*x)**3*sec(c + d*x), x) + Integral(3*sin(c + d*x)
**3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(sin(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx = \frac{2a^3\cos(dx+c)^3 + 9a^3\cos(dx+c)^2 + 12a^3\cos(dx+c) - 12a^3\log(\cos(dx+c)) + \frac{3(6a^3\cos(dx+c)+a^3)}{\cos(dx+c)^2}}{6d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="maxima")
[Out] 1/6*(2*a^3*cos(d*x + c)^3 + 9*a^3*cos(d*x + c)^2 + 12*a^3*cos(d*x + c) - 12
*a^3*log(cos(d*x + c)) + 3*(6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx = -\frac{2a^3\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{6a^3\cos(dx+c)+a^3}{2d\cos(dx+c)^2} + \frac{2a^3d^8\cos(dx+c)^3 + 9a^3d^8\cos(dx+c)^2 + 12a^3d^8\cos(dx+c)}{6d^9}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a^3*cos(d*x + c) + a^3)/(d*
cos(d*x + c)^2) + 1/6*(2*a^3*d^8*cos(d*x + c)^3 + 9*a^3*d^8*cos(d*x + c)^2
+ 12*a^3*d^8*cos(d*x + c))/d^9
```

Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int (a + a\sec(c + dx))^3 \sin^3(c + dx)\, dx$$
$$= \frac{\frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2} + 2\,a^3\,\cos\left(c+d\,x\right) + \frac{3\,a^3\,\cos(c+d\,x)^2}{2} + \frac{a^3\,\cos(c+d\,x)^3}{3} - 2\,a^3\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 2*a^3*cos(c + d*x) + (3*a^3*
cos(c + d*x)^2)/2 + (a^3*cos(c + d*x)^3)/3 - 2*a^3*log(cos(c + d*x)))/d
```

3.42 $\int (a+a\sec (c+dx))^3 \sin (c+dx) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 62

$$\int (a+a\sec (c+dx))^3 \sin (c+dx) \, dx = -\frac{a^3\cos (c+dx)}{d}-\frac{3a^3\log (\cos (c+dx))}{d}+\frac{3a^3\sec (c+dx)}{d}+\frac{a^3\sec ^2(c+dx)}{2d}$$

```
[Out] -a^3*cos(d*x+c)/d-3*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x
+c)^2/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2912, 12, 45}

$$\int (a+a\sec (c+dx))^3 \sin (c+dx) \, dx = -\frac{a^3\cos (c+dx)}{d}+\frac{a^3\sec ^2(c+dx)}{2d}+\frac{3a^3\sec (c+dx)}{d}-\frac{3a^3\log (\cos (c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] -((a^3*Cos[c + d*x])/d) - (3*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x]
)/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^3(-a+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{ad}\\
&= \frac{a^2\text{Subst}\left(\int \frac{(-a+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^2\text{Subst}\left(\int \left(1-\frac{a^3}{x^3}+\frac{3a^2}{x^2}-\frac{3a}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= -\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int (a+a\sec(c+dx))^3\sin(c+dx)\,dx = -\frac{a^3(-4-9\cos(c+dx)+\cos(3(c+dx))+6\log(\cos(c+dx))+\cos(2(c+dx))(-2+6\log(\cos(c+dx))))}{4d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] -1/4*(a^3*(-4 - 9*Cos[c + d*x] + Cos[3*(c + d*x)] + 6*Log[Cos[c + d*x]] + C
os[2*(c + d*x)]*(-2 + 6*Log[Cos[c + d*x]]))*Sec[c + d*x]^2)/d
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)^2}{2}+3\sec(dx+c)+3\ln(\sec(dx+c))-\frac{1}{\sec(dx+c)}\right)}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)^2}{2}+3\sec(dx+c)+3\ln(\sec(dx+c))-\frac{1}{\sec(dx+c)}\right)}{d}$
parts	$-\frac{a^3\cos(dx+c)}{d}+\frac{a^3\sec(dx+c)^2}{2d}+\frac{3a^3\ln(\sec(dx+c))}{d}+\frac{3a^3\sec(dx+c)}{d}$
risch	$3ia^3x-\frac{a^3e^{i(dx+c)}}{2d}-\frac{a^3e^{-i(dx+c)}}{2d}+\frac{6ia^3c}{d}+\frac{2a^3\left(3e^{3i(dx+c)}+e^{2i(dx+c)}+3e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{3a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{4a^3}{d}+\frac{6a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{6a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{3a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{3a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{3a^3\ln\left(1+\mathrm{ta}\right.}{}$
parallelrisch	$\frac{a^3\left(-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)+6\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(2dx+2c)+9\,\mathrm{co}\right.}{2d(1+\cos(2d}$

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^3*(1/2*sec(d*x+c)^2+3*sec(d*x+c)+3*ln(sec(d*x+c))-1/sec(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int (a+a\sec(c+dx))^3\sin(c+dx)\,dx$$
$$=-\frac{2\,a^3\cos\left(dx+c\right)^3+6\,a^3\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-6\,a^3\cos\left(dx+c\right)-a^3}{2\,d\cos\left(dx+c\right)^2}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 6*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 6*a^
3*cos(d*x + c) - a^3)/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^3\sin(c+dx)\,dx = a^3\left(\int 3\sin(c+dx)\sec(c+dx)\,dx + \int 3\sin(c+dx)\sec^2(c+dx)\,dx + \int \sin(c+dx)\sec^3(c+dx)\,dx + \int \sin(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c),x)
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x), x) + Integral(3*sin(c + d*x)*se
c(c + d*x)**2, x) + Integral(sin(c + d*x)*sec(c + d*x)**3, x) + Integral(si
n(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int (a+a\sec(c+dx))^3\sin(c+dx)\,dx = -\frac{2a^3\cos(dx+c)+6a^3\log(\cos(dx+c))-\frac{6a^3}{\cos(dx+c)}-\frac{a^3}{\cos(dx+c)^2}}{2d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="maxima")
[Out] -1/2*(2*a^3*cos(d*x + c) + 6*a^3*log(cos(d*x + c)) - 6*a^3/cos(d*x + c) - a
^3/cos(d*x + c)^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int (a+a\sec(c+dx))^3\sin(c+dx)\,dx = -\frac{a^3\cos(dx+c)}{d} - \frac{3a^3\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{6a^3\cos(dx+c)+a^3}{2d\cos(dx+c)^2}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="giac")
[Out] -a^3*cos(d*x + c)/d - 3*a^3*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a^3*co
s(d*x + c) + a^3)/(d*cos(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int (a+a\sec(c+dx))^3\sin(c+dx)\,dx$$
$$=\frac{a^3\left(3\cos\left(c+d\,x\right)-{\cos\left(c+d\,x\right)}^3-3\,{\cos\left(c+d\,x\right)}^2\,\ln\left(\cos\left(c+d\,x\right)\right)+\frac{1}{2}\right)}{d\,{\cos\left(c+d\,x\right)}^2}$$

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*(3*cos(c + d*x) - cos(c + d*x)^3 - 3*cos(c + d*x)^2*log(cos(c + d*x))
+ 1/2))/(d*cos(c + d*x)^2)
```

3.43 $\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{4a^3\log(1-\cos(c+dx))}{d} - \frac{4a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+dx)}{2d}$$

```
[Out] 4*a^3*ln(1-cos(d*x+c))/d-4*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*
sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2915, 12, 90}

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\sec^2(c+dx)}{2d} + \frac{3a^3\sec(c+dx)}{d} + \frac{4a^3\log(1-\cos(c+dx))}{d} - \frac{4a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (4*a^3*Log[1 - Cos[c + d*x]])/d - (4*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[
c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3 \csc(c+dx)\sec^3(c+dx)\,dx \\
&= \frac{a\text{Subst}\left(\int \frac{a^3(-a+x)^2}{(-a-x)x^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^4\text{Subst}\left(\int \frac{(-a+x)^2}{(-a-x)x^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^4\text{Subst}\left(\int \left(-\frac{a}{x^3}+\frac{3}{x^2}-\frac{4}{ax}+\frac{4}{a(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{4a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{a^3\left(1+6\cos(c+dx)-4\log(\cos(c+dx))-4\cos(2(c+dx))\left(\log(\cos(c+dx))-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\right.}{2d}$$

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + 6*Cos[c + d*x] - 4*Log[Cos[c + d*x]] - 4*Cos[2*(c + d*x)]*(Log[Co
s[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) + 8*Log[Sin[(c + d*x)/2]])*Sec[c + d
*x]^2)/(2*d)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a^3\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+3a^3\ln(\tan(dx+c))+a^3\ln(-\cot(dx+c}{d}$
default	$\frac{a^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a^3\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+3a^3\ln(\tan(dx+c))+a^3\ln(-\cot(dx+c}{d}$
risch	$\frac{2a^3\left(3\,e^{3i(dx+c)}+e^{2i(dx+c)}+3\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{8a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{4a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{6a^3}{d}-\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{8a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{4a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$
parallelrisch	$\frac{-8a^3(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-a^3\left(-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(2dx+2c)+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)\right.}{2d(1+\cos(2dx+2c))}$

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+3*a^3*(1/cos(d*x+c)+ln(-cot(d*x+
c)+csc(d*x+c)))+3*a^3*ln(tan(d*x+c))+a^3*ln(-cot(d*x+c)+csc(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{8\,a^3\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-8\,a^3\cos\left(dx+c\right)^2\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-6\,a^3\cos\left(dx+c\right)}{2\,d\cos\left(dx+c\right)^2}$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
[Out] -1/2*(8*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 8*a^3*cos(d*x + c)^2*log(-1
/2*cos(d*x + c) + 1/2) - 6*a^3*cos(d*x + c) - a^3)/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\csc\left(c+dx\right)\sec\left(c+dx\right)dx + \int 3\csc\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \csc\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int \csc\left(c+dx\right)dx\right)$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**3,x)
[Out] a**3*(Integral(3*csc(c + d*x)*sec(c + d*x), x) + Integral(3*csc(c + d*x)*se
c(c + d*x)**2, x) + Integral(csc(c + d*x)*sec(c + d*x)**3, x) + Integral(cs
c(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{8\,a^3\log\left(\cos\left(dx+c\right)-1\right)-8\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{6\,a^3\cos(dx+c)+a^3}{\cos(dx+c)^2}}{2\,d}$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*(8*a^3*log(cos(d*x + c) - 1) - 8*a^3*log(cos(d*x + c)) + (6*a^3*cos(d*x
 + c) + a^3)/cos(d*x + c)^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(65) = 130$.

Time = 0.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.12

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{2\left(2a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{6a^3+\frac{8a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}\right)}{d}$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a^3*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (6*a^3 + 8*a^3*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 3*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{3\,a^3\,\cos\left(c+d\,x\right)+\frac{a^3}{2}}{d\cos\left(c+d\,x\right)^2}-\frac{8\,a^3\,\mathrm{atanh}(2\,\cos\left(c+d\,x\right)-1)}{d}$$

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x),x)
```

```
[Out] (3*a^3*cos(c + d*x) + a^3/2)/(d*cos(c + d*x)^2) - (8*a^3*atanh(2*cos(c + d*
x) - 1))/d
```

3.44 $\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{2a^4}{d(a-a\cos(c+dx))}+\frac{5a^3\log(1-\cos(c+dx))}{d} - \frac{5a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}$$

```
[Out] -2*a^4/d/(a-a*cos(d*x+c))+5*a^3*ln(1-cos(d*x+c))/d-5*a^3*ln(cos(d*x+c))/d+3
*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 78}

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{2a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d} + \frac{3a^3\sec(c+dx)}{d}+\frac{5a^3\log(1-\cos(c+dx))}{d} - \frac{5a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a^4)/(d*(a - a*Cos[c + d*x])) + (5*a^3*Log[1 - Cos[c + d*x]])/d - (5*a^
3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3 \csc^3(c+dx)\sec^3(c+dx)\,dx\\
&= \frac{a^3\text{Subst}\left(\int \frac{a^3(-a+x)}{(-a-x)^2x^3}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^6\text{Subst}\left(\int \frac{-a+x}{(-a-x)^2x^3}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^6\text{Subst}\left(\int \left(-\frac{1}{ax^3}+\frac{3}{a^2x^2}-\frac{5}{a^3x}+\frac{2}{a^2(a+x)^2}+\frac{5}{a^3(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= -\frac{2a^4}{d(a-a\cos(c+dx))}+\frac{5a^3\log(1-\cos(c+dx))}{d}\\
&\quad -\frac{5a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2\csc^2\left(\frac{1}{2}(c+dx)\right)+10\left(\log(\cos(c+dx))-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right.\right.\right.\right.}{16d}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/16*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(2*Csc[(c + d*x)/2]^2 +
10*(Log[Cos[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) - 6*Sec[c + d*x] - Sec[c +
 d*x]^2))/d
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.50

method	result
risch	$\frac{2a^3\left(5\,e^{5i(dx+c)}-5\,e^{4i(dx+c)}+8\,e^{3i(dx+c)}-5\,e^{2i(dx+c)}+5\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2\left(e^{i(dx+c)}-1\right)^2}+\frac{10a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{5a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{a^3}{d}-\frac{5a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{10a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}$
derivativedivides	$\frac{a^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c))\right)+3a^3\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(-\cot(dx+c)+}{2}\right.}{d}$
default	$\frac{a^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c))\right)+3a^3\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(-\cot(dx+c)+}{2}\right.}{d}$
parallelrisch	$\frac{a^3\left(9\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos(2dx+2c)-56\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos(dx+c)+20\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(2dx+2c)-10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.\right.}{}$

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^3/d/(exp(2*I*(d*x+c))+1)^2/(exp(I*(d*x+c))-1)^2*(5*exp(5*I*(d*x+c))-5*e
xp(4*I*(d*x+c))+8*exp(3*I*(d*x+c))-5*exp(2*I*(d*x+c))+5*exp(I*(d*x+c)))+10/
d*a^3*ln(exp(I*(d*x+c))-1)-5/d*a^3*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.50

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{10\,a^3\cos(dx+c)^2-5\,a^3\cos(dx+c)-a^3-10\left(a^3\cos(dx+c)^3-a^3\cos(dx+c)^2\right)\log(-\cos(dx+c))+}{2\left(d\cos(dx+c)^3-d\cos(dx+c)^2\right)}$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(10*a^3*cos(d*x + c)^2 - 5*a^3*cos(d*x + c) - a^3 - 10*(a^3*cos(d*x + c
)^3 - a^3*cos(d*x + c)^2)*log(-cos(d*x + c)) + 10*(a^3*cos(d*x + c)^3 - a^3
*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - d*cos(d*
x + c)^2)
```

Sympy [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx=a^3\left(\int 3\csc^3(c+dx)\sec(c+dx)\,dx+\int 3\csc^3(c+dx)\sec^2(c+dx)\,dx+\int \csc^3(c+dx)\sec^3(c+dx)\,dx+\int \csc^3(c+dx)\,dx\right)$$

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*csc(c + d*x)**3*sec(c + d*x), x) + Integral(3*csc(c + d*x)
**3*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(csc(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{10\,a^3\log(\cos(dx+c)-1)-10\,a^3\log(\cos(dx+c))+\frac{10\,a^3\cos(dx+c)^2-5\,a^3\cos(dx+c)-a^3}{\cos(dx+c)^3-\cos(dx+c)^2}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(10*a^3*log(cos(d*x + c) - 1) - 10*a^3*log(cos(d*x + c)) + (10*a^3*cos(
d*x + c)^2 - 5*a^3*cos(d*x + c) - a^3)/(cos(d*x + c)^3 - cos(d*x + c)^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(87) = 174$.

Time = 0.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{10\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 10\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{2\left(a^3-\frac{5\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1} + \frac{27\,a^3+\frac{38\,a^3(\cos}{\cos(d}}{(}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(10*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 10*a^3*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 2*(a^3 - 5*a^3*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (27*a
^3 + 38*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 15*a^3*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{-5\,a^3\cos\left(c+d\,x\right)^2+\frac{5\,a^3\,\cos(c+d\,x)}{2}+\frac{a^3}{2}}{d\,\left(\cos\left(c+d\,x\right)^2-\cos\left(c+d\,x\right)^3\right)} - \frac{10\,a^3\,\mathrm{atanh}(2\,\cos\left(c+d\,x\right)-1)}{d}$$

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^3,x)
```

```
[Out] ((5*a^3*cos(c + d*x))/2 + a^3/2 - 5*a^3*cos(c + d*x)^2)/(d*(cos(c + d*x)^2
- cos(c + d*x)^3)) - (10*a^3*atanh(2*cos(c + d*x) - 1))/d
```

3.45 $\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^5}{2d(a-a\cos(c+dx))^2}-\frac{3a^4}{d(a-a\cos(c+dx))}+\frac{6a^3\log(1-\cos(c+dx))}{d}-\frac{6a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}$$

```
[Out] -1/2*a^5/d/(a-a*cos(d*x+c))^2-3*a^4/d/(a-a*cos(d*x+c))+6*a^3*ln(1-cos(d*x+c
))/d-6*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 46}

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^5}{2d(a-a\cos(c+dx))^2}-\frac{3a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{6a^3\log(1-\cos(c+dx))}{d}-\frac{6a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*a^5/(d*(a - a*Cos[c + d*x])^2) - (3*a^4)/(d*(a - a*Cos[c + d*x])) + (6
*a^3*Log[1 - Cos[c + d*x]])/d - (6*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c
+ d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3\csc^5(c+dx)\sec^3(c+dx)\,dx \\
&= \frac{a^5\text{Subst}\left(\int \frac{a^3}{(-a-x)^3x^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^8\text{Subst}\left(\int \frac{1}{(-a-x)^3x^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^8\text{Subst}\left(\int \left(-\frac{1}{a^3x^3}+\frac{3}{a^4x^2}-\frac{6}{a^5x}+\frac{1}{a^3(a+x)^3}+\frac{3}{a^4(a+x)^2}+\frac{6}{a^5(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= -\frac{a^5}{2d(a-a\cos(c+dx))^2}-\frac{3a^4}{d(a-a\cos(c+dx))}+\frac{6a^3\log(1-\cos(c+dx))}{d} \\
&\quad -\frac{6a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)+48\left(\log(\cos(c+dx))-2\right.\right.}{64d}$$

[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]

[Out] -1/64*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^4 + 48*(Log[Cos[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) - 24*Sec[c + d*x] - 4*Sec[c + d*x]^2))/d

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.38

method	result
norman	$\frac{-\frac{a^3}{8d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d}-\frac{23a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4d}+\frac{75a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{12a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$
parallelrisch	$\frac{12\left(\frac{(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{49}{}\right.}{d(1+\cos(2dx+2c))}$
risch	$\frac{4a^3\left(3\,e^{7i(dx+c)}-9\,e^{6i(dx+c)}+13\,e^{5i(dx+c)}-16\,e^{4i(dx+c)}+13\,e^{3i(dx+c)}-9\,e^{2i(dx+c)}+3\,e^{i(dx+c)}\right)}{d\left(e^{i(dx+c)}-1\right)^4\left(e^{2i(dx+c)}+1\right)^2}+\frac{12a^3\ln\left(e^{i(dx+c)}-1\right)}{d}$
derivativedivides	$\frac{a^3\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)^2}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+3a^3\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)}-\frac{}{8\sin}\right.}{}$
default	$\frac{a^3\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)^2}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+3a^3\left(-\frac{1}{4\sin(dx+c)^4\cos(dx+c)}-\frac{}{8\sin}\right.}{}$

[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] (-1/8/d*a^3-3/2*a^3/d*tan(1/2*d*x+1/2*c)^2-23/4*a^3/d*tan(1/2*d*x+1/2*c)^6+75/8*a^3/d*tan(1/2*d*x+1/2*c)^4)/tan(1/2*d*x+1/2*c)^4/(-1+tan(1/2*d*x+1/2*c)^2)^2+12/d*a^3*ln(tan(1/2*d*x+1/2*c))-6/d*a^3*ln(tan(1/2*d*x+1/2*c)-1)-6/d*a^3*ln(tan(1/2*d*x+1/2*c)+1)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.59

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{12\,a^3\cos\left(dx+c\right)^3 - 18\,a^3\cos\left(dx+c\right)^2 + 4\,a^3\cos\left(dx+c\right) + a^3 - 12\,\left(a^3\cos\left(dx+c\right)^4 - 2\,a^3\cos\left(dx+c\right)^3}{2\,\left(d\cos\left(dx+c\right)}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a^3*cos(d*x + c)^3 - 18*a^3*cos(d*x + c)^2 + 4*a^3*cos(d*x + c) + a
^3 - 12*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*lo
g(-cos(d*x + c)) + 12*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(
d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x +
 c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{12\,a^3\log\left(\cos\left(dx+c\right)-1\right) - 12\,a^3\log\left(\cos\left(dx+c\right)\right) + \frac{12\,a^3\cos\left(dx+c\right)^3 - 18\,a^3\cos\left(dx+c\right)^2 + 4\,a^3\cos\left(dx+c\right) + a^3}{\cos\left(dx+c\right)^4 - 2\,\cos\left(dx+c\right)^3 + \cos\left(dx+c\right)^2}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*a^3*log(cos(d*x + c) - 1) - 12*a^3*log(cos(d*x + c)) + (12*a^3*cos(
d*x + c)^3 - 18*a^3*cos(d*x + c)^2 + 4*a^3*cos(d*x + c) + a^3)/(cos(d*x + c
)^4 - 2*cos(d*x + c)^3 + cos(d*x + c)^2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.68

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{48\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-48\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{a^3-\frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{75\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{46\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)^2}}{8\,d}$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(48*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 48*a^3*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^3 - 12*a^3*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 75*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 - 46*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{6\,a^3\cos(c+d\,x)^3-9\,a^3\cos(c+d\,x)^2+2\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{d\,\left(\cos(c+d\,x)^4-2\cos(c+d\,x)^3+\cos(c+d\,x)^2\right)}-\frac{12\,a^3\,\mathrm{atanh}(2\,\cos(c+d\,x)-1)}{d}$$

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^5,x)
```

```
[Out] (2*a^3*cos(c + d*x) + a^3/2 - 9*a^3*cos(c + d*x)^2 + 6*a^3*cos(c + d*x)^3)/
(d*(cos(c + d*x)^2 - 2*cos(c + d*x)^3 + cos(c + d*x)^4)) - (12*a^3*atanh(2*
cos(c + d*x) - 1))/d
```

3.46 $\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^6}{6d(a-a\cos(c+dx))^3} - \frac{7a^5}{8d(a-a\cos(c+dx))^2} - \frac{31a^4}{8d(a-a\cos(c+dx))} + \frac{111a^3\log(1-\cos(c+dx))}{16d} - \frac{7a^3\log(\cos(c+dx))}{d} + \frac{a^3\log(1+\cos(c+dx))}{16d} + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+dx)}{2d}$$

```
[Out] -1/6*a^6/d/(a-a*cos(d*x+c))^3-7/8*a^5/d/(a-a*cos(d*x+c))^2-31/8*a^4/d/(a-a*
cos(d*x+c))+111/16*a^3*ln(1-cos(d*x+c))/d-7*a^3*ln(cos(d*x+c))/d+1/16*a^3*l
n(1+cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

$= \{3957, 2915, 12, 90\}$

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^6}{6d(a-a\cos(c+dx))^3} - \frac{7a^5}{8d(a-a\cos(c+dx))^2} - \frac{31a^4}{8d(a-a\cos(c+dx))} + \frac{a^3\sec^2(c+dx)}{2d} + \frac{3a^3\sec(c+dx)}{d} + \frac{111a^3\log(1-\cos(c+dx))}{16d} - \frac{7a^3\log(\cos(c+dx))}{d} + \frac{a^3\log(\cos(c+dx)+1)}{16d}$$

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/6*a^6/(d*(a - a*Cos[c + d*x])^3) - (7*a^5)/(8*d*(a - a*Cos[c + d*x])^2)
- (31*a^4)/(8*d*(a - a*Cos[c + d*x])) + (111*a^3*Log[1 - Cos[c + d*x]])/(16
*d) - (7*a^3*Log[Cos[c + d*x]])/d + (a^3*Log[1 + Cos[c + d*x]])/(16*d) + (3
*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3\csc^7(c+dx)\sec^3(c+dx)\,dx \\
&= \frac{a^7\text{Subst}\left(\int \frac{a^3}{(-a-x)^4x^3(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^{10}\text{Subst}\left(\int \frac{1}{(-a-x)^4x^3(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^{10}\text{Subst}\left(\int \left(-\frac{1}{16a^7(a-x)}-\frac{1}{a^5x^3}+\frac{3}{a^6x^2}-\frac{7}{a^7x}+\frac{1}{2a^4(a+x)^4}+\frac{7}{4a^5(a+x)^3}+\frac{31}{8a^6(a+x)^2}+\frac{111}{16a^7(a+x)}\right)\,dx, x,}{d} \\
&= -\frac{a^6}{6d(a-a\cos(c+dx))^3}-\frac{7a^5}{8d(a-a\cos(c+dx))^2}-\frac{31a^4}{8d(a-a\cos(c+dx))} \\
&\quad+\frac{111a^3\log(1-\cos(c+dx))}{16d}-\frac{7a^3\log(\cos(c+dx))}{d} \\
&\quad+\frac{a^3\log(1+\cos(c+dx))}{16d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$
\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(186\csc^2\left(\frac{1}{2}(c+dx)\right)+21\csc^4\left(\frac{1}{2}(c+dx)\right)+2\csc^6\left(\frac{1}{2}(c+dx)\right)\right.}{}
$$

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/768*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(186*Csc[(c + d*x)/2]^2
 + 21*Csc[(c + d*x)/2]^4 + 2*Csc[(c + d*x)/2]^6 - 12*(Log[Cos[(c + d*x)/2]]
 - 56*Log[Cos[c + d*x]] + 111*Log[Sin[(c + d*x)/2]] + 24*Sec[c + d*x] + 4*S
ec[c + d*x]^2)))/d
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{79\left(\frac{112(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{79}+\frac{112(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{79}+\frac{222(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{79}\right.}{16d(1+\cos(2dx+2c}$
norman	$\frac{-\frac{a^3}{48d}-\frac{23a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{96d}-\frac{91a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{48d}-\frac{103a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{16d}+\frac{339a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{32d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{111a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-$
risch	$\frac{a^3\left(165\,e^{9i(dx+c)}-822\,e^{8i(dx+c)}+1852\,e^{7i(dx+c)}-2754\,e^{6i(dx+c)}+3182\,e^{5i(dx+c)}-2754\,e^{4i(dx+c)}+1852\,e^{3i(dx+c)}-822\,e^{2i}\right.}{12d\left(e^{i(dx+c)}-1\right)^6\left(e^{2i(dx+c)}+1\right)^2}$
derivativedivides	$a^3\left(-\frac{1}{6\sin(dx+c)^6\cos(dx+c)^2}-\frac{1}{3\sin(dx+c)^4\cos(dx+c)^2}+\frac{1}{\sin(dx+c)^2\cos(dx+c)^2}-\frac{2}{\sin(dx+c)^2}+4\ln(\tan(dx+c))\right)+3a^3\left(-\frac{}{6\sin(}\right.$
default	$a^3\left(-\frac{1}{6\sin(dx+c)^6\cos(dx+c)^2}-\frac{1}{3\sin(dx+c)^4\cos(dx+c)^2}+\frac{1}{\sin(dx+c)^2\cos(dx+c)^2}-\frac{2}{\sin(dx+c)^2}+4\ln(\tan(dx+c))\right)+3a^3\left(-\frac{}{6\sin(}\right.$

```
[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 79/16*(112/79*(-cos(2*d*x+2*c)-1)*ln(tan(1/2*d*x+1/2*c)-1)+112/79*(-cos(2*d
*x+2*c)-1)*ln(tan(1/2*d*x+1/2*c)+1)+222/79*(1+cos(2*d*x+2*c))*ln(tan(1/2*d*
x+1/2*c))+cot(1/2*d*x+1/2*c)^2*csc(1/2*d*x+1/2*c)^4*(cos(d*x+c)-161/237*cos
(2*d*x+2*c)+71/237*cos(3*d*x+3*c)-449/7584*cos(4*d*x+4*c)-4319/7584))*a^3/d
/(1+cos(2*d*x+2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(148) = 296$.

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.89

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{330\,a^3\cos(dx+c)^4-822\,a^3\cos(dx+c)^3+596\,a^3\cos(dx+c)^2-72\,a^3\cos(dx+c)-24\,a^3-336\,(a^3\cos}{}$$

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(330*a^3*cos(d*x + c)^4 - 822*a^3*cos(d*x + c)^3 + 596*a^3*cos(d*x + c
)^2 - 72*a^3*cos(d*x + c) - 24*a^3 - 336*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*
x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x + c)^2)*log(-cos(d*x + c)) +
3*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*c
os(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 333*(a^3*cos(d*x + c)^5 - 3*a^
3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x + c)^2)*log(-1/2*cos(
d*x + c) + 1/2))/(d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^
3 - d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c))**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{3\,a^3\log\left(\cos\left(dx+c\right)+1\right)+333\,a^3\log\left(\cos\left(dx+c\right)-1\right)-336\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(165\,a^3\cos(dx+c)^4-411\right.}{\cos(dx+c)}}{48\,d}$$

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] 1/48*(3*a^3*log(cos(d*x + c) + 1) + 333*a^3*log(cos(d*x + c) - 1) - 336*a^3
*log(cos(d*x + c)) + 2*(165*a^3*cos(d*x + c)^4 - 411*a^3*cos(d*x + c)^3 + 2
98*a^3*cos(d*x + c)^2 - 36*a^3*cos(d*x + c) - 12*a^3)/(cos(d*x + c)^5 - 3*c
os(d*x + c)^4 + 3*cos(d*x + c)^3 - cos(d*x + c)^2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.55

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{666\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-672\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(2\,a^3-\frac{27\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{234\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{1221\,a^3(\cdots}{(\cos(\cdots}\right.}{(\cos(dx+c)-1)^3}}{96\,d}$$

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] 1/96*(666*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 672*a^3*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a^3 - 27*a^3*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 234*a^3*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 - 1221*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x +
c) + 1)^3/(cos(d*x + c) - 1)^3 + 48*(33*a^3 + 50*a^3*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 21*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{111\,a^3\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{16\,d}+\frac{a^3\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{16\,d} + \frac{-\frac{55\,a^3\,\cos(c+d\,x)^4}{8}+\frac{137\,a^3\,\cos(c+d\,x)^3}{8}-\frac{149\,a^3\,\cos(c+d\,x)^2}{12}+\frac{3\,a^3\,\cos(c+d\,x)}{2}+\frac{a^3}{2}}{d\,\left(-\cos\left(c+d\,x\right)^5+3\,\cos\left(c+d\,x\right)^4-3\,\cos\left(c+d\,x\right)^3+\cos\left(c+d\,x\right)^2\right)} - \frac{7\,a^3\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^7,x)
```

```
[Out] (111*a^3*log(cos(c + d*x) - 1))/(16*d) + (a^3*log(cos(c + d*x) + 1))/(16*d)
 + ((3*a^3*cos(c + d*x))/2 + a^3/2 - (149*a^3*cos(c + d*x)^2)/12 + (137*a^3
*cos(c + d*x)^3)/8 - (55*a^3*cos(c + d*x)^4)/8)/(d*(cos(c + d*x)^2 - 3*cos(
c + d*x)^3 + 3*cos(c + d*x)^4 - cos(c + d*x)^5)) - (7*a^3*log(cos(c + d*x))
)/d
```

3.47 $\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 202

$$\begin{aligned}\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx = & -\frac{a^7}{16d(a-a\cos(c+dx))^4}-\frac{a^6}{3d(a-a\cos(c+dx))^3} \\ & -\frac{39a^5}{32d(a-a\cos(c+dx))^2} \\ & -\frac{75a^4}{16d(a-a\cos(c+dx))}-\frac{a^4}{32d(a+a\cos(c+dx))} \\ & +\frac{501a^3\log(1-\cos(c+dx))}{64d} \\ & -\frac{8a^3\log(\cos(c+dx))}{d}+\frac{11a^3\log(1+\cos(c+dx))}{64d} \\ & +\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}\end{aligned}$$

```
[Out] -1/16*a^7/d/(a-a*cos(d*x+c))^4-1/3*a^6/d/(a-a*cos(d*x+c))^3-39/32*a^5/d/(a-
a*cos(d*x+c))^2-75/16*a^4/d/(a-a*cos(d*x+c))-1/32*a^4/d/(a+a*cos(d*x+c))+50
1/64*a^3*ln(1-cos(d*x+c))/d-8*a^3*ln(cos(d*x+c))/d+11/64*a^3*ln(1+cos(d*x+c
))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^7}{16d(a-a\cos(c+dx))^4}-\frac{a^6}{3d(a-a\cos(c+dx))^3} -\frac{39a^5}{32d(a-a\cos(c+dx))^2}-\frac{75a^4}{16d(a-a\cos(c+dx))} -\frac{a^4}{32d(a\cos(c+dx)+a)}+\frac{a^3\sec^2(c+dx)}{2d} +\frac{3a^3\sec(c+dx)}{d}+\frac{501a^3\log(1-\cos(c+dx))}{64d} -\frac{8a^3\log(\cos(c+dx))}{d}+\frac{11a^3\log(\cos(c+dx)+1)}{64d}$$

```
[In] Int[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/16*a^7/(d*(a - a*Cos[c + d*x])^4) - a^6/(3*d*(a - a*Cos[c + d*x])^3) - (
39*a^5)/(32*d*(a - a*Cos[c + d*x])^2) - (75*a^4)/(16*d*(a - a*Cos[c + d*x])
) - a^4/(32*d*(a + a*Cos[c + d*x])) + (501*a^3*Log[1 - Cos[c + d*x]])/(64*d
) - (8*a^3*Log[Cos[c + d*x]])/d + (11*a^3*Log[1 + Cos[c + d*x]])/(64*d) + (
3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a - a\cos(c+dx))^3 \csc^9(c+dx)\sec^3(c+dx)\,dx \\
&= \frac{a^9 \text{Subst}\left(\int \frac{a^3}{(-a-x)^5 x^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^{12} \text{Subst}\left(\int \frac{1}{(-a-x)^5 x^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^{12} \text{Subst}\left(\int \left(-\frac{1}{32a^8(a-x)^2} - \frac{11}{64a^9(a-x)} - \frac{1}{a^7x^3} + \frac{3}{a^8x^2} - \frac{8}{a^9x} + \frac{1}{4a^5(a+x)^5} + \frac{1}{a^6(a+x)^4} + \frac{39}{16a^7(a+x)^3} + \frac{}{16a^8}\right.\right.}{d} \\
&= -\frac{a^7}{16d(a-a\cos(c+dx))^4} - \frac{a^6}{3d(a-a\cos(c+dx))^3} - \frac{39a^5}{32d(a-a\cos(c+dx))^2} \\
&\quad - \frac{75a^4}{16d(a-a\cos(c+dx))} - \frac{a^4}{32d(a+a\cos(c+dx))} + \frac{501a^3\log(1-\cos(c+dx))}{64d} \\
&\quad - \frac{8a^3\log(\cos(c+dx))}{d} + \frac{11a^3\log(1+\cos(c+dx))}{64d} + \frac{3a^3\sec(c+dx)}{d} \\
&\quad + \frac{a^3\sec^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(1800\csc^2\left(\frac{1}{2}(c+dx)\right)+234\csc^4\left(\frac{1}{2}(c+dx)\right)+32\csc^6\left(\frac{1}{2}(c+dx\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/6144*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(1800*Csc[(c + d*x)/2]
^2 + 234*Csc[(c + d*x)/2]^4 + 32*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8
- 12*(22*Log[Cos[(c + d*x)/2]] - 512*Log[Cos[c + d*x]] + 1002*Log[Sin[(c +
d*x)/2]] - Sec[(c + d*x)/2]^2 + 192*Sec[c + d*x] + 32*Sec[c + d*x]^2)))/d
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.04

method	result
norman	$\frac{-\frac{a^3}{256d}-\frac{19a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{384d}-\frac{263a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{768d}-\frac{431a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{192d}-\frac{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{64d}-\frac{451a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{64d}+\frac{749a^3\tan(}{64d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$
parallelrisch	$-\frac{a^3\left(2048\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2048\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2\ln(\right.}{}$
risch	$\frac{a^3\left(735\,e^{13i(dx+c)}-3642\,e^{12i(dx+c)}+6662\,e^{11i(dx+c)}-4650\,e^{10i(dx+c)}-1983\,e^{9i(dx+c)}+8868\,e^{8i(dx+c)}-12748\,e^{7i(dx+c)}+\right.}{48d\left(e^{i(dx+c)}-1\right)^8\left(e^{i(dx+c)}+1\right)^2\left(e^{2i(\right.}}$
derivativedivides	$\frac{a^3\left(-\frac{1}{8\sin(dx+c)^8\cos(dx+c)^2}-\frac{5}{24\sin(dx+c)^6\cos(dx+c)^2}-\frac{5}{12\sin(dx+c)^4\cos(dx+c)^2}+\frac{5}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)^2}+5\right.}{}$
default	$\frac{a^3\left(-\frac{1}{8\sin(dx+c)^8\cos(dx+c)^2}-\frac{5}{24\sin(dx+c)^6\cos(dx+c)^2}-\frac{5}{12\sin(dx+c)^4\cos(dx+c)^2}+\frac{5}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)^2}+5\right.}{}$

```
[In] int(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/256/d*a^3-19/384*a^3/d*tan(1/2*d*x+1/2*c)^2-263/768*a^3/d*tan(1/2*d*x+1
/2*c)^4-431/192*a^3/d*tan(1/2*d*x+1/2*c)^6-1/64/d*a^3*tan(1/2*d*x+1/2*c)^14
-451/64*a^3/d*tan(1/2*d*x+1/2*c)^10+749/64*a^3/d*tan(1/2*d*x+1/2*c)^8)/tan(
1/2*d*x+1/2*c)^8/(-1+tan(1/2*d*x+1/2*c)^2)^2+501/32/d*a^3*ln(tan(1/2*d*x+1/
2*c))-8/d*a^3*ln(tan(1/2*d*x+1/2*c)-1)-8/d*a^3*ln(tan(1/2*d*x+1/2*c)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(190) = 380.

Time = 0.29 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.07

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{1470\,a^3\cos(dx+c)^6-3642\,a^3\cos(dx+c)^5+1126\,a^3\cos(dx+c)^4+3390\,a^3\cos(dx+c)^3-2752\,a^3\cos(}{}$$

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/192*(1470*a^3*cos(d*x + c)^6 - 3642*a^3*cos(d*x + c)^5 + 1126*a^3*cos(d*x
 + c)^4 + 3390*a^3*cos(d*x + c)^3 - 2752*a^3*cos(d*x + c)^2 + 288*a^3*cos(d
*x + c) + 96*a^3 - 1536*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6 + 2*a^3*
cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x
+ c)^2)*log(-cos(d*x + c)) + 33*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6
+ 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*
```

```
cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 1503*(a^3*cos(d*x + c)^7 - 3*
a^3*cos(d*x + c)^6 + 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*co
s(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x
 + c)^7 - 3*d*cos(d*x + c)^6 + 2*d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 - 3*
d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**9*(a+a*sec(d*x+c))**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{33\,a^3\log\left(\cos\left(dx+c\right)+1\right)+1503\,a^3\log\left(\cos\left(dx+c\right)-1\right)-1536\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(735\,a^3\cos\left(dx+c\right)^6\right.}{}}{192\,d}$$

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] 1/192*(33*a^3*log(cos(d*x + c) + 1) + 1503*a^3*log(cos(d*x + c) - 1) - 1536
*a^3*log(cos(d*x + c)) + 2*(735*a^3*cos(d*x + c)^6 - 1821*a^3*cos(d*x + c)^
5 + 563*a^3*cos(d*x + c)^4 + 1695*a^3*cos(d*x + c)^3 - 1376*a^3*cos(d*x + c
)^2 + 144*a^3*cos(d*x + c) + 48*a^3)/(cos(d*x + c)^7 - 3*cos(d*x + c)^6 + 2
*cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 3*cos(d*x + c)^3 + cos(d*x + c)^2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.45

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{6012\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-6144\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{\left(3\,a^3-\frac{44\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+34\right.}{}}{768}$$

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(6012*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 6144*a^
3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 12*a^3*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - (3*a^3 - 44*a^3*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + 348*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2376*a^3*(cos
(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 12525*a^3*(cos(d*x + c) - 1)^4/(cos
(d*x + c) + 1)^4)*(cos(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4 + 1536*(9*a^3 +
 14*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^3*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{501\,a^3\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{64\,d}+\frac{11\,a^3\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{64\,d}$$
$$+\frac{\frac{245\,a^3\,\cos(c+d\,x)^6}{32}-\frac{607\,a^3\,\cos(c+d\,x)^5}{32}+\frac{563\,a^3\,\cos(c+d\,x)^4}{96}+\frac{565\,a^3\,\cos(c+d\,x)^3}{32}-\frac{43\,a^3\,\cos(c+d\,x)^2}{3}+\frac{3\,a^3\,\cos(c+d\,x)}{2}+\frac{a^3}{2}}{d\,\left(\cos\left(c+d\,x\right)^7-3\cos\left(c+d\,x\right)^6+2\cos\left(c+d\,x\right)^5+2\cos\left(c+d\,x\right)^4-3\cos\left(c+d\,x\right)^3+\cos\left(c+d\,x\right)^2\right)}$$
$$-\frac{8\,a^3\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^9,x)
```

```
[Out] (501*a^3*log(cos(c + d*x) - 1))/(64*d) + (11*a^3*log(cos(c + d*x) + 1))/(64
*d) + ((3*a^3*cos(c + d*x))/2 + a^3/2 - (43*a^3*cos(c + d*x)^2)/3 + (565*a^
3*cos(c + d*x)^3)/32 + (563*a^3*cos(c + d*x)^4)/96 - (607*a^3*cos(c + d*x)^
5)/32 + (245*a^3*cos(c + d*x)^6)/32)/(d*(cos(c + d*x)^2 - 3*cos(c + d*x)^3
+ 2*cos(c + d*x)^4 + 2*cos(c + d*x)^5 - 3*cos(c + d*x)^6 + cos(c + d*x)^7))
 - (8*a^3*log(cos(c + d*x)))/d
```

3.48 $\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 210

$$\begin{aligned}\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx = & -\frac{805a^3x}{128}-\frac{a^3\text{arctanh}(\sin(c+dx))}{2d}\\ & +\frac{603a^3\cos(c+dx)\sin(c+dx)}{128d}\\ & -\frac{293a^3\cos^3(c+dx)\sin(c+dx)}{192d}\\ & -\frac{a^3\cos^5(c+dx)\sin(c+dx)}{48d}\\ & +\frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d}-\frac{a^3\sin^3(c+dx)}{3d}\\ & -\frac{2a^3\sin^5(c+dx)}{5d}-\frac{3a^3\sin^7(c+dx)}{7d}\\ & +\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

```
[Out] -805/128*a^3*x-1/2*a^3*arctanh(sin(d*x+c))/d+603/128*a^3*cos(d*x+c)*sin(d*x
+c)/d-293/192*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/48*a^3*cos(d*x+c)^5*sin(d*x+c
)/d+1/8*a^3*cos(d*x+c)^7*sin(d*x+c)/d-1/3*a^3*sin(d*x+c)^3/d-2/5*a^3*sin(d*
x+c)^5/d-3/7*a^3*sin(d*x+c)^7/d+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d
*x+c)/d
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3957, 2951, 2717, 2715, 8, 2713, 3855, 3852, 3853}

$$\begin{aligned}\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx = &-\frac{a^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{3a^3\sin^7(c+dx)}{7d}\\&-\frac{2a^3\sin^5(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{3d}\\&+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}\\&-\frac{a^3\sin(c+dx)\cos^5(c+dx)}{48d}\\&-\frac{293a^3\sin(c+dx)\cos^3(c+dx)}{192d}\\&+\frac{603a^3\sin(c+dx)\cos(c+dx)}{128d}\\&+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{805a^3x}{128}\end{aligned}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^8,x]
```

```
[Out] (-805*a^3*x)/128 - (a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (603*a^3*Cos[c + d*x
]*Sin[c + d*x])/(128*d) - (293*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) - (
a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a^3*Cos[c + d*x]^7*Sin[c + d*x])
/(8*d) - (a^3*Sin[c + d*x]^3)/(3*d) - (2*a^3*Sin[c + d*x]^5)/(5*d) - (3*a^3
*Sin[c + d*x]^7)/(7*d) + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c + d*x]*Tan[c +
 d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int (-a - a\cos(c+dx))^3 \sin^5(c+dx)\tan^3(c+dx)\,dx$$

$$= -\frac{\int \left(11a^{11} + 6a^{11}\cos(c+dx) - 14a^{11}\cos^2(c+dx) - 14a^{11}\cos^3(c+dx) + 6a^{11}\cos^4(c+dx) + 11\right.}{}$$

$$= -11a^3x - a^3\int \cos^6(c+dx)\,dx + a^3\int \cos^8(c+dx)\,dx - a^3\int \sec(c+dx)\,dx$$
$$+ a^3\int \sec^3(c+dx)\,dx + \left(3a^3\right)\int \cos^7(c+dx)\,dx + \left(3a^3\right)\int \sec^2(c+dx)\,dx$$
$$- \left(6a^3\right)\int \cos(c+dx)\,dx - \left(6a^3\right)\int \cos^4(c+dx)\,dx - \left(11a^3\right)\int \cos^5(c+dx)\,dx$$
$$+ \left(14a^3\right)\int \cos^2(c+dx)\,dx + \left(14a^3\right)\int \cos^3(c+dx)\,dx$$

$$= -11a^3x - \frac{a^3\text{arctanh}(\sin(c+dx))}{d} - \frac{6a^3\sin(c+dx)}{d} + \frac{7a^3\cos(c+dx)\sin(c+dx)}{d}$$
$$- \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{2d} - \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d}$$
$$+ \frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$
$$+ \frac{1}{2}a^3\int \sec(c+dx)\,dx - \frac{1}{6}\left(5a^3\right)\int \cos^4(c+dx)\,dx + \frac{1}{8}\left(7a^3\right)\int \cos^6(c+dx)\,dx$$
$$- \frac{1}{2}\left(9a^3\right)\int \cos^2(c+dx)\,dx + \left(7a^3\right)\int 1\,dx - \frac{\left(3a^3\right)\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d}$$
$$- \frac{\left(3a^3\right)\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx, x, -\sin(c+dx)\right)}{d}$$
$$+ \frac{\left(11a^3\right)\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -\sin(c+dx)\right)}{d}$$
$$- \frac{\left(14a^3\right)\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{d}$$

$$= -4a^3x - \frac{a^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{19a^3\cos(c+dx)\sin(c+dx)}{4d}$$
$$- \frac{41a^3\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^3\cos^5(c+dx)\sin(c+dx)}{48d}$$
$$+ \frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{a^3\sin^3(c+dx)}{3d} - \frac{2a^3\sin^5(c+dx)}{5d}$$
$$- \frac{3a^3\sin^7(c+dx)}{7d} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$
$$- \frac{1}{8}\left(5a^3\right)\int \cos^2(c+dx)\,dx + \frac{1}{48}\left(35a^3\right)\int \cos^4(c+dx)\,dx - \frac{1}{4}\left(9a^3\right)\int 1\,dx$$

$$
\begin{aligned}
&= -\frac{25a^3x}{4} - \frac{a^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{71a^3\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad - \frac{293a^3\cos^3(c+dx)\sin(c+dx)}{192d} - \frac{a^3\cos^5(c+dx)\sin(c+dx)}{48d} \\
&\quad + \frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{a^3\sin^3(c+dx)}{3d} - \frac{2a^3\sin^5(c+dx)}{5d} \\
&\quad - \frac{3a^3\sin^7(c+dx)}{7d} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} \\
&\quad - \frac{1}{16}(5a^3)\int 1\,dx + \frac{1}{64}(35a^3)\int \cos^2(c+dx)\,dx \\
&= -\frac{105a^3x}{16} - \frac{a^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{603a^3\cos(c+dx)\sin(c+dx)}{128d} \\
&\quad - \frac{293a^3\cos^3(c+dx)\sin(c+dx)}{192d} - \frac{a^3\cos^5(c+dx)\sin(c+dx)}{48d} \\
&\quad + \frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{a^3\sin^3(c+dx)}{3d} \\
&\quad - \frac{2a^3\sin^5(c+dx)}{5d} - \frac{3a^3\sin^7(c+dx)}{7d} + \frac{3a^3\tan(c+dx)}{d} \\
&\quad + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{128}(35a^3)\int 1\,dx \\
&= -\frac{805a^3x}{128} - \frac{a^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{603a^3\cos(c+dx)\sin(c+dx)}{128d} \\
&\quad - \frac{293a^3\cos^3(c+dx)\sin(c+dx)}{192d} - \frac{a^3\cos^5(c+dx)\sin(c+dx)}{48d} \\
&\quad + \frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{a^3\sin^3(c+dx)}{3d} - \frac{2a^3\sin^5(c+dx)}{5d} \\
&\quad - \frac{3a^3\sin^7(c+dx)}{7d} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.74

$$
\begin{aligned}
&\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx \\
&= \frac{a^3\sec^2(c+dx)\,(-1352400c - 1352400dx - 215040\text{arctanh}(\sin(c+dx))\cos^2(c+dx) - 1352400(c+dx)}{}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^8,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(-1352400*c - 1352400*d*x - 215040*ArcTanh[Sin[c + d*x]
]*Cos[c + d*x]^2 - 1352400*(c + d*x)*Cos[2*(c + d*x)] + 173600*Sin[c + d*x]
 + 1052520*Sin[2*(c + d*x)] - 11648*Sin[3*(c + d*x)] + 175280*Sin[4*(c + d*
x)] + 22784*Sin[5*(c + d*x)] - 18095*Sin[6*(c + d*x)] - 6288*Sin[7*(c + d*x
)] + 770*Sin[8*(c + d*x)] + 720*Sin[9*(c + d*x)] + 105*Sin[10*(c + d*x)]))/
(430080*d)
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.94

method	result
parallelrisch	$\frac{a^3\left(107520(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-107520(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-1352400dx\cos(2dx+2c\right.}{}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^9}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^7}{2}+\frac{7\sin(dx+c)^5}{10}+\frac{7\sin(dx+c)^3}{6}+\frac{7\sin(dx+c)}{2}-\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^9}{\cos(dx+c)}+\left(\mathrm{si}\right.\right.}{}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^9}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^7}{2}+\frac{7\sin(dx+c)^5}{10}+\frac{7\sin(dx+c)^3}{6}+\frac{7\sin(dx+c)}{2}-\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^9}{\cos(dx+c)}+\left(\mathrm{si}\right.\right.}{}$
parts	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^9}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^7}{2}+\right.}{}$
risch	$-\frac{805a^3x}{128}+\frac{127ia^3\mathrm{e}^{-2i(dx+c)}}{128d}-\frac{127ia^3\mathrm{e}^{2i(dx+c)}}{128d}-\frac{ia^3\left(\mathrm{e}^{3i(dx+c)}-6\,\mathrm{e}^{2i(dx+c)}-\mathrm{e}^{i(dx+c)}-6\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{67ia^3\mathrm{e}^{3i(dx+c)}}{384d}+$

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/215040*a^3*(107520*(1+cos(2*d*x+2*c))*ln(tan(1/2*d*x+1/2*c)-1)-107520*(1+
cos(2*d*x+2*c))*ln(tan(1/2*d*x+1/2*c)+1)-1352400*d*x*cos(2*d*x+2*c)-1352400
*d*x+22784*sin(5*d*x+5*c)-18095*sin(6*d*x+6*c)-6288*sin(7*d*x+7*c)+770*sin(
8*d*x+8*c)+720*sin(9*d*x+9*c)+105*sin(10*d*x+10*c)+173600*sin(d*x+c)+105252
0*sin(2*d*x+2*c)-11648*sin(3*d*x+3*c)+175280*sin(4*d*x+4*c))/d/(1+cos(2*d*x
+2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97

$$\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx =$$
$$-\frac{84525\,a^3dx\cos(dx+c)^2+3360\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1)-3360\,a^3\cos(dx+c)^2\log(-\sin(}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/13440*(84525*a^3*d*x*cos(d*x + c)^2 + 3360*a^3*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - 3360*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - (1680*a^3*co
s(d*x + c)^9 + 5760*a^3*cos(d*x + c)^8 - 280*a^3*cos(d*x + c)^7 - 22656*a^3
*cos(d*x + c)^6 - 20510*a^3*cos(d*x + c)^5 + 32512*a^3*cos(d*x + c)^4 + 633
```

```
15*a^3*cos(d*x + c)^3 - 15616*a^3*cos(d*x + c)^2 + 40320*a^3*cos(d*x + c) +
 6720*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**8,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.39

$$\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx = -\frac{1536\left(30\sin\left(dx+c\right)^7+42\sin\left(dx+c\right)^5+70\sin\left(dx+c\right)^3-105\log\left(\sin\left(dx+c\right)+1\right)+105\log\left(\sin\right.\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="maxima")
[Out] -1/107520*(1536*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3
- 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))
*a^3 - 1792*(12*sin(d*x + c)^5 + 40*sin(d*x + c)^3 - 30*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 18
0*sin(d*x + c))*a^3 - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(
8*d*x + 8*c) + 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^3 + 6720*(105
*d*x + 105*c - (87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x + c))/(
tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*x + c)
)*a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.16

$$\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx =$$

$$-\frac{84525\,(dx+c)a^3+6720\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6720\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{13440\left(5\,a^3\right.}{}}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/13440*(84525*(d*x + c)*a^3 + 6720*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1))
 - 6720*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 13440*(5*a^3*tan(1/2*d*x +
 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*
(44205*a^3*tan(1/2*d*x + 1/2*c)^15 + 303065*a^3*tan(1/2*d*x + 1/2*c)^13 + 8
41981*a^3*tan(1/2*d*x + 1/2*c)^11 + 1123793*a^3*tan(1/2*d*x + 1/2*c)^9 + 48
7983*a^3*tan(1/2*d*x + 1/2*c)^7 - 490749*a^3*tan(1/2*d*x + 1/2*c)^5 - 26722
5*a^3*tan(1/2*d*x + 1/2*c)^3 - 44205*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^2 + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 14.84 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.52

$$\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx$$

$$=\frac{-\frac{741\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{64}-\frac{12469\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{192}-\frac{5027\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{40}-\frac{19211\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{420}+\frac{199977\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{1120}+\frac{87}{}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-28\tan\right.}$$

$$-\frac{a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{805\,a^3\,x}{128}$$

```
[In] int(sin(c + d*x)^8*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((4967*a^3*tan(c/2 + (d*x)/2)^3)/64 + (6243*a^3*tan(c/2 + (d*x)/2)^5)/40 +
(10233*a^3*tan(c/2 + (d*x)/2)^7)/140 + (877061*a^3*tan(c/2 + (d*x)/2)^9)/33
60 + (199977*a^3*tan(c/2 + (d*x)/2)^11)/1120 - (19211*a^3*tan(c/2 + (d*x)/2
)^13)/420 - (5027*a^3*tan(c/2 + (d*x)/2)^15)/40 - (12469*a^3*tan(c/2 + (d*x
)/2)^17)/192 - (741*a^3*tan(c/2 + (d*x)/2)^19)/64 + (869*a^3*tan(c/2 + (d*x
)/2))/64)/(d*(6*tan(c/2 + (d*x)/2)^2 + 13*tan(c/2 + (d*x)/2)^4 + 8*tan(c/2
+ (d*x)/2)^6 - 14*tan(c/2 + (d*x)/2)^8 - 28*tan(c/2 + (d*x)/2)^10 - 14*tan(
c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + 13*tan(c/2 + (d*x)/2)^16 + 6*
tan(c/2 + (d*x)/2)^18 + tan(c/2 + (d*x)/2)^20 + 1)) - (a^3*atanh(tan(c/2 +
(d*x)/2)))/d - (805*a^3*x)/128
```

3.49 $\int (a+a\sec(c+dx))^3\sin^6(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 182

$$\begin{aligned}\int (a+a\sec(c+dx))^3\sin^6(c+dx)\,dx = &-\frac{85a^3x}{16}+\frac{a^3\operatorname{arctanh}(\sin(c+dx))}{2d}\\ &-\frac{a^3\sin(c+dx)}{d}+\frac{43a^3\cos(c+dx)\sin(c+dx)}{16d}\\ &-\frac{5a^3\cos^3(c+dx)\sin(c+dx)}{24d}\\ &-\frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d}\\ &-\frac{2a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^5(c+dx)}{5d}\\ &+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

```
[Out] -85/16*a^3*x+1/2*a^3*arctanh(sin(d*x+c))/d-a^3*sin(d*x+c)/d+43/16*a^3*cos(d
*x+c)*sin(d*x+c)/d-5/24*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a^3*cos(d*x+c)^5*
sin(d*x+c)/d-2/3*a^3*sin(d*x+c)^3/d-3/5*a^3*sin(d*x+c)^5/d+3*a^3*tan(d*x+c)
/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

$= \{3957, 2951, 2717, 2715, 8, 2713, 3852, 3853, 3855\}$

$$\int (a+a\sec(c+dx))^3\sin^6(c+dx)\,dx = \frac{a^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{3a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}-\frac{a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}-\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{43a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{85a^3x}{16}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] (-85*a^3*x)/16 + (a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (a^3*Sin[c + d*x])/d +
 (43*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (5*a^3*Cos[c + d*x]^3*Sin[c +
d*x])/(24*d) - (a^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^3*Sin[c + d*x
]^3)/(3*d) - (3*a^3*Sin[c + d*x]^5)/(5*d) + (3*a^3*Tan[c + d*x])/d + (a^3*S
ec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3\sin^3(c+dx)\tan^3(c+dx)\,dx \\
&= -\frac{\int \left(8a^9+6a^9\cos(c+dx)-6a^9\cos^2(c+dx)-8a^9\cos^3(c+dx)+3a^9\cos^5(c+dx)+a^9\cos^6(c-\right.}{a^6} \\
&= -8a^3x-a^3\int\cos^6(c+dx)\,dx+a^3\int\sec^3(c+dx)\,dx \\
&\quad-\left(3a^3\right)\int\cos^5(c+dx)\,dx+\left(3a^3\right)\int\sec^2(c+dx)\,dx-\left(6a^3\right)\int\cos(c+dx)\,dx \\
&\quad+\left(6a^3\right)\int\cos^2(c+dx)\,dx+\left(8a^3\right)\int\cos^3(c+dx)\,dx
\end{aligned}
$$

$$= -8a^3x - \frac{6a^3\sin(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin(c+dx)}{d} - \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a^3\int\sec(c+dx)\,dx - \frac{1}{6}(5a^3)\int\cos^4(c+dx)\,dx + (3a^3)\int 1\,dx - \frac{(3a^3)\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} + \frac{(3a^3)\,\text{Subst}\left(\int(1-2x^2+x^4)\,dx, x, -\sin(c+dx)\right)}{d} - \frac{(8a^3)\,\text{Subst}\left(\int(1-x^2)\,dx, x, -\sin(c+dx)\right)}{d}$$

$$= -5a^3x + \frac{a^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{a^3\sin(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin(c+dx)}{d} - \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{2a^3\sin^3(c+dx)}{3d} - \frac{3a^3\sin^5(c+dx)}{5d} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{8}(5a^3)\int\cos^2(c+dx)\,dx$$

$$= -5a^3x + \frac{a^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{a^3\sin(c+dx)}{d} + \frac{43a^3\cos(c+dx)\sin(c+dx)}{16d} - \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{2a^3\sin^3(c+dx)}{3d} - \frac{3a^3\sin^5(c+dx)}{5d} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{16}(5a^3)\int 1\,dx$$

$$= -\frac{85a^3x}{16} + \frac{a^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{a^3\sin(c+dx)}{d} + \frac{43a^3\cos(c+dx)\sin(c+dx)}{16d} - \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{2a^3\sin^3(c+dx)}{3d} - \frac{3a^3\sin^5(c+dx)}{5d} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.75

$$\int(a+a\sec(c+dx))^3\sin^6(c+dx)\,dx = -\frac{a^3\sec^2(c+dx)\left(10200c+10200dx-1920\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+10200(c+dx)\cos(2(c+\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] -1/3840*(a^3*Sec[c + d*x]^2*(10200*c + 10200*d*x - 1920*ArcTanh[Sin[c + d*x
]]*Cos[c + d*x]^2 + 10200*(c + d*x)*Cos[2*(c + d*x)] - 460*Sin[c + d*x] - 8
```

```
145*Sin[2*(c + d*x)] + 1156*Sin[3*(c + d*x)] - 1120*Sin[4*(c + d*x)] - 268*
Sin[5*(c + d*x)] + 55*Sin[6*(c + d*x)] + 36*Sin[7*(c + d*x)] + 5*Sin[8*(c +
 d*x)]))/d
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09

method	result
parallelrisch	$\frac{a^3\left(-10200dx\cos(2dx+2c)-960\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)+960\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)-10200dx+4\right.}{}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^7}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{2}+\frac{5\sin(dx+c)^3}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^7}{\cos(dx+c)}+\left(\sin(dx+c)^5+\right.\right.}{}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^7}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{2}+\frac{5\sin(dx+c)^3}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^7}{\cos(dx+c)}+\left(\sin(dx+c)^5+\right.\right.}{}$
parts	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^7}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{2}+\frac{5\sin(dx+c)^3}{6}+\right.}{d}$
risch	$-\frac{85a^3x}{16}+\frac{17ia^3\mathrm{e}^{-3i(dx+c)}}{96d}+\frac{81ia^3\mathrm{e}^{-2i(dx+c)}}{128d}-\frac{81ia^3\mathrm{e}^{2i(dx+c)}}{128d}+\frac{15ia^3\mathrm{e}^{i(dx+c)}}{16d}-\frac{ia^3\left(\mathrm{e}^{3i(dx+c)}-6\,\mathrm{e}^{2i(dx+c}\right.}{d\left(\mathrm{e}^{2i(dx+c)}+\right.}$
norman	$\frac{-\frac{85a^3x}{16}-\frac{85a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4}-\frac{85a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{85a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4}+\frac{425a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}+\frac{85a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{4}-}{}$

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1920*a^3*(-10200*d*x*cos(2*d*x+2*c)-960*ln(tan(1/2*d*x+1/2*c)-1)*cos(2*d*
x+2*c)+960*ln(tan(1/2*d*x+1/2*c)+1)*cos(2*d*x+2*c)-10200*d*x+460*sin(d*x+c)
+8145*sin(2*d*x+2*c)+1120*sin(4*d*x+4*c)-55*sin(6*d*x+6*c)-5*sin(8*d*x+8*c)
-36*sin(7*d*x+7*c)+268*sin(5*d*x+5*c)-1156*sin(3*d*x+3*c)-960*ln(tan(1/2*d*
x+1/2*c)-1)+960*ln(tan(1/2*d*x+1/2*c)+1))/d/(1+cos(2*d*x+2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.97

$$\int(a+a\sec(c+dx))^3\sin^6(c+dx)\,dx =$$

$$-\frac{1275\,a^3dx\cos\left(dx+c\right)^2-60\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+60\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+\right.\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/240*(1275*a^3*d*x*cos(d*x + c)^2 - 60*a^3*cos(d*x + c)^2*log(sin(d*x + c
) + 1) + 60*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + (40*a^3*cos(d*x + c
)^7 + 144*a^3*cos(d*x + c)^6 + 50*a^3*cos(d*x + c)^5 - 448*a^3*cos(d*x + c)
^4 - 645*a^3*cos(d*x + c)^3 + 544*a^3*cos(d*x + c)^2 - 720*a^3*cos(d*x + c)
 - 120*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^3 \sin^6(c + dx)\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.32

$$\int (a + a\sec(c + dx))^3 \sin^6(c + dx)\, dx =$$

$$-\frac{96\left(6\sin(dx+c)^5 + 10\sin(dx+c)^3 - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1) + 30\sin(dx\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] -1/960*(96*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^3 - 5*(4*sin(2*d*x + 2*c)^
3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 80*(4*s
in(d*x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) +
 1) + 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*a^3 + 360*(15*d*x + 15*c
- (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 +
1) - 8*tan(d*x + c))*a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.16

$$\int (a+a\sec(c+dx))^3\sin^6(c+dx)\,dx =$$

$$-\frac{1275\,(dx+c)a^3 - 120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{240\left(5\,a^3\tan(\right.}{(}}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(1275*(d*x + c)*a^3 - 120*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 1
20*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 240*(5*a^3*tan(1/2*d*x + 1/2*c)
^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(795*a^
3*tan(1/2*d*x + 1/2*c)^11 + 4025*a^3*tan(1/2*d*x + 1/2*c)^9 + 7614*a^3*tan(
1/2*d*x + 1/2*c)^7 + 5634*a^3*tan(1/2*d*x + 1/2*c)^5 - 345*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 315*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/
d
```

Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.43

$$\int (a+a\sec(c+dx))^3\sin^6(c+dx)\,dx = \frac{a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{85\,a^3\,x}{16}$$

$$+\frac{-\frac{93\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{8} - \frac{1039\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{24} - \frac{4319\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{120} + \frac{6169\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{120} + \frac{3933\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{40} + \cdots}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16} + 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} + 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} - 4\tan \cdots\right.}$$

```
[In] int(sin(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*atanh(tan(c/2 + (d*x)/2)))/d - (85*a^3*x)/16 + ((277*a^3*tan(c/2 + (d*
x)/2)^3)/8 + (997*a^3*tan(c/2 + (d*x)/2)^5)/40 + (3933*a^3*tan(c/2 + (d*x)/
2)^7)/40 + (6169*a^3*tan(c/2 + (d*x)/2)^9)/120 - (4319*a^3*tan(c/2 + (d*x)/
2)^11)/120 - (1039*a^3*tan(c/2 + (d*x)/2)^13)/24 - (93*a^3*tan(c/2 + (d*x)/
2)^15)/8 + (77*a^3*tan(c/2 + (d*x)/2))/8)/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*ta
n(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^6 - 10*tan(c/2 + (d*x)/2)^8 - 4*t
an(c/2 + (d*x)/2)^10 + 4*tan(c/2 + (d*x)/2)^12 + 4*tan(c/2 + (d*x)/2)^14 +
tan(c/2 + (d*x)/2)^16 + 1))
```

3.50 $\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx = -\frac{33a^3x}{8}+\frac{3a^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{2a^3\sin(c+dx)}{d}+\frac{7a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a^3\sin^3(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] -33/8*a^3*x+3/2*a^3*arctanh(sin(d*x+c))/d-2*a^3*sin(d*x+c)/d+7/8*a^3*cos(d*
x+c)*sin(d*x+c)/d+1/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d+3*a^
3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3957, 2951, 2717, 2715, 8, 2713, 3855, 3852, 3853}

$$\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx = \frac{3a^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{a^3\sin^3(c+dx)}{d}-\frac{2a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{33a^3x}{8}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (-33*a^3*x)/8 + (3*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (2*a^3*Sin[c + d*x])/
d + (7*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3\sin(c+dx)\tan^3(c+dx)\,dx \\
&= -\frac{\int \left(5a^7+5a^7\cos(c+dx)-a^7\cos^2(c+dx)-3a^7\cos^3(c+dx)-a^7\cos^4(c+dx)-a^7\sec(c+dx)\right.}{a^4} \\
&= -5a^3x+a^3\int\cos^2(c+dx)\,dx+a^3\int\cos^4(c+dx)\,dx \\
&\quad +a^3\int\sec(c+dx)\,dx+a^3\int\sec^3(c+dx)\,dx+\left(3a^3\right)\int\cos^3(c+dx)\,dx \\
&\quad +\left(3a^3\right)\int\sec^2(c+dx)\,dx-\left(5a^3\right)\int\cos(c+dx)\,dx \\
&= -5a^3x+\frac{a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{5a^3\sin(c+dx)}{d} \\
&\quad +\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d} \\
&\quad +\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a^3\int 1\,dx+\frac{1}{2}a^3\int\sec(c+dx)\,dx \\
&\quad +\frac{1}{4}\left(3a^3\right)\int\cos^2(c+dx)\,dx-\frac{\left(3a^3\right)\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d} \\
&\quad -\frac{\left(3a^3\right)\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d} \\
&= -\frac{9a^3x}{2}+\frac{3a^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{2a^3\sin(c+dx)}{d} \\
&\quad +\frac{7a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a^3\sin^3(c+dx)}{d} \\
&\quad +\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{8}\left(3a^3\right)\int 1\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{33a^3x}{8} + \frac{3a^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{2a^3\sin(c+dx)}{d} \\
&\quad + \frac{7a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d} \\
&\quad - \frac{a^3\sin^3(c+dx)}{d} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$
\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx
$$
$$
= \frac{a^3\sec^2(c+dx)\left(-264c-264dx+192\text{arctanh}(\sin(c+dx))\cos^2(c+dx)-264(c+dx)\cos(2(c+dx))-\right.}{}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(-264*c - 264*d*x + 192*ArcTanh[Sin[c + d*x]]*Cos[c + d
*x]^2 - 264*(c + d*x)*Cos[2*(c + d*x)] - 16*Sin[c + d*x] + 225*Sin[2*(c + d
*x)] - 72*Sin[3*(c + d*x)] + 18*Sin[4*(c + d*x)] + 8*Sin[5*(c + d*x)] + Sin
[6*(c + d*x)]))/(128*d)
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{\left((-12\cos(2dx+2c)-12)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(12\cos(2dx+2c)+12)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-33dx\cos(2dx+2c)-33dx+\text{s}\right.}{8d(1+\cos(2dx+2c))}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\text{co}\right.}{}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\text{co}\right.}{}$
parts	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\text{tan}}{2}\right.}{d}$
risch	$-\frac{33a^3x}{8}-\frac{ia^3e^{3i(dx+c)}}{8d}-\frac{ia^3e^{2i(dx+c)}}{4d}+\frac{11ia^3e^{i(dx+c)}}{8d}-\frac{11ia^3e^{-i(dx+c)}}{8d}+\frac{ia^3e^{-2i(dx+c)}}{4d}+\frac{ia^3e^{-3i(dx+c)}}{8d}$
norman	$\frac{-\frac{33a^3x}{8}-\frac{33a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4}+\frac{33a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8}+\frac{33a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}+\frac{33a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}-\frac{33a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{4}-\frac{33a^3}{}}{\left(-1+\tan\left(\frac{dx}{2}\right.\right.}$

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((-12*cos(2*d*x+2*c)-12)*ln(tan(1/2*d*x+1/2*c)-1)+(12*cos(2*d*x+2*c)+12
)*ln(tan(1/2*d*x+1/2*c)+1)-33*d*x*cos(2*d*x+2*c)-33*d*x+sin(5*d*x+5*c)+1/8*
sin(6*d*x+6*c)-2*sin(d*x+c)+225/8*sin(2*d*x+2*c)-9*sin(3*d*x+3*c)+9/4*sin(4
*d*x+4*c))*a^3/d/(1+cos(2*d*x+2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

$$\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx =$$
$$-\frac{33\,a^3dx\cos\left(dx+c\right)^2-6\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+6\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/8*(33*a^3*d*x*cos(d*x + c)^2 - 6*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1
) + 6*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - (2*a^3*cos(d*x + c)^5 + 8
*a^3*cos(d*x + c)^4 + 7*a^3*cos(d*x + c)^3 - 24*a^3*cos(d*x + c)^2 + 24*a^3
*cos(d*x + c) + 4*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx = a^3\left(\int 3\sin^4\left(c+dx\right)\sec\left(c+dx\right)dx + \int 3\sin^4\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \sin^4\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int \sin^4\left(c+dx\right)dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**4,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)**4*sec(c + d*x), x) + Integral(3*sin(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(sin(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.32

$$\int (a + a\sec(c + dx))^3 \sin^4(c + dx)\,dx =$$

$$-\frac{16\left(2\,\sin\left(dx + c\right)^3 - 3\,\log\left(\sin\left(dx + c\right) + 1\right) + 3\,\log\left(\sin\left(dx + c\right) - 1\right) + 6\,\sin\left(dx + c\right)\right)a^3 - (12\,dx + \cdots}{\cdots}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/32*(16*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*
x + 2*c))*a^3 + 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan
(d*x + c))*a^3 + 8*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x
 + c) + 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.30

$$\int (a + a\sec(c + dx))^3 \sin^4(c + dx)\,dx =$$

$$-\frac{33\,(dx + c)a^3 - 12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + 12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{8\left(5\,a^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\cdots\right.\right.}{\left(\tan\left(\frac{1}{2}\cdots\right.\right.}\cdots}{8\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/8*(33*(d*x + c)*a^3 - 12*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 12*a^3
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 8*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a
^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(25*a^3*tan(1/2
*d*x + 1/2*c)^7 + 81*a^3*tan(1/2*d*x + 1/2*c)^5 + 79*a^3*tan(1/2*d*x + 1/2*
c)^3 + 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.48

$$\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx=\frac{3\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{33\,a^3\,x}{8}$$
$$+\frac{-\frac{45\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{4}-\frac{83\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4}+\frac{25\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2}+\frac{79\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{2}+\frac{27\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{4}+\frac{21\,a^3\,\tan\left(\frac{c}{2}\right.}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+2\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (3*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (33*a^3*x)/8 + ((27*a^3*tan(c/2 + (d*
x)/2)^3)/4 + (79*a^3*tan(c/2 + (d*x)/2)^5)/2 + (25*a^3*tan(c/2 + (d*x)/2)^7
)/2 - (83*a^3*tan(c/2 + (d*x)/2)^9)/4 - (45*a^3*tan(c/2 + (d*x)/2)^11)/4 +
(21*a^3*tan(c/2 + (d*x)/2))/4)/(d*(2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)
/2)^4 - 4*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 2*tan(c/2 + (d*x)/2
)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.51 $\int (a+a\sec(c+dx))^3 \sin^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx = -\frac{5a^3x}{2}+\frac{5a^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{3a^3\sin(c+dx)}{d} - \frac{a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] -5/2*a^3*x+5/2*a^3*arctanh(sin(d*x+c))/d-3*a^3*sin(d*x+c)/d-1/2*a^3*cos(d*x
+c)*sin(d*x+c)/d+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3957, 2951, 2717, 2715, 8, 3855, 3852, 3853}

$$\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx = \frac{5a^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{3a^3\sin(c+dx)}{d} + \frac{3a^3\tan(c+dx)}{d} - \frac{a^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{a^3\tan(c+dx)\sec(c+dx)}{2d} - \frac{5a^3x}{2}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (-5*a^3*x)/2 + (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^3*Sin[c + d*x])/d
 - (a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a^3*Tan[c + d*x])/d + (a^3*Se
c[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3 \sec(c+dx)\tan^2(c+dx)\,dx \\
&= -\frac{\int (2a^5+3a^5\cos(c+dx)+a^5\cos^2(c+dx)-2a^5\sec(c+dx)-3a^5\sec^2(c+dx)-a^5\sec^3(c+a}{a^2} \\
&= -2a^3x-a^3\int \cos^2(c+dx)\,dx+a^3\int \sec^3(c+dx)\,dx \\
&\quad +\left(2a^3\right)\int \sec(c+dx)\,dx-\left(3a^3\right)\int \cos(c+dx)\,dx+\left(3a^3\right)\int \sec^2(c+dx)\,dx \\
&= -2a^3x+\frac{2a^3\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{3a^3\sin(c+dx)}{d} \\
&\quad -\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}a^3\int 1\,dx \\
&\quad +\frac{1}{2}a^3\int \sec(c+dx)\,dx-\frac{\left(3a^3\right)\operatorname{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d} \\
&= -\frac{5a^3x}{2}+\frac{5a^3\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{3a^3\sin(c+dx)}{d} \\
&\quad -\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 300 vs. $2(98) = 196$.

Time = 2.20 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.06

$$\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx$$
$$= \frac{1}{32}a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-10x - \frac{10\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{10\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{12\cos(dx)\sin(c)}{d} - \frac{\cos(2dx)\sin(2c)}{d} - \frac{12\cos(c)\sin(dx)}{d} - \frac{\cos(2c)\sin(2dx)}{d} + \frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{12\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{12\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-10*x - (10*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/d + (10*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/
d - (12*Cos[d*x]*Sin[c])/d - (Cos[2*d*x]*Sin[2*c])/d - (12*Cos[c]*Sin[d*x])
/d - (Cos[2*c]*Sin[2*d*x])/d + 1/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2
) + (12*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (12*Sin[(d*x)/
2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/32
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3(\tan(dx+c)-dx-c)+3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan}{d}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3(\tan(dx+c)-dx-c)+3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan}{d}$
parts	$\frac{a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{3a^3(-\sin(dx+c)+\ln}{}$
parallelrisch	$\frac{a^3\left(-20(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-20dx\cos(2dx+2c)+20\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)-20dx+20\ln\left(\right.\right.}{8d(1+\cos(2dx+2c))}$
norman	$\frac{-\frac{5a^3x}{2}+5a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\frac{5a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{2}+\frac{18a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{10a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+$
risch	$-\frac{5a^3x}{2}+\frac{ia^3\mathrm{e}^{2i(dx+c)}}{8d}+\frac{3ia^3\mathrm{e}^{i(dx+c)}}{2d}-\frac{3ia^3\mathrm{e}^{-i(dx+c)}}{2d}-\frac{ia^3\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{ia^3\left(\mathrm{e}^{3i(dx+c)}-6\,\mathrm{e}^{2i(dx+c)}-\mathrm{e}^{i(dx+}\right.}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}$

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+3*a^3*(tan(d*x+c)-d*x-c)+3*a^3*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*
x+c)))+a^3*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx = -\frac{10\,a^3dx\cos(dx+c)^2-5\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1)+5\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+}{4\,d\cos(dx+c)^2}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(10*a^3*d*x*cos(d*x + c)^2 - 5*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1
) + 5*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c)^3 + 6
*a^3*cos(d*x + c)^2 - 6*a^3*cos(d*x + c) - a^3)*sin(d*x + c))/(d*cos(d*x +
c)^2)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx = a^3\left(\int 3\sin^2{(c+dx)}\sec{(c+dx)}\,dx + \int 3\sin^2{(c+dx)}\sec^2{(c+dx)}\,dx + \int \sin^2{(c+dx)}\sec^3{(c+dx)}\,dx + \int \sin^2{(c+dx)}\,dx\right)$$

[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**2,x)

[Out] a**3*(Integral(3*sin(c + d*x)**2*sec(c + d*x), x) + Integral(3*sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2*sec(c + d*x)**3, x) + Integral(sin(c + d*x)**2, x))

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.30

$$\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx = \frac{(2\,dx+2\,c-\sin{(2\,dx+2\,c)})a^3-12\,(dx+c-\tan{(dx+c)})a^3-a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log{(\sin{(dx+c)}+1)}-\right.}{4\,d}$$

[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^3 - 12*(d*x + c - tan(d*x + c))*a^3 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx =$$

$$-\frac{5\,(dx+c)a^3-5\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+5\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}}{2\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)*a^3 - 5*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 5*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^3*
tan(1/2*d*x + 1/2*c)^3)/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx = \frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{5\,a^3\,x}{2}+\frac{18\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+1\right)}$$

```
[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*x)/2 + (18*a^3*tan(c/2 + (d*x)
/2)^3 - 10*a^3*tan(c/2 + (d*x)/2)^7)/(d*(tan(c/2 + (d*x)/2)^8 - 2*tan(c/2 +
 (d*x)/2)^4 + 1))
```

3.52 $\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{9a^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 9/2*a^3*arctanh(sin(d*x+c))/d-4*a^3*sin(d*x+c)/d/(1-cos(d*x+c))+3*a^3*tan(d
*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2951, 2727, 3855, 3852, 8, 3853}

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{9a^3\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{3a^3\tan(c+dx)}{d} - \frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))} + \frac{a^3\tan(c+dx)\sec(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (9*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (4*a^3*Sin[c + d*x])/(d*(1 - Cos[c +
d*x])) + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int (-a - a\cos(c+dx))^3 \csc^2(c+dx)\sec^3(c+dx)\,dx\\ &= a^2\int \left(\frac{4a}{1-\cos(c+dx)} + 4a\sec(c+dx) + 3a\sec^2(c+dx) + a\sec^3(c+dx)\right)\,dx\end{aligned}$$

$$
\begin{aligned}
&= a^3\int \sec^3(c+dx)\,dx + \left(3a^3\right)\int \sec^2(c+dx)\,dx \\
&\quad + \left(4a^3\right)\int \frac{1}{1-\cos(c+dx)}\,dx + \left(4a^3\right)\int \sec(c+dx)\,dx \\
&= \frac{4a^3\text{arctanh}(\sin(c+dx))}{d} - \frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} \\
&\quad + \frac{1}{2}a^3\int \sec(c+dx)\,dx - \frac{\left(3a^3\right)\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d} \\
&= \frac{9a^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))} \\
&\quad + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. $2(80) = 160$.

Time = 1.52 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.05

$$
\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx
$$

$$
= \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+18\log\left(\cos\left(\frac{1}{2}(c+da\right.\right.\right.}{}
$$

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-18*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 18*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 16*Csc[c/2]
*Csc[(c + d*x)/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2)
 - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2) + (12*Sin[d*x])/((Cos[c/2] -
Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]))))/(32*d)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

method	result
parallelrisch	$\frac{9a^3\left((-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{10\left(\cos(dx+c)-\frac{7\cos(2dx+2c)}{5}-\frac{6}{5}\right.}{9}\right.}{2d(1+\cos(2dx+2c))}$
norman	$\frac{-\frac{4a^3}{d}+\frac{15a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{9a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{9a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{9a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{ia^3\left(9\,e^{4i(dx+c)}-7\,e^{3i(dx+c)}+21\,e^{2i(dx+c)}-5\,e^{i(dx+c)}+14\right)}{d\left(e^{i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)^2}+\frac{9a^3\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{9a^3\ln\left(e^{i(dx+c)}-i\right)}{2d}$
derivativedivides	$\frac{a^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^3\left(-\frac{}{\sin(}\right.}{d}$
default	$\frac{a^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^3\left(-\frac{}{\sin(}\right.}{d}$

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 9/2*a^3*((-cos(2*d*x+2*c)-1)*ln(tan(1/2*d*x+1/2*c)-1)+(1+cos(2*d*x+2*c))*ln
(tan(1/2*d*x+1/2*c)+1)+10/9*(cos(d*x+c)-7/5*cos(2*d*x+2*c)-6/5)*cot(1/2*d*x
+1/2*c))/d/(1+cos(2*d*x+2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.52

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{9\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1)\sin(dx+c)-9\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+1)\sin(dx+c}{4\,d\cos(dx+c)^2\sin(dx+c)}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(9*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)*sin(d*x + c) - 9*a^3*cos(d*
x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) - 28*a^3*cos(d*x + c)^3 - 18*a
^3*cos(d*x + c)^2 + 12*a^3*cos(d*x + c) + 2*a^3)/(d*cos(d*x + c)^2*sin(d*x
+ c))
```

Sympy [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\csc^2(c+dx)\sec(c+dx)\,dx + \int 3\csc^2(c+dx)\sec^2(c+dx)\,dx + \int \csc^2(c+dx)\sec^3(c+dx)\,dx + \int \csc^2(c+dx)\,dx\right)$$

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*csc(c + d*x)**2*sec(c + d*x), x) + Integral(3*csc(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(csc(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.71

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\left(\frac{2\left(3\sin(dx+c)^2-2\right)}{\sin(dx+c)^3-\sin(dx+c)} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right) + 6a^3\left(\frac{2}{\sin(dx+c)} - \log(\sin(d\right.}{4d}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a^3*(2*(3*sin(d*x + c)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 6*a^3*(2/sin(d*x + c) - log
(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3*(1/tan(d*x + c) - tan(
d*x + c)) + 4*a^3/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.32

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{9\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{8\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\cdots\right.}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(9*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a^3*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 8*a^3/tan(1/2*d*x + 1/2*c) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^
3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{9\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{9\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a^3}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^2,x)
```

```
[Out] (9*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (9*a^3*tan(c/2 + (d*x)/2)^4 - 15*a^3*
tan(c/2 + (d*x)/2)^2 + 4*a^3)/(d*(tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^5))
```

3.53 $\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{11a^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2} - \frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 11/2*a^3*arctanh(sin(d*x+c))/d-2/3*a^3*sin(d*x+c)/d/(1-cos(d*x+c))^2-17/3*a
^3*sin(d*x+c)/d/(1-cos(d*x+c))+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*
x+c)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3957, 2951, 2729, 2727, 3855, 3852, 8, 3853}

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{11a^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{3a^3\tan(c+dx)}{d} - \frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))} - \frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2} + \frac{a^3\tan(c+dx)\sec(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (11*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (2*a^3*Sin[c + d*x])/(3*d*(1 - Cos[c
 + d*x])^2) - (17*a^3*Sin[c + d*x])/(3*d*(1 - Cos[c + d*x])) + (3*a^3*Tan[c
 + d*x])/d + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a - a\cos(c+dx))^3 \csc^4(c+dx)\sec^3(c+dx)\,dx \\
&= a^4\int \left(\frac{2}{a(1-\cos(c+dx))^2} + \frac{5}{a(1-\cos(c+dx))} + \frac{5\sec(c+dx)}{a} + \frac{3\sec^2(c+dx)}{a} + \frac{\sec^3(c+dx)}{a}\right) dx \\
&= a^3\int \sec^3(c+dx)\,dx + \left(2a^3\right)\int \frac{1}{(1-\cos(c+dx))^2}\,dx + \left(3a^3\right)\int \sec^2(c+dx)\,dx \\
&\quad + \left(5a^3\right)\int \frac{1}{1-\cos(c+dx)}\,dx + \left(5a^3\right)\int \sec(c+dx)\,dx \\
&= \frac{5a^3\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2} - \frac{5a^3\sin(c+dx)}{d(1-\cos(c+dx))} \\
&\quad + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a^3\int \sec(c+dx)\,dx \\
&\quad + \frac{1}{3}\left(2a^3\right)\int \frac{1}{1-\cos(c+dx)}\,dx - \frac{\left(3a^3\right)\operatorname{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d} \\
&= \frac{11a^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2} \\
&\quad - \frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 290 vs. 2(110) = 220.

Time = 6.71 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.64

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-4\cot\left(\frac{c}{2}\right)\csc^2\left(\frac{1}{2}(c+dx)\right) - 66\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-4*Cot[c/2]*Csc[(c + d*x)/2]^
2 - 66*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 66*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] - 2*(-19 + 17*Cos[c + d*x])*Csc[c/2]*Csc[(c + d*x)/2]^3*
Sin[(d*x)/2] + 3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (36*Sin[(d*x)/2]
)/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - 3/(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^2 + (36*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(96*d)
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

method	result
parallelrisch	$-\frac{11a^3\Big((1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{17\csc\left(\frac{dx}{2}+\frac{c}{2}\right)^2\big(\cos(dx+c)-}{}}{2d(1+\cos(2dx+2c))}$
norman	$\frac{-\frac{a^3}{3d}-\frac{16a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}+\frac{56a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}-\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{11a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{11a^3\ln\big(\tan\big(\frac{dx}{2}+\frac{c}{2}}{2d}$
risch	$-\frac{ia^3\left(33\,e^{6i(dx+c)}-99\,e^{5i(dx+c)}+154\,e^{4i(dx+c)}-210\,e^{3i(dx+c)}+161\,e^{2i(dx+c)}-123\,e^{i(dx+c)}+52\right)}{3d\left(e^{i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)^2}-\frac{11a^3\ln\big(e^{i(dx+c)}-}{2d}$
derivativedivides	$\frac{a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)^2}+\frac{5}{6\sin(dx+c)\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)}+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\big(-\frac{1}{3\sin(dx+c)^3\cos(dx}}{}$
default	$\frac{a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)^2}+\frac{5}{6\sin(dx+c)\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)}+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\big(-\frac{1}{3\sin(dx+c)^3\cos(dx}}{}$

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -11/2*a^3*((1+cos(2*d*x+2*c))*ln(tan(1/2*d*x+1/2*c)-1)+(-cos(2*d*x+2*c)-1)*
ln(tan(1/2*d*x+1/2*c)+1)-17/11*csc(1/2*d*x+1/2*c)^2*(cos(d*x+c)-71/102*cos(
2*d*x+2*c)+13/51*cos(3*d*x+3*c)-65/102)*cot(1/2*d*x+1/2*c))/d/(1+cos(2*d*x+
2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.62

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{104\,a^3\cos(dx+c)^4-38\,a^3\cos(dx+c)^3-118\,a^3\cos(dx+c)^2+30\,a^3\cos(dx+c)+6\,a^3-33\,\big(a^3\cos}{12\,\big(d}$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(104*a^3*cos(d*x + c)^4 - 38*a^3*cos(d*x + c)^3 - 118*a^3*cos(d*x + c
)^2 + 30*a^3*cos(d*x + c) + 6*a^3 - 33*(a^3*cos(d*x + c)^3 - a^3*cos(d*x +
c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 33*(a^3*cos(d*x + c)^3 - a^3*cos
(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^3 - d*co
s(d*x + c)^2)*sin(d*x + c))
```

Sympy [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\csc^4(c+dx)\sec(c+dx)\,dx + \int 3\csc^4(c+dx)\sec^2(c+dx)\,dx + \int \csc^4(c+dx)\sec^3(c+dx)\,dx + \int \csc^4(c+dx)\,dx\right)$$

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*csc(c + d*x)**4*sec(c + d*x), x) + Integral(3*csc(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(csc(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.71

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\left(\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2-2\right)}{\sin(dx+c)^5-\sin(dx+c)^3} - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1)\right) + 6a^3\left(\frac{2\left(3\sin(d\right.}{\sin(d}\right.}{}$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(a^3*(2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 -
 sin(d*x + c)^3) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6
*a^3*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3
*log(sin(d*x + c) - 1)) + 12*a^3*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3
*tan(d*x + c)) + 4*(3*tan(d*x + c)^2 + 1)*a^3/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{33\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-33\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{6\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{6\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(33*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 33*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 6*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - 2*(18*a^3*tan(1/2*d*x + 1/2*c)^2 + a^
3)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 18.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{11\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{56\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{16\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{a^3}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^4,x)
```

```
[Out] (11*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((16*a^3*tan(c/2 + (d*x)/2)^2)/3 - (
56*a^3*tan(c/2 + (d*x)/2)^4)/3 + 11*a^3*tan(c/2 + (d*x)/2)^6 + a^3/3)/(d*(t
an(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^7))
```

3.54 $\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{13a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{152a^3\tan(c+dx)}{15d}+\frac{13a^3\sec(c+dx)\tan(c+dx)}{2d}-\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3}-\frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2}-\frac{76a^6\sec(c+dx)\tan(c+dx)}{15d\,(a^3-a^3\cos(c+dx))}$$

```
[Out] 13/2*a^3*arctanh(sin(d*x+c))/d+152/15*a^3*tan(d*x+c)/d+13/2*a^3*sec(d*x+c)*
tan(d*x+c)/d-1/5*a^6*sec(d*x+c)*tan(d*x+c)/d/(a-a*cos(d*x+c))^3-11/15*a^5*s
ec(d*x+c)*tan(d*x+c)/d/(a-a*cos(d*x+c))^2-76/15*a^6*sec(d*x+c)*tan(d*x+c)/d
/(a^3-a^3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{3957, 2948, 2845, 3057, 2827, 3853, 3855, 3852, 8\}$

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^6\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\tan(c+dx)\sec(c+dx)}{15d(a-a\cos(c+dx))^2} + \frac{13a^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{152a^3\tan(c+dx)}{15d} + \frac{13a^3\tan(c+dx)\sec(c+dx)}{2d} - \frac{76a^6\tan(c+dx)\sec(c+dx)}{15d\,(a^3-a^3\cos(c+dx))}$$

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (152*a^3*Tan[c + d*x])/(15*d) + (13*
a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (a^6*Sec[c + d*x]*Tan[c + d*x])/(5*d
*(a - a*Cos[c + d*x])^3) - (11*a^5*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a - a*
Cos[c + d*x])^2) - (76*a^6*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 - a^3*Cos[
c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
```

```
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int (-a - a\cos(c+dx))^3 \csc^6(c+dx)\sec^3(c+dx)\,dx$$

$$= -\left(a^6 \int \frac{\sec^3(c+dx)}{(-a+a\cos(c+dx))^3}\,dx\right)$$

$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{1}{5}a^4\int\frac{(-7a-4a\cos(c+dx))\sec^3(c+dx)}{(-a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{1}{15}a^2\int\frac{(43a^2+33a^2\cos(c+dx))\sec^3(c+dx)}{-a+a\cos(c+dx)}\,dx$$

$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{76a^4\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))} - \frac{1}{15}\int(-195a^3-152a^3\cos(c+dx))\sec^3(c+dx)\,dx$$

$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{76a^4\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))} + \frac{1}{15}(152a^3)\int\sec^2(c+dx)\,dx + (13a^3)\int\sec^3(c+dx)\,dx$$

$$= \frac{13a^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{76a^4\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))} + \frac{1}{2}(13a^3)\int\sec(c+dx)\,dx - \frac{(152a^3)\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{15d}$$

$$= \frac{13a^3\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{152a^3\tan(c+dx)}{15d} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{76a^4\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 353 vs. $2(165) = 330$.

Time = 2.01 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.14

$$\int\csc^6(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(24960\cos^2(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/30720*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^2*(24960
*Cos[c + d*x]^2*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + Csc[c/2]*Csc[(c + d*x)/2]^5*Sec[c]*(-1235*Sin[
(d*x)/2] + 3805*Sin[(3*d*x)/2] + 4329*Sin[c - (d*x)/2] - 1989*Sin[c + (d*x)
/2] - 3575*Sin[2*c + (d*x)/2] + 475*Sin[c + (3*d*x)/2] + 2005*Sin[2*c + (3*
d*x)/2] + 2275*Sin[3*c + (3*d*x)/2] - 2673*Sin[c + (5*d*x)/2] + 105*Sin[2*c
 + (5*d*x)/2] - 1593*Sin[3*c + (5*d*x)/2] - 975*Sin[4*c + (5*d*x)/2] + 1325
*Sin[2*c + (7*d*x)/2] - 255*Sin[3*c + (7*d*x)/2] + 875*Sin[4*c + (7*d*x)/2]
 + 195*Sin[5*c + (7*d*x)/2] - 304*Sin[3*c + (9*d*x)/2] + 90*Sin[4*c + (9*d*
x)/2] - 214*Sin[5*c + (9*d*x)/2])))/d
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{777a^3\left(\frac{520(-\cos(2dx+2c)-1)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{777}+\frac{520(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{777}+\cot\left(\frac{dx}{2}+\frac{c}{2}\right)\csc\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(\cos(d\right.\right.}{80d(1+\cos(2dx+2c))}$
norman	$\frac{-\frac{a^3}{20d}-\frac{17a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{30d}-\frac{97a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{15d}+\frac{131a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{6d}-\frac{51a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{13a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+$
risch	$-\frac{ia^3\left(195\,e^{8i(dx+c)}-975\,e^{7i(dx+c)}+2275\,e^{6i(dx+c)}-3575\,e^{5i(dx+c)}+4329\,e^{4i(dx+c)}-3805\,e^{3i(dx+c)}+2673\,e^{2i(dx+c)}-132\right.}{15d\left(e^{i(dx+c)}-1\right)^5\left(e^{2i(dx+c)}+1\right)^2}$
derivativedivides	$\frac{a^3\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)^2}-\frac{7}{15\sin(dx+c)^3\cos(dx+c)^2}+\frac{7}{6\sin(dx+c)\cos(dx+c)^2}-\frac{7}{2\sin(dx+c)}+\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3}{}$
default	$\frac{a^3\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)^2}-\frac{7}{15\sin(dx+c)^3\cos(dx+c)^2}+\frac{7}{6\sin(dx+c)\cos(dx+c)^2}-\frac{7}{2\sin(dx+c)}+\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3}{}$

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 777/80*a^3*(520/777*(-cos(2*d*x+2*c)-1)*ln(tan(1/2*d*x+1/2*c)-1)+520/777*(1
+cos(2*d*x+2*c))*ln(tan(1/2*d*x+1/2*c)+1)+cot(1/2*d*x+1/2*c)*csc(1/2*d*x+1/
2*c)^4*(cos(d*x+c)-174/259*cos(2*d*x+2*c)+239/777*cos(3*d*x+3*c)-152/2331*c
os(4*d*x+4*c)-1354/2331))/d/(1+cos(2*d*x+2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.36

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{608\,a^3\cos(dx+c)^5 - 826\,a^3\cos(dx+c)^4 - 476\,a^3\cos(dx+c)^3 + 868\,a^3\cos(dx+c)^2 - 120\,a^3\cos(dx$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(608*a^3*cos(d*x + c)^5 - 826*a^3*cos(d*x + c)^4 - 476*a^3*cos(d*x +
c)^3 + 868*a^3*cos(d*x + c)^2 - 120*a^3*cos(d*x + c) - 30*a^3 - 195*(a^3*co
s(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1)*sin(d*x + c) + 195*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*co
s(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d
*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.38

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{a^3\left(\frac{2\left(105\,\sin(dx+c)^6-70\,\sin(dx+c)^4-14\,\sin(dx+c)^2-6\right)}{\sin(dx+c)^7-\sin(dx+c)^5} - 105\,\log(\sin(dx+c)+1) + 105\,\log(\sin(dx+c)-1)\right)$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(a^3*(2*(105*sin(d*x + c)^6 - 70*sin(d*x + c)^4 - 14*sin(d*x + c)^2 -
 6)/(sin(d*x + c)^7 - sin(d*x + c)^5) - 105*log(sin(d*x + c) + 1) + 105*log
(sin(d*x + c) - 1)) + 6*a^3*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/s
in(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 36*a
^3*((15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x +
 c)) + 4*(15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^3/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.85

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{390\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-390\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{60\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{60\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 390*a^3*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 60*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (465*a^3*tan(1/2*d*x + 1/2*c)^4 +
 40*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.82

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{13\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{51\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\frac{262\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{388\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}+\frac{34\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{a^3}{5}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^6,x)
```

```
[Out] (13*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((34*a^3*tan(c/2 + (d*x)/2)^2)/15 +
(388*a^3*tan(c/2 + (d*x)/2)^4)/15 - (262*a^3*tan(c/2 + (d*x)/2)^6)/3 + 51*a
^3*tan(c/2 + (d*x)/2)^8 + a^3/5)/(d*(4*tan(c/2 + (d*x)/2)^5 - 8*tan(c/2 + (
d*x)/2)^7 + 4*tan(c/2 + (d*x)/2)^9))
```

3.55 $\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 192

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15a^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(c+dx)}{d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{15a^3\csc(c+dx)}{2d} - \frac{5a^3\csc^3(c+dx)}{2d} - \frac{3a^3\csc^5(c+dx)}{2d} - \frac{15a^3\csc^7(c+dx)}{14d} + \frac{a^3\csc^7(c+dx)\sec^2(c+dx)}{2d} + \frac{3a^3\tan(c+dx)}{d}$$

```
[Out] 15/2*a^3*arctanh(sin(d*x+c))/d-13*a^3*cot(d*x+c)/d-7*a^3*cot(d*x+c)^3/d-3*a
^3*cot(d*x+c)^5/d-4/7*a^3*cot(d*x+c)^7/d-15/2*a^3*csc(d*x+c)/d-5/2*a^3*csc(
d*x+c)^3/d-3/2*a^3*csc(d*x+c)^5/d-15/14*a^3*csc(d*x+c)^7/d+1/2*a^3*csc(d*x+
c)^7*sec(d*x+c)^2/d+3*a^3*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{3957, 2952, 3852, 2701, 308, 213, 2700, 276, 294\}$

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15a^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{3a^3\tan(c+dx)}{d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{3a^3\cot^5(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} - \frac{13a^3\cot(c+dx)}{d} - \frac{15a^3\csc^7(c+dx)}{14d} - \frac{3a^3\csc^5(c+dx)}{2d} - \frac{5a^3\csc^3(c+dx)}{2d} - \frac{15a^3\csc(c+dx)}{2d} + \frac{a^3\csc^7(c+dx)\sec^2(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (13*a^3*Cot[c + d*x])/d - (7*a^3*Cot
[c + d*x]^3)/d - (3*a^3*Cot[c + d*x]^5)/d - (4*a^3*Cot[c + d*x]^7)/(7*d) -
(15*a^3*Csc[c + d*x])/(2*d) - (5*a^3*Csc[c + d*x]^3)/(2*d) - (3*a^3*Csc[c +
 d*x]^5)/(2*d) - (15*a^3*Csc[c + d*x]^7)/(14*d) + (a^3*Csc[c + d*x]^7*Sec[c
 + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3\csc^8(c+dx)\sec^3(c+dx)\,dx\\
&= \int \left(a^3\csc^8(c+dx)+3a^3\csc^8(c+dx)\sec(c+dx)+3a^3\csc^8(c+dx)\sec^2(c+dx)\right.\\
&\qquad \left.+a^3\csc^8(c+dx)\sec^3(c+dx)\right)\,dx\\
&= a^3\int \csc^8(c+dx)\,dx+a^3\int \csc^8(c+dx)\sec^3(c+dx)\,dx\\
&\quad +\left(3a^3\right)\int \csc^8(c+dx)\sec(c+dx)\,dx+\left(3a^3\right)\int \csc^8(c+dx)\sec^2(c+dx)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a^3\text{Subst}\left(\int \frac{x^{10}}{(-1+x^2)^2}\,dx, x, \csc(c+dx)\right)}{d}\\
&\quad - \frac{a^3\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx, x, \cot(c+dx)\right)}{d}\\
&\quad - \frac{(3a^3)\,\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}\\
&\quad + \frac{(3a^3)\,\text{Subst}\left(\int \frac{(1+x^2)^4}{x^8}\,dx, x, \tan(c+dx)\right)}{d}\\
&= -\frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(c+dx)}{5d}\\
&\quad - \frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3\csc^7(c+dx)\sec^2(c+dx)}{2d}\\
&\quad + \frac{(3a^3)\,\text{Subst}\left(\int \left(1+\frac{1}{x^8}+\frac{4}{x^6}+\frac{6}{x^4}+\frac{4}{x^2}\right)\,dx, x, \tan(c+dx)\right)}{d}\\
&\quad - \frac{(3a^3)\,\text{Subst}\left(\int \left(1+x^2+x^4+x^6+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{d}\\
&\quad - \frac{(9a^3)\,\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{2d}\\
&= -\frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(c+dx)}{d}\\
&\quad - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{3a^3\csc(c+dx)}{d} - \frac{a^3\csc^3(c+dx)}{d}\\
&\quad - \frac{3a^3\csc^5(c+dx)}{5d} - \frac{3a^3\csc^7(c+dx)}{7d} + \frac{a^3\csc^7(c+dx)\sec^2(c+dx)}{2d}\\
&\quad + \frac{3a^3\tan(c+dx)}{d} - \frac{(3a^3)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}\\
&\quad - \frac{(9a^3)\,\text{Subst}\left(\int \left(1+x^2+x^4+x^6+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{2d}\\
&= \frac{3a^3\text{arctanh}(\sin(c+dx))}{d} - \frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d}\\
&\quad - \frac{3a^3\cot^5(c+dx)}{d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{15a^3\csc(c+dx)}{2d} - \frac{5a^3\csc^3(c+dx)}{2d}\\
&\quad - \frac{3a^3\csc^5(c+dx)}{2d} - \frac{15a^3\csc^7(c+dx)}{14d} + \frac{a^3\csc^7(c+dx)\sec^2(c+dx)}{2d}\\
&\quad + \frac{3a^3\tan(c+dx)}{d} - \frac{(9a^3)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{2d}
\end{aligned}
$$

$$= \frac{15a^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(c+dx)}{d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{15a^3\csc(c+dx)}{2d} - \frac{5a^3\csc^3(c+dx)}{2d} - \frac{3a^3\csc^5(c+dx)}{2d} - \frac{15a^3\csc^7(c+dx)}{14d} + \frac{a^3\csc^7(c+dx)\sec^2(c+dx)}{2d} + \frac{3a^3\tan(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 430 vs. $2(192) = 384$.

Time = 3.79 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.24

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\cos(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3\left(-860160\cos^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c-\right.\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(-860160*Cos[c +
d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 860160*Cos[c + d*x]^2*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 8*Csc[2*c]*Csc[(c + d*x)/2]^6*Csc[c
 + d*x]*(5264*Sin[2*c] - 9580*Sin[d*x] + 8480*Sin[2*d*x] + 2776*Sin[c - d*x
] - 6080*Sin[c + d*x] + 8816*Sin[2*(c + d*x)] - 7904*Sin[3*(c + d*x)] + 486
4*Sin[4*(c + d*x)] - 1824*Sin[5*(c + d*x)] + 304*Sin[6*(c + d*x)] - 9580*Si
n[2*c + d*x] - 10024*Sin[3*c + d*x] + 13891*Sin[c + 2*d*x] + 7720*Sin[2*(c
+ 2*d*x)] + 13891*Sin[3*c + 2*d*x] + 10080*Sin[4*c + 2*d*x] - 10060*Sin[c +
 3*d*x] - 12454*Sin[2*c + 3*d*x] - 12454*Sin[4*c + 3*d*x] - 6580*Sin[5*c +
3*d*x] + 7664*Sin[3*c + 4*d*x] + 7664*Sin[5*c + 4*d*x] + 2520*Sin[6*c + 4*d
*x] - 3420*Sin[3*c + 5*d*x] - 2874*Sin[4*c + 5*d*x] - 2874*Sin[6*c + 5*d*x]
 - 420*Sin[7*c + 5*d*x] + 640*Sin[4*c + 6*d*x] + 479*Sin[5*c + 6*d*x] + 479
*Sin[7*c + 6*d*x])))/(917504*d)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00

method	result
norman	$\frac{-\frac{a^3}{112d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{28d}-\frac{85a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{112d}-\frac{15a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2d}+\frac{395a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{16d}-\frac{57a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{4d}-\frac{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{16d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$
risch	$-\frac{ia^3\left(105\,e^{11i(dx+c)}-630\,e^{10i(dx+c)}+1645\,e^{9i(dx+c)}-2520\,e^{8i(dx+c)}+2506\,e^{7i(dx+c)}-1316\,e^{6i(dx+c)}-694\,e^{5i(dx+c)}+21\right.}{7d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)\left(e^{2i(dx+c)}+1\right)^2}$
parallelrisch	$\frac{5\sec\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\left(-\frac{329}{40}+\frac{21\left(\frac{\sin(6dx+6c)}{16}+\frac{29\sin(2dx+2c)}{16}-\frac{3\sin(5dx+5c)}{8}-\frac{5\sin(dx+c)}{4}+\sin(4dx+4c)-\frac{13\sin(3dx+3c)}{8}\right)\ln\left(\tan\left(\right.\right.}{2}\right.}{}$
derivativedivides	$\frac{a^3\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)^2}-\frac{9}{35\sin(dx+c)^5\cos(dx+c)^2}-\frac{3}{5\sin(dx+c)^3\cos(dx+c)^2}+\frac{3}{2\sin(dx+c)\cos(dx+c)^2}-\frac{9}{2\sin(dx+c)}+\frac{9\ln(s}{}\right.}{}$
default	$\frac{a^3\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)^2}-\frac{9}{35\sin(dx+c)^5\cos(dx+c)^2}-\frac{3}{5\sin(dx+c)^3\cos(dx+c)^2}+\frac{3}{2\sin(dx+c)\cos(dx+c)^2}-\frac{9}{2\sin(dx+c)}+\frac{9\ln(s}{}\right.}{}$

```
[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/112/d*a^3-3/28*a^3/d*tan(1/2*d*x+1/2*c)^2-85/112*a^3/d*tan(1/2*d*x+1/2*
c)^4-15/2*a^3/d*tan(1/2*d*x+1/2*c)^6+395/16*a^3/d*tan(1/2*d*x+1/2*c)^8-57/4
*a^3/d*tan(1/2*d*x+1/2*c)^10-1/16*a^3/d*tan(1/2*d*x+1/2*c)^12)/tan(1/2*d*x+
1/2*c)^7/(-1+tan(1/2*d*x+1/2*c)^2)^2-15/2/d*a^3*ln(tan(1/2*d*x+1/2*c)-1)+15
/2/d*a^3*ln(tan(1/2*d*x+1/2*c)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.45

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{320\,a^3\cos\left(dx+c\right)^6-750\,a^3\cos\left(dx+c\right)^5+170\,a^3\cos\left(dx+c\right)^4+720\,a^3\cos\left(dx+c\right)^3-520\,a^3\cos\left(dx+\right.}{}$$

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/28*(320*a^3*cos(d*x + c)^6 - 750*a^3*cos(d*x + c)^5 + 170*a^3*cos(d*x +
c)^4 + 720*a^3*cos(d*x + c)^3 - 520*a^3*cos(d*x + c)^2 + 42*a^3*cos(d*x + c
) + 14*a^3 - 105*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x
 + c)^3 - a^3*cos(d*x + c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a^3
*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x
 + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^5 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^3 - d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c))**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.40

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{a^3\left(\frac{2\left(315\,\sin(dx+c)^8-210\,\sin(dx+c)^6-42\,\sin(dx+c)^4-18\,\sin(dx+c)^2-10\right)}{\sin(dx+c)^9-\sin(dx+c)^7}-315\,\log\left(\sin\left(dx+c\right)+1\right)+315\,\log\left(\sin\left(\right.\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] -1/140*(a^3*(2*(315*sin(d*x + c)^8 - 210*sin(d*x + c)^6 - 42*sin(d*x + c)^4
 - 18*sin(d*x + c)^2 - 10)/(sin(d*x + c)^9 - sin(d*x + c)^7) - 315*log(sin(
d*x + c) + 1) + 315*log(sin(d*x + c) - 1)) + 2*a^3*(2*(105*sin(d*x + c)^6 +
 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2 + 15)/sin(d*x + c)^7 - 105*log(sin(d
*x + c) + 1) + 105*log(sin(d*x + c) - 1)) + 12*a^3*((140*tan(d*x + c)^6 + 7
0*tan(d*x + c)^4 + 28*tan(d*x + c)^2 + 5)/tan(d*x + c)^7 - 35*tan(d*x + c))
 + 4*(35*tan(d*x + c)^6 + 35*tan(d*x + c)^4 + 21*tan(d*x + c)^2 + 5)*a^3/ta
n(d*x + c)^7)/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{840\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-840\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{112\left(5\,a^3\,\mathrm{ta}\right.}{}}{112\,d}$$

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/112*(840*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 840*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 7*a^3*tan(1/2*d*x + 1/2*c) - 112*(5*a^3*tan(1/2*d*x +
 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (1
050*a^3*tan(1/2*d*x + 1/2*c)^6 + 112*a^3*tan(1/2*d*x + 1/2*c)^4 + 14*a^3*ta
n(1/2*d*x + 1/2*c)^2 + a^3)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B] (verification not implemented)

Time = 16.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{230\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 396\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 120\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + \frac{85\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{7} + \frac{12\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{7} + \frac{a^3}{7}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11} - 32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 + 16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\right)} - \frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^8,x)
```

```
[Out] (15*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((12*a^3*tan(c/2 + (d*x)/2)^2)/7 + (
85*a^3*tan(c/2 + (d*x)/2)^4)/7 + 120*a^3*tan(c/2 + (d*x)/2)^6 - 396*a^3*tan
(c/2 + (d*x)/2)^8 + 230*a^3*tan(c/2 + (d*x)/2)^10 + a^3/7)/(d*(16*tan(c/2 +
 (d*x)/2)^7 - 32*tan(c/2 + (d*x)/2)^9 + 16*tan(c/2 + (d*x)/2)^11)) - (a^3*t
an(c/2 + (d*x)/2))/(16*d)
```

3.56 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 232

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{17a^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{16a^3\cot(c+dx)}{d} - \frac{34a^3\cot^3(c+dx)}{3d} - \frac{36a^3\cot^5(c+dx)}{5d} - \frac{19a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+dx)}{9d} - \frac{17a^3\csc(c+dx)}{2d} - \frac{17a^3\csc^3(c+dx)}{6d} - \frac{17a^3\csc^5(c+dx)}{10d} - \frac{17a^3\csc^7(c+dx)}{14d} - \frac{17a^3\csc^9(c+dx)}{18d} + \frac{a^3\csc^9(c+dx)\sec^2(c+dx)}{2d} + \frac{3a^3\tan(c+dx)}{d}$$

```
[Out] 17/2*a^3*arctanh(sin(d*x+c))/d-16*a^3*cot(d*x+c)/d-34/3*a^3*cot(d*x+c)^3/d-
36/5*a^3*cot(d*x+c)^5/d-19/7*a^3*cot(d*x+c)^7/d-4/9*a^3*cot(d*x+c)^9/d-17/2
*a^3*csc(d*x+c)/d-17/6*a^3*csc(d*x+c)^3/d-17/10*a^3*csc(d*x+c)^5/d-17/14*a^
3*csc(d*x+c)^7/d-17/18*a^3*csc(d*x+c)^9/d+1/2*a^3*csc(d*x+c)^9*sec(d*x+c)^2
/d+3*a^3*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3957, 2952, 3852, 2701, 308, 213, 2700, 276, 294}

$$\begin{aligned}\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx=&\ \frac{17a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)}{d}\\&-\frac{4a^3\cot^9(c+dx)}{9d}-\frac{19a^3\cot^7(c+dx)}{7d}\\&-\frac{36a^3\cot^5(c+dx)}{5d}-\frac{34a^3\cot^3(c+dx)}{3d}\\&-\frac{16a^3\cot(c+dx)}{d}-\frac{17a^3\csc^9(c+dx)}{18d}\\&-\frac{17a^3\csc^7(c+dx)}{14d}-\frac{17a^3\csc^5(c+dx)}{10d}\\&-\frac{17a^3\csc^3(c+dx)}{6d}-\frac{17a^3\csc(c+dx)}{2d}\\&+\frac{a^3\csc^9(c+dx)\sec^2(c+dx)}{2d}\end{aligned}$$

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (17*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (16*a^3*Cot[c + d*x])/d - (34*a^3*Co
t[c + d*x]^3)/(3*d) - (36*a^3*Cot[c + d*x]^5)/(5*d) - (19*a^3*Cot[c + d*x]^
7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d) - (17*a^3*Csc[c + d*x])/(2*d) - (17
*a^3*Csc[c + d*x]^3)/(6*d) - (17*a^3*Csc[c + d*x]^5)/(10*d) - (17*a^3*Csc[c
 + d*x]^7)/(14*d) - (17*a^3*Csc[c + d*x]^9)/(18*d) + (a^3*Csc[c + d*x]^9*Se
c[c + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int (-a - a\cos(c+dx))^3 \csc^{10}(c+dx)\sec^3(c+dx)\,dx$$

$$
\begin{aligned}
&= \int \left(a^3 \csc^{10}(c+dx) + 3a^3 \csc^{10}(c+dx)\sec(c+dx) + 3a^3 \csc^{10}(c+dx)\sec^2(c+dx)\right.\\
&\qquad\left. + a^3 \csc^{10}(c+dx)\sec^3(c+dx)\right)\, dx\\
&= a^3 \int \csc^{10}(c+dx)\, dx + a^3 \int \csc^{10}(c+dx)\sec^3(c+dx)\, dx\\
&\quad + \left(3a^3\right) \int \csc^{10}(c+dx)\sec(c+dx)\, dx + \left(3a^3\right) \int \csc^{10}(c+dx)\sec^2(c+dx)\, dx\\
&= -\frac{a^3 \text{Subst}\left(\int \frac{x^{12}}{(-1+x^2)^2}\, dx, x, \csc(c+dx)\right)}{d}\\
&\quad - \frac{a^3 \text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\, dx, x, \cot(c+dx)\right)}{d}\\
&\quad - \frac{\left(3a^3\right) \text{Subst}\left(\int \frac{x^{10}}{-1+x^2}\, dx, x, \csc(c+dx)\right)}{d}\\
&\quad + \frac{\left(3a^3\right) \text{Subst}\left(\int \frac{(1+x^2)^5}{x^{10}}\, dx, x, \tan(c+dx)\right)}{d}\\
&= -\frac{a^3 \cot(c+dx)}{d} - \frac{4a^3 \cot^3(c+dx)}{3d} - \frac{6a^3 \cot^5(c+dx)}{5d}\\
&\quad - \frac{4a^3 \cot^7(c+dx)}{7d} - \frac{a^3 \cot^9(c+dx)}{9d} + \frac{a^3 \csc^9(c+dx)\sec^2(c+dx)}{2d}\\
&\quad + \frac{\left(3a^3\right) \text{Subst}\left(\int \left(1+\frac{1}{x^{10}}+\frac{5}{x^8}+\frac{10}{x^6}+\frac{10}{x^4}+\frac{5}{x^2}\right)\, dx, x, \tan(c+dx)\right)}{d}\\
&\quad - \frac{\left(3a^3\right) \text{Subst}\left(\int \left(1+x^2+x^4+x^6+x^8+\frac{1}{-1+x^2}\right)\, dx, x, \csc(c+dx)\right)}{d}\\
&\quad - \frac{\left(11a^3\right) \text{Subst}\left(\int \frac{x^{10}}{-1+x^2}\, dx, x, \csc(c+dx)\right)}{2d}\\
&= -\frac{16a^3 \cot(c+dx)}{d} - \frac{34a^3 \cot^3(c+dx)}{3d} - \frac{36a^3 \cot^5(c+dx)}{5d} - \frac{19a^3 \cot^7(c+dx)}{7d}\\
&\quad - \frac{4a^3 \cot^9(c+dx)}{9d} - \frac{3a^3 \csc(c+dx)}{d} - \frac{a^3 \csc^3(c+dx)}{d} - \frac{3a^3 \csc^5(c+dx)}{5d}\\
&\quad - \frac{3a^3 \csc^7(c+dx)}{7d} - \frac{a^3 \csc^9(c+dx)}{3d} + \frac{a^3 \csc^9(c+dx)\sec^2(c+dx)}{2d}\\
&\quad + \frac{3a^3 \tan(c+dx)}{d} - \frac{\left(3a^3\right) \text{Subst}\left(\int \frac{1}{-1+x^2}\, dx, x, \csc(c+dx)\right)}{d}\\
&\quad - \frac{\left(11a^3\right) \text{Subst}\left(\int \left(1+x^2+x^4+x^6+x^8+\frac{1}{-1+x^2}\right)\, dx, x, \csc(c+dx)\right)}{2d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3a^3\text{arctanh}(\sin(c+dx))}{d} - \frac{16a^3\cot(c+dx)}{d} - \frac{34a^3\cot^3(c+dx)}{3d} \\
&\quad - \frac{36a^3\cot^5(c+dx)}{5d} - \frac{19a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+dx)}{9d} \\
&\quad - \frac{17a^3\csc(c+dx)}{2d} - \frac{17a^3\csc^3(c+dx)}{6d} - \frac{17a^3\csc^5(c+dx)}{10d} \\
&\quad - \frac{17a^3\csc^7(c+dx)}{14d} - \frac{17a^3\csc^9(c+dx)}{18d} + \frac{a^3\csc^9(c+dx)\sec^2(c+dx)}{2d} \\
&\quad + \frac{3a^3\tan(c+dx)}{d} - \frac{(11a^3)\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{2d} \\
&= \frac{17a^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{16a^3\cot(c+dx)}{d} - \frac{34a^3\cot^3(c+dx)}{3d} \\
&\quad - \frac{36a^3\cot^5(c+dx)}{5d} - \frac{19a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+dx)}{9d} - \frac{17a^3\csc(c+dx)}{2d} \\
&\quad - \frac{17a^3\csc^3(c+dx)}{6d} - \frac{17a^3\csc^5(c+dx)}{10d} - \frac{17a^3\csc^7(c+dx)}{14d} \\
&\quad - \frac{17a^3\csc^9(c+dx)}{18d} + \frac{a^3\csc^9(c+dx)\sec^2(c+dx)}{2d} + \frac{3a^3\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1000 vs. $2(232) = 464$.

Time = 12.60 (sec) , antiderivative size = 1000, normalized size of antiderivative = 4.31

$$
\begin{aligned}
&\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx \\
&= -\frac{9833\cos^3(c+dx)\cot\left(\frac{c}{2}\right)\csc^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{80640d} \\
&\quad-\frac{979\cos^3(c+dx)\cot\left(\frac{c}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{53760d} \\
&\quad-\frac{5\cos^3(c+dx)\cot\left(\frac{c}{2}\right)\csc^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{2016d} \\
&\quad-\frac{\cos^3(c+dx)\cot\left(\frac{c}{2}\right)\csc^8\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{4608d} \\
&\quad-\frac{17\cos^3(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{16d} \\
&\quad+\frac{17\cos^3(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3}{16d} \\
&\quad+\frac{197147\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{161280d} \\
&\quad+\frac{9833\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{80640d} \\
&\quad+\frac{979\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\csc^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{53760d} \\
&\quad+\frac{5\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\csc^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{2016d} \\
&\quad+\frac{\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\csc^9\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{4608d} \\
&\quad-\frac{35\cos^3(c+dx)\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{1536d} \\
&\quad-\frac{\cos^3(c+dx)\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin\left(\frac{dx}{2}\right)}{1536d} \\
&\quad+\frac{\cos(c+dx)\sec(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sin(dx)}{16d} \\
&\quad+\frac{\cos^2(c+dx)\sec(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3(\sin(c)+6\sin(dx))}{16d} \\
&\quad-\frac{\cos^3(c+dx)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\tan\left(\frac{c}{2}\right)}{1536d}
\end{aligned}
$$

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-9833*Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^6*(a
 + a*Sec[c + d*x])^3)/(80640*d) - (979*Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d
*x)/2]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(53760*d) - (5*Cos[c
+ d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^6*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
```

```
d*x])^3)/(2016*d) - (Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^8*Sec[c/2 +
 (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(4608*d) - (17*Cos[c + d*x]^3*Log[Cos[c
/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x
])^3)/(16*d) + (17*Cos[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/
2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(16*d) + (197147*Cos[c + d
*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])
^3*Sin[(d*x)/2])/(161280*d) + (9833*Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)
/2]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(80640*d) +
 (979*Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^5*Sec[c/2 + (d*x)/2]^6*(a
+ a*Sec[c + d*x])^3*Sin[(d*x)/2])/(53760*d) + (5*Cos[c + d*x]^3*Csc[c/2]*Cs
c[c/2 + (d*x)/2]^7*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2]
)/(2016*d) + (Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^9*Sec[c/2 + (d*x)/
2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(4608*d) - (35*Cos[c + d*x]^3*Sec
[c/2]*Sec[c/2 + (d*x)/2]^7*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(1536*d) -
(Cos[c + d*x]^3*Sec[c/2]*Sec[c/2 + (d*x)/2]^9*(a + a*Sec[c + d*x])^3*Sin[(d
*x)/2])/(1536*d) + (Cos[c + d*x]*Sec[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
 d*x])^3*Sin[d*x])/(16*d) + (Cos[c + d*x]^2*Sec[c]*Sec[c/2 + (d*x)/2]^6*(a
+ a*Sec[c + d*x])^3*(Sin[c] + 6*Sin[d*x]))/(16*d) - (Cos[c + d*x]^3*Sec[c/2
 + (d*x)/2]^8*(a + a*Sec[c + d*x])^3*Tan[c/2])/(1536*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.75 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{i a^3 \left(5355\,\mathrm{e}^{15 i (dx+c)}-32130\,\mathrm{e}^{14 i (dx+c)}+73185\,\mathrm{e}^{13 i (dx+c)}-64260\,\mathrm{e}^{12 i (dx+c)}-34629\,\mathrm{e}^{11 i (dx+c)}+157794\,\mathrm{e}^{10 i (dx+c)}-20\right.}{3}$
parallelrisch	$-\frac{1673 \sec\left(\frac{dx}{2}+\frac{c}{2}\right)^3 \left(\frac{31314}{11711}+\frac{765\left(\frac{\sin(8dx+8c)}{14}-\frac{3\sin(7dx+7c)}{7}-\sin(5dx+5c)+\sin(6dx+6c)-\frac{\sin(4dx+4c)}{7}+\frac{13\sin(3dx+3c)}{7}+\frac{17}{}\right.}{478}\right.}{}$
derivativedivides	$\frac{a^3\left(-\frac{1}{9\sin(dx+c)^9\cos(dx+c)^2}-\frac{11}{63\sin(dx+c)^7\cos(dx+c)^2}-\frac{11}{35\sin(dx+c)^5\cos(dx+c)^2}-\frac{11}{15\sin(dx+c)^3\cos(dx+c)^2}+\frac{11}{6\sin(dx+c)\,\mathrm{co}}\right.}{}$
default	$\frac{a^3\left(-\frac{1}{9\sin(dx+c)^9\cos(dx+c)^2}-\frac{11}{63\sin(dx+c)^7\cos(dx+c)^2}-\frac{11}{35\sin(dx+c)^5\cos(dx+c)^2}-\frac{11}{15\sin(dx+c)^3\cos(dx+c)^2}+\frac{11}{6\sin(dx+c)\,\mathrm{co}}\right.}{}$

```
[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*I*a^3*(5355*exp(15*I*(d*x+c))-32130*exp(14*I*(d*x+c))+73185*exp(13*I
*(d*x+c))-64260*exp(12*I*(d*x+c))-34629*exp(11*I*(d*x+c))+157794*exp(10*I*(
d*x+c))-207111*exp(9*I*(d*x+c))+125256*exp(8*I*(d*x+c))+62713*exp(7*I*(d*x+
c))-175518*exp(6*I*(d*x+c))+171707*exp(5*I*(d*x+c))-80132*exp(4*I*(d*x+c))-
37919*exp(3*I*(d*x+c))+78974*exp(2*I*(d*x+c))-42261*exp(I*(d*x+c))+7936)/d/
(exp(I*(d*x+c))-1)^9/(exp(I*(d*x+c))+1)^3/(exp(2*I*(d*x+c))+1)^2-17/2*a^3/d
```

```
*ln(exp(I*(d*x+c))-I)+17/2*a^3/d*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.62

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{15872\,a^3\cos\left(dx+c\right)^8 - 36906\,a^3\cos\left(dx+c\right)^7 - 8322\,a^3\cos\left(dx+c\right)^6 + 73402\,a^3\cos\left(dx+c\right)^5 - 33342\,}{}$$

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/1260*(15872*a^3*cos(d*x + c)^8 - 36906*a^3*cos(d*x + c)^7 - 8322*a^3*cos
(d*x + c)^6 + 73402*a^3*cos(d*x + c)^5 - 33342*a^3*cos(d*x + c)^4 - 34746*a
^3*cos(d*x + c)^3 + 26702*a^3*cos(d*x + c)^2 - 1890*a^3*cos(d*x + c) - 630*
a^3 - 5355*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6 + 2*a^3*cos(d*x + c)^
5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(s
in(d*x + c) + 1)*sin(d*x + c) + 5355*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x +
c)^6 + 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 +
 a^3*cos(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^
7 - 3*d*cos(d*x + c)^6 + 2*d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 - 3*d*cos(
d*x + c)^3 + d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.33

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{a^3\left(\frac{2\left(3465\,\sin(dx+c)^{10}-2310\,\sin(dx+c)^8-462\,\sin(dx+c)^6-198\,\sin(dx+c)^4-110\,\sin(dx+c)^2-70\right)}{\sin(dx+c)^{11}-\sin(dx+c)^9} - 3465\,\log\left(\sin\left(dx+c\right)+1\right)}{}$$

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] -1/1260*(a^3*(2*(3465*sin(d*x + c)^10 - 2310*sin(d*x + c)^8 - 462*sin(d*x +
 c)^6 - 198*sin(d*x + c)^4 - 110*sin(d*x + c)^2 - 70)/(sin(d*x + c)^11 - si
n(d*x + c)^9) - 3465*log(sin(d*x + c) + 1) + 3465*log(sin(d*x + c) - 1)) +
6*a^3*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4 + 45*
sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315*log(s
in(d*x + c) - 1)) + 60*a^3*((315*tan(d*x + c)^8 + 210*tan(d*x + c)^6 + 126*
tan(d*x + c)^4 + 45*tan(d*x + c)^2 + 7)/tan(d*x + c)^9 - 63*tan(d*x + c)) +
 4*(315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*tan(d*x + c)^4 + 180*tan(
d*x + c)^2 + 35)*a^3/tan(d*x + c)^9)/d
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.87

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 171360\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 171360\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] -1/20160*(105*a^3*tan(1/2*d*x + 1/2*c)^3 - 171360*a^3*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) + 171360*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 3780*a^3*ta
n(1/2*d*x + 1/2*c) + 20160*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*
x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + (220185*a^3*tan(1/2*d*x + 1/2*
c)^8 + 26880*a^3*tan(1/2*d*x + 1/2*c)^6 + 4347*a^3*tan(1/2*d*x + 1/2*c)^4 +
 540*a^3*tan(1/2*d*x + 1/2*c)^2 + 35*a^3)/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [B] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{17\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{192\,d} - \frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$

$$-\frac{1019\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - \frac{5282\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{3} + \frac{8132\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{15} + \frac{6242\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{105} + \frac{3302\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{315} + }{d\,\left(64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13} - 128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11} + 64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\right)}$$

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^10,x)
```

```
[Out] (17*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^3)/(192*d) -
 (3*a^3*tan(c/2 + (d*x)/2))/(16*d) - ((94*a^3*tan(c/2 + (d*x)/2)^2)/63 + (3
302*a^3*tan(c/2 + (d*x)/2)^4)/315 + (6242*a^3*tan(c/2 + (d*x)/2)^6)/105 + (
8132*a^3*tan(c/2 + (d*x)/2)^8)/15 - (5282*a^3*tan(c/2 + (d*x)/2)^10)/3 + 10
19*a^3*tan(c/2 + (d*x)/2)^12 + a^3/9)/(d*(64*tan(c/2 + (d*x)/2)^9 - 128*tan
(c/2 + (d*x)/2)^11 + 64*tan(c/2 + (d*x)/2)^13))
```

3.57 $\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^3(c+dx)}{3ad} - \frac{3\cos^5(c+dx)}{5ad} + \frac{3\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{\sin^8(c+dx)}{8ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d-3/5*cos(d*x+c)^5/a/d+3/7*cos(d*x+c)^7/a/d-1/9*cos(d*x+
c)^9/a/d+1/8*sin(d*x+c)^8/a/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2914, 2644, 30, 2645, 276}

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin^8(c+dx)}{8ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{3\cos^7(c+dx)}{7ad} - \frac{3\cos^5(c+dx)}{5ad} + \frac{\cos^3(c+dx)}{3ad}$$

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) - (3*Cos[c + d*x]^5)/(5*a*d) + (3*Cos[c + d*x]^7)/(7
*a*d) - Cos[c + d*x]^9/(9*a*d) + Sin[c + d*x]^8/(8*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin^9(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\int \cos(c+dx)\sin^7(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin^7(c+dx)\,dx}{a} \\
&= \frac{\operatorname{Subst}\left(\int x^7\,dx,x,\sin(c+dx)\right)}{ad}+\frac{\operatorname{Subst}\left(\int x^2\left(1-x^2\right)^3\,dx,x,\cos(c+dx)\right)}{ad} \\
&= \frac{\sin^8(c+dx)}{8ad}+\frac{\operatorname{Subst}\left(\int \left(x^2-3x^4+3x^6-x^8\right)\,dx,x,\cos(c+dx)\right)}{ad}
\end{aligned}
$$

$$= \frac{\cos^3(c+dx)}{3ad} - \frac{3\cos^5(c+dx)}{5ad} + \frac{3\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{\sin^8(c+dx)}{8ad}$$

Mathematica [A] (verified)

Time = 3.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.68

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{(4258+6995\cos(c+dx)+3650\cos(2(c+dx))+1085\cos(3(c+dx))+140\cos(4(c+dx)))\sin^{10}\left(\frac{1}{2}(c+\right.}{315ad}$$

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((4258 + 6995*Cos[c + d*x] + 3650*Cos[2*(c + d*x)] + 1085*Cos[3*(c + d*x)]
+ 140*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]^10)/(315*a*d)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{\cos(dx+c)^8}{8}+\frac{3\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{2}-\frac{3\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4}{4}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$
default	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{\cos(dx+c)^8}{8}+\frac{3\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{2}-\frac{3\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4}{4}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$
parallelrisch	$\frac{8820\cos(4dx+4c)+17640\cos(dx+c)+27409+315\cos(8dx+8c)+900\cos(7dx+7c)-2520\cos(6dx+6c)-2016\cos(5dx+5c}{322560da}$
norman	$\frac{\frac{32}{315ad}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{da}+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{35da}+\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{35da}+\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{15da}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{5da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^9}$
risch	$\frac{7\cos(dx+c)}{128ad}-\frac{\cos(9dx+9c)}{2304ad}+\frac{\cos(8dx+8c)}{1024ad}+\frac{5\cos(7dx+7c)}{1792ad}-\frac{\cos(6dx+6c)}{128ad}-\frac{\cos(5dx+5c)}{160ad}+\frac{7\cos(4dx+4c}{256ad}$

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/9*cos(d*x+c)^9+1/8*cos(d*x+c)^8+3/7*cos(d*x+c)^7-1/2*cos(d*x+c)^6
-3/5*cos(d*x+c)^5+3/4*cos(d*x+c)^4+1/3*cos(d*x+c)^3-1/2*cos(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{280\cos(dx+c)^9 - 315\cos(dx+c)^8 - 1080\cos(dx+c)^7 + 1260\cos(dx+c)^6 + 1512\cos(dx+c)^5 -}{2520\,ad}$$

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 1080*cos(d*x + c)^7 + 12
60*cos(d*x + c)^6 + 1512*cos(d*x + c)^5 - 1890*cos(d*x + c)^4 - 840*cos(d*x
 + c)^3 + 1260*cos(d*x + c)^2)/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{280\cos(dx+c)^9 - 315\cos(dx+c)^8 - 1080\cos(dx+c)^7 + 1260\cos(dx+c)^6 + 1512\cos(dx+c)^5 -}{2520\,ad}$$

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 1080*cos(d*x + c)^7 + 12
60*cos(d*x + c)^6 + 1512*cos(d*x + c)^5 - 1890*cos(d*x + c)^4 - 840*cos(d*x
 + c)^3 + 1260*cos(d*x + c)^2)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.55

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{32\left(\frac{9(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{36(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{84(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{126(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{630(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-1\right)}{315\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9}$$

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 32/315*(9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 36*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 84*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 126*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 630*(cos(d*x + c) - 1)^5/(cos(d*x
 + c) + 1)^5 - 1)/(a*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\cos(c+d\,x)^2}{2\,a}-\frac{\cos(c+d\,x)^3}{3\,a}-\frac{3\cos(c+d\,x)^4}{4\,a}+\frac{3\cos(c+d\,x)^5}{5\,a}+\frac{\cos(c+d\,x)^6}{2\,a}-\frac{3\cos(c+d\,x)^7}{7\,a}-\frac{\cos(c+d\,x)^8}{8\,a}+\frac{\cos(c+d\,x)^9}{9\,a}}{d}$$

```
[In] int(sin(c + d*x)^9/(a + a/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - (3*cos(c + d*x)^4)/(4*a) +
(3*cos(c + d*x)^5)/(5*a) + cos(c + d*x)^6/(2*a) - (3*cos(c + d*x)^7)/(7*a)
- cos(c + d*x)^8/(8*a) + cos(c + d*x)^9/(9*a))/d
```

3.58 $\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\cos^3(c+dx)}{3ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^6(c+dx)}{6ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d-2/5*cos(d*x+c)^5/a/d+1/7*cos(d*x+c)^7/a/d+1/6*sin(d*x+
c)^6/a/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2914, 2644, 30, 2645, 276}

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\sin^6(c+dx)}{6ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) - (2*Cos[c + d*x]^5)/(5*a*d) + Cos[c + d*x]^7/(7*a*d
) + Sin[c + d*x]^6/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin^7(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\int \cos(c+dx)\sin^5(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a} \\
&= \frac{\text{Subst}\left(\int x^5\,dx, x, \sin(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx, x, \cos(c+dx)\right)}{ad} \\
&= \frac{\sin^6(c+dx)}{6ad} + \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \cos(c+dx)\right)}{ad} \\
&= \frac{\cos^3(c+dx)}{3ad} - \frac{2\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} + \frac{\sin^6(c+dx)}{6ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$
\begin{aligned}
&\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx \\
&= \frac{4(123+197\cos(c+dx)+85\cos(2(c+dx))+15\cos(3(c+dx)))\sin^8\left(\frac{1}{2}(c+dx)\right)}{105ad}
\end{aligned}
$$

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x]),x]
[Out] (4*(123 + 197*Cos[c + d*x] + 85*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sin
[(c + d*x)/2]^8)/(105*a*d)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{6}-\frac{2\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$
default	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{6}-\frac{2\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$
parallelrisch	$\frac{-525\cos(2dx+2c)+862+210\cos(4dx+4c)+525\cos(dx+c)+35\cos(3dx+3c)-63\cos(5dx+5c)-35\cos(6dx+6c)+15\cos(7c}{6720da}$
norman	$\frac{\frac{16}{105ad}+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{15da}+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{5da}+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3da}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7}$
risch	$\frac{5\cos(dx+c)}{64ad}+\frac{\cos(7dx+7c)}{448ad}-\frac{\cos(6dx+6c)}{192ad}-\frac{3\cos(5dx+5c)}{320ad}+\frac{\cos(4dx+4c)}{32ad}+\frac{\cos(3dx+3c)}{192ad}-\frac{5\cos(2dx+2c)}{64ad}$

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/7*cos(d*x+c)^7-1/6*cos(d*x+c)^6-2/5*cos(d*x+c)^5+1/2*cos(d*x+c)^4+
1/3*cos(d*x+c)^3-1/2*cos(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$
\begin{aligned}
&\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx \\
&= \frac{30\cos(dx+c)^7-35\cos(dx+c)^6-84\cos(dx+c)^5+105\cos(dx+c)^4+70\cos(dx+c)^3-105\cos(dx}{210\,ad}
\end{aligned}
$$

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(30*cos(d*x + c)^7 - 35*cos(d*x + c)^6 - 84*cos(d*x + c)^5 + 105*cos(
d*x + c)^4 + 70*cos(d*x + c)^3 - 105*cos(d*x + c)^2)/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{30\cos(dx+c)^7 - 35\cos(dx+c)^6 - 84\cos(dx+c)^5 + 105\cos(dx+c)^4 + 70\cos(dx+c)^3 - 105\cos(}{210\,ad}$$

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/210*(30*cos(d*x + c)^7 - 35*cos(d*x + c)^6 - 84*cos(d*x + c)^5 + 105*cos(
d*x + c)^4 + 70*cos(d*x + c)^3 - 105*cos(d*x + c)^2)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{16\left(\frac{7(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{21(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{35(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{140(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - 1\right)}{105\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^7}$$

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 16/105*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 21*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 35*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 140*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1)/(a*d*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 1)^7)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$=-\frac{\frac{\cos(c+d\,x)^2}{2\,a}-\frac{\cos(c+d\,x)^3}{3\,a}-\frac{\cos(c+d\,x)^4}{2\,a}+\frac{2\cos(c+d\,x)^5}{5\,a}+\frac{\cos(c+d\,x)^6}{6\,a}-\frac{\cos(c+d\,x)^7}{7\,a}}{d}$$

```
[In] int(sin(c + d*x)^7/(a + a/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - cos(c + d*x)^4/(2*a) + (2*c
os(c + d*x)^5)/(5*a) + cos(c + d*x)^6/(6*a) - cos(c + d*x)^7/(7*a))/d
```

3.59 $\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^3(c+dx)}{3ad} - \frac{\cos^5(c+dx)}{5ad} + \frac{\sin^4(c+dx)}{4ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d-1/5*cos(d*x+c)^5/a/d+1/4*sin(d*x+c)^4/a/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2914, 2644, 30, 2645, 14}

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin^4(c+dx)}{4ad} - \frac{\cos^5(c+dx)}{5ad} + \frac{\cos^3(c+dx)}{3ad}$$

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) + Sin[c + d*x]^4/(4*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin^5(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= \frac{\int \cos(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int x^3\,dx, x, \sin(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int x^2(1-x^2)\,dx, x, \cos(c+dx)\right)}{ad}\\
&= \frac{\sin^4(c+dx)}{4ad} + \frac{\text{Subst}\left(\int (x^2-x^4)\,dx, x, \cos(c+dx)\right)}{ad}\\
&= \frac{\cos^3(c+dx)}{3ad} - \frac{\cos^5(c+dx)}{5ad} + \frac{\sin^4(c+dx)}{4ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{2(13+21\cos(c+dx)+6\cos(2(c+dx)))\sin^6\left(\frac{1}{2}(c+dx)\right)}{15ad}$$

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x]),x]
[Out] (2*(13 + 21*Cos[c + d*x] + 6*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]^6)/(15*a*d)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{4}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$	49
default	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{4}+\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$	49
parallelrisch	$\frac{-60\cos(2dx+2c)+109+15\cos(4dx+4c)+60\cos(dx+c)+10\cos(3dx+3c)-6\cos(5dx+5c)}{480da}$	63
norman	$\frac{\frac{4}{15ad}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{da}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3da}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$	83
risch	$\frac{\cos(dx+c)}{8ad}-\frac{\cos(5dx+5c)}{80ad}+\frac{\cos(4dx+4c)}{32ad}+\frac{\cos(3dx+3c)}{48ad}-\frac{\cos(2dx+2c)}{8ad}$	84

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-1/5*cos(d*x+c)^5+1/4*cos(d*x+c)^4+1/3*cos(d*x+c)^3-1/2*cos(d*x+c)^2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{12\cos(dx+c)^5-15\cos(dx+c)^4-20\cos(dx+c)^3+30\cos(dx+c)^2}{60\,ad}$$

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/60*(12*cos(d*x + c)^5 - 15*cos(d*x + c)^4 - 20*cos(d*x + c)^3 + 30*cos(d
*x + c)^2)/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{12\,\cos\left(dx+c\right)^5-15\,\cos\left(dx+c\right)^4-20\,\cos\left(dx+c\right)^3+30\,\cos\left(dx+c\right)^2}{60\,ad}$$

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/60*(12*cos(d*x + c)^5 - 15*cos(d*x + c)^4 - 20*cos(d*x + c)^3 + 30*cos(d
*x + c)^2)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.76

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{4\left(\frac{5\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{10\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{30\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-1\right)}{15\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}$$

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 4/15*(5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 + 30*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d*
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\cos(c+d\,x)^2}{2\,a}-\frac{\cos(c+d\,x)^3}{3\,a}-\frac{\cos(c+d\,x)^4}{4\,a}+\frac{\cos(c+d\,x)^5}{5\,a}}{d}$$

```
[In] int(sin(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - cos(c + d*x)^4/(4*a) + cos(
c + d*x)^5/(5*a))/d
```

3.60 $\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d+1/2*sin(d*x+c)^2/a/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2914, 2644, 30, 2645}

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin^2(c+dx)}{2ad}+\frac{\cos^3(c+dx)}{3ad}$$

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) + Sin[c + d*x]^2/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin^3(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\int \cos(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin(c+dx)\,dx}{a} \\
&= \frac{\text{Subst}\left(\int x\,dx, x, \sin(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int x^2\,dx, x, \cos(c+dx)\right)}{ad} \\
&= \frac{\cos^3(c+dx)}{3ad} + \frac{\sin^2(c+dx)}{2ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{2(1+2\cos(c+dx))\sin^4\left(\frac{1}{2}(c+dx)\right)}{3ad}$$

[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x]),x]

[Out] (2*(1 + 2*Cos[c + d*x])*Sin[(c + d*x)/2]^4)/(3*a*d)

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$	29
default	$\frac{\frac{\cos(dx+c)^3}{3}-\frac{\cos(dx+c)^2}{2}}{da}$	29
parallelrisch	$\frac{7+3\cos(dx+c)+\cos(3dx+3c)-3\cos(2dx+2c)}{12da}$	39
risch	$\frac{\cos(dx+c)}{4ad}+\frac{\cos(3dx+3c)}{12ad}-\frac{\cos(2dx+2c)}{4ad}$	50
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{da}+\frac{2}{3ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$	64

[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d/a*(1/3*cos(d*x+c)^3-1/2*cos(d*x+c)^2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{2\,\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)^2}{6\,ad}$$

[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/6*(2*cos(d*x + c)^3 - 3*cos(d*x + c)^2)/(a*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\,\cos\left(dx+c\right)^3 - 3\,\cos\left(dx+c\right)^2}{6\,ad}$$

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/6*(2*cos(d*x + c)^3 - 3*cos(d*x + c)^2)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2\,\cos(dx+c)^3}{d} - \frac{3\,\cos(dx+c)^2}{d}}{6\,a}$$

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 1/6*(2*cos(d*x + c)^3/d - 3*cos(d*x + c)^2/d)/a
```

Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos\left(c+d\,x\right)^2\,\left(2\,\cos\left(c+d\,x\right)-3\right)}{6\,a\,d}$$

```
[In] int(sin(c + d*x)^3/(a + a/cos(c + d*x)),x)
[Out] (cos(c + d*x)^2*(2*cos(c + d*x) - 3))/(6*a*d)
```

3.61 $\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{\cos(c+dx)}{ad}+\frac{\log(1+\cos(c+dx))}{ad}$$

```
[Out] -cos(d*x+c)/a/d+ln(1+cos(d*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2912, 12, 45}

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\log(\cos(c+dx)+1)}{ad}-\frac{\cos(c+dx)}{ad}$$

```
[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Cos[c + d*x]/(a*d)) + Log[1 + Cos[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x}{a(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \frac{x}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^2 d} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a-x}\right)\,dx, x, -a\cos(c+dx)\right)}{a^2 d} \\
&= -\frac{\cos(c+dx)}{ad}+\frac{\log(1+\cos(c+dx))}{ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos(c+dx)-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

```
[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x]),x]
[Out] -((Cos[c + d*x] - 2*Log[Cos[(c + d*x)/2]])/(a*d))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{-\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)+1-\cos(dx+c)}{ad}$	32
derivativedivides	$\frac{\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-\ln(\sec(dx+c))}{da}$	37
default	$\frac{\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-\ln(\sec(dx+c))}{da}$	37
norman	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{da\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}-\frac{\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}{ad}$	58
risch	$-\frac{ix}{a}-\frac{e^{i(dx+c)}}{2da}-\frac{e^{-i(dx+c)}}{2da}-\frac{2ic}{da}+\frac{2\ln\left(e^{i(dx+c)}+1\right)}{da}$	73

```
[In] int(sin(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/a/d*(-ln(sec(1/2*d*x+1/2*c)^2)+1-cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{\cos(dx+c)-\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{ad}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -(cos(d*x + c) - log(1/2*cos(d*x + c) + 1/2))/(a*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\int \frac{\sin(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x)
[Out] Integral(sin(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\cos(dx+c)}{a} - \frac{\log(\cos(dx+c)+1)}{a}}{d}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -(cos(d*x + c)/a - log(cos(d*x + c) + 1)/a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos(dx+c)}{ad} + \frac{\log(|-\cos(dx+c)-1|)}{ad}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] -cos(d*x + c)/(a*d) + log(abs(-cos(d*x + c) - 1))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\ln(\cos(c+d\,x)+1)-\cos(c+d\,x)}{a\,d}$$

```
[In] int(sin(c + d*x)/(a + a/cos(c + d*x)),x)
[Out] (log(cos(c + d*x) + 1) - cos(c + d*x))/(a*d)
```

3.62 $\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{2ad} + \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\csc^2(c+dx)}{2ad}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d+1/2*cot(d*x+c)*csc(d*x+c)/a/d-1/2*csc(d*x+c)^2
/a/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3957, 2785, 2686, 30, 2691, 3855}

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{2ad} - \frac{\csc^2(c+dx)}{2ad} + \frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*ArcTanh[Cos[c + d*x]]/(a*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*a*d) - Cs
c[c + d*x]^2/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^2(c+dx)\,dx}{a} \\
&= \frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \csc(c+dx)\,dx}{2a}-\frac{\mathrm{Subst}\left(\int x\,dx,x,\csc(c+dx)\right)}{ad} \\
&= -\frac{\operatorname{arctanh}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}-\frac{\csc^2(c+dx)}{2ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\left(1+2\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec(c+dx)}{2ad(1+\sec(c+dx))}$$

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x]),x]
[Out] -1/2*((1 + 2*Cos[(c + d*x)/2]^2*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/
2]]))*Sec[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

method	result	size
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$	35
norman	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4da}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}$	39
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{4}-\frac{1}{2(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{4}}{da}$	43
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{4}-\frac{1}{2(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{4}}{da}$	43
risch	$-\frac{e^{i(dx+c)}}{ad\left(e^{i(dx+c)}+1\right)^2}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2da}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2da}$	72

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/4*(-tan(1/2*d*x+1/2*c)^2+2*ln(tan(1/2*d*x+1/2*c)))/d/a
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)+1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2}{4\left(ad\cos(dx+c)+ad\right)}$$

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c) + 1)*l
og(-1/2*cos(d*x + c) + 1/2) + 2)/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x)
[Out] Integral(csc(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\log(\cos(dx+c)+1)}{a} - \frac{\log(\cos(dx+c)-1)}{a} + \frac{2}{a\cos(dx+c)+a}}{4\,d}$$

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/4*(log(cos(d*x + c) + 1)/a - log(cos(d*x + c) - 1)/a + 2/(a*cos(d*x + c)
 + a))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} + \frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{4\,d}$$

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 1/4*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (cos(d*x + c) -
1)/(a*(cos(d*x + c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.57

$$\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{1}{2\,d\,\left(a+a\,\cos\left(c+d\,x\right)\right)}-\frac{\mathrm{atanh}(\cos\left(c+d\,x\right))}{2\,a\,d}$$

```
[In] int(1/(sin(c + d*x)*(a + a/cos(c + d*x))),x)
```

```
[Out] - 1/(2*d*(a + a*cos(c + d*x))) - atanh(cos(c + d*x))/(2*a*d)
```

3.63 $\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\csc^4(c+dx)}{4ad}$$

```
[Out] -1/8*arctanh(cos(d*x+c))/a/d-1/8*cot(d*x+c)*csc(d*x+c)/a/d+1/4*cot(d*x+c)*c
sc(d*x+c)^3/a/d-1/4*csc(d*x+c)^4/a/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2914, 2686, 30, 2691, 3853, 3855}

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\cos(c+dx))}{8ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*ArcTanh[Cos[c + d*x]]/(a*d) - (Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (C
ot[c + d*x]*Csc[c + d*x]^3)/(4*a*d) - Csc[c + d*x]^4/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cot(c+dx)\csc^2(c+dx)}{-a-a\cos(c+dx)}\,dx$$

$$= -\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^4(c+dx)\,dx}{a}$$

$$= \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{\int \csc^3(c+dx)\,dx}{4a} - \frac{\text{Subst}\left(\int x^3\,dx, x, \csc(c+dx)\right)}{ad}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{8ad} + \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\csc^4(c+dx)}{4ad} + \frac{\int \csc(c+dx)\,dx}{8a}$$

$$= -\frac{\text{arctanh}(\cos(c+dx))}{8ad} - \frac{\cot(c+dx)\csc(c+dx)}{8ad} + \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\csc^4(c+dx)}{4ad}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\left(2\cot^2\left(\frac{1}{2}(c+dx)\right) + 4\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right) + \sec^2\left(\frac{1}{2}(c+\right.\right.}{16ad(1+\sec(c+dx))}$$

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/16*((2*Cot[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^2*(Log[Cos[(c + d*x)/2]]
- Log[Sin[(c + d*x)/2]]) + Sec[(c + d*x)/2]^2)*Sec[c + d*x])/(a*d*(1 + Sec[
c + d*x]))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{-\frac{1}{8(\cos(dx+c)+1)^2}-\frac{\ln(\cos(dx+c)+1)}{16}+\frac{1}{8\cos(dx+c)-8}+\frac{\ln(\cos(dx+c)-1)}{16}}{da}$	55
default	$\frac{-\frac{1}{8(\cos(dx+c)+1)^2}-\frac{\ln(\cos(dx+c)+1)}{16}+\frac{1}{8\cos(dx+c)-8}+\frac{\ln(\cos(dx+c)-1)}{16}}{da}$	55
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-2\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32da}$	61
norman	$\frac{-\frac{1}{16ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{32da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}$	79
risch	$\frac{e^{5i(dx+c)}+2\,e^{4i(dx+c)}+10\,e^{3i(dx+c)}+2\,e^{2i(dx+c)}+e^{i(dx+c)}}{4ad\left(e^{i(dx+c)}+1\right)^4\left(e^{i(dx+c)}-1\right)^2}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{8da}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{8da}$	128

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/8/(cos(d*x+c)+1)^2-1/16*ln(cos(d*x+c)+1)+1/8/(cos(d*x+c)-1)+1/16*
ln(cos(d*x+c)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\cos(dx+c)^2 - \left(\cos(dx+c)^3 + \cos(dx+c)^2 - \cos(dx+c) - 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + (\cos(dx+c}{16\left(ad\cos(dx+c)^3 + ad\cos(dx+c)^2 - a\right.}$$

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) -
1)*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x
 + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*cos(d*x + c) + 4)/(a*d*cos(d*x
+ c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)
```

Sympy [F]

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2\left(\cos(dx+c)^2+\cos(dx+c)+2\right)}{a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a} - \frac{\log(\cos(dx+c)+1)}{a} + \frac{\log(\cos(dx+c)-1)}{a}}{16\,d}$$

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(d*x + c)^2 + cos(d*x + c) + 2)/(a*cos(d*x + c)^3 + a*cos(d*x +
 c)^2 - a*cos(d*x + c) - a) - log(cos(d*x + c) + 1)/a + log(cos(d*x + c) -
1)/a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.57

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)}{a(\cos(dx+c)-1)} - \frac{2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} - \frac{\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}}{32\,d}$$

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)/(a*
(cos(d*x + c) - 1)) - 2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a
 - (2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2)/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{atanh}(\cos(c+d\,x))}{8\,a\,d} - \frac{\frac{\cos(c+d\,x)^2}{8}+\frac{\cos(c+d\,x)}{8}+\frac{1}{4}}{d\,\left(-a\cos(c+d\,x)^3-a\cos(c+d\,x)^2+a\,\cos(c+d\,x)+a\right)}$$

```
[In] int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))),x)
```

```
[Out] - atanh(cos(c + d*x))/(8*a*d) - (cos(c + d*x)/8 + cos(c + d*x)^2/8 + 1/4)/(
d*(a + a*cos(c + d*x) - a*cos(c + d*x)^2 - a*cos(c + d*x)^3))
```

3.64 $\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{16ad} - \frac{\cot(c+dx)\csc(c+dx)}{16ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{24ad} + \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\csc^6(c+dx)}{6ad}$$

```
[Out] -1/16*arctanh(cos(d*x+c))/a/d-1/16*cot(d*x+c)*csc(d*x+c)/a/d-1/24*cot(d*x+c
)*csc(d*x+c)^3/a/d+1/6*cot(d*x+c)*csc(d*x+c)^5/a/d-1/6*csc(d*x+c)^6/a/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2914, 2686, 30, 2691, 3853, 3855}

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{16ad} - \frac{\csc^6(c+dx)}{6ad} + \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{24ad} - \frac{\cot(c+dx)\csc(c+dx)}{16ad}$$

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/16*ArcTanh[Cos[c + d*x]]/(a*d) - (Cot[c + d*x]*Csc[c + d*x])/(16*a*d) -
(Cot[c + d*x]*Csc[c + d*x]^3)/(24*a*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a
*d) - Csc[c + d*x]^6/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)\csc^4(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^6(c+dx)\,dx}{a} \\
&= \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} + \frac{\int \csc^5(c+dx)\,dx}{6a} - \frac{\operatorname{Subst}\left(\int x^5\,dx, x, \csc(c+dx)\right)}{ad} \\
&= -\frac{\cot(c+dx)\csc^3(c+dx)}{24ad} + \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\csc^6(c+dx)}{6ad} + \frac{\int \csc^3(c+dx)\,dx}{8a} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{16ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{24ad} \\
&\quad + \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\csc^6(c+dx)}{6ad} + \frac{\int \csc(c+dx)\,dx}{16a} \\
&= -\frac{\operatorname{arctanh}(\cos(c+dx))}{16ad} - \frac{\cot(c+dx)\csc(c+dx)}{16ad} \\
&\quad - \frac{\cot(c+dx)\csc^3(c+dx)}{24ad} + \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\csc^6(c+dx)}{6ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.15

$$
\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+3\csc^4\left(\frac{1}{2}(c+dx)\right)+24\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+c\right.\right.\right.\right.}{192ad(1+\sec(c+dx))}
$$

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/192*(Cos[(c + d*x)/2]^2*(12*Csc[(c + d*x)/2]^2 + 3*Csc[(c + d*x)/2]^4 +
24*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 3*Sec[(c + d*x)/2]^4 +
 2*Sec[(c + d*x)/2]^6)*Sec[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{1}{32(\cos(dx+c)-1)^2}+\frac{1}{16\cos(dx+c)-16}+\frac{\ln(\cos(dx+c)-1)}{32}-\frac{1}{24(\cos(dx+c)+1)^3}-\frac{1}{32(\cos(dx+c)+1)^2}-\frac{\ln(\cos(dx+c)+1)}{32}}{da}$
default	$\frac{-\frac{1}{32(\cos(dx+c)-1)^2}+\frac{1}{16\cos(dx+c)-16}+\frac{\ln(\cos(dx+c)-1)}{32}-\frac{1}{24(\cos(dx+c)+1)^3}-\frac{1}{32(\cos(dx+c)+1)^2}-\frac{\ln(\cos(dx+c)+1)}{32}}{da}$
parallelrisch	$\frac{-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4-12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-18\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2+24\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384da}$
norman	$\frac{-\frac{1}{128ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{64da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{32da}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{128da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{192da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}$
risch	$\frac{3\,\mathrm{e}^{9i(dx+c)}+6\,\mathrm{e}^{8i(dx+c)}-8\,\mathrm{e}^{7i(dx+c)}-22\,\mathrm{e}^{6i(dx+c)}-150\,\mathrm{e}^{5i(dx+c)}-22\,\mathrm{e}^{4i(dx+c)}-8\,\mathrm{e}^{3i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}}{24ad\left(\mathrm{e}^{i(dx+c)}+1\right)^6\left(\mathrm{e}^{i(dx+c)}-1\right)^4}-$

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/32/(cos(d*x+c)-1)^2+1/16/(cos(d*x+c)-1)+1/32*ln(cos(d*x+c)-1)-1/2
4/(cos(d*x+c)+1)^3-1/32/(cos(d*x+c)+1)^2-1/32*ln(cos(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(96) = 192$.

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.05

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{6\cos(dx+c)^4+6\cos(dx+c)^3-10\cos(dx+c)^2-3\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-\right.}{96\,(}$$

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(6*cos(d*x + c)^4 + 6*cos(d*x + c)^3 - 10*cos(d*x + c)^2 - 3*(cos(d*x
+ c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c
) + 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2
*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*log(-1/2*cos(d*x + c
) + 1/2) - 10*cos(d*x + c) - 16)/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 -
 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c)),x)
[Out] Integral(csc(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.23

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2\left(3\cos(dx+c)^4+3\cos(dx+c)^3-5\cos(dx+c)^2-5\cos(dx+c)-8\right)}{a\cos(dx+c)^5+a\cos(dx+c)^4-2a\cos(dx+c)^3-2a\cos(dx+c)^2+a\cos(dx+c)+a}-\frac{3\log(\cos(dx+c)+1)}{a}+\frac{3\log(\cos(dx+c)-1)}{a}}{96\,d}$$

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/96*(2*(3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - 5*cos(d*x
 + c) - 8)/(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*
cos(d*x + c)^2 + a*cos(d*x + c) + a) - 3*log(cos(d*x + c) + 1)/a + 3*log(co
s(d*x + c) - 1)/a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.72

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{3\left(\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2}+\frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{\frac{12a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{9a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2a^2(\cos(dx+c)}{(\cos(dx+c)+}}{a^3}}{384\,d}$$

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 1/384*(3*(6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2) + 1
2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (12*a^2*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 9*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 + 2*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^3)/d
```

Mupad [B] (verification not implemented)

Time = 13.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.08

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\operatorname{atanh}(\cos(c+d\,x))}{16\,a\,d} - \frac{-\frac{\cos(c+d\,x)^4}{16}-\frac{\cos(c+d\,x)^3}{16}+\frac{5\cos(c+d\,x)^2}{48}+\frac{5\,\cos(c+d\,x)}{48}+\frac{1}{6}}{d\,\left(a\cos(c+d\,x)^5+a\cos(c+d\,x)^4-2\,a\cos(c+d\,x)^3-2\,a\cos(c+d\,x)^2+a\,\cos(c+d\,x)+a\right)}$$

```
[In] int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))),x)
```

```
[Out] - atanh(cos(c + d*x))/(16*a*d) - ((5*cos(c + d*x))/48 + (5*cos(c + d*x)^2)/
48 - cos(c + d*x)^3/16 - cos(c + d*x)^4/16 + 1/6)/(d*(a + a*cos(c + d*x) -
2*a*cos(c + d*x)^2 - 2*a*cos(c + d*x)^3 + a*cos(c + d*x)^4 + a*cos(c + d*x)
^5))
```

3.65 $\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{5x}{128a}-\frac{5\cos(c+dx)\sin(c+dx)}{128ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{64ad}+\frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad}+\frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad}+\frac{\sin^7(c+dx)}{7ad}$$

```
[Out] -5/128*x/a-5/128*cos(d*x+c)*sin(d*x+c)/a/d+5/64*cos(d*x+c)^3*sin(d*x+c)/a/d
+5/48*cos(d*x+c)^3*sin(d*x+c)^3/a/d+1/8*cos(d*x+c)^3*sin(d*x+c)^5/a/d+1/7*s
in(d*x+c)^7/a/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2918, 2644, 30, 2648, 2715, 8}

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin^7(c+dx)}{7ad}+\frac{\sin^5(c+dx)\cos^3(c+dx)}{8ad}+\frac{5\sin^3(c+dx)\cos^3(c+dx)}{48ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{64ad}-\frac{5\sin(c+dx)\cos(c+dx)}{128ad}-\frac{5x}{128a}$$

```
[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-5*x)/(128*a) - (5*Cos[c + d*x]*Sin[c + d*x])/(128*a*d) + (5*Cos[c + d*x]^
3*Sin[c + d*x])/(64*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x]^3)/(48*a*d) + (Co
s[c + d*x]^3*Sin[c + d*x]^5)/(8*a*d) + Sin[c + d*x]^7/(7*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin^8(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= \frac{\int \cos(c+dx)\sin^6(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin^6(c+dx)\,dx}{a}\\
&= \frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad}-\frac{5\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{8a}+\frac{\operatorname{Subst}\left(\int x^6\,dx,x,\sin(c+dx)\right)}{ad}\\
&= \frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad}+\frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad}\\
&\quad+\frac{\sin^7(c+dx)}{7ad}-\frac{5\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{16a}\\
&= \frac{5\cos^3(c+dx)\sin(c+dx)}{64ad}+\frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad}\\
&\quad+\frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad}+\frac{\sin^7(c+dx)}{7ad}-\frac{5\int \cos^2(c+dx)\,dx}{64a}\\
&= -\frac{5\cos(c+dx)\sin(c+dx)}{128ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{64ad}\\
&\quad+\frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad}+\frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad}+\frac{\sin^7(c+dx)}{7ad}-\frac{5\int 1\,dx}{128a}\\
&= -\frac{5x}{128a}-\frac{5\cos(c+dx)\sin(c+dx)}{128ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{64ad}\\
&\quad+\frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad}+\frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad}+\frac{\sin^7(c+dx)}{7ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$
\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx
= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(1176c-840dx+1680\sin(c+dx)+336\sin(2(c+dx))-1008\sin(3(c+dx)\right.}{10752}
$$

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1176*c - 840*d*x + 1680*Sin[c + d*x] + 33
6*Sin[2*(c + d*x)] - 1008*Sin[3*(c + d*x)] + 168*Sin[4*(c + d*x)] + 336*Sin
[5*(c + d*x)] - 112*Sin[6*(c + d*x)] - 48*Sin[7*(c + d*x)] + 21*Sin[8*(c +
d*x)] - 1176*Tan[c/2]))/(10752*a*d*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{-840dx+1680\sin(dx+c)-48\sin(7dx+7c)+336\sin(5dx+5c)-1008\sin(3dx+3c)+21\sin(8dx+8c)-112\sin(6dx+6c)+16}{21504da}$
risch	$-\frac{5x}{128a}+\frac{5\sin(dx+c)}{64ad}+\frac{\sin(8dx+8c)}{1024da}-\frac{\sin(7dx+7c)}{448da}-\frac{\sin(6dx+6c)}{192da}+\frac{\sin(5dx+5c)}{64da}+\frac{\sin(4dx+4c)}{128da}-\frac{3\sin}{}$
derivativedivides	$\frac{-\frac{256\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16384}-\frac{115\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{49152}-\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{49152}-\frac{5053\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{344064}-\frac{44099\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{344064}+\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{49152}+\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{8}}}{da}$
default	$\frac{-\frac{256\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16384}-\frac{115\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{49152}-\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{49152}-\frac{5053\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{344064}-\frac{44099\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{344064}+\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{49152}+\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{8}}}{da}$
norman	$\frac{-\frac{5x}{128a}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{115\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{192ad}+\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{192ad}+\frac{5053\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{1344ad}+\frac{44099\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{1344ad}-\frac{383\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{192ad}-}{}$

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21504*(-840*d*x+1680*sin(d*x+c)-48*sin(7*d*x+7*c)+336*sin(5*d*x+5*c)-1008
*sin(3*d*x+3*c)+21*sin(8*d*x+8*c)-112*sin(6*d*x+6*c)+168*sin(4*d*x+4*c)+336
*sin(2*d*x+2*c))/d/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{105\,dx-\left(336\,\cos\left(dx+c\right)^{7}-384\,\cos\left(dx+c\right)^{6}-952\,\cos\left(dx+c\right)^{5}+1152\,\cos\left(dx+c\right)^{4}+826\,\cos\left(d\right.\right.}{2688\,ad}$$

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2688*(105*d*x - (336*cos(d*x + c)^7 - 384*cos(d*x + c)^6 - 952*cos(d*x +
 c)^5 + 1152*cos(d*x + c)^4 + 826*cos(d*x + c)^3 - 1152*cos(d*x + c)^2 - 10
5*cos(d*x + c) + 384)*sin(d*x + c))/(a*d)
```

Sympy [F]

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sin^8(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c)),x)
[Out] Integral(sin(c + d*x)**8/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(113) = 226$.

Time = 0.29 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.88

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2681\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5053\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{44099\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{2681\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{805\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{105\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a+\frac{8a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}} - \frac{10}{}}{1344\,d}$$

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/1344*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 2681*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5053*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 + 44099*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2681*s
in(d*x + c)^11/(cos(d*x + c) + 1)^11 - 805*sin(d*x + c)^13/(cos(d*x + c) +
1)^13 - 105*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a + 8*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 56*a*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
+ 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a*sin(d*x + c)^12/(cos(d*
x + c) + 1)^12 + 8*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a*sin(d*x + c)
^16/(cos(d*x + c) + 1)^16) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)
/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}+805\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+2681\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-44099\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-5053\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-2681}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{8}a}}{2688\,d}$$

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2688*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^15 + 805*tan(1/2*d*x
 + 1/2*c)^13 + 2681*tan(1/2*d*x + 1/2*c)^11 - 44099*tan(1/2*d*x + 1/2*c)^9
- 5053*tan(1/2*d*x + 1/2*c)^7 - 2681*tan(1/2*d*x + 1/2*c)^5 - 805*tan(1/2*d
*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*a
))/d
```

Mupad [B] (verification not implemented)

Time = 16.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}-\frac{115\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}-\frac{383\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{192}+\frac{44099\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{1344}+\frac{5053\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{1344}+\frac{383\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{192}+\cdots}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^8}$$
$$-\frac{5\,x}{128\,a}$$

```
[In] int(sin(c + d*x)^8/(a + a/cos(c + d*x)),x)
```

```
[Out] ((5*tan(c/2 + (d*x)/2))/64 + (115*tan(c/2 + (d*x)/2)^3)/192 + (383*tan(c/2
+ (d*x)/2)^5)/192 + (5053*tan(c/2 + (d*x)/2)^7)/1344 + (44099*tan(c/2 + (d*
x)/2)^9)/1344 - (383*tan(c/2 + (d*x)/2)^11)/192 - (115*tan(c/2 + (d*x)/2)^1
3)/192 - (5*tan(c/2 + (d*x)/2)^15)/64)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^8) -
 (5*x)/(128*a)
```

3.66 $\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{16a}-\frac{\cos(c+dx)\sin(c+dx)}{16ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad}+\frac{\sin^5(c+dx)}{5ad}$$

```
[Out] -1/16*x/a-1/16*cos(d*x+c)*sin(d*x+c)/a/d+1/8*cos(d*x+c)^3*sin(d*x+c)/a/d+1/
6*cos(d*x+c)^3*sin(d*x+c)^3/a/d+1/5*sin(d*x+c)^5/a/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2918, 2644, 30, 2648, 2715, 8}

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/16*x/a - (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c +
d*x])/(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a*d) + Sin[c + d*x]^5/(5
*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos(c+dx)\sin^6(c+dx)}{-a-a\cos(c+dx)}\,dx$$

$$
\begin{aligned}
&= \frac{\int \cos(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a} \\
&= \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{2a} + \frac{\text{Subst}\left(\int x^4\,dx, x, \sin(c+dx)\right)}{ad} \\
&= \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\sin^5(c+dx)}{5ad} - \frac{\int \cos^2(c+dx)\,dx}{8a} \\
&= -\frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} \\
&\quad + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\sin^5(c+dx)}{5ad} - \frac{\int 1\,dx}{16a} \\
&= -\frac{x}{16a} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} \\
&\quad + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\sin^5(c+dx)}{5ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

$$
\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx
= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(75c-60dx+120\sin(c+dx)+15\sin(2(c+dx))-60\sin(3(c+dx))+15\sin\right.}{480ad(1+\sec(c+dx))}
$$

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(75*c - 60*d*x + 120*Sin[c + d*x] + 15*Sin
[2*(c + d*x)] - 60*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*x)] + 12*Sin[5*(c + d
*x)] - 5*Sin[6*(c + d*x)] - 75*Tan[c/2]))/(480*a*d*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{-60dx+120\sin(dx+c)+12\sin(5dx+5c)-60\sin(3dx+3c)-5\sin(6dx+6c)+15\sin(4dx+4c)+15\sin(2dx+2c)}{960da}$
risch	$-\frac{x}{16a}+\frac{\sin(dx+c)}{8ad}-\frac{\sin(6dx+6c)}{192da}+\frac{\sin(5dx+5c)}{80da}+\frac{\sin(4dx+4c)}{64da}-\frac{\sin(3dx+3c)}{16da}+\frac{\sin(2dx+2c)}{64da}$
derivativedivides	$\frac{-\frac{64\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{512}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{1536}-\frac{223\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{1280}-\frac{33\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{1280}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{1536}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{6}}-\frac{\arctan\left(\tan\left(\right.\right.}{8}}{da}$
default	$\frac{-\frac{64\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{512}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{1536}-\frac{223\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{1280}-\frac{33\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{1280}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{1536}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{6}}-\frac{\arctan\left(\tan\left(\right.\right.}{8}}{da}$
norman	$\frac{-\frac{x}{16a}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{24ad}+\frac{33\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{20ad}+\frac{223\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{20ad}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{24ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8ad}-\frac{3x\tan\left(\frac{dx}{2}+\right.}{8a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{6}}$

```
[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/960*(-60*d*x+120*sin(d*x+c)+12*sin(5*d*x+5*c)-60*sin(3*d*x+3*c)-5*sin(6*d
*x+6*c)+15*sin(4*d*x+4*c)+15*sin(2*d*x+2*c))/d/a
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{15\,dx+\left(40\,\cos\left(dx+c\right)^5-48\,\cos\left(dx+c\right)^4-70\,\cos\left(dx+c\right)^3+96\,\cos\left(dx+c\right)^2+15\,\cos\left(dx+c\right)-\right.}{240\,ad}$$

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(15*d*x + (40*cos(d*x + c)^5 - 48*cos(d*x + c)^4 - 70*cos(d*x + c)^3
 + 96*cos(d*x + c)^2 + 15*cos(d*x + c) - 48)*sin(d*x + c))/(a*d)
```

Sympy [F]

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sin^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(89) = 178$.

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.81

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{85\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{198\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1338\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{85\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{15\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a+\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{120\,d}$$

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 85*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 198*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1338*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 - 85*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*sin(d*x +
c)^11/(cos(d*x + c) + 1)^11)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 15*arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.14

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{15(dx+c)}{a}+\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+85\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-1338\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-198\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-85\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-15\tan\left(\frac{1}{2}dx+\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^6 a}}{240\,d}$$

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^11 + 85*tan(1/2*d*x + 1
/2*c)^9 - 1338*tan(1/2*d*x + 1/2*c)^7 - 198*tan(1/2*d*x + 1/2*c)^5 - 85*tan
(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1
)^6*a))/d
```

Mupad [B] (verification not implemented)

Time = 16.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{223\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{33\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6} - \frac{x}{16\,a}$$

```
[In] int(sin(c + d*x)^6/(a + a/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)/8 + (17*tan(c/2 + (d*x)/2)^3)/24 + (33*tan(c/2 + (d*x)/
2)^5)/20 + (223*tan(c/2 + (d*x)/2)^7)/20 - (17*tan(c/2 + (d*x)/2)^9)/24 - t
an(c/2 + (d*x)/2)^11/8)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) - x/(16*a)
```

3.67 $\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{8a}-\frac{\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}$$

```
[Out] -1/8*x/a-1/8*cos(d*x+c)*sin(d*x+c)/a/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a/d+1/3*
sin(d*x+c)^3/a/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2918, 2644, 30, 2648, 2715, 8}

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\sin(c+dx)\cos(c+dx)}{8ad}-\frac{x}{8a}$$

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*x/a - (Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*
x])/(4*a*d) + Sin[c + d*x]^3/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos(c+dx)\sin^4(c+dx)}{-a-a\cos(c+dx)}\,dx$$

$$= \frac{\int \cos(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int \cos^2(c+dx)\,dx}{4a} + \frac{\text{Subst}\left(\int x^2\,dx, x, \sin(c+dx)\right)}{ad}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\sin^3(c+dx)}{3ad} - \frac{\int 1\,dx}{8a}$$

$$= -\frac{x}{8a} - \frac{\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\sin^3(c+dx)}{3ad}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(24\sin(c+dx) - 8\sin(3(c+dx)) + 3\left(4c - 4dx + \sin(4(c+dx)) - 4\tan\left(\frac{c}{2}\right)\right)\right)}{48ad(1+\sec(c+dx))}$$

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x]),x]
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(24*Sin[c + d*x] - 8*Sin[3*(c + d*x)] + 3*
(4*c - 4*d*x + Sin[4*(c + d*x)] - 4*Tan[c/2])))/(48*a*d*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{-12dx+24\sin(dx+c)-8\sin(3dx+3c)+3\sin(4dx+4c)}{96da}$
risch	$-\frac{x}{8a}+\frac{\sin(dx+c)}{4ad}+\frac{\sin(4dx+4c)}{32da}-\frac{\sin(3dx+3c)}{12da}$
derivativedivides	$\frac{-\frac{16\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{64}-\frac{53\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{192}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{192}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
default	$\frac{-\frac{16\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{64}-\frac{53\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{192}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{192}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
norman	$\frac{-\frac{x}{8a}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12ad}+\frac{53\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4ad}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}-\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2a}-}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/96*(-12*d*x+24*sin(d*x+c)-8*sin(3*d*x+3*c)+3*sin(4*d*x+4*c))/d/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3\,dx - \left(6\,\cos\left(dx+c\right)^{3} - 8\,\cos\left(dx+c\right)^{2} - 3\,\cos\left(dx+c\right) + 8\right)\sin\left(dx+c\right)}{24\,ad}$$

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/24*(3*d*x - (6*cos(d*x + c)^3 - 8*cos(d*x + c)^2 - 3*cos(d*x + c) + 8)*s
in(d*x + c))/(a*d)
```

Sympy [F]

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sin^{4}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c)),x)
[Out] Integral(sin(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.68

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{11\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} + \frac{53\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}} - \frac{3\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a + \frac{4\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{6\,a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}} + \frac{4\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}} + \frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}} - \frac{3\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12\,d}$$

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/12*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 53*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos(d*x
```

```
 + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{3\,(dx+c)}{a}+\frac{2\,\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-53\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a}}{24\,d}$$

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(3*(d*x + c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^7 - 53*tan(1/2*d*x + 1/2*c
)^5 - 11*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^4*a))/d
```

Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin\left(c+d\,x\right)}{4\,a\,d}-\frac{x}{8\,a}-\frac{\sin\left(3\,c+3\,d\,x\right)}{12\,a\,d}+\frac{\sin\left(4\,c+4\,d\,x\right)}{32\,a\,d}$$

```
[In] int(sin(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

```
[Out] sin(c + d*x)/(4*a*d) - x/(8*a) - sin(3*c + 3*d*x)/(12*a*d) + sin(4*c + 4*d*
x)/(32*a*d)
```

3.68 $\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{x}{2a}+\frac{\sin(c+dx)}{ad}-\frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

```
[Out] -1/2*x/a+sin(d*x+c)/a/d-1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2918, 2717, 2715, 8}

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x}{2a}$$

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*x/a + Sin[c + d*x]/(a*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin^2(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\int \cos(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\,dx}{a} \\
&= \frac{\sin(c+dx)}{ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int 1\,dx}{2a} \\
&= -\frac{x}{2a} + \frac{\sin(c+dx)}{ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$
\begin{aligned}
&\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx \\
&= -\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-c+2dx-4\sin(c+dx)+\sin(2(c+dx))+\tan\left(\frac{c}{2}\right)\right)}{2ad(1+\sec(c+dx))}
\end{aligned}
$$

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(-c + 2*d*x - 4*Sin[c + d*x] + Sin[2*
(c + d*x)] + Tan[c/2]))/(a*d*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

method	result	size
parallelrisch	$\frac{-2dx+4\sin(dx+c)-\sin(2dx+2c)}{4da}$	33
risch	$-\frac{x}{2a}+\frac{\sin(dx+c)}{ad}-\frac{\sin(2dx+2c)}{4da}$	38
derivativedivides	$\frac{-\frac{4\left(-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	64
default	$\frac{-\frac{4\left(-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	64
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{x}{2a}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	93

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/4*(-2*d*x+4*sin(d*x+c)-sin(2*d*x+2*c))/d/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{dx+(\cos(dx+c)-2)\sin(dx+c)}{2\,ad}$$

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(d*x + (cos(d*x + c) - 2)*sin(d*x + c))/(a*d)
```

Sympy [F]

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sin^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c)),x)
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(40) = 80$.

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.55

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} - \frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/
(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4) - arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{dx+c}{a} - \frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 a}}{2d}$$

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sin(2c+2dx)-4\sin(c+dx)+2dx}{4ad}$$

```
[In] int(sin(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

```
[Out] -(sin(2*c + 2*d*x) - 4*sin(c + d*x) + 2*d*x)/(4*a*d)
```

3.69 $\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d-1/3*csc(d*x+c)^3/a/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2918, 2686, 30, 2687}

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) - Csc[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)\csc(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^3(c+dx)\,dx}{a} \\
&= -\frac{\operatorname{Subst}\left(\int x^2\,dx, x, -\cot(c+dx)\right)}{ad} - \frac{\operatorname{Subst}\left(\int x^2\,dx, x, \csc(c+dx)\right)}{ad} \\
&= \frac{\cot^3(c+dx)}{3ad} - \frac{\csc^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.78

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\csc(c)\csc(2(c+dx))(-6\sin(c)+4\sin(dx)+2\sin(c+dx)+\sin(2(c+dx))+2\sin(c+2dx))}{6ad(1+\sec(c+dx))}$$

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x]),x]
[Out] (Csc[c]*Csc[2*(c + d*x)]*(-6*Sin[c] + 4*Sin[d*x] + 2*Sin[c + d*x] + Sin[2*(
c + d*x)] + 2*Sin[c + 2*d*x]))/(6*a*d*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-3}{12d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a}$	35
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	36
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	36
norman	$\frac{-\frac{1}{4ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{12da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$	41
risch	$-\frac{2i\left(3\,e^{2i(dx+c)}+2\,e^{i(dx+c)}+1\right)}{3ad\left(e^{i(dx+c)}+1\right)^{3}\left(e^{i(dx+c)}-1\right)}$	60

```
[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/12*(-tan(1/2*d*x+1/2*c)^4-3)/d/tan(1/2*d*x+1/2*c)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{\cos\left(dx+c\right)^2+\cos\left(dx+c\right)+1}{3\left(ad\cos\left(dx+c\right)+ad\right)\sin\left(dx+c\right)}$$

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/3*(cos(d*x + c)^2 + cos(d*x + c) + 1)/((a*d*cos(d*x + c) + a*d)*sin(d*x
+ c))
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\int \frac{\csc^2{(c+dx)}}{\sec{(c+dx)}+1}\,dx}{a}$$

```
[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c)),x)
[Out] Integral(csc(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{3\,(\cos(dx+c)+1)}{a\sin(dx+c)}+\frac{\sin(dx+c)^3}{a(\cos(dx+c)+1)^3}}{12\,d}$$

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(cos(d*x + c) + 1)/(a*sin(d*x + c)) + sin(d*x + c)^3/(a*(cos(d*x +
 c) + 1)^3))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{a}+\frac{3}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{12\,d}$$

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(tan(1/2*d*x + 1/2*c)^3/a + 3/(a*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3}{12\,a\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

```
[In] int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^4 + 3)/(12*a*d*tan(c/2 + (d*x)/2))
```

3.70 $\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}-\frac{\csc^5(c+dx)}{5ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+1/5*cot(d*x+c)^5/a/d-1/5*csc(d*x+c)^5/a/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2918, 2686, 30, 2687, 14}

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^5(c+dx)}{5ad}$$

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x]),x]
[Out] Cot[c + d*x]^3/(3*a*d) + Cot[c + d*x]^5/(5*a*d) - Csc[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)\csc^3(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= -\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^5(c+dx)\,dx}{a}\\
&= -\frac{\operatorname{Subst}\left(\int x^4\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,-\cot(c+dx)\right)}{ad}\\
&= -\frac{\csc^5(c+dx)}{5ad}-\frac{\operatorname{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,-\cot(c+dx)\right)}{ad}\\
&= \frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}-\frac{\csc^5(c+dx)}{5ad}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 116 vs. $2(55) = 110$.

Time = 0.68 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.11

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\csc(c)\csc^3(c+dx)\sec(c+dx)(240\sin(c)-96\sin(dx)-54\sin(c+dx)-18\sin(2(c+dx))+18\sin(3(}{960ad(1+\sec(c+d}$$

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/960*(Csc[c]*Csc[c + d*x]^3*Sec[c + d*x]*(240*Sin[c] - 96*Sin[d*x] - 54*S
in[c + d*x] - 18*Sin[2*(c + d*x)] + 18*Sin[3*(c + d*x)] + 9*Sin[4*(c + d*x)
] - 32*Sin[c + 2*d*x] + 32*Sin[2*c + 3*d*x] + 16*Sin[3*c + 4*d*x]))/(a*d*(1
 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-5\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-30\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{240da}$	60
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{16da}$	62
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{16da}$	62
norman	$\frac{-\frac{1}{48ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{8da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{24da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{80da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}$	79
risch	$\frac{4i\left(15\,e^{4i(dx+c)}+6\,e^{3i(dx+c)}+2\,e^{2i(dx+c)}-2\,e^{i(dx+c)}-1\right)}{15ad\left(e^{i(dx+c)}+1\right)^{5}\left(e^{i(dx+c)}-1\right)^{3}}$	82

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*(-3*tan(1/2*d*x+1/2*c)^5-5*cot(1/2*d*x+1/2*c)^3-10*tan(1/2*d*x+1/2*c)
^3-30*cot(1/2*d*x+1/2*c))/d/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{2\cos(dx+c)^4+2\cos(dx+c)^3-3\cos(dx+c)^2-3\cos(dx+c)-3}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad\right)\sin(dx+c)}$$

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 3*cos(d*x +
 c) - 3)/((a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d
)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.75

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}+\frac{5\left(\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{240\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*((10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5)/a + 5*(6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x +
c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{5\left(6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{a^5}}{240\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(5*(6*tan(1/2*d*x + 1/2*c)^2 + 1)/(a*tan(1/2*d*x + 1/2*c)^3) + (3*a^
4*tan(1/2*d*x + 1/2*c)^5 + 10*a^4*tan(1/2*d*x + 1/2*c)^3)/a^5)/d
```

Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+30\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+5}{240\,a\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

```
[In] int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))),x)
```

```
[Out] -(30*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^6 + 3*tan(c/2 + (d*x)/2)^
8 + 5)/(240*a*d*tan(c/2 + (d*x)/2)^3)
```

3.71 $\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\cot^3(c+dx)}{3ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^7(c+dx)}{7ad}-\frac{\csc^7(c+dx)}{7ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+2/5*cot(d*x+c)^5/a/d+1/7*cot(d*x+c)^7/a/d-1/7*csc(d*x+
c)^7/a/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2918, 2686, 30, 2687, 276}

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\cot^7(c+dx)}{7ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^7(c+dx)}{7ad}$$

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + (2*Cot[c + d*x]^5)/(5*a*d) + Cot[c + d*x]^7/(7*a*d
) - Csc[c + d*x]^7/(7*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)\csc^5(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc^6(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^7(c+dx)\,dx}{a} \\
&= -\frac{\text{Subst}\left(\int x^6\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^2\,dx,x,-\cot(c+dx)\right)}{ad} \\
&= -\frac{\csc^7(c+dx)}{7ad}-\frac{\text{Subst}\left(\int \left(x^2+2x^4+x^6\right)\,dx,x,-\cot(c+dx)\right)}{ad} \\
&= \frac{\cot^3(c+dx)}{3ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^7(c+dx)}{7ad}-\frac{\csc^7(c+dx)}{7ad}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. $2(73) = 146$.

Time = 0.83 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.16

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\csc(c)\csc^5(c+dx)\sec(c+dx)(-8960\sin(c)+2560\sin(dx)+1500\sin(c+dx)+375\sin(2(c+dx))-750}{}$$

[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x]),x]

[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]*(-8960*Sin[c] + 2560*Sin[d*x] + 1500*Si
n[c + d*x] + 375*Sin[2*(c + d*x)] - 750*Sin[3*(c + d*x)] - 300*Sin[4*(c + d
*x)] + 150*Sin[5*(c + d*x)] + 75*Sin[6*(c + d*x)] + 640*Sin[c + 2*d*x] - 12
80*Sin[2*c + 3*d*x] - 512*Sin[3*c + 4*d*x] + 256*Sin[4*c + 5*d*x] + 128*Sin
[5*c + 6*d*x]))/(53760*a*d*(1 + Sec[c + d*x]))

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18

method	result	size
parallelrisch	$\frac{-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-84\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-21\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-175\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-140\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-525\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{6720da}$	86
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{4}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{64da}$	88
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{4}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{64da}$	88
risch	$-\frac{16i\left(70\,e^{6i(dx+c)}+20\,e^{5i(dx+c)}+5\,e^{4i(dx+c)}-10\,e^{3i(dx+c)}-4\,e^{2i(dx+c)}+2\,e^{i(dx+c)}+1\right)}{105ad\left(e^{i(dx+c)}+1\right)^{7}\left(e^{i(dx+c)}-1\right)^{5}}$	104
norman	$\frac{-\frac{1}{320ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{448ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{48da}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{64da}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{192da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{80da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}$	117

[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/6720*(-15*tan(1/2*d*x+1/2*c)^7-84*tan(1/2*d*x+1/2*c)^5-21*cot(1/2*d*x+1/2
*c)^5-175*tan(1/2*d*x+1/2*c)^3-140*cot(1/2*d*x+1/2*c)^3-525*cot(1/2*d*x+1/2
*c))/d/a

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(65) = 130$.

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{8\cos(dx+c)^6+8\cos(dx+c)^5-20\cos(dx+c)^4-20\cos(dx+c)^3+15\cos(dx+c)^2+15\cos(d}{105\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4-2\,ad\cos(dx+c)^3-2\,ad\cos(dx+c)^2+ad\cos(dx+c)+ad\right.}$$

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/105*(8*cos(d*x + c)^6 + 8*cos(d*x + c)^5 - 20*cos(d*x + c)^4 - 20*cos(d*
x + c)^3 + 15*cos(d*x + c)^2 + 15*cos(d*x + c) + 15)/((a*d*cos(d*x + c)^5 +
 a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos
(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c)),x)
[Out] Integral(csc(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.86

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\frac{175\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a}+\frac{7\left(\frac{20\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{75\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)(\cos(dx+c)+1)^5}{a\sin(dx+c)^5}}{6720\,d}$$

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/6720*((175*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a + 7*(20*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3)*
(cos(d*x + c) + 1)^5/(a*sin(d*x + c)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{7\left(75\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+20\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}+\frac{15\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+84\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+175\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{a^7}}{6720\,d}$$

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(7*(75*tan(1/2*d*x + 1/2*c)^4 + 20*tan(1/2*d*x + 1/2*c)^2 + 3)/(a*t
an(1/2*d*x + 1/2*c)^5) + (15*a^6*tan(1/2*d*x + 1/2*c)^7 + 84*a^6*tan(1/2*d*
x + 1/2*c)^5 + 175*a^6*tan(1/2*d*x + 1/2*c)^3)/a^7)/d
```

Mupad [B] (verification not implemented)

Time = 14.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.10

$$\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{21\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+140\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+525\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+175\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{6720\,a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

```
[In] int(1/(sin(c + d*x)^6*(a + a/cos(c + d*x))),x)
```

```
[Out] -(21*cos(c/2 + (d*x)/2)^12 + 15*sin(c/2 + (d*x)/2)^12 + 84*cos(c/2 + (d*x)/
2)^2*sin(c/2 + (d*x)/2)^10 + 175*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8
+ 525*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 140*cos(c/2 + (d*x)/2)^10
*sin(c/2 + (d*x)/2)^2)/(6720*a*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)
```

3.72 $\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^3(c+dx)}{3ad}+\frac{3\cot^5(c+dx)}{5ad}+\frac{3\cot^7(c+dx)}{7ad}+\frac{\cot^9(c+dx)}{9ad}-\frac{\csc^9(c+dx)}{9ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+3/5*cot(d*x+c)^5/a/d+3/7*cot(d*x+c)^7/a/d+1/9*cot(d*x+
c)^9/a/d-1/9*csc(d*x+c)^9/a/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2918, 2686, 30, 2687, 276}

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^9(c+dx)}{9ad}+\frac{3\cot^7(c+dx)}{7ad}+\frac{3\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^9(c+dx)}{9ad}$$

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + (3*Cot[c + d*x]^5)/(5*a*d) + (3*Cot[c + d*x]^7)/(7
*a*d) + Cot[c + d*x]^9/(9*a*d) - Csc[c + d*x]^9/(9*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)\csc^7(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc^8(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^9(c+dx)\,dx}{a} \\
&= -\frac{\text{Subst}\left(\int x^8\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2(1+x^2)^3\,dx,x,-\cot(c+dx)\right)}{ad} \\
&= -\frac{\csc^9(c+dx)}{9ad}-\frac{\text{Subst}\left(\int \left(x^2+3x^4+3x^6+x^8\right)\,dx,x,-\cot(c+dx)\right)}{ad}
\end{aligned}
$$

$$= \frac{\cot^3(c+dx)}{3ad} + \frac{3\cot^5(c+dx)}{5ad} + \frac{3\cot^7(c+dx)}{7ad} + \frac{\cot^9(c+dx)}{9ad} - \frac{\csc^9(c+dx)}{9ad}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 200 vs. $2(91) = 182$.

Time = 2.07 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.20

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{\csc(c)\csc^7(c+dx)\sec(c+dx)(645120\sin(c) - 143360\sin(dx) - 85750\sin(c+dx) - 17150\sin(2(c+d}{}$$

```
[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/5160960*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]*(645120*Sin[c] - 143360*Sin[
d*x] - 85750*Sin[c + d*x] - 17150*Sin[2*(c + d*x)] + 51450*Sin[3*(c + d*x)]
 + 17150*Sin[4*(c + d*x)] - 17150*Sin[5*(c + d*x)] - 7350*Sin[6*(c + d*x)]
+ 2450*Sin[7*(c + d*x)] + 1225*Sin[8*(c + d*x)] - 28672*Sin[c + 2*d*x] + 86
016*Sin[2*c + 3*d*x] + 28672*Sin[3*c + 4*d*x] - 28672*Sin[4*c + 5*d*x] - 12
288*Sin[5*c + 6*d*x] + 4096*Sin[6*c + 7*d*x] + 2048*Sin[7*c + 8*d*x]))/(a*d
*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.23

method	result
parallelrisch	$\frac{-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9-270\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-45\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7-882\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-378\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5-1470\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-14}{80640da}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{6}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}-\frac{}{3\tan}}{256da}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{6}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}-\frac{}{3\tan}}{256da}$
risch	$\frac{32i\left(315\,e^{8i(dx+c)}+70\,e^{7i(dx+c)}+14\,e^{6i(dx+c)}-42\,e^{5i(dx+c)}-14\,e^{4i(dx+c)}+14\,e^{3i(dx+c)}+6\,e^{2i(dx+c)}-2\,e^{i(dx+c)}-1\right)}{315ad\left(e^{i(dx+c)}+1\right)^9\left(e^{i(dx+c)}-1\right)^7}$
norman	$\frac{-\frac{1}{1792ad}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{640ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{896ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}}{2304ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{640da}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{384da}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{128da}-\frac{7\tan\left(\frac{dx}{2}\right.}{384}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}$

```
[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/80640*(-35*tan(1/2*d*x+1/2*c)^9-270*tan(1/2*d*x+1/2*c)^7-45*cot(1/2*d*x+1
/2*c)^7-882*tan(1/2*d*x+1/2*c)^5-378*cot(1/2*d*x+1/2*c)^5-1470*tan(1/2*d*x+
1/2*c)^3-1470*cot(1/2*d*x+1/2*c)^3-4410*cot(1/2*d*x+1/2*c))/d/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(81) = 162$.

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.95

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{16\cos(dx+c)^8+16\cos(dx+c)^7-56\cos(dx+c)^6-56\cos(dx+c)^5+70\cos(dx+c)^4+70\cos(}{315\left(ad\cos(dx+c)^7+ad\cos(dx+c)^6-3\,ad\cos(dx+c)^5-3\,ad\cos(dx+c)^4+3\,ad\cos(dx+c)^3+3}$$

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/315*(16*cos(d*x + c)^8 + 16*cos(d*x + c)^7 - 56*cos(d*x + c)^6 - 56*cos(
d*x + c)^5 + 70*cos(d*x + c)^4 + 70*cos(d*x + c)^3 - 35*cos(d*x + c)^2 - 35
*cos(d*x + c) - 35)/((a*d*cos(d*x + c)^7 + a*d*cos(d*x + c)^6 - 3*a*d*cos(d
*x + c)^5 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^3 + 3*a*d*cos(d*x + c
)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\csc^8(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**8/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(81) = 162$.

Time = 0.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.93

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\frac{1470\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{882\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a}+\frac{3\left(\frac{126\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{490\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1470\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+15\right)(\cos(dx+c)+1}{a\sin(dx+c)^7}}{80640\,d}$$

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80640*((1470*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 882*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 + 270*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9)/a + 3*(126*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 490*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1470*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 15)*(cos(d*x + c) + 1)^7/(a*sin(d*x + c)^7))/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.45

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{3\left(1470\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+490\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+126\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}+\frac{35\,a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+270\,a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+882\,a^8\tan\left(\frac{1}{2}\right.}{a^9}}{80640\,d}$$

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(3*(1470*tan(1/2*d*x + 1/2*c)^6 + 490*tan(1/2*d*x + 1/2*c)^4 + 126
*tan(1/2*d*x + 1/2*c)^2 + 15)/(a*tan(1/2*d*x + 1/2*c)^7) + (35*a^8*tan(1/2*
d*x + 1/2*c)^9 + 270*a^8*tan(1/2*d*x + 1/2*c)^7 + 882*a^8*tan(1/2*d*x + 1/2
*c)^5 + 1470*a^8*tan(1/2*d*x + 1/2*c)^3)/a^9)/d
```

Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.21

$$\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+378\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1470\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4410\cos\left(\frac{c}{2}\right.}{}$$

```
[In] int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))),x)
```

```
[Out] -(45*cos(c/2 + (d*x)/2)^16 + 35*sin(c/2 + (d*x)/2)^16 + 270*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^14 + 882*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^1
2 + 1470*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 4410*cos(c/2 + (d*x)/
2)^10*sin(c/2 + (d*x)/2)^6 + 1470*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^
4 + 378*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2)/(80640*a*d*cos(c/2 + (d
*x)/2)^9*sin(c/2 + (d*x)/2)^7)
```

3.73 $\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^3(c+dx)}{3ad} + \frac{4\cot^5(c+dx)}{5ad} + \frac{6\cot^7(c+dx)}{7ad} + \frac{4\cot^9(c+dx)}{9ad} + \frac{\cot^{11}(c+dx)}{11ad} - \frac{\csc^{11}(c+dx)}{11ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+4/5*cot(d*x+c)^5/a/d+6/7*cot(d*x+c)^7/a/d+4/9*cot(d*x+
c)^9/a/d+1/11*cot(d*x+c)^11/a/d-1/11*csc(d*x+c)^11/a/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2918, 2686, 30, 2687, 276}

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^{11}(c+dx)}{11ad} + \frac{4\cot^9(c+dx)}{9ad} + \frac{6\cot^7(c+dx)}{7ad} + \frac{4\cot^5(c+dx)}{5ad} + \frac{\cot^3(c+dx)}{3ad} - \frac{\csc^{11}(c+dx)}{11ad}$$

```
[In] Int[Csc[c + d*x]^10/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + (4*Cot[c + d*x]^5)/(5*a*d) + (6*Cot[c + d*x]^7)/(7
*a*d) + (4*Cot[c + d*x]^9)/(9*a*d) + Cot[c + d*x]^11/(11*a*d) - Csc[c + d*x
]^11/(11*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)\csc^9(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^{11}(c+dx)\,dx}{a} \\
&= -\frac{\text{Subst}\left(\int x^{10}\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^4\,dx,x,-\cot(c+dx)\right)}{ad}
\end{aligned}
$$

$$= -\frac{\csc^{11}(c+dx)}{11ad} - \frac{\text{Subst}\left(\int \left(x^2+4x^4+6x^6+4x^8+x^{10}\right)\, dx, x, -\cot(c+dx)\right)}{ad}$$

$$= \frac{\cot^3(c+dx)}{3ad} + \frac{4\cot^5(c+dx)}{5ad} + \frac{6\cot^7(c+dx)}{7ad} + \frac{4\cot^9(c+dx)}{9ad} + \frac{\cot^{11}(c+dx)}{11ad} - \frac{\csc^{11}(c+dx)}{11ad}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 242 vs. $2(109) = 218$.

Time = 3.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.22

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\, dx$$
$$= \frac{\csc(c)\csc^9(c+dx)\sec(c+dx)(-45416448\sin(c)+8257536\sin(dx)+5000940\sin(c+dx)+833490\sin(2(}{}$$

```
[In] Integrate[Csc[c + d*x]^10/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^9*Sec[c + d*x]*(-45416448*Sin[c] + 8257536*Sin[d*x] +
5000940*Sin[c + d*x] + 833490*Sin[2*(c + d*x)] - 3333960*Sin[3*(c + d*x)] -
 952560*Sin[4*(c + d*x)] + 1428840*Sin[5*(c + d*x)] + 535815*Sin[6*(c + d*x
)] - 357210*Sin[7*(c + d*x)] - 158760*Sin[8*(c + d*x)] + 39690*Sin[9*(c + d
*x)] + 19845*Sin[10*(c + d*x)] + 1376256*Sin[c + 2*d*x] - 5505024*Sin[2*c +
 3*d*x] - 1572864*Sin[3*c + 4*d*x] + 2359296*Sin[4*c + 5*d*x] + 884736*Sin[
5*c + 6*d*x] - 589824*Sin[6*c + 7*d*x] - 262144*Sin[7*c + 8*d*x] + 65536*Si
n[8*c + 9*d*x] + 32768*Sin[9*c + 10*d*x]))/(454164480*a*d*(1 + Sec[c + d*x]
))
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

method	result
parallelrisch	$\frac{-315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-385\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-3080\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-3960\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-13365\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-18711\cot\left(\frac{dx}{2}+\right.}{3548160da}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{27\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{48\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-\frac{1}{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-}{1024da}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{27\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{48\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-\frac{1}{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-}{1024da}$
risch	$-\frac{256i\left(1386\,e^{10i(dx+c)}+252\,e^{9i(dx+c)}+42\,e^{8i(dx+c)}-168\,e^{7i(dx+c)}-48\,e^{6i(dx+c)}+72\,e^{5i(dx+c)}+27\,e^{4i(dx+c)}-18\,e^{3i(dx+\right.}{3465ad\left(e^{i(dx+c)}+1\right)^{11}\left(e^{i(dx+c)}-1\right)^{9}}$

```
[In] int(csc(d*x+c)^10/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3548160*(-315*tan(1/2*d*x+1/2*c)^11-385*cot(1/2*d*x+1/2*c)^9-3080*tan(1/2
*d*x+1/2*c)^9-3960*cot(1/2*d*x+1/2*c)^7-13365*tan(1/2*d*x+1/2*c)^7-18711*co
t(1/2*d*x+1/2*c)^5-33264*tan(1/2*d*x+1/2*c)^5-55440*cot(1/2*d*x+1/2*c)^3-48
510*tan(1/2*d*x+1/2*c)^3-145530*cot(1/2*d*x+1/2*c))/d/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(97) = 194$.

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.01

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{128\cos(dx+c)^{10}+128\cos(dx+c)^{9}-576\cos(dx+c)^{8}-576\cos(dx+c)^{7}+1008\cos(dx+c)^{6}+}{3465\left(ad\cos(dx+c)^{9}+ad\cos(dx+c)^{8}-4ad\cos(dx+c)^{7}-4ad\cos(dx+c)^{6}+6ad\cos(dx+\right.}$$

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3465*(128*cos(d*x + c)^10 + 128*cos(d*x + c)^9 - 576*cos(d*x + c)^8 - 57
6*cos(d*x + c)^7 + 1008*cos(d*x + c)^6 + 1008*cos(d*x + c)^5 - 840*cos(d*x
+ c)^4 - 840*cos(d*x + c)^3 + 315*cos(d*x + c)^2 + 315*cos(d*x + c) + 315)/
((a*d*cos(d*x + c)^9 + a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^7 - 4*a*d*co
s(d*x + c)^6 + 6*a*d*cos(d*x + c)^5 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x
+ c)^3 - 4*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**10/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(97) = 194$.

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.98

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\frac{48510\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{33264\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{13365\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3080\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{315\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a}+\frac{11\left(\frac{360\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1701\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5040\,\sin(dx+}{(\cos(dx+c)+}\right.}{a\sin(dx+}}{3548160\,d}$$

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3548160*((48510*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 33264*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 + 13365*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3080*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 315*sin(d*x + c)^11/(cos(d*x + c) + 1
)^11)/a + 11*(360*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1701*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 5040*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 13230*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 35)*(cos(d*x + c) + 1)^9/(a*sin(d*x + c
)^9))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.48

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{11\left(13230\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+5040\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+1701\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9}+\frac{315\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3080\,a^{10}\,\mathrm{t}}{}}{3548160\,d}$$

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3548160*(11*(13230*tan(1/2*d*x + 1/2*c)^8 + 5040*tan(1/2*d*x + 1/2*c)^6
+ 1701*tan(1/2*d*x + 1/2*c)^4 + 360*tan(1/2*d*x + 1/2*c)^2 + 35)/(a*tan(1/2
*d*x + 1/2*c)^9) + (315*a^10*tan(1/2*d*x + 1/2*c)^11 + 3080*a^10*tan(1/2*d*
x + 1/2*c)^9 + 13365*a^10*tan(1/2*d*x + 1/2*c)^7 + 33264*a^10*tan(1/2*d*x +
 1/2*c)^5 + 48510*a^10*tan(1/2*d*x + 1/2*c)^3)/a^11)/d
```

Mupad [B] (verification not implemented)

Time = 15.91 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx =$$

$$-\frac{63\,\cos\left(c+d\,x\right)+\frac{21\,\cos\left(2\,c+2\,d\,x\right)}{2}-42\,\cos\left(3\,c+3\,d\,x\right)-12\,\cos\left(4\,c+4\,d\,x\right)+18\,\cos\left(5\,c+5\,d\,x\right)+\frac{27\,\mathrm{co}}{}}{3548160\,a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}\right.}$$

```
[In] int(1/(sin(c + d*x)^10*(a + a/cos(c + d*x))),x)
```

```
[Out] -(63*cos(c + d*x) + (21*cos(2*c + 2*d*x))/2 - 42*cos(3*c + 3*d*x) - 12*cos(
4*c + 4*d*x) + 18*cos(5*c + 5*d*x) + (27*cos(6*c + 6*d*x))/4 - (9*cos(7*c +
 7*d*x))/2 - 2*cos(8*c + 8*d*x) + cos(9*c + 9*d*x)/2 + cos(10*c + 10*d*x)/4
 + 693/2)/(3548160*a*d*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^9)
```

3.74 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\begin{aligned}\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx =& \frac{4(a-a\cos(c+dx))^6}{3a^8d}-\frac{4(a-a\cos(c+dx))^7}{a^9d}\\&+\frac{19(a-a\cos(c+dx))^8}{4a^{10}d}-\frac{25(a-a\cos(c+dx))^9}{9a^{11}d}\\&+\frac{4(a-a\cos(c+dx))^{10}}{5a^{12}d}-\frac{(a-a\cos(c+dx))^{11}}{11a^{13}d}\end{aligned}$$

```
[Out] 4/3*(a-a*cos(d*x+c))^6/a^8/d-4*(a-a*cos(d*x+c))^7/a^9/d+19/4*(a-a*cos(d*x+c
))^8/a^10/d-25/9*(a-a*cos(d*x+c))^9/a^11/d+4/5*(a-a*cos(d*x+c))^10/a^12/d-1
/11*(a-a*cos(d*x+c))^11/a^13/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$\begin{aligned}\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx =& -\frac{(a-a\cos(c+dx))^{11}}{11a^{13}d}+\frac{4(a-a\cos(c+dx))^{10}}{5a^{12}d}\\&-\frac{25(a-a\cos(c+dx))^9}{9a^{11}d}+\frac{19(a-a\cos(c+dx))^8}{4a^{10}d}\\&-\frac{4(a-a\cos(c+dx))^7}{a^9d}+\frac{4(a-a\cos(c+dx))^6}{3a^8d}\end{aligned}$$

```
[In] Int[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(a - a*Cos[c + d*x])^6)/(3*a^8*d) - (4*(a - a*Cos[c + d*x])^7)/(a^9*d) +
 (19*(a - a*Cos[c + d*x])^8)/(4*a^10*d) - (25*(a - a*Cos[c + d*x])^9)/(9*a^
11*d) + (4*(a - a*Cos[c + d*x])^10)/(5*a^12*d) - (a - a*Cos[c + d*x])^11/(1
1*a^13*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^{11}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^5x^2(-a+x)^3}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^{11}d} \\
&= \frac{\text{Subst}\left(\int (-a-x)^5x^2(-a+x)^3\,dx, x, -a\cos(c+dx)\right)}{a^{13}d} \\
&= \frac{\text{Subst}\left(\int \left(-8a^5(-a-x)^5-28a^4(-a-x)^6-38a^3(-a-x)^7-25a^2(-a-x)^8-8a(-a-x)^9-\right.\right.}{a^{13}d} \\
&= \frac{4(a-a\cos(c+dx))^6}{3a^8d}-\frac{4(a-a\cos(c+dx))^7}{a^9d}+\frac{19(a-a\cos(c+dx))^8}{4a^{10}d} \\
&\quad -\frac{25(a-a\cos(c+dx))^9}{9a^{11}d}+\frac{4(a-a\cos(c+dx))^{10}}{5a^{12}d}-\frac{(a-a\cos(c+dx))^{11}}{11a^{13}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.90 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.53

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{4(2360+4038\cos(c+dx)+2586\cos(2(c+dx))+1189\cos(3(c+dx))+342\cos(4(c+dx))+45\cos(5(c+}{495a^2d}$$

```
[In] Integrate[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(2360 + 4038*Cos[c + d*x] + 2586*Cos[2*(c + d*x)] + 1189*Cos[3*(c + d*x)
] + 342*Cos[4*(c + d*x)] + 45*Cos[5*(c + d*x)])*Sin[(c + d*x)/2]^12)/(495*a
^2*d)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^{11}}{11}-\frac{\cos(dx+c)^{10}}{5}-\frac{2\cos(dx+c)^{9}}{9}+\frac{3\cos(dx+c)^{8}}{4}-\cos(dx+c)^{6}+\frac{2\cos(dx+c)^{5}}{5}+\frac{\cos(dx+c)^{4}}{2}-\frac{\cos(dx+c)^{3}}{3}}{d\,a^2}$
default	$\frac{\frac{\cos(dx+c)^{11}}{11}-\frac{\cos(dx+c)^{10}}{5}-\frac{2\cos(dx+c)^{9}}{9}+\frac{3\cos(dx+c)^{8}}{4}-\cos(dx+c)^{6}+\frac{2\cos(dx+c)^{5}}{5}+\frac{\cos(dx+c)^{4}}{2}-\frac{\cos(dx+c)^{3}}{3}}{d\,a^2}$
parallelrisch	$\frac{-42470-3960\cos(4dx+4c)-990\cos(3dx+3c)+45\cos(11dx+11c)-34650\cos(dx+c)+990\cos(8dx+8c)-1485\cos(7dx+7c}{506880a^2d}$
risch	$-\frac{35\cos(dx+c)}{512a^2d}+\frac{\cos(11dx+11c)}{11264d\,a^2}-\frac{\cos(10dx+10c)}{2560d\,a^2}+\frac{\cos(9dx+9c)}{9216d\,a^2}+\frac{\cos(8dx+8c)}{512d\,a^2}-\frac{3\cos(7dx+7c)}{1024d\,a^2}-\frac{\cos(6dx}{512d}$

```
[In] int(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/11*cos(d*x+c)^11-1/5*cos(d*x+c)^10-2/9*cos(d*x+c)^9+3/4*cos(d*x+
c)^8-cos(d*x+c)^6+2/5*cos(d*x+c)^5+1/2*cos(d*x+c)^4-1/3*cos(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{180\,\cos(dx+c)^{11}-396\,\cos(dx+c)^{10}-440\,\cos(dx+c)^{9}+1485\,\cos(dx+c)^{8}-1980\,\cos(dx+c)^{6}+79}{1980\,a^2d}$$

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1980*(180*cos(d*x + c)^11 - 396*cos(d*x + c)^10 - 440*cos(d*x + c)^9 + 14
85*cos(d*x + c)^8 - 1980*cos(d*x + c)^6 + 792*cos(d*x + c)^5 + 990*cos(d*x
+ c)^4 - 660*cos(d*x + c)^3)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**11/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{180\,\cos\left(dx+c\right)^{11} - 396\,\cos\left(dx+c\right)^{10} - 440\,\cos\left(dx+c\right)^{9} + 1485\,\cos\left(dx+c\right)^{8} - 1980\,\cos\left(dx+c\right)^{6} + \cdots}{1980\,a^2 d}$$

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/1980*(180*cos(d*x + c)^11 - 396*cos(d*x + c)^10 - 440*cos(d*x + c)^9 + 14
85*cos(d*x + c)^8 - 1980*cos(d*x + c)^6 + 792*cos(d*x + c)^5 + 990*cos(d*x
+ c)^4 - 660*cos(d*x + c)^3)/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{64\left(\frac{11\,(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{55\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{165\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{330\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{462\,(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} + \frac{198\,(\cos(dx+c}{(\cos(dx+c)} \cdots\right.}{495\,a^2 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^{11}}$$

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -64/495*(11*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 55*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 165*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 330
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 462*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 + 198*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 990*(cos(
d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^11)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\cos(c+d\,x)^3}{3\,a^2}-\frac{\cos(c+d\,x)^4}{2\,a^2}-\frac{2\cos(c+d\,x)^5}{5\,a^2}+\frac{\cos(c+d\,x)^6}{a^2}-\frac{3\cos(c+d\,x)^8}{4\,a^2}+\frac{2\cos(c+d\,x)^9}{9\,a^2}+\frac{\cos(c+d\,x)^{10}}{5\,a^2}-\frac{\cos(c+d\,x)^{11}}{11\,a^2}}{d}$$

```
[In] int(sin(c + d*x)^11/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)^3/(3*a^2) - cos(c + d*x)^4/(2*a^2) - (2*cos(c + d*x)^5)/(5*a
^2) + cos(c + d*x)^6/a^2 - (3*cos(c + d*x)^8)/(4*a^2) + (2*cos(c + d*x)^9)/
(9*a^2) + cos(c + d*x)^10/(5*a^2) - cos(c + d*x)^11/(11*a^2))/d
```

3.75 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2} dx = \frac{4(a-a\cos(c+dx))^5}{5a^7d} - \frac{2(a-a\cos(c+dx))^6}{a^8d} + \frac{13(a-a\cos(c+dx))^7}{7a^9d} - \frac{3(a-a\cos(c+dx))^8}{4a^{10}d} + \frac{(a-a\cos(c+dx))^9}{9a^{11}d}$$

```
[Out] 4/5*(a-a*cos(d*x+c))^5/a^7/d-2*(a-a*cos(d*x+c))^6/a^8/d+13/7*(a-a*cos(d*x+c
))^7/a^9/d-3/4*(a-a*cos(d*x+c))^8/a^10/d+1/9*(a-a*cos(d*x+c))^9/a^11/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2} dx = \frac{(a-a\cos(c+dx))^9}{9a^{11}d} - \frac{3(a-a\cos(c+dx))^8}{4a^{10}d} + \frac{13(a-a\cos(c+dx))^7}{7a^9d} - \frac{2(a-a\cos(c+dx))^6}{a^8d} + \frac{4(a-a\cos(c+dx))^5}{5a^7d}$$

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(a - a*Cos[c + d*x])^5)/(5*a^7*d) - (2*(a - a*Cos[c + d*x])^6)/(a^8*d) +
 (13*(a - a*Cos[c + d*x])^7)/(7*a^9*d) - (3*(a - a*Cos[c + d*x])^8)/(4*a^10
*d) + (a - a*Cos[c + d*x])^9/(9*a^11*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^9(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^4x^2(-a+x)^2}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^9d} \\
&= \frac{\text{Subst}\left(\int (-a-x)^4x^2(-a+x)^2\,dx, x, -a\cos(c+dx)\right)}{a^{11}d} \\
&= \frac{\text{Subst}\left(\int \left(4a^4(-a-x)^4+12a^3(-a-x)^5+13a^2(-a-x)^6+6a(-a-x)^7+(-a-x)^8\right)\,dx, x, -a\right.}{a^{11}d} \\
&= \frac{4(a-a\cos(c+dx))^5}{5a^7d}-\frac{2(a-a\cos(c+dx))^6}{a^8d}+\frac{13(a-a\cos(c+dx))^7}{7a^9d} \\
&\quad -\frac{3(a-a\cos(c+dx))^8}{4a^{10}d}+\frac{(a-a\cos(c+dx))^9}{9a^{11}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.54

$$
\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx
= \frac{2(992+1615\cos(c+dx)+970\cos(2(c+dx))+385\cos(3(c+dx))+70\cos(4(c+dx)))\sin^{10}\left(\frac{1}{2}(c+dx)\right)}{315a^2d}
$$

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*(992 + 1615*Cos[c + d*x] + 970*Cos[2*(c + d*x)] + 385*Cos[3*(c + d*x)] +
 70*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]^10)/(315*a^2*d)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{\cos(dx+c)^8}{4}+\frac{\cos(dx+c)^7}{7}-\frac{2\cos(dx+c)^6}{3}+\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$
default	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{\cos(dx+c)^8}{4}+\frac{\cos(dx+c)^7}{7}-\frac{2\cos(dx+c)^6}{3}+\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$
parallelrisch	$\frac{-\frac{64}{315}-\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{3}-\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{35}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{15}-\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{5}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{35}}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^9a^2}$
norman	$\frac{-\frac{64}{315ad}-\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{3ad}-\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{35da}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{35da}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{15da}-\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{5da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^9a}$
risch	$-\frac{13\cos(dx+c)}{128a^2d}-\frac{\cos(9dx+9c)}{2304d\,a^2}+\frac{\cos(8dx+8c)}{512d\,a^2}-\frac{3\cos(7dx+7c)}{1792d\,a^2}-\frac{\cos(6dx+6c)}{192d\,a^2}+\frac{\cos(5dx+5c)}{80d\,a^2}-\frac{\cos(4dx+}{128d\,a^2}$

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/9*cos(d*x+c)^9+1/4*cos(d*x+c)^8+1/7*cos(d*x+c)^7-2/3*cos(d*x+c)
^6+1/5*cos(d*x+c)^5+1/2*cos(d*x+c)^4-1/3*cos(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{140\,\cos\left(dx+c\right)^9 - 315\,\cos\left(dx+c\right)^8 - 180\,\cos\left(dx+c\right)^7 + 840\,\cos\left(dx+c\right)^6 - 252\,\cos\left(dx+c\right)^5 - 630}{1260\,a^2 d}$$

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/1260*(140*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 180*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 630*cos(d*x + c)^4 + 420*cos(d*x + c
)^3)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{140\,\cos\left(dx+c\right)^9 - 315\,\cos\left(dx+c\right)^8 - 180\,\cos\left(dx+c\right)^7 + 840\,\cos\left(dx+c\right)^6 - 252\,\cos\left(dx+c\right)^5 - 630}{1260\,a^2 d}$$

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/1260*(140*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 180*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 630*cos(d*x + c)^4 + 420*cos(d*x + c
)^3)/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.24

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{64\left(\frac{9\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{36\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{84\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{126\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{210\,(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}-1\right)}{315\,a^2d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9}$$

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -64/315*(9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 36*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 84*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 126*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 210*(cos(d*x + c) - 1)^6/(cos(d*
x + c) + 1)^6 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.84

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\cos(c+d\,x)^4}{2\,a^2}-\frac{\cos(c+d\,x)^3}{3\,a^2}+\frac{\cos(c+d\,x)^5}{5\,a^2}-\frac{2\cos(c+d\,x)^6}{3\,a^2}+\frac{\cos(c+d\,x)^7}{7\,a^2}+\frac{\cos(c+d\,x)^8}{4\,a^2}-\frac{\cos(c+d\,x)^9}{9\,a^2}}{d}$$

```
[In] int(sin(c + d*x)^9/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^4/(2*a^2) - cos(c + d*x)^3/(3*a^2) + cos(c + d*x)^5/(5*a^2) -
 (2*cos(c + d*x)^6)/(3*a^2) + cos(c + d*x)^7/(7*a^2) + cos(c + d*x)^8/(4*a^
2) - cos(c + d*x)^9/(9*a^2))/d
```

3.76 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx=-\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^6(c+dx)}{3a^2d}+\frac{\cos^7(c+dx)}{7a^2d}$$

```
[Out] -1/3*cos(d*x+c)^3/a^2/d+1/2*cos(d*x+c)^4/a^2/d-1/3*cos(d*x+c)^6/a^2/d+1/7*c
os(d*x+c)^7/a^2/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 76}

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\frac{\cos^7(c+dx)}{7a^2d}-\frac{\cos^6(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cos[c + d*x]^3/(a^2*d) + Cos[c + d*x]^4/(2*a^2*d) - Cos[c + d*x]^6/(3*
a^2*d) + Cos[c + d*x]^7/(7*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^7(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\
&= \frac{\operatorname{Subst}\left(\int \frac{(-a-x)^3x^2(-a+x)}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^7d}\\
&= \frac{\operatorname{Subst}\left(\int (-a-x)^3x^2(-a+x)\,dx, x, -a\cos(c+dx)\right)}{a^9d}\\
&= \frac{\operatorname{Subst}\left(\int \left(a^4x^2+2a^3x^3-2ax^5-x^6\right)\,dx, x, -a\cos(c+dx)\right)}{a^9d}\\
&= -\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^6(c+dx)}{3a^2d}+\frac{\cos^7(c+dx)}{7a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$
\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{4(17\cos(c+dx)+10\cos(2(c+dx))+3(4+\cos(3(c+dx))))\sin^8\left(\frac{1}{2}(c+dx)\right)}{21a^2d}
$$

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
[Out] (4*(17*Cos[c + d*x] + 10*Cos[2*(c + d*x)] + 3*(4 + Cos[3*(c + d*x)]))*Sin[(
c + d*x)/2]^8)/(21*a^2*d)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{3}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$	49
default	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{3}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$	49
parallelrisch	$\frac{126\cos(2dx+2c)-368-14\cos(6dx+6c)-231\cos(dx+c)-49\cos(3dx+3c)+21\cos(5dx+5c)+3\cos(7dx+7c)}{1344a^2d}$	74
risch	$-\frac{11\cos(dx+c)}{64a^2d}+\frac{\cos(7dx+7c)}{448d\,a^2}-\frac{\cos(6dx+6c)}{96d\,a^2}+\frac{\cos(5dx+5c)}{64d\,a^2}-\frac{7\cos(3dx+3c)}{192d\,a^2}+\frac{3\cos(2dx+2c)}{32d\,a^2}$	101
norman	$\frac{-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{da}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{da}-\frac{8}{21ad}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3da}-\frac{40\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{3da}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{3da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{7}a}$	124

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/7*cos(d*x+c)^7-1/3*cos(d*x+c)^6+1/2*cos(d*x+c)^4-1/3*cos(d*x+c)^
3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{6\,\cos\left(dx+c\right)^7-14\,\cos\left(dx+c\right)^6+21\,\cos\left(dx+c\right)^4-14\,\cos\left(dx+c\right)^3}{42\,a^2d}$$

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/42*(6*cos(d*x + c)^7 - 14*cos(d*x + c)^6 + 21*cos(d*x + c)^4 - 14*cos(d*x
 + c)^3)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{6\,\cos\left(dx+c\right)^7 - 14\,\cos\left(dx+c\right)^6 + 21\,\cos\left(dx+c\right)^4 - 14\,\cos\left(dx+c\right)^3}{42\,a^2d}$$

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/42*(6*cos(d*x + c)^7 - 14*cos(d*x + c)^6 + 21*cos(d*x + c)^4 - 14*cos(d*x
 + c)^3)/(a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(65) = 130.

Time = 0.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.93

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{8\left(\frac{7\,(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{21\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{35\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{14\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{42\,(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - 1\right)}{21\,a^2d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^7}$$

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -8/21*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 21*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 35*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 14*(cos
(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 42*(cos(d*x + c) - 1)^5/(cos(d*x +
c) + 1)^5 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\cos(c+d\,x)^3}{3\,a^2} - \frac{\cos(c+d\,x)^4}{2\,a^2} + \frac{\cos(c+d\,x)^6}{3\,a^2} - \frac{\cos(c+d\,x)^7}{7\,a^2}}{d}$$

```
[In] int(sin(c + d*x)^7/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)^3/(3*a^2) - cos(c + d*x)^4/(2*a^2) + cos(c + d*x)^6/(3*a^2)
- cos(c + d*x)^7/(7*a^2))/d
```

3.77 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx=-\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^5(c+dx)}{5a^2d}$$

```
[Out] -1/3*cos(d*x+c)^3/a^2/d+1/2*cos(d*x+c)^4/a^2/d-1/5*cos(d*x+c)^5/a^2/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 45}

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx=-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cos[c + d*x]^3/(a^2*d) + Cos[c + d*x]^4/(2*a^2*d) - Cos[c + d*x]^5/(5*
a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\cos^2(c+dx)\sin^5(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\ &= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^2}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^5d} \\ &= \frac{\text{Subst}\left(\int (-a-x)^2x^2\,dx, x, -a\cos(c+dx)\right)}{a^7d} \\ &= \frac{\text{Subst}\left(\int \left(a^2x^2+2ax^3+x^4\right)\,dx, x, -a\cos(c+dx)\right)}{a^7d} \\ &= -\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^5(c+dx)}{5a^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{4(4+3\cos(c+dx)+3\cos(2(c+dx)))\sin^6\left(\frac{1}{2}(c+dx)\right)}{15a^2d}$$

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
[Out] (4*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]^6)/(15*a^2*d)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$	39
default	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^4}{2}-\frac{\cos(dx+c)^3}{3}}{d\,a^2}$	39
parallelrisch	$\frac{60\cos(2dx+2c)-203-35\cos(3dx+3c)-3\cos(5dx+5c)-90\cos(dx+c)+15\cos(4dx+4c)}{240a^2d}$	63
risch	$-\frac{3\cos(dx+c)}{8a^2d}-\frac{\cos(5dx+5c)}{80d\,a^2}+\frac{\cos(4dx+4c)}{16d\,a^2}-\frac{7\cos(3dx+3c)}{48d\,a^2}+\frac{\cos(2dx+2c)}{4d\,a^2}$	84
norman	$\frac{-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{da}-\frac{16}{15ad}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{da}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3da}-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5 a}$	105

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(-1/5*cos(d*x+c)^5+1/2*cos(d*x+c)^4-1/3*cos(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{6\,\cos\left(dx+c\right)^5-15\,\cos\left(dx+c\right)^4+10\,\cos\left(dx+c\right)^3}{30\,a^2d}$$

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -1/30*(6*cos(d*x + c)^5 - 15*cos(d*x + c)^4 + 10*cos(d*x + c)^3)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{6\,\cos\left(dx+c\right)^5-15\,\cos\left(dx+c\right)^4+10\,\cos\left(dx+c\right)^3}{30\,a^2d}$$

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/30*(6*cos(d*x + c)^5 - 15*cos(d*x + c)^4 + 10*cos(d*x + c)^3)/(a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(49) = 98.

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.16

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{8\left(\frac{10\left(\cos(dx+c)-1\right)}{\cos(dx+c)+1}-\frac{20\left(\cos(dx+c)-1\right)^2}{\left(\cos(dx+c)+1\right)^2}+\frac{15\left(\cos(dx+c)-1\right)^3}{\left(\cos(dx+c)+1\right)^3}-\frac{15\left(\cos(dx+c)-1\right)^4}{\left(\cos(dx+c)+1\right)^4}-2\right)}{15\,a^2d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}$$

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -8/15*(10*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 20*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 15*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 15*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2)/(a^2*d*((cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 1)^5)
```

Mupad [B] (verification not implemented)

Time = 13.97 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos\left(c+d\,x\right)^3\,\left(6\cos\left(c+d\,x\right)^2-15\,\cos\left(c+d\,x\right)+10\right)}{30\,a^2\,d}$$

```
[In] int(sin(c + d*x)^5/(a + a/cos(c + d*x))^2,x)
[Out] -(cos(c + d*x)^3*(6*cos(c + d*x)^2 - 15*cos(c + d*x) + 10))/(30*a^2*d)
```

3.78 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\cos(c+dx)}{a^2d}-\frac{\cos^2(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{3a^2d}-\frac{2\log(1+\cos(c+dx))}{a^2d}$$

```
[Out] 2*cos(d*x+c)/a^2/d-cos(d*x+c)^2/a^2/d+1/3*cos(d*x+c)^3/a^2/d-2*ln(1+cos(d*x
+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 78}

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{2\log(\cos(c+dx)+1)}{a^2d}$$

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[c + d*x])/(a^2*d) - Cos[c + d*x]^2/(a^2*d) + Cos[c + d*x]^3/(3*a^2*d
) - (2*Log[1 + Cos[c + d*x]])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)x^2}{a^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)x^2}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \left(-2a^2+\frac{2a^3}{a-x}-2ax-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{a^5d}\\
&= \frac{2\cos(c+dx)}{a^2d}-\frac{\cos^2(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{3a^2d}-\frac{2\log(1+\cos(c+dx))}{a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{-22+27\cos(c+dx)-6\cos(2(c+dx))+\cos(3(c+dx))-48\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{12a^2d}$$

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
[Out] (-22 + 27*Cos[c + d*x] - 6*Cos[2*(c + d*x)] + Cos[3*(c + d*x)] - 48*Log[Cos
[(c + d*x)/2]])/(12*a^2*d)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{\cos(dx+c)^3}{3}-\cos(dx+c)^2+2\cos(dx+c)-2\ln(\cos(dx+c)+1)}{d\,a^2}$	48
default	$\frac{\frac{\cos(dx+c)^3}{3}-\cos(dx+c)^2+2\cos(dx+c)-2\ln(\cos(dx+c)+1)}{d\,a^2}$	48
parallelrisch	$\frac{24\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+34+\cos(3dx+3c)-6\cos(2dx+2c)+27\cos(dx+c)}{12a^2d}$	53
norman	$\frac{\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{da}+\frac{14}{3ad}+\frac{12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3 a}+\frac{2\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^2d}$	90
risch	$\frac{2ix}{a^2}+\frac{9\,e^{i(dx+c)}}{8a^2d}+\frac{9\,e^{-i(dx+c)}}{8a^2d}+\frac{4ic}{a^2d}-\frac{4\ln\left(e^{i(dx+c)}+1\right)}{a^2d}+\frac{\cos(3dx+3c)}{12d\,a^2}-\frac{\cos(2dx+2c)}{2d\,a^2}$	107

```
[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/3*cos(d*x+c)^3-cos(d*x+c)^2+2*cos(d*x+c)-2*ln(cos(d*x+c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos(dx+c)^3-3\,\cos(dx+c)^2+6\,\cos(dx+c)-6\,\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)}{3\,a^2d}$$

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 6*cos(d*x + c) - 6*log(1/2*cos(d*x
 + c) + 1/2))/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\cos(dx+c)^3-3\cos(dx+c)^2+6\cos(dx+c)}{a^2}-\frac{6\log(\cos(dx+c)+1)}{a^2}}{3\,d}$$

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 6*cos(d*x + c))/a^2 - 6*log(cos(d
*x + c) + 1)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\log\left(|-\cos\left(dx+c\right)-1|\right)}{a^2d} + \frac{a^4d^2\cos\left(dx+c\right)^3-3\,a^4d^2\cos\left(dx+c\right)^2+6\,a^4d^2\cos\left(dx+c\right)}{3\,a^6d^3}$$

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*log(abs(-cos(d*x + c) - 1))/(a^2*d) + 1/3*(a^4*d^2*cos(d*x + c)^3 - 3*a^
4*d^2*cos(d*x + c)^2 + 6*a^4*d^2*cos(d*x + c))/(a^6*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\,\ln(\cos(c+d\,x)+1)}{a^2}-\frac{2\,\cos(c+d\,x)}{a^2}+\frac{\cos(c+d\,x)^2}{a^2}-\frac{\cos(c+d\,x)^3}{3\,a^2}}{d}$$

```
[In] int(sin(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -((2*log(cos(c + d*x) + 1))/a^2 - (2*cos(c + d*x))/a^2 + cos(c + d*x)^2/a^2
 - cos(c + d*x)^3/(3*a^2))/d
```

3.79 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2} \, dx = -\frac{\cos(c+dx)}{a^2d}+\frac{1}{d\,(a^2+a^2\cos(c+dx))}+\frac{2\log(1+\cos(c+dx))}{a^2d}$$

```
[Out] -cos(d*x+c)/a^2/d+1/d/(a^2+a^2*cos(d*x+c))+2*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2912, 12, 45}

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2} \, dx = -\frac{\cos(c+dx)}{a^2d}+\frac{1}{d\,(a^2\cos(c+dx)+a^2)}+\frac{2\log(\cos(c+dx)+1)}{a^2d}$$

```
[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]/(a^2*d)) + 1/(d*(a^2 + a^2*Cos[c + d*x])) + (2*Log[1 + Cos[c
 + d*x]])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{a^2(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{(a-x)^2}-\frac{2a}{a-x}\right)\,dx, x, -a\cos(c+dx)\right)}{a^3d} \\
&= -\frac{\cos(c+dx)}{a^2d}+\frac{1}{d\,(a^2+a^2\cos(c+dx))}+\frac{2\log(1+\cos(c+dx))}{a^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\left(-3+\cos(2(c+dx))-8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^2d}$$

```
[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*((-3 + Cos[2*(c + d*x)] - 8*Log[Cos[(c + d*x)/2]] - 8*Cos[c + d*x]*Log
[Cos[(c + d*x)/2]])*Sec[(c + d*x)/2]^2)/(a^2*d)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{1-4\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)-2\cos(dx+c)+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2a^{2}d}$	44
derivativedivides	$\frac{-\frac{1}{1+\sec(dx+c)}+2\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-2\ln(\sec(dx+c))}{d\,a^{2}}$	51
default	$\frac{-\frac{1}{1+\sec(dx+c)}+2\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-2\ln(\sec(dx+c))}{d\,a^{2}}$	51
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2da}-\frac{5}{2ad}}{a\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}-\frac{2\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}{a^{2}d}$	71
risch	$-\frac{2ix}{a^{2}}-\frac{e^{i(dx+c)}}{2a^{2}d}-\frac{e^{-i(dx+c)}}{2a^{2}d}-\frac{4ic}{a^{2}d}+\frac{2\,e^{i(dx+c)}}{a^{2}d\left(e^{i(dx+c)}+1\right)^{2}}+\frac{4\ln\left(e^{i(dx+c)}+1\right)}{a^{2}d}$	103

```
[In] int(sin(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1-4*ln(sec(1/2*d*x+1/2*c)^2)-2*cos(d*x+c)+sec(1/2*d*x+1/2*c)^2)/a^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos\left(dx+c\right)^{2}-2\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\cos\left(dx+c\right)-1}{a^{2}d\cos\left(dx+c\right)+a^{2}d}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c)^2 - 2*(cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + cos(d
*x + c) - 1)/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))**2,x)
[Out] Integral(sin(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{1}{a^2\cos(dx+c)+a^2} - \frac{\cos(dx+c)}{a^2} + \frac{2\log(\cos(dx+c)+1)}{a^2}}{d}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] (1/(a^2*cos(d*x + c) + a^2) - cos(d*x + c)/a^2 + 2*log(cos(d*x + c) + 1)/a^
2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos(dx+c)}{a^2d} + \frac{2\log(|-\cos(dx+c)-1|)}{a^2d} + \frac{1}{a^2d(\cos(dx+c)+1)}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -cos(d*x + c)/(a^2*d) + 2*log(abs(-cos(d*x + c) - 1))/(a^2*d) + 1/(a^2*d*(c
os(d*x + c) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{a^2\,d}-\frac{{\cos\left(c+d\,x\right)}^2-2}{a^2\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

```
[In] int(sin(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*log(cos(c + d*x) + 1))/(a^2*d) - (cos(c + d*x)^2 - 2)/(a^2*d*(cos(c + d*
x) + 1))
```

3.80 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{4a^2d} + \frac{1}{4d(a+a\cos(c+dx))^2} - \frac{3}{4d\,(a^2+a^2\cos(c+dx))}$$

```
[Out] -1/4*arctanh(cos(d*x+c))/a^2/d+1/4/d/(a+a*cos(d*x+c))^2-3/4/d/(a^2+a^2*cos(
d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3957, 2915, 12, 90, 212}

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{4a^2d} - \frac{3}{4d\,(a^2\cos(c+dx)+a^2)} + \frac{1}{4d(a\cos(c+dx)+a)^2}$$

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*ArcTanh[Cos[c + d*x]]/(a^2*d) + 1/(4*d*(a + a*Cos[c + d*x])^2) - 3/(4*
d*(a^2 + a^2*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos(c+dx)\cot(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{a\text{Subst}\left(\int \frac{x^2}{a^2(-a-x)(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(-a-x)(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{2(a-x)^3}-\frac{3}{4(a-x)^2}+\frac{1}{4(a^2-x^2)}\right)\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{1}{4d(a+a\cos(c+dx))^2}-\frac{3}{4d\left(a^2+a^2\cos(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, -a\cos(c+dx)\right)}{4ad} \\
&= -\frac{\operatorname{arctanh}(\cos(c+dx))}{4a^2d}+\frac{1}{4d(a+a\cos(c+dx))^2}-\frac{3}{4d\left(a^2+a^2\cos(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\left(-1+6\cos^2\left(\frac{1}{2}(c+dx)\right)+4\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec^2(c+}{4a^2d(1+\sec(c+dx))^2}$$

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*((-1 + 6*Cos[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/
2]] - Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a^2d}$	46
derivativedivides	$\frac{\frac{1}{4(\cos(dx+c)+1)^2}-\frac{3}{4(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{8}+\frac{\ln(\cos(dx+c)-1)}{8}}{d\,a^2}$	55
default	$\frac{\frac{1}{4(\cos(dx+c)+1)^2}-\frac{3}{4(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{8}+\frac{\ln(\cos(dx+c)-1)}{8}}{d\,a^2}$	55
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16da}}{a}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2d}$	63
risch	$-\frac{3\,e^{3i(dx+c)}+4\,e^{2i(dx+c)}+3\,e^{i(dx+c)}}{2a^2d\left(e^{i(dx+c)}+1\right)^4}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{4a^2d}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{4a^2d}$	97

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(tan(1/2*d*x+1/2*c)^4-4*tan(1/2*d*x+1/2*c)^2+4*ln(tan(1/2*d*x+1/2*c)))
/a^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.77

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log}{8\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*((cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 6*cos(d
*x + c) + 4)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\,(3\,\cos(dx+c)+2)}{a^2\,\cos(dx+c)^2+2\,a^2\,\cos(dx+c)+a^2}+\frac{\log(\cos(dx+c)+1)}{a^2}-\frac{\log(\cos(dx+c)-1)}{a^2}}{8\,d}$$

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(3*cos(d*x + c) + 2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2
) + log(cos(d*x + c) + 1)/a^2 - log(cos(d*x + c) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.45

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{2\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{\frac{4\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^4}}{16\,d}$$

[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] 1/16*(2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (4*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/a^4)/d

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{3\,\cos(c+d\,x)}{4}+\frac{1}{2}}{d\,\left(a^2\cos\left(c+d\,x\right)^2+2\,a^2\,\cos\left(c+d\,x\right)+a^2\right)} - \frac{\mathrm{atanh}(\cos\left(c+d\,x\right))}{4\,a^2\,d}$$

[In] int(1/(sin(c + d*x)*(a + a/cos(c + d*x))^2),x)

[Out] - ((3*cos(c + d*x))/4 + 1/2)/(d*(2*a^2*cos(c + d*x) + a^2 + a^2*cos(c + d*x)^2)) - atanh(cos(c + d*x))/(4*a^2*d)

3.81 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2} dx = -\frac{a+2a\cos(c+dx)}{6d(1-\cos(c+dx))(a+a\cos(c+dx))^3}$$

```
[Out] 1/6*(-a-2*a*cos(d*x+c))/d/(1-cos(d*x+c))/(a+a*cos(d*x+c))^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 82}

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2} dx = -\frac{2a\cos(c+dx)+a}{6d(1-\cos(c+dx))(a\cos(c+dx)+a)^3}$$

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6*(a + 2*a*Cos[c + d*x])/(d*(1 - Cos[c + d*x])*(a + a*Cos[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 82

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(
```

```
n + p + 2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cot^2(c+dx)\csc(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{a^3\text{Subst}\left(\int \frac{x^2}{a^2(-a-x)^2(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a\text{Subst}\left(\int \frac{x^2}{(-a-x)^2(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= -\frac{a+2a\cos(c+dx)}{6d(1-\cos(c+dx))(a+a\cos(c+dx))^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(1+2\cos(c+dx))\csc^2(c+dx)}{6a^2d(1+\cos(c+dx))^2}$$

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
[Out] -1/6*((1 + 2*Cos[c + d*x])*Csc[c + d*x]^2)/(a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-3-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{96d\,a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}$	46
derivativedivides	$\frac{\frac{1}{12(\cos(dx+c)+1)^{3}}-\frac{1}{8(\cos(dx+c)+1)^{2}}-\frac{1}{16(\cos(dx+c)+1)}+\frac{1}{16\cos(dx+c)-16}}{d\,a^{2}}$	57
default	$\frac{\frac{1}{12(\cos(dx+c)+1)^{3}}-\frac{1}{8(\cos(dx+c)+1)^{2}}-\frac{1}{16(\cos(dx+c)+1)}+\frac{1}{16\cos(dx+c)-16}}{d\,a^{2}}$	57
norman	$\frac{-\frac{1}{32ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{16da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{96da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}$	63
risch	$\frac{\frac{8\,e^{5i(dx+c)}}{3}+\frac{8\,e^{4i(dx+c)}}{3}+\frac{8\,e^{3i(dx+c)}}{3}}{a^{2}d\left(e^{i(dx+c)}+1\right)^{6}\left(e^{i(dx+c)}-1\right)^{2}}$	63

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(tan(1/2*d*x+1/2*c)^8-3-6*tan(1/2*d*x+1/2*c)^4)/d/a^2/tan(1/2*d*x+1/2*
c)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\,\cos\left(dx+c\right)+1}{6\,\left(a^{2}d\cos\left(dx+c\right)^{4}+2\,a^{2}d\cos\left(dx+c\right)^{3}-2\,a^{2}d\cos\left(dx+c\right)-a^{2}d\right)}$$

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c) + 1)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2
*a^2*d*cos(d*x + c) - a^2*d)
```

Sympy [F]

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\cos(dx+c)+1}{6\left(a^2\cos(dx+c)^4+2a^2\cos(dx+c)^3-2a^2\cos(dx+c)-a^2\right)d}$$

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] 1/6*(2*cos(d*x + c) + 1)/((a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 - 2*a^
2*cos(d*x + c) - a^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(38) = 76$.

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{3(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}+\frac{\frac{6a^4(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a^4(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^6}}{96d}$$

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] 1/96*(3*(cos(d*x + c) + 1)/(a^2*(cos(d*x + c) - 1)) + (6*a^4*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^
6)/d
```

Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= -\frac{\frac{\cos(c+d\,x)}{3}+\frac{1}{6}}{d\,\left(-a^2\cos\left(c+d\,x\right)^4-2\,a^2\cos\left(c+d\,x\right)^3+2\,a^2\,\cos\left(c+d\,x\right)+a^2\right)}$$

```
[In] int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))^2),x)
[Out] -(cos(c + d*x)/3 + 1/6)/(d*(2*a^2*cos(c + d*x) + a^2 - 2*a^2*cos(c + d*x)^3
 - a^2*cos(c + d*x)^4))
```

3.82 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\begin{aligned}\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\operatorname{arctanh}(\cos(c+dx))}{64a^2d} - \frac{1}{64d(a-a\cos(c+dx))^2} \\ &\quad + \frac{a^2}{32d(a+a\cos(c+dx))^4} - \frac{a}{48d(a+a\cos(c+dx))^3} \\ &\quad - \frac{1}{32d(a+a\cos(c+dx))^2} - \frac{1}{64d\left(a^2-a^2\cos(c+dx)\right)} \\ &\quad - \frac{1}{32d\left(a^2+a^2\cos(c+dx)\right)}\end{aligned}$$

```
[Out] 1/64*arctanh(cos(d*x+c))/a^2/d-1/64/d/(a-a*cos(d*x+c))^2+1/32*a^2/d/(a+a*co
s(d*x+c))^4-1/48*a/d/(a+a*cos(d*x+c))^3-1/32/d/(a+a*cos(d*x+c))^2-1/64/d/(a
^2-a^2*cos(d*x+c))-1/32/d/(a^2+a^2*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2915, 12, 90, 212}

$$\begin{aligned}\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\operatorname{arctanh}(\cos(c+dx))}{64a^2d} + \frac{a^2}{32d(a\cos(c+dx)+a)^4} \\ &\quad - \frac{1}{64d\left(a^2-a^2\cos(c+dx)\right)} \\ &\quad - \frac{1}{32d\left(a^2\cos(c+dx)+a^2\right)} - \frac{a}{48d(a\cos(c+dx)+a)^3} \\ &\quad - \frac{1}{64d(a-a\cos(c+dx))^2} - \frac{1}{32d(a\cos(c+dx)+a)^2}\end{aligned}$$

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(64*a^2*d) - 1/(64*d*(a - a*Cos[c + d*x])^2) + a^2/(3
2*d*(a + a*Cos[c + d*x])^4) - a/(48*d*(a + a*Cos[c + d*x])^3) - 1/(32*d*(a
+ a*Cos[c + d*x])^2) - 1/(64*d*(a^2 - a^2*Cos[c + d*x])) - 1/(32*d*(a^2 + a
^2*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^2(c+dx)\csc^3(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$

$$= \frac{a^5\text{Subst}\left(\int \frac{x^2}{a^2(-a-x)^3(-a+x)^5}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^3 \text{Subst}\left(\int \frac{x^2}{(-a-x)^3(-a+x)^5} \, dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^3 \text{Subst}\left(\int \left(\frac{1}{8a(a-x)^5} - \frac{1}{16a^2(a-x)^4} - \frac{1}{16a^3(a-x)^3} - \frac{1}{32a^4(a-x)^2} + \frac{1}{32a^3(a+x)^3} + \frac{1}{64a^4(a+x)^2} - \frac{1}{64a^4(a^2-x^2)}\right) dx}{d}$$

$$= -\frac{1}{64d(a-a\cos(c+dx))^2} + \frac{a^2}{32d(a+a\cos(c+dx))^4} - \frac{a}{48d(a+a\cos(c+dx))^3} - \frac{1}{32d(a+a\cos(c+dx))^2} - \frac{1}{64d(a^2-a^2\cos(c+dx))} - \frac{1}{32d(a^2+a^2\cos(c+dx))} - \frac{\text{Subst}\left(\int \frac{1}{a^2-x^2} \, dx, x, -a\cos(c+dx)\right)}{64ad}$$

$$= \frac{\text{arctanh}(\cos(c+dx))}{64a^2d} - \frac{1}{64d(a-a\cos(c+dx))^2} + \frac{a^2}{32d(a+a\cos(c+dx))^4} - \frac{a}{48d(a+a\cos(c+dx))^3} - \frac{1}{32d(a+a\cos(c+dx))^2} - \frac{1}{64d(a^2-a^2\cos(c+dx))} - \frac{1}{32d(a^2+a^2\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.04

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2} \, dx = -\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right) + 6\csc^4\left(\frac{1}{2}(c+dx)\right) + 24\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.\right.}{384a^2d(1+}$$

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(Cos[(c + d*x)/2]^4*(12*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4 +
24*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]) + 24*Sec[(c + d*x)/2]^2
 + 12*Sec[(c + d*x)/2]^4 + 4*Sec[(c + d*x)/2]^6 - 3*Sec[(c + d*x)/2]^8)*Sec
[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-6\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-24\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-48\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-24\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{1536a^{2}d}$
derivativedivides	$\frac{-\frac{1}{64(\cos(dx+c)-1)^{2}}+\frac{1}{64\cos(dx+c)-64}-\frac{\ln(\cos(dx+c)-1)}{128}+\frac{1}{32(\cos(dx+c)+1)^{4}}-\frac{1}{48(\cos(dx+c)+1)^{3}}-\frac{1}{32(\cos(dx+c)+1)^{2}}-\frac{}{32(\cos(}}{d\,a^{2}}$
default	$\frac{-\frac{1}{64(\cos(dx+c)-1)^{2}}+\frac{1}{64\cos(dx+c)-64}-\frac{\ln(\cos(dx+c)-1)}{128}+\frac{1}{32(\cos(dx+c)+1)^{4}}-\frac{1}{48(\cos(dx+c)+1)^{3}}-\frac{1}{32(\cos(dx+c)+1)^{2}}-\frac{}{32(\cos(}}{d\,a^{2}}$
norman	$\frac{-\frac{1}{256ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{512ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{64da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{32da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{256da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{192da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a^{2}d}$
risch	$-\frac{3\,e^{11i(dx+c)}+12\,e^{10i(dx+c)}+7\,e^{9i(dx+c)}-32\,e^{8i(dx+c)}+566\,e^{7i(dx+c)}+424\,e^{6i(dx+c)}+566\,e^{5i(dx+c)}-32\,e^{4i(dx+c)}+7\,e}{96a^{2}d\left(e^{i(dx+c)}+1\right)^{8}\left(e^{i(dx+c)}-1\right)^{4}}$

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1536*(3*tan(1/2*d*x+1/2*c)^8+8*tan(1/2*d*x+1/2*c)^6-6*cot(1/2*d*x+1/2*c)^
4-6*tan(1/2*d*x+1/2*c)^4-24*cot(1/2*d*x+1/2*c)^2-48*tan(1/2*d*x+1/2*c)^2-24
*ln(tan(1/2*d*x+1/2*c)))/a^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(134) = 268$.

Time = 0.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.94

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{6\cos(dx+c)^5+12\cos(dx+c)^4-4\cos(dx+c)^3-20\cos(dx+c)^2-3\left(\cos(dx+c)^6+2\cos(dx+\right.}{}$$

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(6*cos(d*x + c)^5 + 12*cos(d*x + c)^4 - 4*cos(d*x + c)^3 - 20*cos(d*
x + c)^2 - 3*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*
x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2)
 + 3*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3
 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 70*c
os(d*x + c) + 32)/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*co
s(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
[Out] Integral(csc(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.14

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\left(3\cos(dx+c)^5+6\cos(dx+c)^4-2\cos(dx+c)^3-10\cos(dx+c)^2+35\cos(dx+c)+16\right)}{a^2\cos(dx+c)^6+2a^2\cos(dx+c)^5-a^2\cos(dx+c)^4-4a^2\cos(dx+c)^3-a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2}-\frac{3\log(\cos(dx+c)+1)}{a^2}+\cdots}{384\,d}$$

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/384*(2*(3*cos(d*x + c)^5 + 6*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 10*cos(
d*x + c)^2 + 35*cos(d*x + c) + 16)/(a^2*cos(d*x + c)^6 + 2*a^2*cos(d*x + c)
^5 - a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2 + 2*a^2
*cos(d*x + c) + a^2) - 3*log(cos(d*x + c) + 1)/a^2 + 3*log(cos(d*x + c) - 1
)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.42

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{6\left(\frac{4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a^2(\cos(dx+c)-1)^2}-\frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{\frac{48a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\cdots}{a^8}}{1536\,d}$$

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] 1/1536*(6*(4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a^2*(cos(d*x + c) - 1)^2)
- 12*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (48*a^6*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 6*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 8*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3*a^6*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4)/a^8)/d
```

Mupad [B] (verification not implemented)

Time = 13.64 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.04

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\operatorname{atanh}(\cos(c+d\,x))}{64\,a^2\,d} - \frac{\frac{\cos(c+d\,x)^5}{64}+\frac{\cos(c+d\,x)^4}{32}-\frac{\cos(c+d\,x)^3}{96}-\frac{5\cos(c+d\,x)^2}{96}+\frac{35\cos(c+d\,x)}{192}+\frac{1}{12}}{d\,\left(a^2\cos(c+d\,x)^6+2\,a^2\cos(c+d\,x)^5-a^2\cos(c+d\,x)^4-4\,a^2\cos(c+d\,x)^3-a^2\cos(c+d\,x)^2+2\,a\right.}$$

```
[In] int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))^2),x)
```

```
[Out] atanh(cos(c + d*x))/(64*a^2*d) - ((35*cos(c + d*x))/192 - (5*cos(c + d*x)^2
)/96 - cos(c + d*x)^3/96 + cos(c + d*x)^4/32 + cos(c + d*x)^5/64 + 1/12)/(d
*(2*a^2*cos(c + d*x) + a^2 - a^2*cos(c + d*x)^2 - 4*a^2*cos(c + d*x)^3 - a^
2*cos(c + d*x)^4 + 2*a^2*cos(c + d*x)^5 + a^2*cos(c + d*x)^6))
```

3.83 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{11x}{128a^2} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d} - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{2\sin^5(c+dx)}{5a^2d} + \frac{2\sin^7(c+dx)}{7a^2d}$$

```
[Out] 11/128*x/a^2+11/128*cos(d*x+c)*sin(d*x+c)/a^2/d-7/64*cos(d*x+c)^3*sin(d*x+c
)/a^2/d-1/16*cos(d*x+c)^5*sin(d*x+c)/a^2/d-1/6*cos(d*x+c)^3*sin(d*x+c)^3/a^
2/d-1/8*cos(d*x+c)^5*sin(d*x+c)^3/a^2/d-2/5*sin(d*x+c)^5/a^2/d+2/7*sin(d*x+
c)^7/a^2/d
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3957, 2954, 2952, 2648, 2715, 8, 2644, 14}

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\sin^7(c+dx)}{7a^2d} - \frac{2\sin^5(c+dx)}{5a^2d} - \frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d} - \frac{\sin(c+dx)\cos^5(c+dx)}{16a^2d} - \frac{\sin^3(c+dx)\cos^3(c+dx)}{6a^2d} - \frac{7\sin(c+dx)\cos^3(c+dx)}{64a^2d} + \frac{11\sin(c+dx)\cos(c+dx)}{128a^2d} + \frac{11x}{128a^2}$$

```
[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (11*x)/(128*a^2) + (11*Cos[c + d*x]*Sin[c + d*x])/(128*a^2*d) - (7*Cos[c +
d*x]^3*Sin[c + d*x])/(64*a^2*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(16*a^2*d)
- (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a^2*d) - (Cos[c + d*x]^5*Sin[c + d*x]^
3)/(8*a^2*d) - (2*Sin[c + d*x]^5)/(5*a^2*d) + (2*Sin[c + d*x]^7)/(7*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^8(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\sin^4(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(a^2\cos^2(c+dx)\sin^4(c+dx)-2a^2\cos^3(c+dx)\sin^4(c+dx)+a^2\cos^4(c+dx)\sin^4(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a^2}+\frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2} \\
&\quad -\frac{2\int \cos^3(c+dx)\sin^4(c+dx)\,dx}{a^2} \\
&= -\frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d}-\frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d}+\frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{8a^2} \\
&\quad +\frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{2a^2}-\frac{2\text{Subst}\left(\int x^4(1-x^2)\,dx,x,\sin(c+dx)\right)}{a^2d} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d}-\frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} \\
&\quad -\frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d}-\frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d}+\frac{\int \cos^4(c+dx)\,dx}{16a^2} \\
&\quad +\frac{\int \cos^2(c+dx)\,dx}{8a^2}-\frac{2\text{Subst}\left(\int (x^4-x^6)\,dx,x,\sin(c+dx)\right)}{a^2d} \\
&= \frac{\cos(c+dx)\sin(c+dx)}{16a^2d}-\frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d}-\frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} \\
&\quad -\frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d}-\frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} \\
&\quad -\frac{2\sin^5(c+dx)}{5a^2d}+\frac{2\sin^7(c+dx)}{7a^2d}+\frac{3\int \cos^2(c+dx)\,dx}{64a^2}+\frac{\int 1\,dx}{16a^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x}{16a^2} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} \\
&\quad - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d} \\
&\quad - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{2\sin^5(c+dx)}{5a^2d} + \frac{2\sin^7(c+dx)}{7a^2d} + \frac{3\int 1\,dx}{128a^2} \\
&= \frac{11x}{128a^2} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} \\
&\quad - \frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d} - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{2\sin^5(c+dx)}{5a^2d} + \frac{2\sin^7(c+dx)}{7a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

$$
\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(9240dx - 10080\sin(c+dx) - 1680\sin(2(c+dx)) + 3360\sin(3(c+dx)) - \right.}{26880}
$$

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(9240*d*x - 10080*Sin[c + d*x] - 1680*Si
n[2*(c + d*x)] + 3360*Sin[3*(c + d*x)] - 2520*Sin[4*(c + d*x)] + 672*Sin[5*
(c + d*x)] + 560*Sin[6*(c + d*x)] - 480*Sin[7*(c + d*x)] + 105*Sin[8*(c + d
*x)] + 980*Tan[c/2]))/(26880*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.59

method	result
parallelrisch	$\frac{9240dx-10080\sin(dx+c)+105\sin(8dx+8c)+560\sin(6dx+6c)-2520\sin(4dx+4c)-1680\sin(2dx+2c)-480\sin(7dx+7c)+}{107520a^2d}$
derivativedivides	$\frac{\frac{128\left(-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192}-\frac{253\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24576}-\frac{4213\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{122880}-\frac{55583\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{860160}+\frac{31007\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{860160}-\frac{20363\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{122880}+\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^8}}{a^2d}$
default	$\frac{\frac{128\left(-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192}-\frac{253\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24576}-\frac{4213\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{122880}-\frac{55583\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{860160}+\frac{31007\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{860160}-\frac{20363\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{122880}+\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^8}}{a^2d}$
risch	$\frac{11x}{128a^2}-\frac{3\sin(dx+c)}{32a^2d}+\frac{\sin(8dx+8c)}{1024a^2d}-\frac{\sin(7dx+7c)}{224a^2d}+\frac{\sin(6dx+6c)}{192a^2d}+\frac{\sin(5dx+5c)}{160a^2d}-\frac{3\sin(4dx+4c)}{128a^2d}+\frac{\sin(3dx}{32a}$
norman	$\frac{\frac{11x}{128a}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{253\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{192ad}-\frac{4213\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{960ad}-\frac{55583\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6720ad}+\frac{31007\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{6720ad}-\frac{20363\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{960ad}}{}$

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/107520*(9240*d*x-10080*sin(d*x+c)+105*sin(8*d*x+8*c)+560*sin(6*d*x+6*c)-2
520*sin(4*d*x+4*c)-1680*sin(2*d*x+2*c)-480*sin(7*d*x+7*c)+672*sin(5*d*x+5*c
)+3360*sin(3*d*x+3*c))/a^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$=\frac{1155\,dx+\left(1680\,\cos\left(dx+c\right)^7-3840\,\cos\left(dx+c\right)^6-280\,\cos\left(dx+c\right)^5+6144\,\cos\left(dx+c\right)^4-3710\,\cos\left(c\right.\right.}{13440\,a^2d}$$

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/13440*(1155*d*x + (1680*cos(d*x + c)^7 - 3840*cos(d*x + c)^6 - 280*cos(d*
x + c)^5 + 6144*cos(d*x + c)^4 - 3710*cos(d*x + c)^3 - 768*cos(d*x + c)^2 +
 1155*cos(d*x + c) - 1536)*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(151) = 302$.

Time = 0.29 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.26

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\frac{1155\sin(dx+c)}{\cos(dx+c)+1}+\frac{8855\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{29491\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{55583\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{31007\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{142541\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{8855\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{1155\sin(dx+c)}{(\cos(dx+c)+1)}}{a^2+\frac{8a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^2\sin(dx+c)}{(\cos(dx+c)+1}}}{6720\,d}$$

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6720*((1155*sin(d*x + c)/(cos(d*x + c) + 1) + 8855*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 29491*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 55583*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 - 31007*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 1
42541*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 8855*sin(d*x + c)^13/(cos(d*x
 + c) + 1)^13 - 1155*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^2 + 8*a^2*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 + 56*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^2*
sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^2*sin(d*x + c)^14/(cos(d*x + c)
 + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 1155*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{1155\,(dx+c)}{a^2}+\frac{2\left(1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}+8855\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-142541\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+31007\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-55583\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^8a^2}}{13440\,d}$$

[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] 1/13440*(1155*(d*x + c)/a^2 + 2*(1155*tan(1/2*d*x + 1/2*c)^15 + 8855*tan(1/2*d*x + 1/2*c)^13 - 142541*tan(1/2*d*x + 1/2*c)^11 + 31007*tan(1/2*d*x + 1/2*c)^9 - 55583*tan(1/2*d*x + 1/2*c)^7 - 29491*tan(1/2*d*x + 1/2*c)^5 - 8855*tan(1/2*d*x + 1/2*c)^3 - 1155*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*a^2))/d

Mupad [B] (verification not implemented)

Time = 16.85 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.80

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{11\,x}{128\,a^2} - \frac{-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64} - \frac{253\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192} + \frac{20363\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{960} - \frac{31007\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{6720} + \frac{55583\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{6720} + \frac{4213\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{960}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^8}$$

[In] int(sin(c + d*x)^8/(a + a/cos(c + d*x))^2,x)

[Out] (11*x)/(128*a^2) - ((11*tan(c/2 + (d*x)/2))/64 + (253*tan(c/2 + (d*x)/2)^3)/192 + (4213*tan(c/2 + (d*x)/2)^5)/960 + (55583*tan(c/2 + (d*x)/2)^7)/6720 - (31007*tan(c/2 + (d*x)/2)^9)/6720 + (20363*tan(c/2 + (d*x)/2)^11)/960 - (253*tan(c/2 + (d*x)/2)^13)/192 - (11*tan(c/2 + (d*x)/2)^15)/64)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^8)

3.84 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2} \, dx = \frac{3x}{16a^2} - \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d}$$

```
[Out] 3/16*x/a^2-3/16*cos(d*x+c)*sin(d*x+c)/a^2/d-1/8*cos(d*x+c)*sin(d*x+c)^3/a^2
/d-1/6*(a-a*cos(d*x+c))^3*sin(d*x+c)^3/a^5/d-1/10*sin(d*x+c)^5/a^2/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2954, 2949, 2748, 2715, 8}

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2} \, dx = -\frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d} - \frac{\sin^3(c+dx)\cos(c+dx)}{8a^2d} - \frac{3\sin(c+dx)\cos(c+dx)}{16a^2d} + \frac{3x}{16a^2}$$

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*x)/(16*a^2) - (3*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d) - (Cos[c + d*x]*S
in[c + d*x]^3)/(8*a^2*d) - ((a - a*Cos[c + d*x])^3*Sin[c + d*x]^3)/(6*a^5*d
) - Sin[c + d*x]^5/(10*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2949

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(2*b*f*g*(m + 1))), x] + Dist[a/(2*g^
2), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Fr
eeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^6(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\sin^2(c+dx)\,dx}{a^4} \\
&= -\frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\int(-a+a\cos(c+dx))\sin^4(c+dx)\,dx}{2a^3}
\end{aligned}
$$

$$= -\frac{(a - a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d} + \frac{\int \sin^4(c+dx)\,dx}{2a^2}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a - a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d} + \frac{3\int \sin^2(c+dx)\,dx}{8a^2}$$

$$= -\frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a - a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d} + \frac{3\int 1\,dx}{16a^2}$$

$$= \frac{3x}{16a^2} - \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a - a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(360dx - 480\sin(c+dx) + 30\sin(2(c+dx)) + 80\sin(3(c+dx)) - 90\sin(4\right.}{480a^2d(1+\sec(c+dx))^2}$$

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(360*d*x - 480*Sin[c + d*x] + 30*Sin[2*(
c + d*x)] + 80*Sin[3*(c + d*x)] - 90*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)]
 - 10*Sin[6*(c + d*x)] + 25*Tan[c/2]))/(480*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{180dx-240\sin(dx+c)-5\sin(6dx+6c)-45\sin(4dx+4c)+15\sin(2dx+2c)+24\sin(5dx+5c)+40\sin(3dx+3c)}{960a^2d}$
risch	$\frac{3x}{16a^2}-\frac{\sin(dx+c)}{4a^2d}-\frac{\sin(6dx+6c)}{192a^2d}+\frac{\sin(5dx+5c)}{40a^2d}-\frac{3\sin(4dx+4c)}{64a^2d}+\frac{\sin(3dx+3c)}{24a^2d}+\frac{\sin(2dx+2c)}{64a^2d}$
derivativedivides	$\frac{\frac{32\left(\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{256}-\frac{205\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{768}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{640}-\frac{99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{640}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{256}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{256}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6}+\frac{3\arctan\left(\tan\left(\right.\right.}{8}}{a^2d}$
default	$\frac{\frac{32\left(\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{256}-\frac{205\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{768}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{640}-\frac{99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{640}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{256}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{256}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6}+\frac{3\arctan\left(\tan\left(\right.\right.}{8}}{a^2d}$
norman	$\frac{\frac{3x}{16a}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8ad}-\frac{99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20ad}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{20ad}-\frac{205\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{24ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8ad}+\frac{9x\tan\left(\frac{dx}{2}+\right.}{8a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6a}$

```
[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/960*(180*d*x-240*sin(d*x+c)-5*sin(6*d*x+6*c)-45*sin(4*d*x+4*c)+15*sin(2*d
*x+2*c)+24*sin(5*d*x+5*c)+40*sin(3*d*x+3*c))/a^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$=\frac{45\,dx-\left(40\,\cos\left(dx+c\right)^5-96\,\cos\left(dx+c\right)^4+50\,\cos\left(dx+c\right)^3+32\,\cos\left(dx+c\right)^2-45\,\cos\left(dx+c\right)+64\right)}{240\,a^2d}$$

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(45*d*x - (40*cos(d*x + c)^5 - 96*cos(d*x + c)^4 + 50*cos(d*x + c)^3
+ 32*cos(d*x + c)^2 - 45*cos(d*x + c) + 64)*sin(d*x + c))/(a^2*d)
```

Sympy [F]

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
[Out] Integral(sin(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(95) = 190$.

Time = 0.29 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.81

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}+\frac{255\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{594\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{174\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{1025\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{45\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^2+\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{120\,d}$$

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/120*((45*sin(d*x + c)/(cos(d*x + c) + 1) + 255*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 594*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 174*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 1025*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 45*sin(d*x
 + c)^11/(cos(d*x + c) + 1)^11)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) +
1)^12) - 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{45(dx+c)}{a^2}+\frac{2\left(45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-1025\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-174\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-594\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-255\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-45\tan\left(\frac{1}{2}dx+\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^6a^2}}{240\,d}$$

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(45*(d*x + c)/a^2 + 2*(45*tan(1/2*d*x + 1/2*c)^11 - 1025*tan(1/2*d*x
+ 1/2*c)^9 - 174*tan(1/2*d*x + 1/2*c)^7 - 594*tan(1/2*d*x + 1/2*c)^5 - 255*
tan(1/2*d*x + 1/2*c)^3 - 45*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
+ 1)^6*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{3\,x}{16\,a^2} - \frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{205\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{99\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{a^2\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^6}$$

```
[In] int(sin(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (3*x)/(16*a^2) - ((3*tan(c/2 + (d*x)/2))/8 + (17*tan(c/2 + (d*x)/2)^3)/8 +
(99*tan(c/2 + (d*x)/2)^5)/20 + (29*tan(c/2 + (d*x)/2)^7)/20 + (205*tan(c/2
+ (d*x)/2)^9)/24 - (3*tan(c/2 + (d*x)/2)^11)/8)/(a^2*d*(tan(c/2 + (d*x)/2)^
2 + 1)^6)
```

3.85 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{7x}{8a^2}-\frac{2\sin(c+dx)}{a^2d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{2\sin^3(c+dx)}{3a^2d}$$

```
[Out] 7/8*x/a^2-2*sin(d*x+c)/a^2/d+7/8*cos(d*x+c)*sin(d*x+c)/a^2/d+1/4*cos(d*x+c)
^3*sin(d*x+c)/a^2/d+2/3*sin(d*x+c)^3/a^2/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2948, 2836, 2715, 8, 2713}

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\sin^3(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{7x}{8a^2}$$

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*x)/(8*a^2) - (2*Sin[c + d*x])/(a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])/(8
*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^2*d) + (2*Sin[c + d*x]^3)/(3*a
^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\cos^2(c+dx)-2a^2\cos^3(c+dx)+a^2\cos^4(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int \cos^2(c+dx)\,dx}{a^2}+\frac{\int \cos^4(c+dx)\,dx}{a^2}-\frac{2\int \cos^3(c+dx)\,dx}{a^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} + \frac{\int 1\,dx}{2a^2} \\
&\quad + \frac{3\int \cos^2(c+dx)\,dx}{4a^2} + \frac{2\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{a^2d} \\
&= \frac{x}{2a^2} - \frac{2\sin(c+dx)}{a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{8a^2d} \\
&\quad + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} + \frac{2\sin^3(c+dx)}{3a^2d} + \frac{3\int 1\,dx}{8a^2} \\
&= \frac{7x}{8a^2} - \frac{2\sin(c+dx)}{a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} + \frac{2\sin^3(c+dx)}{3a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$
\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx
= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(84dx - 144\sin(c+dx) + 48\sin(2(c+dx)) - 16\sin(3(c+dx)) + 3\sin(4(c
}{24a^2d(1+\sec(c+dx))^2}
$$

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(84*d*x - 144*Sin[c + d*x] + 48*Sin[2*(c
 + d*x)] - 16*Sin[3*(c + d*x)] + 3*Sin[4*(c + d*x)] + 2*Tan[c/2]))/(24*a^2*
d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{84dx-144\sin(dx+c)-16\sin(3dx+3c)+3\sin(4dx+4c)+48\sin(2dx+2c)}{96a^2d}$
risch	$\frac{7x}{8a^2}-\frac{3\sin(dx+c)}{2a^2d}+\frac{\sin(4dx+4c)}{32a^2d}-\frac{\sin(3dx+3c)}{6a^2d}+\frac{\sin(2dx+2c)}{2a^2d}$
derivativedivides	$\frac{\frac{8\left(-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{32}-\frac{83\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{96}-\frac{77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{96}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}+\frac{7\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^2d}$
default	$\frac{\frac{8\left(-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{32}-\frac{83\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{96}-\frac{77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{96}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}+\frac{7\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^2d}$
norman	$\frac{\frac{7x}{8a}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12ad}-\frac{83\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12ad}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4ad}+\frac{7x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}+\frac{21x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4a}+\frac{7x\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4a}$

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(84*d*x-144*sin(d*x+c)-16*sin(3*d*x+3*c)+3*sin(4*d*x+4*c)+48*sin(2*d*x
+2*c))/a^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{21\,dx+\left(6\,\cos\left(dx+c\right)^3-16\,\cos\left(dx+c\right)^2+21\,\cos\left(dx+c\right)-32\right)\sin\left(dx+c\right)}{24\,a^2d}$$

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(21*d*x + (6*cos(d*x + c)^3 - 16*cos(d*x + c)^2 + 21*cos(d*x + c) - 32
)*sin(d*x + c))/(a^2*d)
```

Sympy [F]

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
[Out] Integral(sin(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(79) = 158$.

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.37

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{83\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{21\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{12\,d}$$

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 83*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^2 + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6 + a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 21*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{21(dx+c)}{a^2}-\frac{2\left(75\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+83\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+77\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4a^2}}{24\,d}$$

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] 1/24*(21*(d*x + c)/a^2 - 2*(75*tan(1/2*d*x + 1/2*c)^7 + 83*tan(1/2*d*x + 1/
2*c)^5 + 77*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^4*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 17.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{7\,x}{8\,a^2} - \frac{\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{83\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{77\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

```
[In] int(sin(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (7*x)/(8*a^2) - ((7*tan(c/2 + (d*x)/2))/4 + (77*tan(c/2 + (d*x)/2)^3)/12 +
(83*tan(c/2 + (d*x)/2)^5)/12 + (25*tan(c/2 + (d*x)/2)^7)/4)/(a^2*d*(tan(c/2
 + (d*x)/2)^2 + 1)^4)
```

3.86 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx=-\frac{5x}{2a^2}+\frac{2\sin(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(1+\cos(c+dx))}$$

```
[Out] -5/2*x/a^2+2*sin(d*x+c)/a^2/d-1/2*cos(d*x+c)*sin(d*x+c)/a^2/d+2*sin(d*x+c)/
a^2/d/(1+cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3957, 2953, 3029, 2788, 2717, 2715, 8, 2727}

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\frac{2\sin(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{5x}{2a^2}$$

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-5*x)/(2*a^2) + (2*Sin[c + d*x])/(a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*
a^2*d) + (2*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\
&= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))}{-a-a\cos(c+dx)}\,dx}{a^2}\\
&= \frac{\int (-a+a\cos(c+dx))^2\cot^2(c+dx)\,dx}{a^4}\\
&= \frac{\int \left(-2+2\cos(c+dx)-\cos^2(c+dx)+\frac{2}{1+\cos(c+dx)}\right)dx}{a^2}\\
&= -\frac{2x}{a^2}-\frac{\int \cos^2(c+dx)\,dx}{a^2}+\frac{2\int \cos(c+dx)\,dx}{a^2}+\frac{2\int \frac{1}{1+\cos(c+dx)}\,dx}{a^2}\\
&= -\frac{2x}{a^2}+\frac{2\sin(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{\int 1\,dx}{2a^2}\\
&= -\frac{5x}{2a^2}+\frac{2\sin(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(1+\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$
\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(60dx\cos\left(\frac{dx}{2}\right)+60dx\cos\left(c+\frac{dx}{2}\right)-119\sin\left(\frac{dx}{2}\right)-25\sin\left(c+\frac{dx}{2}\right)-21\sin\left(c-\right.\right.}{48a^2d}
$$

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/48*(Sec[c/2]*Sec[(c + d*x)/2]*(60*d*x*Cos[(d*x)/2] + 60*d*x*Cos[c + (d*x
)/2] - 119*Sin[(d*x)/2] - 25*Sin[c + (d*x)/2] - 21*Sin[c + (3*d*x)/2] - 21*
Sin[2*c + (3*d*x)/2] + 3*Sin[2*c + (5*d*x)/2] + 3*Sin[3*c + (5*d*x)/2]))/(a
^2*d)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{(15-\cos(2dx+2c)+6\cos(dx+c))\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-10dx}{4a^2d}$	45
derivativedivides	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-5\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	73
default	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-5\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	73
risch	$-\frac{5x}{2a^2}-\frac{i e^{i(dx+c)}}{a^2d}+\frac{i e^{-i(dx+c)}}{a^2d}+\frac{4i}{a^2d\left(e^{i(dx+c)}+1\right)}-\frac{\sin(2dx+2c)}{4a^2d}$	83
norman	$\frac{-\frac{5x}{2a}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}-\frac{5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}-\frac{5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2a}$	116

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((15-cos(2*d*x+2*c)+6*cos(d*x+c))*tan(1/2*d*x+1/2*c)-10*d*x)/a^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{5\,dx\cos(dx+c)+5\,dx+\left(\cos(dx+c)^2-3\,\cos(dx+c)-8\right)\sin(dx+c)}{2\left(a^2d\cos(dx+c)+a^2d\right)}$$

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(5*d*x*cos(d*x + c) + 5*d*x + (cos(d*x + c)^2 - 3*cos(d*x + c) - 8)*si
n(d*x + c))/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(65) = 130$.

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.03

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} - \frac{5\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} + \frac{2\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{d}$$

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] ((3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4) - 5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 2*sin(
d*x + c)/(a^2*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{5(dx+c)}{a^2} - \frac{4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2} - \frac{2\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 a^2}}{2d}$$

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -1/2*(5*(d*x + c)/a^2 - 4*tan(1/2*d*x + 1/2*c)/a^2 - 2*(5*tan(1/2*d*x + 1/2
*c)^3 + 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(c+dx)+10\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

```
[In] int(sin(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*sin(c/2 + (d*x)/2) - 5*cos(c/2 + (d*x)/2)*(c + d*x) + 10*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2) - 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/(2*a^
2*d*cos(c/2 + (d*x)/2))
```

3.87 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cot^3(c+dx)}{3a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-2/5*cot(d*x+c)^5/a^2/d-2/3*csc(d*x+c)^3/a^2/d+2/5*c
sc(d*x+c)^5/a^2/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2790, 2687, 30, 2686, 14}

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\cot^5(c+dx)}{5a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d} - \frac{2\csc^3(c+dx)}{3a^2d}$$

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cot[c + d*x]^3/(a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (2*Csc[c + d*x
]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \left(a^2\cot^4(c+dx)\csc^2(c+dx)-2a^2\cot^3(c+dx)\csc^3(c+dx)+a^2\cot^2(c+dx)\csc^4(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a^2} \\
&\quad -\frac{2\int \cot^3(c+dx)\csc^3(c+dx)\,dx}{a^2} \\
&= \frac{\operatorname{Subst}\left(\int x^4\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad +\frac{2\operatorname{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{a^2d}
\end{aligned}
$$

$$= -\frac{\cot^5(c+dx)}{5a^2d} + \frac{\text{Subst}\left(\int \left(x^2+x^4\right)\,dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, \csc(c+dx)\right)}{a^2d}$$

$$= -\frac{\cot^3(c+dx)}{3a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\csc(c)\csc(c+dx)\sec^2(c+dx)(-80\sin(c)+80\sin(dx)+55\sin(c+dx)+44\sin(2(c+dx))+11\sin(3(c+}{240a^2d(1+\sec(c+dx))^2}$$

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]*Sec[c + d*x]^2*(-80*Sin[c] + 80*Sin[d*x] + 55*Sin[c +
d*x] + 44*Sin[2*(c + d*x)] + 11*Sin[3*(c + d*x)] - 60*Sin[2*c + d*x] + 16*S
in[c + 2*d*x] + 4*Sin[2*c + 3*d*x]))/(240*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-15\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{120a^2d}$	58
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8d\,a^2}$	60
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8d\,a^2}$	60
norman	$\frac{-\frac{1}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{24da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{40da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a}$	82
risch	$-\frac{2i\left(15\,\mathrm{e}^{4i(dx+c)}+20\,\mathrm{e}^{3i(dx+c)}+20\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}+1\right)}{15a^2d\left(\mathrm{e}^{i(dx+c)}+1\right)^5\left(\mathrm{e}^{i(dx+c)}-1\right)}$	82

```
[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(3*tan(1/2*d*x+1/2*c)^5-5*tan(1/2*d*x+1/2*c)^3-15*cot(1/2*d*x+1/2*c)-
15*tan(1/2*d*x+1/2*c))/a^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos\left(dx+c\right)^3+2\,\cos\left(dx+c\right)^2+8\,\cos\left(dx+c\right)+4}{15\,\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)\sin\left(dx+c\right)}$$

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + 8*cos(d*x + c) + 4)/((a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}+\frac{15\,(\cos(dx+c)+1)}{a^2\sin(dx+c)}}{120\,d}$$

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 + 15*(cos(d*x + c) +
1)/(a^2*sin(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{15}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - \frac{3\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{10}}}{120\,d}$$

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -1/120*(15/(a^2*tan(1/2*d*x + 1/2*c)) - (3*a^8*tan(1/2*d*x + 1/2*c)^5 - 5*a
^8*tan(1/2*d*x + 1/2*c)^3 - 15*a^8*tan(1/2*d*x + 1/2*c))/a^10)/d
```

Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+14\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3}{120\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

```
[In] int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))^2),x)
[Out] -(14*cos(c/2 + (d*x)/2)^2 - 4*cos(c/2 + (d*x)/2)^4 + 8*cos(c/2 + (d*x)/2)^6
 - 3)/(120*a^2*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2))
```

3.88 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx=-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-3/5*cot(d*x+c)^5/a^2/d-2/7*cot(d*x+c)^7/a^2/d-2/5*c
sc(d*x+c)^5/a^2/d+2/7*csc(d*x+c)^7/a^2/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2954, 2952, 2687, 14, 2686, 276}

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx=-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}$$

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cot[c + d*x]^3/(a^2*d) - (3*Cot[c + d*x]^5)/(5*a^2*d) - (2*Cot[c + d*x
]^7)/(7*a^2*d) - (2*Csc[c + d*x]^5)/(5*a^2*d) + (2*Csc[c + d*x]^7)/(7*a^2*d
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int(-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^6(c+dx)\,dx}{a^4} \\
&= \frac{\int\left(a^2\cot^4(c+dx)\csc^4(c+dx)-2a^2\cot^3(c+dx)\csc^5(c+dx)+a^2\cot^2(c+dx)\csc^6(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int\cot^4(c+dx)\csc^4(c+dx)\,dx}{a^2}+\frac{\int\cot^2(c+dx)\csc^6(c+dx)\,dx}{a^2} \\
&\quad -\frac{2\int\cot^3(c+dx)\csc^5(c+dx)\,dx}{a^2} \\
&= \frac{\mathrm{Subst}\left(\int x^4(1+x^2)\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad +\frac{\mathrm{Subst}\left(\int x^2(1+x^2)^2\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad +\frac{2\mathrm{Subst}\left(\int x^4(-1+x^2)\,dx,x,\csc(c+dx)\right)}{a^2d} \\
&= \frac{\mathrm{Subst}\left(\int(x^4+x^6)\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad +\frac{\mathrm{Subst}\left(\int(x^2+2x^4+x^6)\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad +\frac{2\mathrm{Subst}\left(\int(-x^4+x^6)\,dx,x,\csc(c+dx)\right)}{a^2d} \\
&= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.64

$$
\int\frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\csc(c)\csc^3(c+dx)\sec^2(c+dx)(1344\sin(c)-1456\sin(dx)-714\sin(c+dx)-408\sin(2(c+dx))+153
$$

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/13440*(Csc[c]*Csc[c + d*x]^3*Sec[c + d*x]^2*(1344*Sin[c] - 1456*Sin[d*x]
 - 714*Sin[c + d*x] - 408*Sin[2*(c + d*x)] + 153*Sin[3*(c + d*x)] + 204*Sin
[4*(c + d*x)] + 51*Sin[5*(c + d*x)] + 1680*Sin[2*c + d*x] + 128*Sin[c + 2*d
*x] - 48*Sin[2*c + 3*d*x] - 64*Sin[3*c + 4*d*x] - 16*Sin[4*c + 5*d*x]))/(a^
2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-70\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-35\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-210\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-105\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{3360a^{2}d}$	84
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{32d\,a^{2}}$	86
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{32d\,a^{2}}$	86
risch	$\frac{4i\left(105\,{\rm e}^{6i(dx+c)}+84\,{\rm e}^{5i(dx+c)}+91\,{\rm e}^{4i(dx+c)}-8\,{\rm e}^{3i(dx+c)}+3\,{\rm e}^{2i(dx+c)}+4\,{\rm e}^{i(dx+c)}+1\right)}{105a^{2}d\left({\rm e}^{i(dx+c)}+1\right)^{7}\left({\rm e}^{i(dx+c)}-1\right)^{3}}$	104
norman	$\frac{-\frac{1}{96ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{32da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{16da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{48da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{160da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{224da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}$	120

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3360*(15*tan(1/2*d*x+1/2*c)^7+21*tan(1/2*d*x+1/2*c)^5-70*tan(1/2*d*x+1/2*
c)^3-35*cot(1/2*d*x+1/2*c)^3-210*tan(1/2*d*x+1/2*c)-105*cot(1/2*d*x+1/2*c))
/a^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\cos\left(dx+c\right)^{5}+4\cos\left(dx+c\right)^{4}-\cos\left(dx+c\right)^{3}-6\cos\left(dx+c\right)^{2}+24\cos\left(dx+c\right)+12}{105\left(a^{2}d\cos\left(dx+c\right)^{4}+2\,a^{2}d\cos\left(dx+c\right)^{3}-2\,a^{2}d\cos\left(dx+c\right)-a^{2}d\right)\sin\left(dx+c\right)}$$

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*cos(d*x + c)^5 + 4*cos(d*x + c)^4 - cos(d*x + c)^3 - 6*cos(d*x + c
)^2 + 24*cos(d*x + c) + 12)/((a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3
 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\frac{210\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{70\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2}+\frac{35\left(\frac{3\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^3}{a^2\sin(dx+c)^3}}{3360\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*((210*sin(d*x + c)/(cos(d*x + c) + 1) + 70*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^2 + 35*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(c
os(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.15

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{35\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{15\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+21\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-70\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-210\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{14}}}{3360\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3360*(35*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^2*tan(1/2*d*x + 1/2*c)^3) - (
15*a^12*tan(1/2*d*x + 1/2*c)^7 + 21*a^12*tan(1/2*d*x + 1/2*c)^5 - 70*a^12*t
an(1/2*d*x + 1/2*c)^3 - 210*a^12*tan(1/2*d*x + 1/2*c))/a^14)/d
```

Mupad [B] (verification not implemented)

Time = 13.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-96\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+54\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-15}{3360\,a^2\,d\,\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

```
[In] int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(54*cos(c/2 + (d*x)/2)^2 + 4*cos(c/2 + (d*x)/2)^4 + 24*cos(c/2 + (d*x)/2)^
6 - 96*cos(c/2 + (d*x)/2)^8 + 64*cos(c/2 + (d*x)/2)^10 - 15)/(3360*a^2*d*(c
os(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) - cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*
x)/2)))
```

3.89 $\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cot^3(c+dx)}{3a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}-\frac{2\csc^7(c+dx)}{7a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-4/5*cot(d*x+c)^5/a^2/d-5/7*cot(d*x+c)^7/a^2/d-2/9*c
ot(d*x+c)^9/a^2/d-2/7*csc(d*x+c)^7/a^2/d+2/9*csc(d*x+c)^9/a^2/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2954, 2952, 2687, 276, 2686, 14}

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\cot^9(c+dx)}{9a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{2\csc^7(c+dx)}{7a^2d}$$

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cot[c + d*x]^3/(a^2*d) - (4*Cot[c + d*x]^5)/(5*a^2*d) - (5*Cot[c + d*x
]^7)/(7*a^2*d) - (2*Cot[c + d*x]^9)/(9*a^2*d) - (2*Csc[c + d*x]^7)/(7*a^2*d
) + (2*Csc[c + d*x]^9)/(9*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot^2(c+dx)\csc^4(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int (-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^8(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(a^2\cot^4(c+dx)\csc^6(c+dx)-2a^2\cot^3(c+dx)\csc^7(c+dx)+a^2\cot^2(c+dx)\csc^8(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int \cot^4(c+dx)\csc^6(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^8(c+dx)\,dx}{a^2} \\
&\quad -\frac{2\int \cot^3(c+dx)\csc^7(c+dx)\,dx}{a^2} \\
&= \frac{\text{Subst}\left(\int x^4(1+x^2)^2\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad +\frac{\text{Subst}\left(\int x^2(1+x^2)^3\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad +\frac{2\text{Subst}\left(\int x^6(-1+x^2)\,dx,x,\csc(c+dx)\right)}{a^2d} \\
&= \frac{\text{Subst}\left(\int (x^4+2x^6+x^8)\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad +\frac{\text{Subst}\left(\int (x^2+3x^4+3x^6+x^8)\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad +\frac{2\text{Subst}\left(\int (-x^6+x^8)\,dx,x,\csc(c+dx)\right)}{a^2d} \\
&= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{5\cot^7(c+dx)}{7a^2d} \\
&\quad -\frac{2\cot^9(c+dx)}{9a^2d}-\frac{2\csc^7(c+dx)}{7a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.75

$$
\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx
= \frac{\csc(c)\csc^5(c+dx)\sec^2(c+dx)(-61440\sin(c)+84480\sin(dx)+25875\sin(c+dx)+11500\sin(2(c+dx))
$$

```
[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]^2*(-61440*Sin[c] + 84480*Sin[d*x] + 25875*Sin[c + d*x] + 11500*Sin[2*(c + d*x)] - 10925*Sin[3*(c + d*x)] - 9200*Sin[4*(c + d*x)] + 575*Sin[5*(c + d*x)] + 2300*Sin[6*(c + d*x)] + 575*Sin[7*(c + d*x)] - 107520*Sin[2*c + d*x] - 10240*Sin[c + 2*d*x] + 9728*Sin[2*c + 3*d*x] + 8192*Sin[3*c + 4*d*x] - 512*Sin[4*c + 5*d*x] - 2048*Sin[5*c + 6*d*x] - 512*Sin[6*c + 7*d*x]))/(1290240*a^2*d*(1 + Sec[c + d*x])^2)

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

method	result
parallelrisch	$\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+135\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+63\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-63\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-525\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-315\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-1575\,\mathrm{t}}{40320a^{2}d}$
derivativedivides	$\frac{-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-5\tan\left(\frac{dx}{2}\right.}{128d\,a^{2}}$
default	$\frac{-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-5\tan\left(\frac{dx}{2}\right.}{128d\,a^{2}}$
risch	$-\frac{16i\left(210\,\mathrm{e}^{8i(dx+c)}+120\,\mathrm{e}^{7i(dx+c)}+165\,\mathrm{e}^{6i(dx+c)}-20\,\mathrm{e}^{5i(dx+c)}+19\,\mathrm{e}^{4i(dx+c)}+16\,\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}-1\right)}{315a^{2}d\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}\left(\mathrm{e}^{i(dx+c)}-1\right)^{5}}$
norman	$\frac{-\frac{1}{640ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{896ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{1152ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{128da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{128da}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{128da}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{384da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{640da}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}$

[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/40320*(35*tan(1/2*d*x+1/2*c)^9+135*tan(1/2*d*x+1/2*c)^7+63*tan(1/2*d*x+1/2*c)^5-63*cot(1/2*d*x+1/2*c)^5-525*tan(1/2*d*x+1/2*c)^3-315*cot(1/2*d*x+1/2*c)^3-1575*tan(1/2*d*x+1/2*c)-315*cot(1/2*d*x+1/2*c))/a^2/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.55

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{8\cos(dx+c)^7+16\cos(dx+c)^6-12\cos(dx+c)^5-40\cos(dx+c)^4-5\cos(dx+c)^3+30\cos}{315\left(a^2d\cos(dx+c)^6+2a^2d\cos(dx+c)^5-a^2d\cos(dx+c)^4-4a^2d\cos(dx+c)^3-a^2d\cos(dx+c)^2+\right.}$$

[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] 1/315*(8*cos(d*x + c)^7 + 16*cos(d*x + c)^6 - 12*cos(d*x + c)^5 - 40*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + 30*cos(d*x + c)^2 - 40*cos(d*x + c) - 20)/((a

```
^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^4 - 4*a^2
*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*si
n(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\csc^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
[Out] Integral(csc(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.60

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\frac{1575\sin(dx+c)}{\cos(dx+c)+1}+\frac{525\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{135\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^2}+\frac{63\left(\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)(\cos(dx+c)+1)^{5}}{a^2\sin(dx+c)^5}}{40320\,d}$$

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/40320*((1575*sin(d*x + c)/(cos(d*x + c) + 1) + 525*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 135*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2 + 63*(
5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + 1)*(cos(d*x + c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{63\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}-\frac{35a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+135a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+63a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-525a^{16}\tan\left(\frac{1}{2}dx\right.}{a^{18}}}{40320\,d}$$

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/40320*(63*(5*tan(1/2*d*x + 1/2*c)^4 + 5*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^2
*tan(1/2*d*x + 1/2*c)^5) - (35*a^16*tan(1/2*d*x + 1/2*c)^9 + 135*a^16*tan(1
/2*d*x + 1/2*c)^7 + 63*a^16*tan(1/2*d*x + 1/2*c)^5 - 525*a^16*tan(1/2*d*x +
 1/2*c)^3 - 1575*a^16*tan(1/2*d*x + 1/2*c))/a^18)/d
```

Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{375\,\cos(c+d\,x)}{8}-\frac{5\,\cos(2\,c+2\,d\,x)}{2}+\frac{19\,\cos(3\,c+3\,d\,x)}{8}+2\,\cos\left(4\,c+4\,d\,x\right)-\frac{\cos(5\,c+5\,d\,x)}{8}-\frac{\cos(6\,c+6\,d\,x)}{2}-\frac{\cos(7\,c+7\,d\,x)}{8}}{40320\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

```
[In] int(1/(sin(c + d*x)^6*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -((375*cos(c + d*x))/8 - (5*cos(2*c + 2*d*x))/2 + (19*cos(3*c + 3*d*x))/8 +
 2*cos(4*c + 4*d*x) - cos(5*c + 5*d*x)/8 - cos(6*c + 6*d*x)/2 - cos(7*c + 7
*d*x)/8 + 15)/(40320*a^2*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5)
```

3.90 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2} dx = -\frac{\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{a^2d} - \frac{9\cot^7(c+dx)}{7a^2d} - \frac{7\cot^9(c+dx)}{9a^2d} - \frac{2\cot^{11}(c+dx)}{11a^2d} - \frac{2\csc^9(c+dx)}{9a^2d} + \frac{2\csc^{11}(c+dx)}{11a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-cot(d*x+c)^5/a^2/d-9/7*cot(d*x+c)^7/a^2/d-7/9*cot(d
*x+c)^9/a^2/d-2/11*cot(d*x+c)^11/a^2/d-2/9*csc(d*x+c)^9/a^2/d+2/11*csc(d*x+
c)^11/a^2/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2954, 2952, 2687, 276, 2686, 14}

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2} dx = -\frac{2\cot^{11}(c+dx)}{11a^2d} - \frac{7\cot^9(c+dx)}{9a^2d} - \frac{9\cot^7(c+dx)}{7a^2d} - \frac{\cot^5(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{2\csc^{11}(c+dx)}{11a^2d} - \frac{2\csc^9(c+dx)}{9a^2d}$$

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cot[c + d*x]^3/(a^2*d) - Cot[c + d*x]^5/(a^2*d) - (9*Cot[c + d*x]^7)/(
7*a^2*d) - (7*Cot[c + d*x]^9)/(9*a^2*d) - (2*Cot[c + d*x]^11)/(11*a^2*d) -
(2*Csc[c + d*x]^9)/(9*a^2*d) + (2*Csc[c + d*x]^11)/(11*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot^2(c+dx)\csc^6(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int(-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a^4} \\
&= \frac{\int\left(a^2\cot^4(c+dx)\csc^8(c+dx)-2a^2\cot^3(c+dx)\csc^9(c+dx)+a^2\cot^2(c+dx)\csc^{10}(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int\cot^4(c+dx)\csc^8(c+dx)\,dx}{a^2}+\frac{\int\cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a^2} \\
&\quad-\frac{2\int\cot^3(c+dx)\csc^9(c+dx)\,dx}{a^2} \\
&= \frac{\mathrm{Subst}\left(\int x^4\left(1+x^2\right)^3\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad+\frac{\mathrm{Subst}\left(\int x^2\left(1+x^2\right)^4\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad+\frac{2\mathrm{Subst}\left(\int x^8\left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{a^2d} \\
&= \frac{\mathrm{Subst}\left(\int\left(x^4+3x^6+3x^8+x^{10}\right)\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad+\frac{\mathrm{Subst}\left(\int\left(x^2+4x^4+6x^6+4x^8+x^{10}\right)\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&\quad+\frac{2\mathrm{Subst}\left(\int\left(-x^8+x^{10}\right)\,dx,x,\csc(c+dx)\right)}{a^2d} \\
&= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{a^2d}-\frac{9\cot^7(c+dx)}{7a^2d}-\frac{7\cot^9(c+dx)}{9a^2d} \\
&\quad-\frac{2\cot^{11}(c+dx)}{11a^2d}-\frac{2\csc^9(c+dx)}{9a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.86

$$
\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\csc(c)\csc^7(c+dx)\sec^2(c+dx)(630784\sin(c)-1103872\sin(dx)-218834\sin(c+dx)-79576\sin(2(c+
$$

```
[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/22708224*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]^2*(630784*Sin[c] - 1103872*
Sin[d*x] - 218834*Sin[c + d*x] - 79576*Sin[2*(c + d*x)] + 119364*Sin[3*(c +
 d*x)] + 79576*Sin[4*(c + d*x)] - 28420*Sin[5*(c + d*x)] - 34104*Sin[6*(c +
 d*x)] - 1421*Sin[7*(c + d*x)] + 5684*Sin[8*(c + d*x)] + 1421*Sin[9*(c + d*
x)] + 1419264*Sin[2*c + d*x] + 114688*Sin[c + 2*d*x] - 172032*Sin[2*c + 3*d
*x] - 114688*Sin[3*c + 4*d*x] + 40960*Sin[4*c + 5*d*x] + 49152*Sin[5*c + 6*
d*x] + 2048*Sin[6*c + 7*d*x] - 8192*Sin[7*c + 8*d*x] - 2048*Sin[8*c + 9*d*x
]))/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{}{3\tan}}{512d\,a^{2}}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{}{3\tan}}{512d\,a^{2}}$
parallelrisch	$\frac{63\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+385\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+792\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-99\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-693\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-3234\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-18}{354816a^{2}d}$
risch	$\frac{32i\left(693\,\mathrm{e}^{10i(dx+c)}+308\,\mathrm{e}^{9i(dx+c)}+539\,\mathrm{e}^{8i(dx+c)}-56\,\mathrm{e}^{7i(dx+c)}+84\,\mathrm{e}^{6i(dx+c)}+56\,\mathrm{e}^{5i(dx+c)}-20\,\mathrm{e}^{4i(dx+c)}-24\,\mathrm{e}^{3i(dx+c)}-\right.}{693a^{2}d\left(\mathrm{e}^{i(dx+c)}+1\right)^{11}\left(\mathrm{e}^{i(dx+c)}-1\right)^{7}}$

```
[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/512/d/a^2*(1/11*tan(1/2*d*x+1/2*c)^11+5/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/
2*d*x+1/2*c)^7-14/3*tan(1/2*d*x+1/2*c)^3-14*tan(1/2*d*x+1/2*c)-1/7/tan(1/2*
d*x+1/2*c)^7-1/tan(1/2*d*x+1/2*c)^5-8/3/tan(1/2*d*x+1/2*c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.63

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{16\,\cos\left(dx+c\right)^{9}+32\,\cos\left(dx+c\right)^{8}-40\,\cos\left(dx+c\right)^{7}-112\,\cos\left(dx+c\right)^{6}+14\,\cos\left(dx+c\right)^{5}+140\,\cos\left(\right.}{693\,\left(a^{2}d\cos\left(dx+c\right)^{8}+2\,a^{2}d\cos\left(dx+c\right)^{7}-2\,a^{2}d\cos\left(dx+c\right)^{6}-6\,a^{2}d\cos\left(dx+c\right)^{5}+6\,a^{2}d\cos\left(dx+\right.\right.}$$

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/693*(16*cos(d*x + c)^9 + 32*cos(d*x + c)^8 - 40*cos(d*x + c)^7 - 112*cos(
d*x + c)^6 + 14*cos(d*x + c)^5 + 140*cos(d*x + c)^4 + 35*cos(d*x + c)^3 - 7
```

```
0*cos(d*x + c)^2 + 56*cos(d*x + c) + 28)/((a^2*d*cos(d*x + c)^8 + 2*a^2*d*c
os(d*x + c)^7 - 2*a^2*d*cos(d*x + c)^6 - 6*a^2*d*cos(d*x + c)^5 + 6*a^2*d*c
os(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(
d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.39

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\frac{9702\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{3234\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{792\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{385\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{63\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^2}+\frac{33\left(\frac{21\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{56\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)(\cos(dx+c)+1}{a^2\sin(dx+c)^7}}{354816\,d}$$

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/354816*((9702*sin(d*x + c)/(cos(d*x + c) + 1) + 3234*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 - 792*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 385*sin(d*x +
c)^9/(cos(d*x + c) + 1)^9 - 63*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^2 +
 33*(21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 56*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 3)*(cos(d*x + c) + 1)^7/(a^2*sin(d*x + c)^7))/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{33\left(56\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}-\frac{63\,a^{20}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+385\,a^{20}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+792\,a^{20}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-3234\,a^{20}\tan}{a^{22}}}{354816\,d}$$

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/354816*(33*(56*tan(1/2*d*x + 1/2*c)^4 + 21*tan(1/2*d*x + 1/2*c)^2 + 3)/(
a^2*tan(1/2*d*x + 1/2*c)^7) - (63*a^20*tan(1/2*d*x + 1/2*c)^11 + 385*a^20*t
an(1/2*d*x + 1/2*c)^9 + 792*a^20*tan(1/2*d*x + 1/2*c)^7 - 3234*a^20*tan(1/2
*d*x + 1/2*c)^3 - 9702*a^20*tan(1/2*d*x + 1/2*c))/a^22)/d
```

Mupad [B] (verification not implemented)

Time = 14.01 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.61

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{99\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+693\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1848\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+9702\cos\left(\frac{c}{2}\right.}{}$$

```
[In] int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(99*cos(c/2 + (d*x)/2)^18 - 63*sin(c/2 + (d*x)/2)^18 - 385*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^16 - 792*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^1
4 + 3234*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^10 + 9702*cos(c/2 + (d*x)/
2)^10*sin(c/2 + (d*x)/2)^8 + 1848*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^
4 + 693*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^2)/(354816*a^2*d*cos(c/2 +
 (d*x)/2)^11*sin(c/2 + (d*x)/2)^7)
```

3.91 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3} dx = \frac{2(a-a\cos(c+dx))^6}{3a^9d} - \frac{16(a-a\cos(c+dx))^7}{7a^{10}d} + \frac{25(a-a\cos(c+dx))^8}{8a^{11}d} - \frac{19(a-a\cos(c+dx))^9}{9a^{12}d} + \frac{7(a-a\cos(c+dx))^{10}}{10a^{13}d} - \frac{(a-a\cos(c+dx))^{11}}{11a^{14}d}$$

```
[Out] 2/3*(a-a*cos(d*x+c))^6/a^9/d-16/7*(a-a*cos(d*x+c))^7/a^10/d+25/8*(a-a*cos(d
*x+c))^8/a^11/d-19/9*(a-a*cos(d*x+c))^9/a^12/d+7/10*(a-a*cos(d*x+c))^10/a^1
3/d-1/11*(a-a*cos(d*x+c))^11/a^14/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3} dx = -\frac{(a-a\cos(c+dx))^{11}}{11a^{14}d} + \frac{7(a-a\cos(c+dx))^{10}}{10a^{13}d} - \frac{19(a-a\cos(c+dx))^9}{9a^{12}d} + \frac{25(a-a\cos(c+dx))^8}{8a^{11}d} - \frac{16(a-a\cos(c+dx))^7}{7a^{10}d} + \frac{2(a-a\cos(c+dx))^6}{3a^9d}$$

```
[In] Int[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*(a - a*Cos[c + d*x])^6)/(3*a^9*d) - (16*(a - a*Cos[c + d*x])^7)/(7*a^10*
d) + (25*(a - a*Cos[c + d*x])^8)/(8*a^11*d) - (19*(a - a*Cos[c + d*x])^9)/(
9*a^12*d) + (7*(a - a*Cos[c + d*x])^10)/(10*a^13*d) - (a - a*Cos[c + d*x])^
11/(11*a^14*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin^{11}(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^5x^3(-a+x)^2}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^{11}d} \\
&= \frac{\text{Subst}\left(\int (-a-x)^5x^3(-a+x)^2\,dx, x, -a\cos(c+dx)\right)}{a^{14}d} \\
&= \frac{\text{Subst}\left(\int \left(-4a^5(-a-x)^5-16a^4(-a-x)^6-25a^3(-a-x)^7-19a^2(-a-x)^8-7a(-a-x)^9-\right.\right.}{a^{14}d} \\
&= \frac{2(a-a\cos(c+dx))^6}{3a^9d}-\frac{16(a-a\cos(c+dx))^7}{7a^{10}d}+\frac{25(a-a\cos(c+dx))^8}{8a^{11}d} \\
&\quad -\frac{19(a-a\cos(c+dx))^9}{9a^{12}d}+\frac{7(a-a\cos(c+dx))^{10}}{10a^{13}d}-\frac{(a-a\cos(c+dx))^{11}}{11a^{14}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.90 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{-1615571+2273040\cos(c+dx)-1496880\cos(2(c+dx))+535920\cos(3(c+dx))+110880\cos(4(c+dx\ldots}{}$$

```
[In] Integrate[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-1615571 + 2273040*Cos[c + d*x] - 1496880*Cos[2*(c + d*x)] + 535920*Cos[3*
(c + d*x)] + 110880*Cos[4*(c + d*x)] - 293832*Cos[5*(c + d*x)] + 212520*Cos
[6*(c + d*x)] - 67320*Cos[7*(c + d*x)] - 27720*Cos[8*(c + d*x)] + 40040*Cos
[9*(c + d*x)] - 16632*Cos[10*(c + d*x)] + 2520*Cos[11*(c + d*x)])/(28385280
*a^3*d)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^{11}}{11}-\frac{3\cos(dx+c)^{10}}{10}+\frac{\cos(dx+c)^{9}}{9}+\frac{5\cos(dx+c)^{8}}{8}-\frac{5\cos(dx+c)^{7}}{7}-\frac{\cos(dx+c)^{6}}{6}+\frac{3\cos(dx+c)^{5}}{5}-\frac{\cos(dx+c)^{4}}{4}}{d\,a^3}$
default	$\frac{\frac{\cos(dx+c)^{11}}{11}-\frac{3\cos(dx+c)^{10}}{10}+\frac{\cos(dx+c)^{9}}{9}+\frac{5\cos(dx+c)^{8}}{8}-\frac{5\cos(dx+c)^{7}}{7}-\frac{\cos(dx+c)^{6}}{6}+\frac{3\cos(dx+c)^{5}}{5}-\frac{\cos(dx+c)^{4}}{4}}{d\,a^3}$
parallelrisch	$\frac{13860\cos(4dx+4c)+66990\cos(3dx+3c)+315\cos(11dx+11c)+284130\cos(dx+c)-3465\cos(8dx+8c)-8415\cos(7dx+7c)+\ldots}{3548160a^{\ldots}}$
risch	$\frac{41\cos(dx+c)}{512a^3d}+\frac{\cos(11dx+11c)}{11264d\,a^3}-\frac{3\cos(10dx+10c)}{5120d\,a^3}+\frac{13\cos(9dx+9c)}{9216d\,a^3}-\frac{\cos(8dx+8c)}{1024d\,a^3}-\frac{17\cos(7dx+7c)}{7168d\,a^3}+\frac{23\ldots}{3\ldots}$

```
[In] int(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/11*cos(d*x+c)^11-3/10*cos(d*x+c)^10+1/9*cos(d*x+c)^9+5/8*cos(d*x
+c)^8-5/7*cos(d*x+c)^7-1/6*cos(d*x+c)^6+3/5*cos(d*x+c)^5-1/4*cos(d*x+c)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2520\cos(dx+c)^{11}-8316\cos(dx+c)^{10}+3080\cos(dx+c)^{9}+17325\cos(dx+c)^{8}-19800\cos(dx+c)\ldots}{27720\,a^3d}$$

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/27720*(2520*cos(d*x + c)^11 - 8316*cos(d*x + c)^10 + 3080*cos(d*x + c)^9
+ 17325*cos(d*x + c)^8 - 19800*cos(d*x + c)^7 - 4620*cos(d*x + c)^6 + 16632
*cos(d*x + c)^5 - 6930*cos(d*x + c)^4)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**11/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2520\cos(dx+c)^{11} - 8316\cos(dx+c)^{10} + 3080\cos(dx+c)^{9} + 17325\cos(dx+c)^{8} - 19800\cos(dx+c)^{\cdots}}{27720\,a^3 d}$$

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/27720*(2520*cos(d*x + c)^11 - 8316*cos(d*x + c)^10 + 3080*cos(d*x + c)^9
+ 17325*cos(d*x + c)^8 - 19800*cos(d*x + c)^7 - 4620*cos(d*x + c)^6 + 16632
*cos(d*x + c)^5 - 6930*cos(d*x + c)^4)/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.49

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{32\left(\frac{209(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{1045(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{3135(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{6270(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{8778(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - \frac{13398(\cos\cdots}{(\cos(d\cdots}\cdots\right)}{3465\,a^3 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{11}}$$

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 32/3465*(209*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1045*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 3135*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 -
 6270*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 8778*(cos(d*x + c) - 1)^5
/(cos(d*x + c) + 1)^5 - 13398*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 2
310*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 9240*(cos(d*x + c) - 1)^8/(
cos(d*x + c) + 1)^8 - 19)/(a^3*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1
)^11)
```

Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\cos(c+d\,x)^4}{4\,a^3}-\frac{3\cos(c+d\,x)^5}{5\,a^3}+\frac{\cos(c+d\,x)^6}{6\,a^3}+\frac{5\cos(c+d\,x)^7}{7\,a^3}-\frac{5\cos(c+d\,x)^8}{8\,a^3}-\frac{\cos(c+d\,x)^9}{9\,a^3}+\frac{3\cos(c+d\,x)^{10}}{10\,a^3}-\frac{\cos(c+d\,x)^{11}}{11\,a^3}}{d}$$

```
[In] int(sin(c + d*x)^11/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(6*a
^3) + (5*cos(c + d*x)^7)/(7*a^3) - (5*cos(c + d*x)^8)/(8*a^3) - cos(c + d*x
)^9/(9*a^3) + (3*cos(c + d*x)^10)/(10*a^3) - cos(c + d*x)^11/(11*a^3))/d
```

3.92 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^4(c+dx)}{4a^3d} + \frac{3\cos^5(c+dx)}{5a^3d} - \frac{\cos^6(c+dx)}{3a^3d} - \frac{2\cos^7(c+dx)}{7a^3d} + \frac{3\cos^8(c+dx)}{8a^3d} - \frac{\cos^9(c+dx)}{9a^3d}$$

```
[Out] -1/4*cos(d*x+c)^4/a^3/d+3/5*cos(d*x+c)^5/a^3/d-1/3*cos(d*x+c)^6/a^3/d-2/7*c
os(d*x+c)^7/a^3/d+3/8*cos(d*x+c)^8/a^3/d-1/9*cos(d*x+c)^9/a^3/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 76}

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^9(c+dx)}{9a^3d} + \frac{3\cos^8(c+dx)}{8a^3d} - \frac{2\cos^7(c+dx)}{7a^3d} - \frac{\cos^6(c+dx)}{3a^3d} + \frac{3\cos^5(c+dx)}{5a^3d} - \frac{\cos^4(c+dx)}{4a^3d}$$

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/4*Cos[c + d*x]^4/(a^3*d) + (3*Cos[c + d*x]^5)/(5*a^3*d) - Cos[c + d*x]^6
/(3*a^3*d) - (2*Cos[c + d*x]^7)/(7*a^3*d) + (3*Cos[c + d*x]^8)/(8*a^3*d) -
Cos[c + d*x]^9/(9*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin^9(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^4x^3(-a+x)}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^9d} \\
&= \frac{\text{Subst}\left(\int (-a-x)^4x^3(-a+x)\,dx, x, -a\cos(c+dx)\right)}{a^{12}d} \\
&= \frac{\text{Subst}\left(\int \left(-a^5x^3-3a^4x^4-2a^3x^5+2a^2x^6+3ax^7+x^8\right)\,dx, x, -a\cos(c+dx)\right)}{a^{12}d} \\
&= -\frac{\cos^4(c+dx)}{4a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^6(c+dx)}{3a^3d} \\
&\quad -\frac{2\cos^7(c+dx)}{7a^3d}+\frac{3\cos^8(c+dx)}{8a^3d}-\frac{\cos^9(c+dx)}{9a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{34771 - 52920\cos(c+dx) + 37800\cos(2(c+dx)) - 18480\cos(3(c+dx)) + 3780\cos(4(c+dx)) + 302}{3225}$$

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/322560*(34771 - 52920*Cos[c + d*x] + 37800*Cos[2*(c + d*x)] - 18480*Cos[
3*(c + d*x)] + 3780*Cos[4*(c + d*x)] + 3024*Cos[5*(c + d*x)] - 4200*Cos[6*(
c + d*x)] + 2700*Cos[7*(c + d*x)] - 945*Cos[8*(c + d*x)] + 140*Cos[9*(c + d
*x)])/(a^3*d)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{3\cos(dx+c)^8}{8}-\frac{2\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{3}+\frac{3\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^4}{4}}{d\,a^3}$
default	$\frac{-\frac{\cos(dx+c)^9}{9}+\frac{3\cos(dx+c)^8}{8}-\frac{2\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{3}+\frac{3\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^4}{4}}{d\,a^3}$
parallelrisch	$\frac{-3780\cos(4dx+4c)+18480\cos(3dx+3c)+52920\cos(dx+c)+945\cos(8dx+8c)-2700\cos(7dx+7c)+4200\cos(6dx+6c)-}{322560a^3d}$
risch	$\frac{21\cos(dx+c)}{128a^3d}-\frac{\cos(9dx+9c)}{2304d\,a^3}+\frac{3\cos(8dx+8c)}{1024d\,a^3}-\frac{15\cos(7dx+7c)}{1792d\,a^3}+\frac{5\cos(6dx+6c)}{384d\,a^3}-\frac{3\cos(5dx+5c)}{320d\,a^3}-\frac{3\cos(4}{256}$
norman	$\frac{\frac{128}{315ad}+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{ad}+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{3ad}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{da}+\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{35da}+\frac{512\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{35da}+\frac{512\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{15da}+}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^9 a^2}$

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/9*cos(d*x+c)^9+3/8*cos(d*x+c)^8-2/7*cos(d*x+c)^7-1/3*cos(d*x+c)
^6+3/5*cos(d*x+c)^5-1/4*cos(d*x+c)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{280\,\cos\left(dx+c\right)^9-945\,\cos\left(dx+c\right)^8+720\,\cos\left(dx+c\right)^7+840\,\cos\left(dx+c\right)^6-1512\,\cos\left(dx+c\right)^5+63}{2520\,a^3d}$$

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
[Out] -1/2520*(280*cos(d*x + c)^9 - 945*cos(d*x + c)^8 + 720*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 1512*cos(d*x + c)^5 + 630*cos(d*x + c)^4)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c))**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{280\,\cos\left(dx+c\right)^9-945\,\cos\left(dx+c\right)^8+720\,\cos\left(dx+c\right)^7+840\,\cos\left(dx+c\right)^6-1512\,\cos\left(dx+c\right)^5+63}{2520\,a^3d}$$

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2520*(280*cos(d*x + c)^9 - 945*cos(d*x + c)^8 + 720*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 1512*cos(d*x + c)^5 + 630*cos(d*x + c)^4)/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.70

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{32\left(\frac{36(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{144(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{336(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{504(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{630(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - \frac{105(\cos(dx+c)-}{(\cos(dx+c)+1}\right.}{315\,a^3d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9}$$

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 32/315*(36*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 144*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 336*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 504
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 630*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 - 105*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 315*(cos(
d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 4)/(a^3*d*((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.77

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\cos(c+d\,x)^4}{4\,a^3} - \frac{3\cos(c+d\,x)^5}{5\,a^3} + \frac{\cos(c+d\,x)^6}{3\,a^3} + \frac{2\cos(c+d\,x)^7}{7\,a^3} - \frac{3\cos(c+d\,x)^8}{8\,a^3} + \frac{\cos(c+d\,x)^9}{9\,a^3}}{d}$$

```
[In] int(sin(c + d*x)^9/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(3*a
^3) + (2*cos(c + d*x)^7)/(7*a^3) - (3*cos(c + d*x)^8)/(8*a^3) + cos(c + d*x
)^9/(9*a^3))/d
```

3.93 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3} \, dx = -\frac{\cos^4(c+dx)}{4a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^6(c+dx)}{2a^3d}+\frac{\cos^7(c+dx)}{7a^3d}$$

```
[Out] -1/4*cos(d*x+c)^4/a^3/d+3/5*cos(d*x+c)^5/a^3/d-1/2*cos(d*x+c)^6/a^3/d+1/7*c
os(d*x+c)^7/a^3/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 45}

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3} \, dx = \frac{\cos^7(c+dx)}{7a^3d}-\frac{\cos^6(c+dx)}{2a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^4(c+dx)}{4a^3d}$$

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/4*Cos[c + d*x]^4/(a^3*d) + (3*Cos[c + d*x]^5)/(5*a^3*d) - Cos[c + d*x]^6
/(2*a^3*d) + Cos[c + d*x]^7/(7*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin^7(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^3x^3}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^7d} \\
&= \frac{\text{Subst}\left(\int (-a-x)^3x^3\,dx, x, -a\cos(c+dx)\right)}{a^{10}d} \\
&= \frac{\text{Subst}\left(\int \left(-a^3x^3-3a^2x^4-3ax^5-x^6\right)\,dx, x, -a\cos(c+dx)\right)}{a^{10}d} \\
&= -\frac{\cos^4(c+dx)}{4a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^6(c+dx)}{2a^3d}+\frac{\cos^7(c+dx)}{7a^3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{-2421+4060\cos(c+dx)-3220\cos(2(c+dx))+2100\cos(3(c+dx))-1120\cos(4(c+dx))+476\cos(5}{8960a^3d}$$

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2421 + 4060*Cos[c + d*x] - 3220*Cos[2*(c + d*x)] + 2100*Cos[3*(c + d*x)]
- 1120*Cos[4*(c + d*x)] + 476*Cos[5*(c + d*x)] - 140*Cos[6*(c + d*x)] + 20*
Cos[7*(c + d*x)])/(8960*a^3*d)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{2}+\frac{3\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^4}{4}}{d\,a^3}$
default	$\frac{\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^6}{2}+\frac{3\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^4}{4}}{d\,a^3}$
parallelrisch	$\frac{-805\cos(2dx+2c)+2784-280\cos(4dx+4c)-35\cos(6dx+6c)+1015\cos(dx+c)+525\cos(3dx+3c)+119\cos(5dx+5c)+5\cos}{2240a^3d}$
risch	$\frac{29\cos(dx+c)}{64a^3d}+\frac{\cos(7dx+7c)}{448d\,a^3}-\frac{\cos(6dx+6c)}{64d\,a^3}+\frac{17\cos(5dx+5c)}{320d\,a^3}-\frac{\cos(4dx+4c)}{8d\,a^3}+\frac{15\cos(3dx+3c)}{64d\,a^3}-\frac{23\cos(2dx}{64d\,a^3}$
norman	$\frac{\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{ad}+\frac{52\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{da}+\frac{52}{35ad}+\frac{56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{da}+\frac{24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{da}+\frac{52\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{5da}+\frac{156\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{5da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{7}a^{2}}$

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(1/7*cos(d*x+c)^7-1/2*cos(d*x+c)^6+3/5*cos(d*x+c)^5-1/4*cos(d*x+c)^
4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{20\cos(dx+c)^7-70\cos(dx+c)^6+84\cos(dx+c)^5-35\cos(dx+c)^4}{140\,a^3d}$$

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
[Out] 1/140*(20*cos(d*x + c)^7 - 70*cos(d*x + c)^6 + 84*cos(d*x + c)^5 - 35*cos(d
*x + c)^4)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c))**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{20\cos(dx+c)^7-70\cos(dx+c)^6+84\cos(dx+c)^5-35\cos(dx+c)^4}{140\,a^3d}$$

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/140*(20*cos(d*x + c)^7 - 70*cos(d*x + c)^6 + 84*cos(d*x + c)^5 - 35*cos(d
*x + c)^4)/(a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(65) = 130$.

Time = 0.39 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.23

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{4\left(\frac{91(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{273(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{455(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{490(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{210(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-\frac{140(\cos(dx+c)-1\cdots}{(\cos(dx+c)+1)^6}\cdots\right.}{35\,a^3d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}$$

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 4/35*(91*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 273*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 455*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 490*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 210*(cos(d*x + c) - 1)^5/(cos(d*
x + c) + 1)^5 - 140*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 13)/(a^3*d*
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)
```

Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\cos(c+d\,x)^4}{4\,a^3}-\frac{3\cos(c+d\,x)^5}{5\,a^3}+\frac{\cos(c+d\,x)^6}{2\,a^3}-\frac{\cos(c+d\,x)^7}{7\,a^3}}{d}$$

```
[In] int(sin(c + d*x)^7/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(2*a
^3) - cos(c + d*x)^7/(7*a^3))/d
```

3.94 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{4\cos(c+dx)}{a^3d}+\frac{2\cos^2(c+dx)}{a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{3\cos^4(c+dx)}{4a^3d}-\frac{\cos^5(c+dx)}{5a^3d}+\frac{4\log(1+\cos(c+dx))}{a^3d}$$

```
[Out] -4*cos(d*x+c)/a^3/d+2*cos(d*x+c)^2/a^3/d-4/3*cos(d*x+c)^3/a^3/d+3/4*cos(d*x
+c)^4/a^3/d-1/5*cos(d*x+c)^5/a^3/d+4*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^5(c+dx)}{5a^3d}+\frac{3\cos^4(c+dx)}{4a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{2\cos^2(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{4\log(\cos(c+dx)+1)}{a^3d}$$

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-4*Cos[c + d*x])/(a^3*d) + (2*Cos[c + d*x]^2)/(a^3*d) - (4*Cos[c + d*x]^3)
/(3*a^3*d) + (3*Cos[c + d*x]^4)/(4*a^3*d) - Cos[c + d*x]^5/(5*a^3*d) + (4*L
og[1 + Cos[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin^5(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^3}{a^3(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{a^5d} \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^3}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^8d} \\
&= \frac{\text{Subst}\left(\int \left(4a^4-\frac{4a^5}{a-x}+4a^3x+4a^2x^2+3ax^3+x^4\right)\,dx, x, -a\cos(c+dx)\right)}{a^8d} \\
&= -\frac{4\cos(c+dx)}{a^3d}+\frac{2\cos^2(c+dx)}{a^3d}-\frac{4\cos^3(c+dx)}{3a^3d} \\
&\quad+\frac{3\cos^4(c+dx)}{4a^3d}-\frac{\cos^5(c+dx)}{5a^3d}+\frac{4\log(1+\cos(c+dx))}{a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{3857-4920\cos(c+dx)+1320\cos(2(c+dx))-380\cos(3(c+dx))+90\cos(4(c+dx))-12\cos(5(c+dx)}{960a^3d}$$

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3857 - 4920*Cos[c + d*x] + 1320*Cos[2*(c + d*x)] - 380*Cos[3*(c + d*x)] +
90*Cos[4*(c + d*x)] - 12*Cos[5*(c + d*x)] + 7680*Log[Cos[(c + d*x)/2]])/(96
0*a^3*d)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4}{4}-\frac{4\cos(dx+c)^3}{3}+2\cos(dx+c)^2-4\cos(dx+c)+4\ln(\cos(dx+c)+1)}{d\,a^3}$
default	$\frac{-\frac{\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4}{4}-\frac{4\cos(dx+c)^3}{3}+2\cos(dx+c)^2-4\cos(dx+c)+4\ln(\cos(dx+c)+1)}{d\,a^3}$
parallelrisch	$\frac{-1920\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-3361-6\cos(5dx+5c)+45\cos(4dx+4c)-190\cos(3dx+3c)+660\cos(2dx+2c)-2460\cos(dx+c)}{480a^3d}$
norman	$\frac{-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{da}-\frac{166}{15ad}-\frac{78\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{da}-\frac{154\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3da}-\frac{278\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5a^2}-\frac{4\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^3d}$
risch	$-\frac{4ix}{a^3}-\frac{41\,\mathrm{e}^{i(dx+c)}}{16a^3d}-\frac{41\,\mathrm{e}^{-i(dx+c)}}{16a^3d}-\frac{8ic}{a^3d}+\frac{8\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{a^3d}-\frac{\cos(5dx+5c)}{80d\,a^3}+\frac{3\cos(4dx+4c)}{32d\,a^3}-\frac{19\cos(3dx}{48d\,a^3}$

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/5*cos(d*x+c)^5+3/4*cos(d*x+c)^4-4/3*cos(d*x+c)^3+2*cos(d*x+c)^2
-4*cos(d*x+c)+4*ln(cos(d*x+c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{12\,\cos\left(dx+c\right)^5 - 45\,\cos\left(dx+c\right)^4 + 80\,\cos\left(dx+c\right)^3 - 120\,\cos\left(dx+c\right)^2 + 240\,\cos\left(dx+c\right) - 240\,\mathrm{log}}{60\,a^3 d}$$

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 45*cos(d*x + c)^4 + 80*cos(d*x + c)^3 - 120*cos(
d*x + c)^2 + 240*cos(d*x + c) - 240*log(1/2*cos(d*x + c) + 1/2))/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{12\,\cos(dx+c)^5 - 45\,\cos(dx+c)^4 + 80\,\cos(dx+c)^3 - 120\,\cos(dx+c)^2 + 240\,\cos(dx+c)}{a^3} - \frac{240\,\log(\cos(dx+c)+1)}{a^3}}{60\,d}$$

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((12*cos(d*x + c)^5 - 45*cos(d*x + c)^4 + 80*cos(d*x + c)^3 - 120*cos
(d*x + c)^2 + 240*cos(d*x + c))/a^3 - 240*log(cos(d*x + c) + 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.69

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{\frac{85(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{20(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{200(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{205(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{137(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-29}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{15\,d}$$

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/15*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (85*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 20*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - 200*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 205*(cos(d*x + c)
 - 1)^4/(cos(d*x + c) + 1)^4 - 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^
5 - 29)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{4\ln(\cos(c+dx)+1)}{a^3}-\frac{4\cos(c+dx)}{a^3}+\frac{2\cos(c+dx)^2}{a^3}-\frac{4\cos(c+dx)^3}{3a^3}+\frac{3\cos(c+dx)^4}{4a^3}-\frac{\cos(c+dx)^5}{5a^3}}{d}$$

```
[In] int(sin(c + d*x)^5/(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((4*log(cos(c + d*x) + 1))/a^3 - (4*cos(c + d*x))/a^3 + (2*cos(c + d*x)^2)/
a^3 - (4*cos(c + d*x)^3)/(3*a^3) + (3*cos(c + d*x)^4)/(4*a^3) - cos(c + d*x
)^5/(5*a^3))/d
```

3.95 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{5\cos(c+dx)}{a^3d} - \frac{3\cos^2(c+dx)}{2a^3d} + \frac{\cos^3(c+dx)}{3a^3d} - \frac{2}{d\,(a^3+a^3\cos(c+dx))} - \frac{7\log(1+\cos(c+dx))}{a^3d}$$

```
[Out] 5*cos(d*x+c)/a^3/d-3/2*cos(d*x+c)^2/a^3/d+1/3*cos(d*x+c)^3/a^3/d-2/d/(a^3+a
^3*cos(d*x+c))-7*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 78}

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos^3(c+dx)}{3a^3d} - \frac{3\cos^2(c+dx)}{2a^3d} + \frac{5\cos(c+dx)}{a^3d} - \frac{2}{d\,(a^3\cos(c+dx)+a^3)} - \frac{7\log(\cos(c+dx)+1)}{a^3d}$$

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (5*Cos[c + d*x])/(a^3*d) - (3*Cos[c + d*x]^2)/(2*a^3*d) + Cos[c + d*x]^3/(3
*a^3*d) - 2/(d*(a^3 + a^3*Cos[c + d*x])) - (7*Log[1 + Cos[c + d*x]])/(a^3*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)x^3}{a^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)x^3}{(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^6d}\\
&= \frac{\text{Subst}\left(\int \left(-5a^2-\frac{2a^4}{(a-x)^2}+\frac{7a^3}{a-x}-3ax-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{a^6d}\\
&= \frac{5\cos(c+dx)}{a^3d}-\frac{3\cos^2(c+dx)}{2a^3d}+\frac{\cos^3(c+dx)}{3a^3d}\\
&\quad -\frac{2}{d\left(a^3+a^3\cos(c+dx)\right)}-\frac{7\log(1+\cos(c+dx))}{a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(389-184\cos(2(c+dx))+28\cos(3(c+dx))-4\cos(4(c+dx))+1344\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.}{24a^3d(1+\cos(c+dx))^3}$$

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/24*(Cos[(c + d*x)/2]^4*(389 - 184*Cos[2*(c + d*x)] + 28*Cos[3*(c + d*x)]
 - 4*Cos[4*(c + d*x)] + 1344*Log[Cos[(c + d*x)/2]] + Cos[c + d*x]*(-19 + 13
44*Log[Cos[(c + d*x)/2]])))/(a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^3}{3}-\frac{3\cos(dx+c)^2}{2}+5\cos(dx+c)-7\ln(\cos(dx+c)+1)-\frac{2}{\cos(dx+c)+1}}{d\,a^3}$
default	$\frac{\frac{\cos(dx+c)^3}{3}-\frac{3\cos(dx+c)^2}{2}+5\cos(dx+c)-7\ln(\cos(dx+c)+1)-\frac{2}{\cos(dx+c)+1}}{d\,a^3}$
parallelrisch	$\frac{-12\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2+63\cos(dx+c)+\cos(3dx+3c)-9\cos(2dx+2c)+84\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+121}{12a^3d}$
norman	$\frac{\frac{34\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{da}+\frac{41}{3ad}+\frac{24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{da}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3a^2}+\frac{7\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^3d}$
risch	$\frac{7ix}{a^3}+\frac{21\,e^{i(dx+c)}}{8a^3d}+\frac{21\,e^{-i(dx+c)}}{8a^3d}+\frac{14ic}{a^3d}-\frac{4\,e^{i(dx+c)}}{a^3d\left(e^{i(dx+c)}+1\right)^2}-\frac{14\ln\left(e^{i(dx+c)}+1\right)}{a^3d}+\frac{\cos(3dx+3c)}{12d\,a^3}-\frac{3\cos(2}{4}$

```
[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/3*cos(d*x+c)^3-3/2*cos(d*x+c)^2+5*cos(d*x+c)-7*ln(cos(d*x+c)+1)-
2/(cos(d*x+c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{4\cos(dx+c)^4 - 14\cos(dx+c)^3 + 42\cos(dx+c)^2 - 84(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + 69}{12\,(a^3 d\cos(dx+c) + a^3 d)}$$

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(d*x + c)^4 - 14*cos(d*x + c)^3 + 42*cos(d*x + c)^2 - 84*(cos(d*
x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 69*cos(d*x + c) - 15)/(a^3*d*cos(
d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{12}{a^3\cos(dx+c)+a^3} - \frac{2\cos(dx+c)^3 - 9\cos(dx+c)^2 + 30\cos(dx+c)}{a^3} + \frac{42\log(\cos(dx+c)+1)}{a^3}}{6\,d}$$

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(12/(a^3*cos(d*x + c) + a^3) - (2*cos(d*x + c)^3 - 9*cos(d*x + c)^2 +
30*cos(d*x + c))/a^3 + 42*log(cos(d*x + c) + 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{7\,\log\left(|-\cos\left(dx+c\right)-1|\right)}{a^3d} - \frac{2}{a^3d(\cos\left(dx+c\right)+1)}$$
$$+ \frac{2\,a^6d^5\cos\left(dx+c\right)^3 - 9\,a^6d^5\cos\left(dx+c\right)^2 + 30\,a^6d^5\cos\left(dx+c\right)}{6\,a^9d^6}$$

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -7*log(abs(-cos(d*x + c) - 1))/(a^3*d) - 2/(a^3*d*(cos(d*x + c) + 1)) + 1/6
*(2*a^6*d^5*cos(d*x + c)^3 - 9*a^6*d^5*cos(d*x + c)^2 + 30*a^6*d^5*cos(d*x
+ c))/(a^9*d^6)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{\frac{2}{a^3\,\cos(c+d\,x)+a^3} + \frac{7\,\ln(\cos(c+d\,x)+1)}{a^3} - \frac{5\,\cos(c+d\,x)}{a^3} + \frac{3\cos(c+d\,x)^2}{2\,a^3} - \frac{\cos(c+d\,x)^3}{3\,a^3}}{d}$$

```
[In] int(sin(c + d*x)^3/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(2/(a^3*cos(c + d*x) + a^3) + (7*log(cos(c + d*x) + 1))/a^3 - (5*cos(c + d
*x))/a^3 + (3*cos(c + d*x)^2)/(2*a^3) - cos(c + d*x)^3/(3*a^3))/d
```

3.96 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3} \, dx = -\frac{\cos(c+dx)}{a^3d} - \frac{1}{2ad(a+a\cos(c+dx))^2} + \frac{3}{d(a^3+a^3\cos(c+dx))} + \frac{3\log(1+\cos(c+dx))}{a^3d}$$

```
[Out] -cos(d*x+c)/a^3/d-1/2/a/d/(a+a*cos(d*x+c))^2+3/d/(a^3+a^3*cos(d*x+c))+3*ln(
1+cos(d*x+c))/a^3/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2912, 12, 45}

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3} \, dx = -\frac{\cos(c+dx)}{a^3d} + \frac{3}{d(a^3\cos(c+dx)+a^3)} + \frac{3\log(\cos(c+dx)+1)}{a^3d} - \frac{1}{2ad(a\cos(c+dx)+a)^2}$$

```
[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]/(a^3*d)) - 1/(2*a*d*(a + a*Cos[c + d*x])^2) + 3/(d*(a^3 + a^
3*Cos[c + d*x])) + (3*Log[1 + Cos[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^3}{a^3(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \frac{x^3}{(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^4d} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{a^3}{(a-x)^3}+\frac{3a^2}{(a-x)^2}-\frac{3a}{a-x}\right)\,dx, x, -a\cos(c+dx)\right)}{a^4d} \\
&= -\frac{\cos(c+dx)}{a^3d}-\frac{1}{2ad(a+a\cos(c+dx))^2}+\frac{3}{d\,(a^3+a^3\cos(c+dx))}+\frac{3\log(1+\cos(c+dx))}{a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.37

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(21-2\cos(3(c+dx))+72\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\cos(2(c+dx))\left(-5+24\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.\right.}{4a^3d(1+\cos(c+dx))^3}$$

[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x])^3,x]

[Out] (Cos[(c + d*x)/2]^2*(21 - 2*Cos[3*(c + d*x)] + 72*Log[Cos[(c + d*x)/2]] + C
os[2*(c + d*x)]*(-5 + 24*Log[Cos[(c + d*x)/2]]) + Cos[c + d*x]*(22 + 96*Log
[Cos[(c + d*x)/2]])))/(4*a^3*d*(1 + Cos[c + d*x])^3)

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{-29-24\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^4-8\cos(dx+c)+12\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^3d}$	59
derivativedivides	$\frac{-\frac{1}{2(1+\sec(dx+c))^2}-\frac{2}{1+\sec(dx+c)}+3\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-3\ln(\sec(dx+c))}{d\,a^3}$	63
default	$\frac{-\frac{1}{2(1+\sec(dx+c))^2}-\frac{2}{1+\sec(dx+c)}+3\ln(1+\sec(dx+c))-\frac{1}{\sec(dx+c)}-3\ln(\sec(dx+c))}{d\,a^3}$	63
norman	$\frac{\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8da}-\frac{13}{4ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a^2}-\frac{3\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^3d}$	90
risch	$-\frac{3ix}{a^3}-\frac{e^{i(dx+c)}}{2a^3d}-\frac{e^{-i(dx+c)}}{2a^3d}-\frac{6ic}{a^3d}+\frac{6\,e^{3i(dx+c)}+10\,e^{2i(dx+c)}+6\,e^{i(dx+c)}}{a^3d\left(e^{i(dx+c)}+1\right)^4}+\frac{6\ln\left(e^{i(dx+c)}+1\right)}{a^3d}$	128

[In] int(sin(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/8*(-29-24*ln(sec(1/2*d*x+1/2*c)^2)-sec(1/2*d*x+1/2*c)^4-8*cos(d*x+c)+12*s
ec(1/2*d*x+1/2*c)^2)/a^3/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{2\cos(dx+c)^3+4\cos(dx+c)^2-6\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-4\cos(dx+c)-5}{2\left(a^3d\cos(dx+c)^2+2a^3d\cos(dx+c)+a^3d\right)}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - 6*(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 4*cos(d*x + c) - 5)/(a^3*d*cos(d*x +
 c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sin(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{6\cos(dx+c)+5}{a^3\cos(dx+c)^2+2a^3\cos(dx+c)+a^3}-\frac{2\cos(dx+c)}{a^3}+\frac{6\log(\cos(dx+c)+1)}{a^3}}{2d}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((6*cos(d*x + c) + 5)/(a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) + a^3) -
 2*cos(d*x + c)/a^3 + 6*log(cos(d*x + c) + 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos\left(dx+c\right)}{a^3d} + \frac{3\,\log\left(\left|-\cos\left(dx+c\right)-1\right|\right)}{a^3d} + \frac{6\,\cos\left(dx+c\right)+5}{2\,a^3d(\cos\left(dx+c\right)+1)^2}$$

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^3*d) + 3*log(abs(-cos(d*x + c) - 1))/(a^3*d) + 1/2*(6*cos(
d*x + c) + 5)/(a^3*d*(cos(d*x + c) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{a^3\,d} - \frac{\cos\left(c+d\,x\right)}{a^3\,d} + \frac{3\,\cos\left(c+d\,x\right)+\frac{5}{2}}{a^3\,d\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}$$

```
[In] int(sin(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (3*log(cos(c + d*x) + 1))/(a^3*d) - cos(c + d*x)/(a^3*d) + (3*cos(c + d*x)
+ 5/2)/(a^3*d*(cos(c + d*x) + 1)^2)
```

3.97 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3} dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{8a^3d} - \frac{1}{6d(a+a\cos(c+dx))^3} + \frac{5}{8ad(a+a\cos(c+dx))^2} - \frac{7}{8d(a^3+a^3\cos(c+dx))}$$

```
[Out] -1/8*arctanh(cos(d*x+c))/a^3/d-1/6/d/(a+a*cos(d*x+c))^3+5/8/a/d/(a+a*cos(d*
x+c))^2-7/8/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3957, 2915, 12, 90, 212}

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3} dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{8a^3d} - \frac{7}{8d(a^3\cos(c+dx)+a^3)} + \frac{5}{8ad(a\cos(c+dx)+a)^2} - \frac{1}{6d(a\cos(c+dx)+a)^3}$$

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/8*ArcTanh[Cos[c + d*x]]/(a^3*d) - 1/(6*d*(a + a*Cos[c + d*x])^3) + 5/(8*
a*d*(a + a*Cos[c + d*x])^2) - 7/(8*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^2(c+dx)\cot(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= \frac{a\text{Subst}\left(\int \frac{x^3}{a^3(-a-x)(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x^3}{(-a-x)(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{a^2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a^2}{2(a-x)^4}+\frac{5a}{4(a-x)^3}-\frac{7}{8(a-x)^2}+\frac{1}{8(a^2-x^2)}\right)dx, x, -a\cos(c+dx)\right)}{a^2d} \\
&= -\frac{1}{6d(a+a\cos(c+dx))^3}+\frac{5}{8ad(a+a\cos(c+dx))^2} \\
&\quad -\frac{7}{8d\left(a^3+a^3\cos(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, -a\cos(c+dx)\right)}{8a^2d}
\end{aligned}
$$

$$
= -\frac{\operatorname{arctanh}(\cos(c+dx))}{8a^3d} - \frac{1}{6d(a+a\cos(c+dx))^3} + \frac{5}{8ad(a+a\cos(c+dx))^2} - \frac{7}{8d\left(a^3+a^3\cos(c+dx)\right)}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$
\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\left(2-15\cos^2\left(\frac{1}{2}(c+dx)\right)+42\cos^4\left(\frac{1}{2}(c+dx)\right)+12\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\right.\right.\right.\right.}{12a^3d(1+\sec(c+dx))^3}
$$

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/12*((2 - 15*Cos[(c + d*x)/2]^2 + 42*Cos[(c + d*x)/2]^4 + 12*Cos[(c + d*x
)/2]^6*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^3)/(a^
3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96a^3d}$	61
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{16}-\frac{1}{6(\cos(dx+c)+1)^3}+\frac{5}{8(\cos(dx+c)+1)^2}-\frac{7}{8(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{16}}{d\,a^3}$	67
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{16}-\frac{1}{6(\cos(dx+c)+1)^3}+\frac{5}{8(\cos(dx+c)+1)^2}-\frac{7}{8(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{16}}{d\,a^3}$	67
norman	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{16da}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{32da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{48da}}{a^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^3d}$	82
risch	$-\frac{21\,e^{5i(dx+c)}+54\,e^{4i(dx+c)}+82\,e^{3i(dx+c)}+54\,e^{2i(dx+c)}+21\,e^{i(dx+c)}}{12a^3d\left(e^{i(dx+c)}+1\right)^6}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{8a^3d}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{8a^3d}$	119

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(-2*tan(1/2*d*x+1/2*c)^6+9*tan(1/2*d*x+1/2*c)^4-18*tan(1/2*d*x+1/2*c)^
2+12*ln(tan(1/2*d*x+1/2*c)))/a^3/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(74) = 148$.

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{42\,\cos\left(dx+c\right)^2 + 3\,\left(\cos\left(dx+c\right)^3 + 3\,\cos\left(dx+c\right)^2 + 3\,\cos\left(dx+c\right) + 1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right) + \frac{1}{2}\right) - 3\,(}{48\,\left(a^3 d\cos\left(dx+c\right)^3 + 3\,a^3 d\cos\left(dx\right.\right.}$$

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(42*cos(d*x + c)^2 + 3*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
 + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^3 + 3*cos(d*x + c)
^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 54*cos(d*x + c) + 2
0)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
a^3*d)
```

Sympy [F]

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\left(21\,\cos(dx+c)^2+27\,\cos(dx+c)+10\right)}{a^3\,\cos(dx+c)^3+3\,a^3\,\cos(dx+c)^2+3\,a^3\,\cos(dx+c)+a^3} + \frac{3\,\log(\cos(dx+c)+1)}{a^3} - \frac{3\,\log(\cos(dx+c)-1)}{a^3}}{48\,d}$$

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(21*cos(d*x + c)^2 + 27*cos(d*x + c) + 10)/(a^3*cos(d*x + c)^3 + 3
*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) + 3*log(cos(d*x + c) + 1)/a
^3 - 3*log(cos(d*x + c) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.38

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\frac{18\,a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2\,a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^9}}{96\,d}$$

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (18*a^6*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^6*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 + 2*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^9)/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\frac{7\cos(c+d\,x)^2}{8}+\frac{9\,\cos(c+d\,x)}{8}+\frac{5}{12}}{d\,\left(a^3\cos\left(c+d\,x\right)^3+3\,a^3\cos\left(c+d\,x\right)^2+3\,a^3\,\cos\left(c+d\,x\right)+a^3\right)}\\ &\quad-\frac{\operatorname{atanh}(\cos\left(c+d\,x\right))}{8\,a^3\,d}\end{aligned}$$

```
[In] int(1/(sin(c + d*x)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] - ((9*cos(c + d*x))/8 + (7*cos(c + d*x)^2)/8 + 5/12)/(d*(3*a^3*cos(c + d*x)
 + a^3 + 3*a^3*cos(c + d*x)^2 + a^3*cos(c + d*x)^3)) - atanh(cos(c + d*x))/
(8*a^3*d)
```

3.98 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\operatorname{arctanh}(\cos(c+dx))}{32a^3d} - \frac{a}{16d(a+a\cos(c+dx))^4} + \frac{1}{6d(a+a\cos(c+dx))^3} - \frac{3}{32ad(a+a\cos(c+dx))^2} - \frac{1}{32d\,(a^3-a^3\cos(c+dx))} - \frac{1}{16d\,(a^3+a^3\cos(c+dx))}$$

```
[Out] 1/32*arctanh(cos(d*x+c))/a^3/d-1/16*a/d/(a+a*cos(d*x+c))^4+1/6/d/(a+a*cos(d
*x+c))^3-3/32/a/d/(a+a*cos(d*x+c))^2-1/32/d/(a^3-a^3*cos(d*x+c))-1/16/d/(a^
3+a^3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2786, 90, 212}

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\operatorname{arctanh}(\cos(c+dx))}{32a^3d} - \frac{1}{32d\,(a^3-a^3\cos(c+dx))} - \frac{1}{16d\,(a^3\cos(c+dx)+a^3)} - \frac{a}{16d(a\cos(c+dx)+a)^4} + \frac{1}{6d(a\cos(c+dx)+a)^3} - \frac{3}{32ad(a\cos(c+dx)+a)^2}$$

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(32*a^3*d) - a/(16*d*(a + a*Cos[c + d*x])^4) + 1/(6*d
*(a + a*Cos[c + d*x])^3) - 3/(32*a*d*(a + a*Cos[c + d*x])^2) - 1/(32*d*(a^3
 - a^3*Cos[c + d*x])) - 1/(16*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^3}{(-a-x)^2(-a+x)^5}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a}{4(a-x)^5}+\frac{1}{2(a-x)^4}-\frac{3}{16a(a-x)^3}-\frac{1}{16a^2(a-x)^2}+\frac{1}{32a^2(a+x)^2}-\frac{1}{32a^2(a^2-x^2)}\right)\,dx, x, -a\cos(c+\right.}{d} \\
&= -\frac{a}{16d(a+a\cos(c+dx))^4}+\frac{1}{6d(a+a\cos(c+dx))^3} \\
&\quad -\frac{3}{32ad(a+a\cos(c+dx))^2}-\frac{1}{32d\left(a^3-a^3\cos(c+dx)\right)} \\
&\quad -\frac{1}{16d\left(a^3+a^3\cos(c+dx)\right)}-\frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, -a\cos(c+dx)\right)}{32a^2d}
\end{aligned}
$$

$$= \frac{\operatorname{arctanh}(\cos(c+dx))}{32a^3d} - \frac{a}{16d(a+a\cos(c+dx))^4} + \frac{1}{6d(a+a\cos(c+dx))^3} - \frac{3}{32ad(a+a\cos(c+dx))^2} - \frac{1}{32d\left(a^3-a^3\cos(c+dx)\right)} - \frac{1}{16d\left(a^3+a^3\cos(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+24\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+24\sec^2\left(\frac{1}{2}\right.\right.}{96a^3d(1+\sec(c+d}$$

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/96*(Cos[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 + 24*(-Log[Cos[(c + d*x)/2
]] + Log[Sin[(c + d*x)/2]]) + 24*Sec[(c + d*x)/2]^2 + 18*Sec[(c + d*x)/2]^4
 - 16*Sec[(c + d*x)/2]^6 + 3*Sec[(c + d*x)/2]^8)*Sec[c + d*x]^3)/(a^3*d*(1
+ Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-12\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-24\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768a^3d}$
derivativedivides	$\frac{\frac{1}{32\cos(dx+c)-32}-\frac{\ln(\cos(dx+c)-1)}{64}-\frac{1}{16(\cos(dx+c)+1)^4}+\frac{1}{6(\cos(dx+c)+1)^3}-\frac{3}{32(\cos(dx+c)+1)^2}-\frac{1}{16(\cos(dx+c)+1)}+\frac{\ln(\cos(dx+c)-}{64}}{d\,a^3}$
default	$\frac{\frac{1}{32\cos(dx+c)-32}-\frac{\ln(\cos(dx+c)-1)}{64}-\frac{1}{16(\cos(dx+c)+1)^4}+\frac{1}{6(\cos(dx+c)+1)^3}-\frac{3}{32(\cos(dx+c)+1)^2}-\frac{1}{16(\cos(dx+c)+1)}+\frac{\ln(\cos(dx+c)-}{64}}{d\,a^3}$
norman	$\frac{-\frac{1}{64ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{32da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{64da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{192da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{256da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a^3d}$
risch	$-\frac{3\,e^{9i(dx+c)}+18\,e^{8i(dx+c)}-88\,e^{7i(dx+c)}-162\,e^{6i(dx+c)}-310\,e^{5i(dx+c)}-162\,e^{4i(dx+c)}-88\,e^{3i(dx+c)}+18\,e^{2i(dx+c)}+3\,e^{i(d}}{48a^3d\left(e^{i(dx+c)}+1\right)^8\left(e^{i(dx+c)}-1\right)^2}$

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/768*(-3*tan(1/2*d*x+1/2*c)^8+4*tan(1/2*d*x+1/2*c)^6+12*tan(1/2*d*x+1/2*c)
^4-24*tan(1/2*d*x+1/2*c)^2-12*cot(1/2*d*x+1/2*c)^2-24*ln(tan(1/2*d*x+1/2*c)
))/a^3/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(115) = 230$.

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.90

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{6\cos(dx+c)^4+18\cos(dx+c)^3-50\cos(dx+c)^2-3\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+\right.}{19}$$

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(6*cos(d*x + c)^4 + 18*cos(d*x + c)^3 - 50*cos(d*x + c)^2 - 3*(cos(d
*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(
d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^5 + 3*cos(d*x +
 c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*log(-1/2*
cos(d*x + c) + 1/2) - 54*cos(d*x + c) - 16)/(a^3*d*cos(d*x + c)^5 + 3*a^3*d
*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d
*cos(d*x + c) - a^3*d)
```

Sympy [F]

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\left(3\cos(dx+c)^4+9\cos(dx+c)^3-25\cos(dx+c)^2-27\cos(dx+c)-8\right)}{a^3\cos(dx+c)^5+3a^3\cos(dx+c)^4+2a^3\cos(dx+c)^3-2a^3\cos(dx+c)^2-3a^3\cos(dx+c)-a^3}-\frac{3\log(\cos(dx+c)+1)}{a^3}+\frac{3\log(\cos(dx+c)-1)}{a^3}}{192\,d}$$

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(2*(3*cos(d*x + c)^4 + 9*cos(d*x + c)^3 - 25*cos(d*x + c)^2 - 27*cos
(d*x + c) - 8)/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 2*a^3*cos(d*x +
 c)^3 - 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x + c) - a^3) - 3*log(cos(d*x +
c) + 1)/a^3 + 3*log(cos(d*x + c) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.44

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{12\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^3(\cos(dx+c)-1)} - \frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} + \frac{\frac{24a^9(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12a^9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{4a^9(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{3a^9(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}}{a^{12}}}{768\,d}$$

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(12*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
^3*(cos(d*x + c) - 1)) - 12*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1
))/a^3 + (24*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a^9*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 - 4*a^9*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 - 3*a^9*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^12)/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\mathrm{atanh}(\cos(c+d\,x))}{32\,a^3\,d} - \frac{-\frac{\cos(c+d\,x)^4}{32}-\frac{3\cos(c+d\,x)^3}{32}+\frac{25\cos(c+d\,x)^2}{96}+\frac{9\,\cos(c+d\,x)}{32}+\frac{1}{12}}{d\,\left(-a^3\cos(c+d\,x)^5-3\,a^3\cos(c+d\,x)^4-2\,a^3\cos(c+d\,x)^3+2\,a^3\cos(c+d\,x)^2+3\,a^3\,\cos(c+d\,x)+a^3\right)}$$

```
[In] int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))^3),x)
```

```
[Out] atanh(cos(c + d*x))/(32*a^3*d) - ((9*cos(c + d*x))/32 + (25*cos(c + d*x)^2)
/96 - (3*cos(c + d*x)^3)/32 - cos(c + d*x)^4/32 + 1/12)/(d*(3*a^3*cos(c + d
*x) + a^3 + 2*a^3*cos(c + d*x)^2 - 2*a^3*cos(c + d*x)^3 - 3*a^3*cos(c + d*x
)^4 - a^3*cos(c + d*x)^5))
```

3.99 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3} dx = \frac{3\text{arctanh}(\cos(c+dx))}{128a^3d} - \frac{1}{128ad(a-a\cos(c+dx))^2} - \frac{a^2}{40d(a+a\cos(c+dx))^5} + \frac{3a}{64d(a+a\cos(c+dx))^4} - \frac{1}{64ad(a+a\cos(c+dx))^2} - \frac{3}{128d(a^3+a^3\cos(c+dx))}$$

```
[Out] 3/128*arctanh(cos(d*x+c))/a^3/d-1/128/a/d/(a-a*cos(d*x+c))^2-1/40*a^2/d/(a+
a*cos(d*x+c))^5+3/64*a/d/(a+a*cos(d*x+c))^4-1/64/a/d/(a+a*cos(d*x+c))^2-3/1
28/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2915, 12, 90, 212}

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3} dx = \frac{3\text{arctanh}(\cos(c+dx))}{128a^3d} - \frac{3}{128d(a^3\cos(c+dx)+a^3)} - \frac{a^2}{40d(a\cos(c+dx)+a)^5} + \frac{3a}{64d(a\cos(c+dx)+a)^4} - \frac{1}{128ad(a-a\cos(c+dx))^2} - \frac{1}{64ad(a\cos(c+dx)+a)^2}$$

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(128*a^3*d) - 1/(128*a*d*(a - a*Cos[c + d*x])^2)
- a^2/(40*d*(a + a*Cos[c + d*x])^5) + (3*a)/(64*d*(a + a*Cos[c + d*x])^4) -
 1/(64*a*d*(a + a*Cos[c + d*x])^2) - 3/(128*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$

$$= \frac{a^5\text{Subst}\left(\int \frac{x^3}{a^3(-a-x)^3(-a+x)^6}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^2\text{Subst}\left(\int \frac{x^3}{(-a-x)^3(-a+x)^6}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^2 \text{Subst}\left(\int \left(-\frac{1}{8(a-x)^6} + \frac{3}{16a(a-x)^5} - \frac{1}{32a^3(a-x)^3} - \frac{3}{128a^4(a-x)^2} + \frac{1}{64a^3(a+x)^3} - \frac{3}{128a^4(a^2-x^2)}\right) dx, x, -a \mathrm{c}\right.}{d}$$

$$= -\frac{1}{128ad(a-a\cos(c+dx))^2} - \frac{a^2}{40d(a+a\cos(c+dx))^5} + \frac{3a}{64d(a+a\cos(c+dx))^4} - \frac{1}{64ad(a+a\cos(c+dx))^2} - \frac{3}{128d\,(a^3+a^3\cos(c+dx))} - \frac{3\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, -a\cos(c+dx)\right)}{128a^2d}$$

$$= \frac{3\text{arctanh}(\cos(c+dx))}{128a^3d} - \frac{1}{128ad(a-a\cos(c+dx))^2} - \frac{a^2}{40d(a+a\cos(c+dx))^5} + \frac{3a}{64d(a+a\cos(c+dx))^4} - \frac{1}{64ad(a+a\cos(c+dx))^2} - \frac{3}{128d\,(a^3+a^3\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 3.78 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\left(4-15\cos^2\left(\frac{1}{2}(c+dx)\right)+60\cos^8\left(\frac{1}{2}(c+dx)\right)+10\cos^6\left(\frac{1}{2}(c+dx)\right)\left(2+\cot^4\left(\frac{1}{2}(c+dx)\right)\right)-120\cos^1\right.}{640a^3d(1+\sec(c-}$$

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/640*((4 - 15*Cos[(c + d*x)/2]^2 + 60*Cos[(c + d*x)/2]^8 + 10*Cos[(c + d*
x)/2]^6*(2 + Cot[(c + d*x)/2]^4) - 120*Cos[(c + d*x)/2]^10*(Log[Cos[(c + d*
x)/2]] - Log[Sin[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3)/(a^3*d*
(1 + Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-\frac{1}{40(\cos(dx+c)+1)^5}+\frac{3}{64(\cos(dx+c)+1)^4}-\frac{1}{64(\cos(dx+c)+1)^2}-\frac{3}{128(\cos(dx+c)+1)}+\frac{3\ln(\cos(dx+c)+1)}{256}-\frac{1}{128(\cos(dx+c)-1)^2}-\frac{3\ln(c}{}}{d\,a^3}$
default	$\frac{-\frac{1}{40(\cos(dx+c)+1)^5}+\frac{3}{64(\cos(dx+c)+1)^4}-\frac{1}{64(\cos(dx+c)+1)^2}-\frac{3}{128(\cos(dx+c)+1)}+\frac{3\ln(\cos(dx+c)+1)}{256}-\frac{1}{128(\cos(dx+c)-1)^2}-\frac{3\ln(c}{}}{d\,a^3}$
parallelrisch	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+30\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-10\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-60\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-20\cot\left(\frac{dx}{2}\right.}{5120a^3d}$
norman	$\frac{-\frac{1}{512ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{1024ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{1280ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{256da}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{256da}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{512da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{256da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a^2}-\frac{3\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{128a^3d}$
risch	$-\frac{15\,\mathrm{e}^{13i(dx+c)}+90\,\mathrm{e}^{12i(dx+c)}+170\,\mathrm{e}^{11i(dx+c)}-30\,\mathrm{e}^{10i(dx+c)}+1521\,\mathrm{e}^{9i(dx+c)}+1476\,\mathrm{e}^{8i(dx+c)}+3756\,\mathrm{e}^{7i(dx+c)}+1476\,\mathrm{e}^{6i(}}{320a^3d\left(\mathrm{e}^{i(dx+c)}+1\right)^{10}\left(\mathrm{e}^{i(dx+c)}-1\right)^4}$

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/40/(cos(d*x+c)+1)^5+3/64/(cos(d*x+c)+1)^4-1/64/(cos(d*x+c)+1)^2
-3/128/(cos(d*x+c)+1)+3/256*ln(cos(d*x+c)+1)-1/128/(cos(d*x+c)-1)^2-3/256*l
n(cos(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(117) = 234$.

Time = 0.27 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.48

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{30\,\cos\left(dx+c\right)^6+90\,\cos\left(dx+c\right)^5+40\,\cos\left(dx+c\right)^4-120\,\cos\left(dx+c\right)^3+122\,\cos\left(dx+c\right)^2-15\,(\cos}{}$$

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/1280*(30*cos(d*x + c)^6 + 90*cos(d*x + c)^5 + 40*cos(d*x + c)^4 - 120*co
s(d*x + c)^3 + 122*cos(d*x + c)^2 - 15*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 +
 cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*
cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^7 + 3*cos(
d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x
 + c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 126*cos(d*x +
c) + 32)/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(d*x + c
)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c))**3,x)

[Out] Integral(csc(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/a**3

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.47

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\left(15\cos(dx+c)^6+45\cos(dx+c)^5+20\cos(dx+c)^4-60\cos(dx+c)^3+61\cos(dx+c)^2+63\cos(dx+c)+16\right)}{a^3\cos(dx+c)^7+3a^3\cos(dx+c)^6+a^3\cos(dx+c)^5-5a^3\cos(dx+c)^4-5a^3\cos(dx+c)^3+a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3} - \frac{15\log(\cos(dx+}{a^3}}{1280\,d}$$

[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] -1/1280*(2*(15*cos(d*x + c)^6 + 45*cos(d*x + c)^5 + 20*cos(d*x + c)^4 - 60*cos(d*x + c)^3 + 61*cos(d*x + c)^2 + 63*cos(d*x + c) + 16)/(a^3*cos(d*x + c)^7 + 3*a^3*cos(d*x + c)^6 + a^3*cos(d*x + c)^5 - 5*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) - 15*log(cos(d*x + c) + 1)/a^3 + 15*log(cos(d*x + c) - 1)/a^3)/d

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.81

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{10\left(\frac{2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a^3(\cos(dx+c)-1)^2} - \frac{60\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} + \frac{\frac{60a^{12}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{30a^{12}(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{20a^{12}(c}{(\cos(}}{5120\,d}$$

[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")

```
[Out] 1/5120*(10*(2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a^3*(cos(d*x + c) - 1)^2)
 - 60*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (60*a^12*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 30*a^12*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 20*a^12*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 5*a^12*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 4*a^12*(cos(d*x + c) - 1)^5/(cos(d
*x + c) + 1)^5)/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.35

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\,\mathrm{atanh}(\cos\left(c+d\,x\right))}{128\,a^3\,d} - \frac{\frac{3\cos(c+d\,x)^6}{128}+\frac{9\cos(c+d\,x)^5}{128}+\frac{\cos(c+d\,x)^4}{32}-\frac{3\cos(c+d\,x)^3}{32}+\frac{61\cos(c+d\,x)^2}{640}+\frac{63\cos(c+}{640}}{d\,\left(a^3\cos\left(c+d\,x\right)^7+3\,a^3\cos\left(c+d\,x\right)^6+a^3\cos\left(c+d\,x\right)^5-5\,a^3\cos\left(c+d\,x\right)^4-5\,a^3\cos\left(c+d\,x\right)^3+a^3}$$

```
[In] int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (3*atanh(cos(c + d*x)))/(128*a^3*d) - ((63*cos(c + d*x))/640 + (61*cos(c +
d*x)^2)/640 - (3*cos(c + d*x)^3)/32 + cos(c + d*x)^4/32 + (9*cos(c + d*x)^5
)/128 + (3*cos(c + d*x)^6)/128 + 1/40)/(d*(3*a^3*cos(c + d*x) + a^3 + a^3*c
os(c + d*x)^2 - 5*a^3*cos(c + d*x)^3 - 5*a^3*cos(c + d*x)^4 + a^3*cos(c + d
*x)^5 + 3*a^3*cos(c + d*x)^6 + a^3*cos(c + d*x)^7))
```

3.100 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{29x}{128a^3} - \frac{29\cos(c+dx)\sin(c+dx)}{128a^3d} - \frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d} + \frac{23\cos^5(c+dx)\sin(c+dx)}{48a^3d} + \frac{\cos^7(c+dx)\sin(c+dx)}{8a^3d} + \frac{4\sin^3(c+dx)}{3a^3d} - \frac{7\sin^5(c+dx)}{5a^3d} + \frac{3\sin^7(c+dx)}{7a^3d}$$

```
[Out] -29/128*x/a^3-29/128*cos(d*x+c)*sin(d*x+c)/a^3/d-29/192*cos(d*x+c)^3*sin(d*
x+c)/a^3/d+23/48*cos(d*x+c)^5*sin(d*x+c)/a^3/d+1/8*cos(d*x+c)^7*sin(d*x+c)/
a^3/d+4/3*sin(d*x+c)^3/a^3/d-7/5*sin(d*x+c)^5/a^3/d+3/7*sin(d*x+c)^7/a^3/d
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3957, 2954, 2952, 2644, 14, 2648, 2715, 8, 276}

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\sin^7(c+dx)}{7a^3d} - \frac{7\sin^5(c+dx)}{5a^3d} + \frac{4\sin^3(c+dx)}{3a^3d} + \frac{\sin(c+dx)\cos^7(c+dx)}{8a^3d} + \frac{23\sin(c+dx)\cos^5(c+dx)}{48a^3d} - \frac{29\sin(c+dx)\cos^3(c+dx)}{192a^3d} - \frac{29\sin(c+dx)\cos(c+dx)}{128a^3d} - \frac{29x}{128a^3}$$

[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]

[Out] (-29*x)/(128*a^3) - (29*Cos[c + d*x]*Sin[c + d*x])/(128*a^3*d) - (29*Cos[c + d*x]^3*Sin[c + d*x])/(192*a^3*d) + (23*Cos[c + d*x]^5*Sin[c + d*x])/(48*a^3*d) + (Cos[c + d*x]^7*Sin[c + d*x])/(8*a^3*d) + (4*Sin[c + d*x]^3)/(3*a^3*d) - (7*Sin[c + d*x]^5)/(5*a^3*d) + (3*Sin[c + d*x]^7)/(7*a^3*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
```

```
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin^8(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int \cos^3(c+dx)(-a+a\cos(c+dx))^3\sin^2(c+dx)\,dx}{a^6} \\
&= \\
&\quad -\frac{\int \left(-a^3\cos^3(c+dx)\sin^2(c+dx)+3a^3\cos^4(c+dx)\sin^2(c+dx)-3a^3\cos^5(c+dx)\sin^2(c+dx\right)}{a^6} \\
&= \frac{\int \cos^3(c+dx)\sin^2(c+dx)\,dx}{a^3}-\frac{\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{a^3} \\
&\quad -\frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{a^3}+\frac{3\int \cos^5(c+dx)\sin^2(c+dx)\,dx}{a^3} \\
&= \frac{\cos^5(c+dx)\sin(c+dx)}{2a^3d}+\frac{\cos^7(c+dx)\sin(c+dx)}{8a^3d}-\frac{\int \cos^6(c+dx)\,dx}{8a^3}-\frac{\int \cos^4(c+dx)\,dx}{2a^3} \\
&\quad +\frac{\operatorname{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\sin(c+dx)\right)}{a^3d}+\frac{3\operatorname{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(c+dx)\right)}{a^3d} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{8a^3d}+\frac{23\cos^5(c+dx)\sin(c+dx)}{48a^3d} \\
&\quad +\frac{\cos^7(c+dx)\sin(c+dx)}{8a^3d}-\frac{5\int \cos^4(c+dx)\,dx}{48a^3} \\
&\quad -\frac{3\int \cos^2(c+dx)\,dx}{8a^3}+\frac{\operatorname{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\sin(c+dx)\right)}{a^3d} \\
&\quad +\frac{3\operatorname{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sin(c+dx)\right)}{a^3d}
\end{aligned}$$

$$= -\frac{3\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d} + \frac{23\cos^5(c+dx)\sin(c+dx)}{48a^3d} + \frac{\cos^7(c+dx)\sin(c+dx)}{8a^3d} + \frac{4\sin^3(c+dx)}{3a^3d} - \frac{7\sin^5(c+dx)}{5a^3d} + \frac{3\sin^7(c+dx)}{7a^3d} - \frac{5\int\cos^2(c+dx)\,dx}{64a^3} - \frac{3\int 1\,dx}{16a^3}$$

$$= -\frac{3x}{16a^3} - \frac{29\cos(c+dx)\sin(c+dx)}{128a^3d} - \frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d} + \frac{23\cos^5(c+dx)\sin(c+dx)}{48a^3d} + \frac{\cos^7(c+dx)\sin(c+dx)}{8a^3d} + \frac{4\sin^3(c+dx)}{3a^3d} - \frac{7\sin^5(c+dx)}{5a^3d} + \frac{3\sin^7(c+dx)}{7a^3d} - \frac{5\int 1\,dx}{128a^3}$$

$$= -\frac{29x}{128a^3} - \frac{29\cos(c+dx)\sin(c+dx)}{128a^3d} - \frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d} + \frac{23\cos^5(c+dx)\sin(c+dx)}{48a^3d} + \frac{\cos^7(c+dx)\sin(c+dx)}{8a^3d} + \frac{4\sin^3(c+dx)}{3a^3d} - \frac{7\sin^5(c+dx)}{5a^3d} + \frac{3\sin^7(c+dx)}{7a^3d}$$

Mathematica [A] (verified)

Time = 5.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-24360dx + 38640\sin(c+dx) - 6720\sin(2(c+dx)) - 3920\sin(3(c+dx))\right.}{1344\ldots}$$

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-24360*d*x + 38640*Sin[c + d*x] - 6720*
Sin[2*(c + d*x)] - 3920*Sin[3*(c + d*x)] + 5880*Sin[4*(c + d*x)] - 4368*Sin
[5*(c + d*x)] + 2240*Sin[6*(c + d*x)] - 720*Sin[7*(c + d*x)] + 105*Sin[8*(c
 + d*x)] + 294*Tan[c/2]))/(13440*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{-24360dx+38640\sin(dx+c)+105\sin(8dx+8c)+2240\sin(6dx+6c)+5880\sin(4dx+4c)-6720\sin(2dx+2c)-720\sin(7dx-}{107520a^3d}$
risch	$-\frac{29x}{128a^3}+\frac{23\sin(dx+c)}{64a^3d}+\frac{\sin(8dx+8c)}{1024a^3d}-\frac{3\sin(7dx+7c)}{448a^3d}+\frac{\sin(6dx+6c)}{48a^3d}-\frac{13\sin(5dx+5c)}{320a^3d}+\frac{7\sin(4dx+4c)}{128a^3d}$
derivativedivides	$-\frac{\frac{64\left(-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}-\frac{667\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12288}-\frac{11107\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{61440}-\frac{146537\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{430080}-\frac{72669\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{143360}-\frac{1759\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^8}}{a^3d}$
default	$-\frac{\frac{64\left(-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}-\frac{667\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12288}-\frac{11107\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{61440}-\frac{146537\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{430080}-\frac{72669\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{143360}-\frac{1759\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20480}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^8}}{a^3d}$
norman	$\frac{-\frac{29x}{128a}+\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{667\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{192ad}+\frac{11107\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{960ad}+\frac{146537\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6720ad}+\frac{72669\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{2240ad}+\frac{1759\tan\left(\frac{dx}{2}+\right.}{320ad}}{}$

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/107520*(-24360*d*x+38640*sin(d*x+c)+105*sin(8*d*x+8*c)+2240*sin(6*d*x+6*c
)+5880*sin(4*d*x+4*c)-6720*sin(2*d*x+2*c)-720*sin(7*d*x+7*c)-4368*sin(5*d*x
+5*c)-3920*sin(3*d*x+3*c))/a^3/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.58

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{3045\,dx-\left(1680\,\cos\left(dx+c\right)^7-5760\,\cos\left(dx+c\right)^6+6440\,\cos\left(dx+c\right)^5-1536\,\cos\left(dx+c\right)^4-2030\right.}{13440\,a^3d}$$

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/13440*(3045*d*x - (1680*cos(d*x + c)^7 - 5760*cos(d*x + c)^6 + 6440*cos(
d*x + c)^5 - 1536*cos(d*x + c)^4 - 2030*cos(d*x + c)^3 + 2432*cos(d*x + c)^
2 - 3045*cos(d*x + c) + 4864)*sin(d*x + c))/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(141) = 282$.

Time = 0.28 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.41

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{\frac{3045\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{23345\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{77749\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{146537\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{218007\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{36939\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{120015\,\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{3045\,\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a^3+\frac{8\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70\,a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56\,a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28\,a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8\,a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^3\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}}{6720\,d}$$

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6720*((3045*sin(d*x + c)/(cos(d*x + c) + 1) + 23345*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 77749*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 146537*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 218007*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 +
 36939*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 120015*sin(d*x + c)^13/(cos(
d*x + c) + 1)^13 - 3045*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^3 + 8*a^3
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 + 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a
^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^3*sin(d*x + c)^14/(cos(d*x +
 c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 3045*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{3045\,(dx+c)}{a^3}+\frac{2\left(3045\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-120015\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-36939\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-218007\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-146537\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^8a^3}}{13440\,d}$$

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/13440*(3045*(d*x + c)/a^3 + 2*(3045*tan(1/2*d*x + 1/2*c)^15 - 120015*tan
(1/2*d*x + 1/2*c)^13 - 36939*tan(1/2*d*x + 1/2*c)^11 - 218007*tan(1/2*d*x +
 1/2*c)^9 - 146537*tan(1/2*d*x + 1/2*c)^7 - 77749*tan(1/2*d*x + 1/2*c)^5 -
23345*tan(1/2*d*x + 1/2*c)^3 - 3045*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^8*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{-\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{1143\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\frac{1759\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{320}+\frac{72669\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{2240}+\frac{146537\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{6720}+\frac{11107\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{960}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^8}$$
$$-\frac{29\,x}{128\,a^3}$$

```
[In] int(sin(c + d*x)^8/(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((29*tan(c/2 + (d*x)/2))/64 + (667*tan(c/2 + (d*x)/2)^3)/192 + (11107*tan(c
/2 + (d*x)/2)^5)/960 + (146537*tan(c/2 + (d*x)/2)^7)/6720 + (72669*tan(c/2
+ (d*x)/2)^9)/2240 + (1759*tan(c/2 + (d*x)/2)^11)/320 + (1143*tan(c/2 + (d*
x)/2)^13)/64 - (29*tan(c/2 + (d*x)/2)^15)/64)/(a^3*d*(tan(c/2 + (d*x)/2)^2
+ 1)^8) - (29*x)/(128*a^3)
```

3.101 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{23x}{16a^3} + \frac{4\sin(c+dx)}{a^3d} - \frac{23\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d} - \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d} - \frac{7\sin^3(c+dx)}{3a^3d} + \frac{3\sin^5(c+dx)}{5a^3d}$$

```
[Out] -23/16*x/a^3+4*sin(d*x+c)/a^3/d-23/16*cos(d*x+c)*sin(d*x+c)/a^3/d-23/24*cos
(d*x+c)^3*sin(d*x+c)/a^3/d-1/6*cos(d*x+c)^5*sin(d*x+c)/a^3/d-7/3*sin(d*x+c)
^3/a^3/d+3/5*sin(d*x+c)^5/a^3/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2948, 2836, 2713, 2715, 8}

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\sin^5(c+dx)}{5a^3d} - \frac{7\sin^3(c+dx)}{3a^3d} + \frac{4\sin(c+dx)}{a^3d} - \frac{\sin(c+dx)\cos^5(c+dx)}{6a^3d} - \frac{23\sin(c+dx)\cos^3(c+dx)}{24a^3d} - \frac{23\sin(c+dx)\cos(c+dx)}{16a^3d} - \frac{23x}{16a^3}$$

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-23*x)/(16*a^3) + (4*Sin[c + d*x])/(a^3*d) - (23*Cos[c + d*x]*Sin[c + d*x]
)/(16*a^3*d) - (23*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^3*d) - (Cos[c + d*x]^
5*Sin[c + d*x])/(6*a^3*d) - (7*Sin[c + d*x]^3)/(3*a^3*d) + (3*Sin[c + d*x]^
5)/(5*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos^3(c+dx)\sin^6(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$

$$
\begin{aligned}
&= -\frac{\int \cos^3(c+dx)(-a+a\cos(c+dx))^3\,dx}{a^6} \\
&= -\frac{\int \left(-a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)-3a^3\cos^5(c+dx)+a^3\cos^6(c+dx)\right)\,dx}{a^6} \\
&= \frac{\int \cos^3(c+dx)\,dx}{a^3}-\frac{\int \cos^6(c+dx)\,dx}{a^3}-\frac{3\int \cos^4(c+dx)\,dx}{a^3}+\frac{3\int \cos^5(c+dx)\,dx}{a^3} \\
&= -\frac{3\cos^3(c+dx)\sin(c+dx)}{4a^3d}-\frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d}-\frac{5\int \cos^4(c+dx)\,dx}{6a^3} \\
&\quad -\frac{9\int \cos^2(c+dx)\,dx}{4a^3}-\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{a^3d} \\
&= \frac{4\sin(c+dx)}{a^3d}-\frac{9\cos(c+dx)\sin(c+dx)}{8a^3d}-\frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d} \\
&\quad -\frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d}-\frac{7\sin^3(c+dx)}{3a^3d} \\
&\quad +\frac{3\sin^5(c+dx)}{5a^3d}-\frac{5\int \cos^2(c+dx)\,dx}{8a^3}-\frac{9\int 1\,dx}{8a^3} \\
&= -\frac{9x}{8a^3}+\frac{4\sin(c+dx)}{a^3d}-\frac{23\cos(c+dx)\sin(c+dx)}{16a^3d}-\frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d} \\
&\quad -\frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d}-\frac{7\sin^3(c+dx)}{3a^3d}+\frac{3\sin^5(c+dx)}{5a^3d}-\frac{5\int 1\,dx}{16a^3} \\
&= -\frac{23x}{16a^3}+\frac{4\sin(c+dx)}{a^3d}-\frac{23\cos(c+dx)\sin(c+dx)}{16a^3d}-\frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d} \\
&\quad -\frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d}-\frac{7\sin^3(c+dx)}{3a^3d}+\frac{3\sin^5(c+dx)}{5a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$
\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx
= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-2760dx+5040\sin(c+dx)-1890\sin(2(c+dx))+760\sin(3(c+dx))-2\right.}{240a^3d(1+\sec(c+dx))^3}
$$

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-2760*d*x + 5040*Sin[c + d*x] - 1890*Si
n[2*(c + d*x)] + 760*Sin[3*(c + d*x)] - 270*Sin[4*(c + d*x)] + 72*Sin[5*(c
+ d*x)] - 10*Sin[6*(c + d*x)] + 9*Tan[c/2]))/(240*a^3*d*(1 + Sec[c + d*x])^
3)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{-1380dx+2520\sin(dx+c)+36\sin(5dx+5c)+380\sin(3dx+3c)-5\sin(6dx+6c)-135\sin(4dx+4c)-945\sin(2dx+2c)}{960a^3d}$
risch	$-\frac{23x}{16a^3}+\frac{21\sin(dx+c)}{8a^3d}-\frac{\sin(6dx+6c)}{192a^3d}+\frac{3\sin(5dx+5c)}{80a^3d}-\frac{9\sin(4dx+4c)}{64a^3d}+\frac{19\sin(3dx+3c)}{48a^3d}-\frac{63\sin(2dx+2c)}{64a^3d}$
derivativedivides	$\frac{-\frac{16\left(-\frac{105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{128}-\frac{211\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{128}-\frac{969\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{320}-\frac{759\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{320}-\frac{391\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{384}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6}-2}{a^3d}$
default	$\frac{-\frac{16\left(-\frac{105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{128}-\frac{211\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{128}-\frac{969\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{320}-\frac{759\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{320}-\frac{391\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{384}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6}-2}{a^3d}$
norman	$\frac{-\frac{23x}{16a}+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{391\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24ad}+\frac{759\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20ad}+\frac{969\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{20ad}+\frac{211\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{8ad}+\frac{105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8ad}-69}{\left(1+\tan\left(\frac{dx}{2}+\right.\right.}$

```
[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/960*(-1380*d*x+2520*sin(d*x+c)+36*sin(5*d*x+5*c)+380*sin(3*d*x+3*c)-5*sin
(6*d*x+6*c)-135*sin(4*d*x+4*c)-945*sin(2*d*x+2*c))/a^3/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.54

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{345\,dx+\left(40\,\cos\left(dx+c\right)^5-144\,\cos\left(dx+c\right)^4+230\,\cos\left(dx+c\right)^3-272\,\cos\left(dx+c\right)^2+345\,\cos\left(dx-\right.\right.}{240\,a^3d}$$

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(345*d*x + (40*cos(d*x + c)^5 - 144*cos(d*x + c)^4 + 230*cos(d*x + c
)^3 - 272*cos(d*x + c)^2 + 345*cos(d*x + c) - 544)*sin(d*x + c))/(a^3*d)
```

Sympy [F]

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sin^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c))**3,x)

[Out] Integral(sin(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/a**3

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(117) = 234$.

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.26

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{\frac{345\sin(dx+c)}{\cos(dx+c)+1}+\frac{1955\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4554\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5814\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3165\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{1575\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^3+\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{345\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{120\,d}$$

[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/120*((345*sin(d*x + c)/(cos(d*x + c) + 1) + 1955*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4554*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5814*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3165*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 1575*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^3 + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 345*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{345(dx+c)}{a^3}-\frac{2\left(1575\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+3165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+5814\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+4554\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1955\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+345\text{ t}\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^6a^3}}{240\,d}$$

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(345*(d*x + c)/a^3 - 2*(1575*tan(1/2*d*x + 1/2*c)^11 + 3165*tan(1/2*
d*x + 1/2*c)^9 + 5814*tan(1/2*d*x + 1/2*c)^7 + 4554*tan(1/2*d*x + 1/2*c)^5
+ 1955*tan(1/2*d*x + 1/2*c)^3 + 345*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^6*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 16.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{105\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{211\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8}+\frac{969\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{759\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{391\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{23\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6} - \frac{23\,x}{16\,a^3}$$

```
[In] int(sin(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((23*tan(c/2 + (d*x)/2))/8 + (391*tan(c/2 + (d*x)/2)^3)/24 + (759*tan(c/2 +
 (d*x)/2)^5)/20 + (969*tan(c/2 + (d*x)/2)^7)/20 + (211*tan(c/2 + (d*x)/2)^9
)/8 + (105*tan(c/2 + (d*x)/2)^11)/8)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) -
 (23*x)/(16*a^3)
```

3.102 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{51x}{8a^3} - \frac{7\sin(c+dx)}{a^3d} + \frac{19\cos(c+dx)\sin(c+dx)}{8a^3d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d} - \frac{4\sin(c+dx)}{a^3d(1+\cos(c+dx))} + \frac{\sin^3(c+dx)}{a^3d}$$

```
[Out] 51/8*x/a^3-7*sin(d*x+c)/a^3/d+19/8*cos(d*x+c)*sin(d*x+c)/a^3/d+1/4*cos(d*x+
c)^3*sin(d*x+c)/a^3/d-4*sin(d*x+c)/a^3/d/(1+cos(d*x+c))+sin(d*x+c)^3/a^3/d
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3957, 2954, 2951, 2727, 2717, 2715, 8, 2713}

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sin^3(c+dx)}{a^3d} - \frac{7\sin(c+dx)}{a^3d} + \frac{\sin(c+dx)\cos^3(c+dx)}{4a^3d} + \frac{19\sin(c+dx)\cos(c+dx)}{8a^3d} - \frac{4\sin(c+dx)}{a^3d(\cos(c+dx)+1)} + \frac{51x}{8a^3}$$

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (51*x)/(8*a^3) - (7*Sin[c + d*x])/(a^3*d) + (19*Cos[c + d*x]*Sin[c + d*x])/
(8*a^3*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^3*d) - (4*Sin[c + d*x])/(a^3
*d*(1 + Cos[c + d*x])) + Sin[c + d*x]^3/(a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin^4(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int \cos(c+dx)(-a+a\cos(c+dx))^3\cot^2(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(4a+\frac{4a}{-1-\cos(c+dx)}-4a\cos(c+dx)+4a\cos^2(c+dx)-3a\cos^3(c+dx)+a\cos^4(c+dx)\right)\,dx}{a^4} \\
&= \frac{4x}{a^3}+\frac{\int \cos^4(c+dx)\,dx}{a^3}-\frac{3\int \cos^3(c+dx)\,dx}{a^3} \\
&\quad +\frac{4\int \frac{1}{-1-\cos(c+dx)}\,dx}{a^3}-\frac{4\int \cos(c+dx)\,dx}{a^3}+\frac{4\int \cos^2(c+dx)\,dx}{a^3} \\
&= \frac{4x}{a^3}-\frac{4\sin(c+dx)}{a^3d}+\frac{2\cos(c+dx)\sin(c+dx)}{a^3d} \\
&\quad +\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}-\frac{4\sin(c+dx)}{a^3d(1+\cos(c+dx))}+\frac{3\int \cos^2(c+dx)\,dx}{4a^3} \\
&\quad +\frac{2\int 1\,dx}{a^3}+\frac{3\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{a^3d} \\
&= \frac{6x}{a^3}-\frac{7\sin(c+dx)}{a^3d}+\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d} \\
&\quad -\frac{4\sin(c+dx)}{a^3d(1+\cos(c+dx))}+\frac{\sin^3(c+dx)}{a^3d}+\frac{3\int 1\,dx}{8a^3} \\
&= \frac{51x}{8a^3}-\frac{7\sin(c+dx)}{a^3d}+\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d} \\
&\quad +\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}-\frac{4\sin(c+dx)}{a^3d(1+\cos(c+dx))}+\frac{\sin^3(c+dx)}{a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.60

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(2040dx\cos\left(\frac{dx}{2}\right)+2040dx\cos\left(c+\frac{dx}{2}\right)-3563\sin\left(\frac{dx}{2}\right)-997\sin\left(c+\frac{dx}{2}\right)-800\,\mathrm{s}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(2040*d*x*Cos[(d*x)/2] + 2040*d*x*Cos[c + (d*x)/
2] - 3563*Sin[(d*x)/2] - 997*Sin[c + (d*x)/2] - 800*Sin[c + (3*d*x)/2] - 80
0*Sin[2*c + (3*d*x)/2] + 160*Sin[2*c + (5*d*x)/2] + 160*Sin[3*c + (5*d*x)/2
] - 35*Sin[3*c + (7*d*x)/2] - 35*Sin[4*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)
/2] + 5*Sin[5*c + (9*d*x)/2]))/(640*a^3*d)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{204dx+(-295+\cos(4dx+4c)-6\cos(3dx+3c)+26\cos(2dx+2c)-134\cos(dx+c))\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a^3d}$
derivativedivides	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\left(-\frac{77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{16}-\frac{149\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{16}-\frac{123\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{16}-\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}+\frac{51\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^3d}$
default	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\left(-\frac{77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{16}-\frac{149\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{16}-\frac{123\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{16}-\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}+\frac{51\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^3d}$
risch	$\frac{51x}{8a^3}+\frac{25i\mathrm{e}^{i(dx+c)}}{8a^3d}-\frac{25i\mathrm{e}^{-i(dx+c)}}{8a^3d}-\frac{8i}{a^3d\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{\sin(4dx+4c)}{32a^3d}-\frac{\sin(3dx+3c)}{4a^3d}+\frac{5\sin(2dx+2c)}{4a^3d}$
norman	$\frac{\frac{51x}{8a}-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{187\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4ad}-\frac{245\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4ad}-\frac{141\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4ad}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{ad}+\frac{51x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}+\frac{153x\,\mathrm{ta}}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4a^2}$

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(204*d*x+(-295+cos(4*d*x+4*c)-6*cos(3*d*x+3*c)+26*cos(2*d*x+2*c)-134*c
os(d*x+c))*tan(1/2*d*x+1/2*c))/a^3/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{51\,dx\cos\left(dx+c\right)+51\,dx+\left(2\,\cos\left(dx+c\right)^4-6\,\cos\left(dx+c\right)^3+11\,\cos\left(dx+c\right)^2-29\,\cos\left(dx+c\right)-80\right)\mathrm{s}}{8\left(a^3d\cos\left(dx+c\right)+a^3d\right)}$$

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(51*d*x*cos(d*x + c) + 51*d*x + (2*cos(d*x + c)^4 - 6*cos(d*x + c)^3 +
11*cos(d*x + c)^2 - 29*cos(d*x + c) - 80)*sin(d*x + c))/(a^3*d*cos(d*x + c)
 + a^3*d)
```

Sympy [F]

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sin^4\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(102) = 204$.

Time = 0.29 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.10

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{\frac{\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{123\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{149\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{77\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3+\frac{4\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{51\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{16\,\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{4\,d}$$

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((35*sin(d*x + c)/(cos(d*x + c) + 1) + 123*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 149*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 77*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/(a^3 + 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^3*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 51*arctan(sin(d*x + c)/(c
os(d*x + c) + 1))/a^3 + 16*sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{51\,(dx+c)}{a^3} - \frac{32\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3} - \frac{2\left(77\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+149\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+123\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a^3}}{8\,d}$$

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(51*(d*x + c)/a^3 - 32*tan(1/2*d*x + 1/2*c)/a^3 - 2*(77*tan(1/2*d*x + 1
/2*c)^7 + 149*tan(1/2*d*x + 1/2*c)^5 + 123*tan(1/2*d*x + 1/2*c)^3 + 35*tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{51\,x}{8\,a^3} - \frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d} - \frac{\frac{77\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{149\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4} + \frac{123\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4} + \frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

```
[In] int(sin(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (51*x)/(8*a^3) - (4*tan(c/2 + (d*x)/2))/(a^3*d) - ((35*tan(c/2 + (d*x)/2))/
4 + (123*tan(c/2 + (d*x)/2)^3)/4 + (149*tan(c/2 + (d*x)/2)^5)/4 + (77*tan(c
/2 + (d*x)/2)^7)/4)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.103 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3} dx = -\frac{11x}{2a^3} + \frac{3\sin(c+dx)}{a^3d} - \frac{\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{2\sin(c+dx)}{3a^3d(1+\cos(c+dx))^2} + \frac{19\sin(c+dx)}{3a^3d(1+\cos(c+dx))}$$

```
[Out] -11/2*x/a^3+3*sin(d*x+c)/a^3/d-1/2*cos(d*x+c)*sin(d*x+c)/a^3/d-2/3*sin(d*x+
c)/a^3/d/(1+cos(d*x+c))^2+19/3*sin(d*x+c)/a^3/d/(1+cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2953, 3045, 2717, 2715, 8, 2729, 2727}

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3} dx = \frac{3\sin(c+dx)}{a^3d} - \frac{\sin(c+dx)\cos(c+dx)}{2a^3d} + \frac{19\sin(c+dx)}{3a^3d(\cos(c+dx)+1)} - \frac{2\sin(c+dx)}{3a^3d(\cos(c+dx)+1)^2} - \frac{11x}{2a^3}$$

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-11*x)/(2*a^3) + (3*Sin[c + d*x])/(a^3*d) - (Cos[c + d*x]*Sin[c + d*x])/(2
*a^3*d) - (2*Sin[c + d*x])/(3*a^3*d*(1 + Cos[c + d*x])^2) + (19*Sin[c + d*x
])/(3*a^3*d*(1 + Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int \frac{\cos^3(c+dx)(-a+a\cos(c+dx))}{(-a-a\cos(c+dx))^2}\,dx}{a^2} \\
&= -\frac{\int \left(\frac{5}{a}-\frac{3\cos(c+dx)}{a}+\frac{\cos^2(c+dx)}{a}+\frac{2}{a(1+\cos(c+dx))^2}-\frac{7}{a(1+\cos(c+dx))}\right)dx}{a^2} \\
&= -\frac{5x}{a^3}-\frac{\int \cos^2(c+dx)\,dx}{a^3}-\frac{2\int \frac{1}{(1+\cos(c+dx))^2}\,dx}{a^3}+\frac{3\int \cos(c+dx)\,dx}{a^3}+\frac{7\int \frac{1}{1+\cos(c+dx)}\,dx}{a^3} \\
&= -\frac{5x}{a^3}+\frac{3\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2\sin(c+dx)}{3a^3d(1+\cos(c+dx))^2} \\
&\quad +\frac{7\sin(c+dx)}{a^3d(1+\cos(c+dx))}-\frac{\int 1\,dx}{2a^3}-\frac{2\int \frac{1}{1+\cos(c+dx)}\,dx}{3a^3} \\
&= -\frac{11x}{2a^3}+\frac{3\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^3d} \\
&\quad -\frac{2\sin(c+dx)}{3a^3d(1+\cos(c+dx))^2}+\frac{19\sin(c+dx)}{3a^3d(1+\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.82

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(1980dx\cos\left(\frac{dx}{2}\right)+1980dx\cos\left(c+\frac{dx}{2}\right)+660dx\cos\left(c+\frac{3dx}{2}\right)+660dx\cos\left(2c+\right.\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/960*(Sec[c/2]*Sec[(c + d*x)/2]^3*(1980*d*x*Cos[(d*x)/2] + 1980*d*x*Cos[c
 + (d*x)/2] + 660*d*x*Cos[c + (3*d*x)/2] + 660*d*x*Cos[2*c + (3*d*x)/2] - 3
216*Sin[(d*x)/2] + 1326*Sin[c + (d*x)/2] - 2012*Sin[c + (3*d*x)/2] - 498*Si
n[2*c + (3*d*x)/2] - 135*Sin[2*c + (5*d*x)/2] - 135*Sin[3*c + (5*d*x)/2] +
15*Sin[3*c + (7*d*x)/2] + 15*Sin[4*c + (7*d*x)/2]))/(a^3*d)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{\frac{275\left(\cos(dx+c)+\frac{24\cos(2dx+2c)}{275}-\frac{3\cos(3dx+3c)}{275}+\frac{232}{275}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{48}-\frac{11dx}{2}}{a^{3}d}$	66
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}-11\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}}$	87
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}-11\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}}$	87
risch	$-\frac{11x}{2a^{3}}+\frac{i\mathrm{e}^{2i(dx+c)}}{8a^{3}d}-\frac{3i\mathrm{e}^{i(dx+c)}}{2a^{3}d}+\frac{3i\mathrm{e}^{-i(dx+c)}}{2a^{3}d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^{3}d}+\frac{2i\left(21\,\mathrm{e}^{2i(dx+c)}+36\,\mathrm{e}^{i(dx+c)}+19\right)}{3a^{3}d\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$	126
norman	$\frac{-\frac{11x}{2a}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{3ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{3ad}-\frac{11x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{a}-\frac{11x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a^{2}}$	135

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 11/48*(25*(cos(d*x+c)+24/275*cos(2*d*x+2*c)-3/275*cos(3*d*x+3*c)+232/275)*t
an(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^2-24*d*x)/a^3/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{33\,dx\cos\left(dx+c\right)^2+66\,dx\cos\left(dx+c\right)+33\,dx+\left(3\,\cos\left(dx+c\right)^3-12\,\cos\left(dx+c\right)^2-71\,\cos\left(dx+c\right)\right.}{6\,\left(a^3d\cos\left(dx+c\right)^2+2\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(33*d*x*cos(d*x + c)^2 + 66*d*x*cos(d*x + c) + 33*d*x + (3*cos(d*x + c
)^3 - 12*cos(d*x + c)^2 - 71*cos(d*x + c) - 52)*sin(d*x + c))/(a^3*d*cos(d*
x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sin^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.69

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{3\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{18\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{33\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3d}$$

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] 1/3*(3*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4) + (18*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x +
c)^3/(cos(d*x + c) + 1)^3)/a^3 - 33*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{33(dx+c)}{a^3}-\frac{6\left(7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a^3}+\frac{2\left(a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-18a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^9}}{6d}$$

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] -1/6*(33*(d*x + c)/a^3 - 6*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) + 2*(a^6*tan(1/2*d*x + 1/2*c)^3 -
18*a^6*tan(1/2*d*x + 1/2*c))/a^9)/d
```

Mupad [B] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.19

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-38\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-42\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\sin}{6\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

```
[In] int(sin(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(2*sin(c/2 + (d*x)/2) - 38*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 42*co
s(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^6*sin(c/2 + (
d*x)/2) + 33*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^3*d*cos(c/2 + (d*x)/2)^3)
```

3.104 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3} dx = \frac{3\cot^5(c+dx)}{5a^3d} + \frac{4\cot^7(c+dx)}{7a^3d} - \frac{\csc^3(c+dx)}{a^3d} + \frac{7\csc^5(c+dx)}{5a^3d} - \frac{4\csc^7(c+dx)}{7a^3d}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+4/7*cot(d*x+c)^7/a^3/d-csc(d*x+c)^3/a^3/d+7/5*csc(d*
x+c)^5/a^3/d-4/7*csc(d*x+c)^7/a^3/d
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3957, 2954, 2952, 2687, 30, 2686, 276, 14}

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3} dx = \frac{4\cot^7(c+dx)}{7a^3d} + \frac{3\cot^5(c+dx)}{5a^3d} - \frac{4\csc^7(c+dx)}{7a^3d} + \frac{7\csc^5(c+dx)}{5a^3d} - \frac{\csc^3(c+dx)}{a^3d}$$

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (4*Cot[c + d*x]^7)/(7*a^3*d) - Csc[c + d*x]^
3/(a^3*d) + (7*Csc[c + d*x]^5)/(5*a^3*d) - (4*Csc[c + d*x]^7)/(7*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\cot^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^5(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^6(c+dx)\csc^2(c+dx)+3a^3\cot^5(c+dx)\csc^3(c+dx)-3a^3\cot^4(c+dx)\csc^4(c+dx)+\right.}{a^6} \\
&= -\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^5(c+dx)\,dx}{a^3} \\
&\quad +\frac{3\int \cot^5(c+dx)\csc^3(c+dx)\,dx}{a^3}-\frac{3\int \cot^4(c+dx)\csc^4(c+dx)\,dx}{a^3} \\
&= -\frac{\text{Subst}\left(\int x^6\,dx,x,-\cot(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int x^4(-1+x^2)\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\text{Subst}\left(\int x^2(-1+x^2)^2\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\text{Subst}\left(\int x^4(1+x^2)\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&= \frac{\cot^7(c+dx)}{7a^3d}-\frac{\text{Subst}\left(\int (-x^4+x^6)\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\text{Subst}\left(\int (x^4+x^6)\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\csc^3(c+dx)}{a^3d}+\frac{7\csc^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\csc(c)\csc(c+dx)\sec^3(c+dx)(-840\sin(c)+448\sin(dx)+602\sin(c+dx)+602\sin(2(c+dx))+258\sin(}{22}$$

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

[Out] (Csc[c]*Csc[c + d*x]*Sec[c + d*x]^3*(-840*Sin[c] + 448*Sin[d*x] + 602*Sin[c + d*x] + 602*Sin[2*(c + d*x)] + 258*Sin[3*(c + d*x)] + 43*Sin[4*(c + d*x)] - 560*Sin[2*c + d*x] + 168*Sin[c + 2*d*x] - 280*Sin[3*c + 2*d*x] - 48*Sin[2*c + 3*d*x] - 8*Sin[3*c + 4*d*x]))/(2240*a^3*d*(1 + Sec[c + d*x])^3)

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-70\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-35\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{560a^{3}d}$	58
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16d\,a^{3}}$	60
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16d\,a^{3}}$	60
norman	$\frac{-\frac{1}{16ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{8da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{40da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{112da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}}$	82
risch	$-\frac{2i\left(35\,{\mathrm e}^{6i(dx+c)}+70\,{\mathrm e}^{5i(dx+c)}+105\,{\mathrm e}^{4i(dx+c)}+56\,{\mathrm e}^{3i(dx+c)}+21\,{\mathrm e}^{2i(dx+c)}-6\,{\mathrm e}^{i(dx+c)}-1\right)}{35a^{3}d\left({\mathrm e}^{i(dx+c)}+1\right)^{7}\left({\mathrm e}^{i(dx+c)}-1\right)}$	104

[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/560*(-5*tan(1/2*d*x+1/2*c)^7+14*tan(1/2*d*x+1/2*c)^5-70*tan(1/2*d*x+1/2*c)-35*cot(1/2*d*x+1/2*c))/a^3/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos\left(dx+c\right)^4+3\,\cos\left(dx+c\right)^3-15\,\cos\left(dx+c\right)^2-18\,\cos\left(dx+c\right)-6}{35\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)\sin\left(dx+c\right)}$$

[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/35*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 - 15*cos(d*x + c)^2 - 18*cos(d*x + c) - 6)/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
[Out] Integral(csc(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\frac{70\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3}+\frac{35(\cos(dx+c)+1)}{a^3\sin(dx+c)}}{560\,d}$$

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] -1/560*((70*sin(d*x + c)/(cos(d*x + c) + 1) - 14*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^3 + 35*(cos(d*x + c) +
 1)/(a^3*sin(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{35}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\frac{5a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-14a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+70a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{21}}}{560\,d}$$

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] -1/560*(35/(a^3*tan(1/2*d*x + 1/2*c)) + (5*a^18*tan(1/2*d*x + 1/2*c)^7 - 14
*a^18*tan(1/2*d*x + 1/2*c)^5 + 70*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{-16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8+8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6+72\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4-34\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2+5}{560\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

```
[In] int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(72*cos(c/2 + (d*x)/2)^4 - 34*cos(c/2 + (d*x)/2)^2 + 8*cos(c/2 + (d*x)/2)^
6 - 16*cos(c/2 + (d*x)/2)^8 + 5)/(560*a^3*d*cos(c/2 + (d*x)/2)^7*sin(c/2 +
(d*x)/2))
```

3.105 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3} dx = \frac{3\cot^5(c+dx)}{5a^3d} + \frac{\cot^7(c+dx)}{a^3d} + \frac{4\cot^9(c+dx)}{9a^3d} - \frac{3\csc^5(c+dx)}{5a^3d} + \frac{\csc^7(c+dx)}{a^3d} - \frac{4\csc^9(c+dx)}{9a^3d}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+cot(d*x+c)^7/a^3/d+4/9*cot(d*x+c)^9/a^3/d-3/5*csc(d*
x+c)^5/a^3/d+csc(d*x+c)^7/a^3/d-4/9*csc(d*x+c)^9/a^3/d
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2954, 2952, 2687, 14, 2686, 276}

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3} dx = \frac{4\cot^9(c+dx)}{9a^3d} + \frac{\cot^7(c+dx)}{a^3d} + \frac{3\cot^5(c+dx)}{5a^3d} - \frac{4\csc^9(c+dx)}{9a^3d} + \frac{\csc^7(c+dx)}{a^3d} - \frac{3\csc^5(c+dx)}{5a^3d}$$

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + Cot[c + d*x]^7/(a^3*d) + (4*Cot[c + d*x]^9)/
(9*a^3*d) - (3*Csc[c + d*x]^5)/(5*a^3*d) + Csc[c + d*x]^7/(a^3*d) - (4*Csc[
c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot^3(c+dx)\csc(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^7(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^6(c+dx)\csc^4(c+dx)+3a^3\cot^5(c+dx)\csc^5(c+dx)-3a^3\cot^4(c+dx)\csc^6(c+dx)+\right.}{a^6} \\
&= -\frac{\int \cot^6(c+dx)\csc^4(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^7(c+dx)\,dx}{a^3} \\
&\quad +\frac{3\int \cot^5(c+dx)\csc^5(c+dx)\,dx}{a^3}-\frac{3\int \cot^4(c+dx)\csc^6(c+dx)\,dx}{a^3} \\
&= -\frac{\operatorname{Subst}\left(\int x^6(-1+x^2)\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{\operatorname{Subst}\left(\int x^6(1+x^2)\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\operatorname{Subst}\left(\int x^4(-1+x^2)^2\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\operatorname{Subst}\left(\int x^4(1+x^2)^2\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&= -\frac{\operatorname{Subst}\left(\int (-x^6+x^8)\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{\operatorname{Subst}\left(\int (x^6+x^8)\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\operatorname{Subst}\left(\int (x^4-2x^6+x^8)\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\operatorname{Subst}\left(\int (x^4+2x^6+x^8)\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{a^3d}+\frac{4\cot^9(c+dx)}{9a^3d} \\
&\quad -\frac{3\csc^5(c+dx)}{5a^3d}+\frac{\csc^7(c+dx)}{a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{\csc(c)\csc^3(2(c+dx))(5376\sin(c)-1152\sin(dx)-1764\sin(c+dx)-1323\sin(2(c+dx))+98\sin(3(c}{}$$

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/5760*(Csc[c]*Csc[2*(c + d*x)]^3*(5376*Sin[c] - 1152*Sin[d*x] - 1764*Sin[
c + d*x] - 1323*Sin[2*(c + d*x)] + 98*Sin[3*(c + d*x)] + 588*Sin[4*(c + d*x
)] + 294*Sin[5*(c + d*x)] + 49*Sin[6*(c + d*x)] + 3456*Sin[2*c + d*x] - 115
2*Sin[c + 2*d*x] + 2880*Sin[3*c + 2*d*x] - 128*Sin[2*c + 3*d*x] - 768*Sin[3
*c + 4*d*x] - 384*Sin[4*c + 5*d*x] - 64*Sin[5*c + 6*d*x]))/(a^3*d*(1 + Sec[
c + d*x])^3)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

method	result	si
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{64d\,a^{3}}$	60
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{64d\,a^{3}}$	60
parallelrisch	$\frac{-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+27\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-135\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-15}{2880d\,a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}$	61
norman	$\frac{-\frac{1}{192ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{576ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{64da}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{320da}}{a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}$	82
risch	$\frac{4i\left(45\,\mathrm{e}^{8i(dx+c)}+54\,\mathrm{e}^{7i(dx+c)}+84\,\mathrm{e}^{6i(dx+c)}+18\,\mathrm{e}^{5i(dx+c)}+18\,\mathrm{e}^{4i(dx+c)}+2\,\mathrm{e}^{3i(dx+c)}+12\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}+1\right)}{45a^{3}d\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}\left(\mathrm{e}^{i(dx+c)}-1\right)^{3}}$	12

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a^3*(-1/9*tan(1/2*d*x+1/2*c)^9+3/5*tan(1/2*d*x+1/2*c)^5-3*tan(1/2*d*
x+1/2*c)-1/3/tan(1/2*d*x+1/2*c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\cos(dx+c)^6+6\cos(dx+c)^5+3\cos(dx+c)^4-7\cos(dx+c)^3+3\cos(dx+c)^2+6\cos(dx}{45\left(a^3d\cos(dx+c)^5+3a^3d\cos(dx+c)^4+2a^3d\cos(dx+c)^3-2a^3d\cos(dx+c)^2-3a^3d\cos(dx+c)-\right.}$$

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/45*(2*cos(d*x + c)^6 + 6*cos(d*x + c)^5 + 3*cos(d*x + c)^4 - 7*cos(d*x +
c)^3 + 3*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)/((a^3*d*cos(d*x + c)^5 + 3*a^
3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^
3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\frac{135\sin(dx+c)}{\cos(dx+c)+1}-\frac{27\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^3}+\frac{15(\cos(dx+c)+1)^3}{a^3\sin(dx+c)^3}}{2880\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2880*((135*sin(d*x + c)/(cos(d*x + c) + 1) - 27*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^3 + 15*(cos(d*x + c)
 + 1)^3/(a^3*sin(d*x + c)^3))/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{15}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3} + \frac{5\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 27\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 135\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{27}}}{2880\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2880*(15/(a^3*tan(1/2*d*x + 1/2*c)^3) + (5*a^24*tan(1/2*d*x + 1/2*c)^9 -
 27*a^24*tan(1/2*d*x + 1/2*c)^5 + 135*a^24*tan(1/2*d*x + 1/2*c))/a^27)/d
```

Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} + 135\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 27\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{2880\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

```
[In] int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(15*cos(c/2 + (d*x)/2)^12 + 5*sin(c/2 + (d*x)/2)^12 - 27*cos(c/2 + (d*x)/2
)^4*sin(c/2 + (d*x)/2)^8 + 135*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4)/(
2880*a^3*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)
```

3.106 $\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 127

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx=&\frac{3\cot^5(c+dx)}{5a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}\\&+\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{3\csc^7(c+dx)}{7a^3d}\\&+\frac{7\csc^9(c+dx)}{9a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}\end{aligned}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+10/7*cot(d*x+c)^7/a^3/d+11/9*cot(d*x+c)^9/a^3/d+4/11
*cot(d*x+c)^11/a^3/d-3/7*csc(d*x+c)^7/a^3/d+7/9*csc(d*x+c)^9/a^3/d-4/11*csc
(d*x+c)^11/a^3/d
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2954, 2952, 2687, 276, 2686, 14}

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx=&\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}\\&+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}\\&+\frac{7\csc^9(c+dx)}{9a^3d}-\frac{3\csc^7(c+dx)}{7a^3d}\end{aligned}$$

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (10*Cot[c + d*x]^7)/(7*a^3*d) + (11*Cot[c +
d*x]^9)/(9*a^3*d) + (4*Cot[c + d*x]^11)/(11*a^3*d) - (3*Csc[c + d*x]^7)/(7*
a^3*d) + (7*Csc[c + d*x]^9)/(9*a^3*d) - (4*Csc[c + d*x]^11)/(11*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot^3(c+dx)\csc^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int(-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^9(c+dx)\,dx}{a^6} \\
&= \frac{\int\left(-a^3\cot^6(c+dx)\csc^6(c+dx)+3a^3\cot^5(c+dx)\csc^7(c+dx)-3a^3\cot^4(c+dx)\csc^8(c+dx)+\right.}{a^6} \\
&= -\frac{\int\cot^6(c+dx)\csc^6(c+dx)\,dx}{a^3}+\frac{\int\cot^3(c+dx)\csc^9(c+dx)\,dx}{a^3} \\
&\quad+\frac{3\int\cot^5(c+dx)\csc^7(c+dx)\,dx}{a^3}-\frac{3\int\cot^4(c+dx)\csc^8(c+dx)\,dx}{a^3} \\
&= -\frac{\text{Subst}\left(\int x^8(-1+x^2)\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad-\frac{\text{Subst}\left(\int x^6(1+x^2)^2\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&\quad-\frac{3\text{Subst}\left(\int x^6(-1+x^2)^2\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad-\frac{3\text{Subst}\left(\int x^4(1+x^2)^3\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&= -\frac{\text{Subst}\left(\int(-x^8+x^{10})\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad-\frac{\text{Subst}\left(\int(x^6+2x^8+x^{10})\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&\quad-\frac{3\text{Subst}\left(\int(x^6-2x^8+x^{10})\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad-\frac{3\text{Subst}\left(\int(x^4+3x^6+3x^8+x^{10})\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d} \\
&\quad-\frac{3\csc^7(c+dx)}{7a^3d}+\frac{7\csc^9(c+dx)}{9a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.76

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$=\frac{\csc(c)\csc^5(c+dx)\sec^3(c+dx)(-3886080\sin(c)+563200\sin(dx)+524150\sin(c+dx)+314490\sin(2(c}{}$$

```
[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]^3*(-3886080*Sin[c] + 563200*Sin[d*x] +
524150*Sin[c + d*x] + 314490*Sin[2*(c + d*x)] - 162010*Sin[3*(c + d*x)] - 2
38250*Sin[4*(c + d*x)] - 47650*Sin[5*(c + d*x)] + 47650*Sin[6*(c + d*x)] +
28590*Sin[7*(c + d*x)] + 4765*Sin[8*(c + d*x)] - 2027520*Sin[2*c + d*x] + 1
486848*Sin[c + 2*d*x] - 2365440*Sin[3*c + 2*d*x] + 452608*Sin[2*c + 3*d*x]
+ 665600*Sin[3*c + 4*d*x] + 133120*Sin[4*c + 5*d*x] - 133120*Sin[5*c + 6*d*
x] - 79872*Sin[6*c + 7*d*x] - 13312*Sin[7*c + 8*d*x]))/(56770560*a^3*d*(1 +
 Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{-315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-770\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+990\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+4158\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-693\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-2310\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{887040a^3d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{}{3\tan(}}{256d\,a^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{}{3\tan(}}{256d\,a^3}$
risch	$-\frac{16i(2310\,e^{10i(dx+c)}+1980\,e^{9i(dx+c)}+3795\,e^{8i(dx+c)}+550\,e^{7i(dx+c)}+1452\,e^{6i(dx+c)}+442\,e^{5i(dx+c)}+650\,e^{4i(dx+c)}+1}{3465a^3d\left(e^{i(dx+c)}+1\right)^{11}\left(e^{i(dx+c)}-1\right)^{5}}$
norman	$\frac{-\frac{1}{1280ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{896ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{1152ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}}{2816ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{384da}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{128da}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{128da}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{1}}{640da}}{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}$

```
[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/887040*(-315*tan(1/2*d*x+1/2*c)^11-770*tan(1/2*d*x+1/2*c)^9+990*tan(1/2*d
*x+1/2*c)^7+4158*tan(1/2*d*x+1/2*c)^5-693*cot(1/2*d*x+1/2*c)^5-2310*cot(1/2
*d*x+1/2*c)^3-20790*tan(1/2*d*x+1/2*c)+6930*cot(1/2*d*x+1/2*c))/a^3/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.50

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{104\,\cos{(dx+c)}^8+312\,\cos{(dx+c)}^7+52\,\cos{(dx+c)}^6-676\,\cos{(dx+c)}^5-585\,\cos{(dx+c)}^4+325}{3465\,\left(a^3d\cos{(dx+c)}^7+3\,a^3d\cos{(dx+c)}^6+a^3d\cos{(dx+c)}^5-5\,a^3d\cos{(dx+c)}^4-5\,a^3d\cos{(dx+c)}^3\right.}$$

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(104*cos(d*x + c)^8 + 312*cos(d*x + c)^7 + 52*cos(d*x + c)^6 - 676*c
os(d*x + c)^5 - 585*cos(d*x + c)^4 + 325*cos(d*x + c)^3 - 25*cos(d*x + c)^2
 - 150*cos(d*x + c) - 50)/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 +
 a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\csc^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.37

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{\frac{\frac{20790\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{4158\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{990\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{770\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{315\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^3}+\frac{231\left(\frac{10\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{30\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)(\cos(dx+c)}{a^3\sin(dx+c)^5}}{887040\,d}$$

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/887040*((20790*sin(d*x + c)/(cos(d*x + c) + 1) - 4158*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 990*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 770*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9 + 315*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^3
 + 231*(10*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 30*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 3)*(cos(d*x + c) + 1)^5/(a^3*sin(d*x + c)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{231\left(30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}-\frac{315\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+770\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-990\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-4158\,a^{30}\,\mathrm{ta}}{a^{33}}}{887040\,d}$$

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/887040*(231*(30*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d*x + 1/2*c)^2 - 3)/(
a^3*tan(1/2*d*x + 1/2*c)^5) - (315*a^30*tan(1/2*d*x + 1/2*c)^11 + 770*a^30*
tan(1/2*d*x + 1/2*c)^9 - 990*a^30*tan(1/2*d*x + 1/2*c)^7 - 4158*a^30*tan(1/
2*d*x + 1/2*c)^5 + 20790*a^30*tan(1/2*d*x + 1/2*c))/a^33)/d
```

Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.58

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{693\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+2310\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-6930\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+20790\,\mathrm{cos}}{}$$

```
[In] int(1/(sin(c + d*x)^6*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(693*cos(c/2 + (d*x)/2)^16 + 315*sin(c/2 + (d*x)/2)^16 + 770*cos(c/2 + (d*
x)/2)^2*sin(c/2 + (d*x)/2)^14 - 990*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)
^12 - 4158*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 20790*cos(c/2 + (d*
x)/2)^10*sin(c/2 + (d*x)/2)^6 - 6930*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/
2)^4 + 2310*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2)/(887040*a^3*d*cos(c
/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5)
```

3.107 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\cot^5(c+dx)}{5a^3d}+\frac{13\cot^7(c+dx)}{7a^3d}+\frac{7\cot^9(c+dx)}{3a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d}+\frac{4\cot^{13}(c+dx)}{13a^3d}-\frac{\csc^9(c+dx)}{3a^3d}+\frac{7\csc^{11}(c+dx)}{11a^3d}-\frac{4\csc^{13}(c+dx)}{13a^3d}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+13/7*cot(d*x+c)^7/a^3/d+7/3*cot(d*x+c)^9/a^3/d+15/11
*cot(d*x+c)^11/a^3/d+4/13*cot(d*x+c)^13/a^3/d-1/3*csc(d*x+c)^9/a^3/d+7/11*c
sc(d*x+c)^11/a^3/d-4/13*csc(d*x+c)^13/a^3/d
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2954, 2952, 2687, 276, 2686, 14}

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{4\cot^{13}(c+dx)}{13a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d}+\frac{7\cot^9(c+dx)}{3a^3d}+\frac{13\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^{13}(c+dx)}{13a^3d}+\frac{7\csc^{11}(c+dx)}{11a^3d}-\frac{\csc^9(c+dx)}{3a^3d}$$

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (13*Cot[c + d*x]^7)/(7*a^3*d) + (7*Cot[c + d
*x]^9)/(3*a^3*d) + (15*Cot[c + d*x]^11)/(11*a^3*d) + (4*Cot[c + d*x]^13)/(1
3*a^3*d) - Csc[c + d*x]^9/(3*a^3*d) + (7*Csc[c + d*x]^11)/(11*a^3*d) - (4*C
sc[c + d*x]^13)/(13*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot^3(c+dx)\csc^5(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^{11}(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^6(c+dx)\csc^8(c+dx)+3a^3\cot^5(c+dx)\csc^9(c+dx)-3a^3\cot^4(c+dx)\csc^{10}(c+dx)+\right.}{a^6} \\
&= -\frac{\int \cot^6(c+dx)\csc^8(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^{11}(c+dx)\,dx}{a^3} \\
&\quad +\frac{3\int \cot^5(c+dx)\csc^9(c+dx)\,dx}{a^3}-\frac{3\int \cot^4(c+dx)\csc^{10}(c+dx)\,dx}{a^3} \\
&= -\frac{\operatorname{Subst}\left(\int x^{10}(-1+x^2)\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{\operatorname{Subst}\left(\int x^6(1+x^2)^3\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\operatorname{Subst}\left(\int x^8(-1+x^2)^2\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\operatorname{Subst}\left(\int x^4(1+x^2)^4\,dx,x,-\cot(c+dx)\right)}{a^3d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(-x^{10}+x^{12}\right)dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{\operatorname{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)dx,x,-\cot(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\operatorname{Subst}\left(\int \left(x^8-2x^{10}+x^{12}\right)dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\operatorname{Subst}\left(\int \left(x^4+4x^6+6x^8+4x^{10}+x^{12}\right)dx,x,-\cot(c+dx)\right)}{a^3d} \\
&= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{13\cot^7(c+dx)}{7a^3d}+\frac{7\cot^9(c+dx)}{3a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d} \\
&\quad +\frac{4\cot^{13}(c+dx)}{13a^3d}-\frac{\csc^9(c+dx)}{3a^3d}+\frac{7\csc^{11}(c+dx)}{11a^3d}-\frac{4\csc^{13}(c+dx)}{13a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.10 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.83

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{\csc(c)\csc^7(c+dx)\sec^3(c+dx)(49201152\sin(c)-6336512\sin(dx)-2764580\sin(c+dx)-1382290\,\mathrm{si}}{}$$

[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]

[Out] -1/984023040*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]^3*(49201152*Sin[c] - 6336512*Sin[d*x] - 2764580*Sin[c + d*x] - 1382290*Sin[2*(c + d*x)] + 1275960*Sin[3*(c + d*x)] + 1336720*Sin[4*(c + d*x)] - 60760*Sin[5*(c + d*x)] - 524055*Sin[6*(c + d*x)] - 167090*Sin[7*(c + d*x)] + 60760*Sin[8*(c + d*x)] + 45570*Sin[9*(c + d*x)] + 7595*Sin[10*(c + d*x)] + 20500480*Sin[2*c + d*x] - 23668736*Sin[c + 2*d*x] + 30750720*Sin[3*c + 2*d*x] - 6537216*Sin[2*c + 3*d*x] - 6848512*Sin[3*c + 4*d*x] + 311296*Sin[4*c + 5*d*x] + 2684928*Sin[5*c + 6*d*x] + 856064*Sin[6*c + 7*d*x] - 311296*Sin[7*c + 8*d*x] - 233472*Sin[8*c + 9*d*x] - 38912*Sin[9*c + 10*d*x]))/(a^3*d*(1 + Sec[c + d*x])^3)

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

method	result
parallelrisch	$\frac{-1155\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}-5460\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-5005\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+17160\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+42042\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-210210\,\mathrm{ta}}{15375360a^3d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{13}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{3}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-}{1024d\,a^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{13}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{3}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-}{1024d\,a^3}$
risch	$\frac{32i\left(15015\,\mathrm{e}^{12i(dx+c)}+10010\,\mathrm{e}^{11i(dx+c)}+24024\,\mathrm{e}^{10i(dx+c)}+3094\,\mathrm{e}^{9i(dx+c)}+11557\,\mathrm{e}^{8i(dx+c)}+3192\,\mathrm{e}^{7i(dx+c)}+3344\,\mathrm{e}^{6i(d}\right.}{15015a^3d\left(\mathrm{e}^{i(dx+c)}+1\right)^{13}\left(\mathrm{e}^{i(dx+c)}-1\right.}$

[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/15375360*(-1155*tan(1/2*d*x+1/2*c)^13-5460*tan(1/2*d*x+1/2*c)^11-5005*tan(1/2*d*x+1/2*c)^9+17160*tan(1/2*d*x+1/2*c)^7+42042*tan(1/2*d*x+1/2*c)^5-210210*tan(1/2*d*x+1/2*c)-2145*cot(1/2*d*x+1/2*c)*(cot(1/2*d*x+1/2*c)^6+28/5*cot(1/2*d*x+1/2*c)^4+7*cot(1/2*d*x+1/2*c)^2-56))/a^3/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.48

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{304\cos(dx+c)^{10}+912\cos(dx+c)^9-152\cos(dx+c)^8-2888\cos(dx+c)^7-1862\cos(dx+c)^6+292\ldots}{15015\left(a^3d\cos(dx+c)^9+3a^3d\cos(dx+c)^8-8a^3d\cos(dx+c)^6-6a^3d\cos(dx+c)^5\ldots\right.}$$

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15015*(304*cos(d*x + c)^10 + 912*cos(d*x + c)^9 - 152*cos(d*x + c)^8 - 28
88*cos(d*x + c)^7 - 1862*cos(d*x + c)^6 + 2926*cos(d*x + c)^5 + 3325*cos(d*
x + c)^4 - 665*cos(d*x + c)^3 - 35*cos(d*x + c)^2 + 210*cos(d*x + c) + 70)/
((a^3*d*cos(d*x + c)^9 + 3*a^3*d*cos(d*x + c)^8 - 8*a^3*d*cos(d*x + c)^6 -
6*a^3*d*cos(d*x + c)^5 + 6*a^3*d*cos(d*x + c)^4 + 8*a^3*d*cos(d*x + c)^3 -
3*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.48

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\frac{210210\sin(dx+c)}{\cos(dx+c)+1}-\frac{42042\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{17160\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{5005\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{5460\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{1155\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}}{a^3}+\frac{429\left(\frac{28\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{35\sin(dx\ldots}{(\cos(dx+c\ldots}\right.}{a\ldots}}{15375360\,d}$$

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15375360*((210210*sin(d*x + c)/(cos(d*x + c) + 1) - 42042*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 17160*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5005*si
```

```
n(d*x + c)^9/(cos(d*x + c) + 1)^9 + 5460*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11 + 1155*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)/a^3 + 429*(28*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 35*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 280*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5)*(cos(d*x + c) + 1)^7/(a^3*sin(d*x + c
)^7))/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{429\left(280\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-28\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-5\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7} - \frac{1155\,a^{36}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+5460\,a^{36}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+5005\,a^{36}\,\mathrm{ta}}{}}{15375360\,d}$$

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15375360*(429*(280*tan(1/2*d*x + 1/2*c)^6 - 35*tan(1/2*d*x + 1/2*c)^4 - 2
8*tan(1/2*d*x + 1/2*c)^2 - 5)/(a^3*tan(1/2*d*x + 1/2*c)^7) - (1155*a^36*tan
(1/2*d*x + 1/2*c)^13 + 5460*a^36*tan(1/2*d*x + 1/2*c)^11 + 5005*a^36*tan(1/
2*d*x + 1/2*c)^9 - 17160*a^36*tan(1/2*d*x + 1/2*c)^7 - 42042*a^36*tan(1/2*d
*x + 1/2*c)^5 + 210210*a^36*tan(1/2*d*x + 1/2*c))/a^39)/d
```

Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.72

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{2145\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}+12012\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+15015\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-12012}{}$$

```
[In] int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(2145*cos(c/2 + (d*x)/2)^20 + 1155*sin(c/2 + (d*x)/2)^20 + 5460*cos(c/2 +
(d*x)/2)^2*sin(c/2 + (d*x)/2)^18 + 5005*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x
)/2)^16 - 17160*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^14 - 42042*cos(c/2
+ (d*x)/2)^8*sin(c/2 + (d*x)/2)^12 + 210210*cos(c/2 + (d*x)/2)^12*sin(c/2 +
 (d*x)/2)^8 - 120120*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^6 + 15015*cos
(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^4 + 12012*cos(c/2 + (d*x)/2)^18*sin(c
/2 + (d*x)/2)^2)/(15375360*a^3*d*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^7
)
```

3.108 $\int (a+a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx = -\frac{ae^{5/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{5/2}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}$$

```
[Out] -a*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+a*e^(5/2)*arctanh((e*sin(
d*x+c))^(1/2)/e^(1/2))/d-2/3*a*e*(e*sin(d*x+c))^(3/2)/d-2/5*a*e*cos(d*x+c)*
(e*sin(d*x+c))^(3/2)/d-6/5*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x
+c))^(1/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3957, 2917, 2644, 327, 335, 304, 209, 212, 2715, 2721, 2719}

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx = -\frac{ae^{5/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{5/2}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{6ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}$$

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a*e^(5/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d) + (a*e^(5/2)*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (6*a*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2
]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (2*a*e*(e*Sin[c + d*x])^
(3/2))/(3*d) - (2*a*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^{5/2}\,dx \\
&= a\int (e\sin(c+dx))^{5/2}\,dx + a\int \sec(c+dx)(e\sin(c+dx))^{5/2}\,dx \\
&= -\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de} + \frac{1}{5}\left(3ae^2\right)\int \sqrt{e\sin(c+dx)}\,dx
\end{aligned}
$$

$$= -\frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{(ae)\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{d} + \frac{\left(3ae^2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{5\sqrt{\sin(c+dx)}}$$

$$= \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{(2ae)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d}$$

$$= \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{(ae^3)\,\text{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} - \frac{(ae^3)\,\text{Subst}\left(\int \frac{1}{e+x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d}$$

$$= -\frac{ae^{5/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{5/2}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx = -\frac{a(e\sin(c+dx))^{5/2}\left(15\arctan\left(\sqrt{\sin(c+dx)}\right) - 15\text{arctanh}\left(\sqrt{\sin(c+dx)}\right) + 18E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right.\right.}{15d\sin^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/15*(a*(e*Sin[c + d*x])^(5/2)*(15*ArcTan[Sqrt[Sin[c + d*x]]] - 15*ArcTanh
[Sqrt[Sin[c + d*x]]] + 18*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + 10*Sin[c +
d*x]^(3/2) + 3*Sqrt[Sin[c + d*x]]*Sin[2*(c + d*x)]))/(d*Sin[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 10.94 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.34

method	result
default	$\frac{-\frac{2ae(e\sin(dx+c))^{\frac{3}{2}}}{3}-a\,e^{\frac{5}{2}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+a\,e^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)-\frac{a\,e^{3}\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\mathrm{Ellip}\right.}{}}{d}$
parts	$-\frac{a\,e^{3}\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\mathrm{si}}\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*a*e*(e*sin(d*x+c))^(3/2)-a*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2
))+a*e^(5/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))-1/5*a*e^3*(6*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1
)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^4+2*sin
(d*x+c)^2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 649, normalized size of antiderivative = 4.13

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx = \left[\frac{72i\sqrt{2}a\sqrt{-i\,e}e^{2}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))}{}\right.$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/120*(72*I*sqrt(2)*a*sqrt(-I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInv
erse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 72*I*sqrt(2)*a*sqrt(I*e)*e^2*w
eierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c
))) - 30*a*sqrt(-e)*e^2*arctan(1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sq
rt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x + c) - e)) + 15*a
*sqrt(-e)*e^2*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x +
c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e)
 + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos
(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(3*a*e^2*cos(d
*x + c) + 5*a*e^2)*sqrt(e*sin(d*x + c))*sin(d*x + c))/d, 1/120*(72*I*sqrt(2
```

```
)*a*sqrt(-I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x
+ c) + I*sin(d*x + c))) - 72*I*sqrt(2)*a*sqrt(I*e)*e^2*weierstrassZeta(4, 0
, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 30*a*e^(5/2)*
arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(
e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e)) + 15*a*e^(5/2)*log((e*cos(d*x +
 c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*si
n(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*
sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2
 - 2)*sin(d*x + c) + 8)) - 16*(3*a*e^2*cos(d*x + c) + 5*a*e^2)*sqrt(e*sin(d
*x + c))*sin(d*x + c))/d]
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{5/2}\,dx = \int (a\sec(dx + c) + a)(e\sin(dx + c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{5/2}\,dx = \int (a\sec(dx + c) + a)(e\sin(dx + c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\sin(c + dx))^{5/2}\,dx = \int (e\,\sin(c + d\,x))^{5/2}\left(a + \frac{a}{\cos(c + d\,x)}\right)\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

3.109 $\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 154

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx=\frac{ae^{3/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{3/2}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}}-\frac{2ae\sqrt{e\sin(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}$$

```
[Out] a*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+a*e^(3/2)*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))/d-2/3*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c
)^(1/2)/d/(e*sin(d*x+c))^(1/2)-2*a*e*(e*sin(d*x+c))^(1/2)/d-2/3*a*e*cos(d*x
+c)*(e*sin(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3957, 2917, 2644, 327, 335, 218, 212, 209, 2715, 2721, 2720}

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx=\frac{ae^{3/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{3/2}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2ae^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2ae\sqrt{e\sin(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}$$

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*e^(3/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (a*e^(3/2)*ArcTanh[Sqr
t[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*a*e*Sqrt[e*Sin[c + d*x]]
)/d - (2*a*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^{3/2}\,dx \\
&= a\int (e\sin(c+dx))^{3/2}\,dx + a\int \sec(c+dx)(e\sin(c+dx))^{3/2}\,dx \\
&= -\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de} + \frac{1}{3}\left(ae^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx
\end{aligned}$$

$$= -\frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{(ae)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\sin(c+dx)\right)}{d} + \frac{\left(ae^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3\sqrt{e\sin(c+dx)}}$$

$$= \frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{(2ae)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d}$$

$$= \frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{(ae^2)\,\text{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} + \frac{(ae^2)\,\text{Subst}\left(\int \frac{1}{e+x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d}$$

$$= \frac{ae^{3/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{3/2}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx = \frac{a(e\sin(c+dx))^{3/2}\left(12\arctan\left(\sqrt{\sin(c+dx)}\right)+6\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)-8\,\text{EllipticF}\left(\frac{1}{4}(-2\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*(e*Sin[c + d*x])^(3/2)*(12*ArcTan[Sqrt[Sin[c + d*x]]] + 6*ArcTanh[Sqrt[S
in[c + d*x]]] - 8*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 3*Log[1 - Sqrt[Sin[
```

```
c + d*x]]] + 3*Log[1 + Sqrt[Sin[c + d*x]]] - 24*Sqrt[Sin[c + d*x]] - 8*Cos[
c + d*x]*Sec[2*(c + d*x)]*Sqrt[Sin[c + d*x]] + 16*Cos[c + d*x]*Sec[2*(c + d
*x)]*Sin[c + d*x]^(5/2)))/(12*d*Sin[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 10.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

method	result
default	$\frac{a e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)+a e^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)-2ae\sqrt{e\sin(dx+c)}-\frac{a e^{2}\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \text{Ellipt}\right.}{3\cos(dx+c)\sqrt{}}}{d}$
parts	$-\frac{a e^{2}\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^{3}+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{a\left(e^{\frac{3}{2}}\ \text{a}\right.}{}$

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))+a*e^(3/2)*arctanh((e*sin(d*
x+c))^(1/2)/e^(1/2))-2*a*e*(e*sin(d*x+c))^(1/2)-1/3*a*e^2*((-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/
2),1/2*2^(1/2))-2*sin(d*x+c)^3+2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2
))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.92

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx = \left[\frac{8\sqrt{2}a\sqrt{-i\,e}e\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+8\sqrt{2}a\sqrt{i\,e}e\text{weierstras}}{}\right.$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(8*sqrt(2)*a*sqrt(-I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 8*sqrt(2)*a*sqrt(I*e)*e*weierstrassPInverse(4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - 6*a*sqrt(-e)*e*arctan(1/4*(cos(d*x + c)^2 - 6*sin(d
*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x +
c) - e)) + 3*a*sqrt(-e)*e*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(
7*cos(d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x +
c))*sqrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x +
```

```
c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(a
*e*cos(d*x + c) + 3*a*e)*sqrt(e*sin(d*x + c)))/d, 1/24*(8*sqrt(2)*a*sqrt(-I
*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 8*sqrt(2)*
a*sqrt(I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 6*
a*e^(3/2)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x +
 c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e)) + 3*a*e^(3/2)*log((e*
cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^
2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d*x + c)^
2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d
*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(a*e*cos(d*x + c) + 3*a*e)*sqrt(e*si
n(d*x + c)))/d]
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx=\int (a\sec{(dx+c)}+a)(e\sin{(dx+c)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx=\int (a\sec{(dx+c)}+a)(e\sin{(dx+c)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx=\int (e\,\sin\left(c+d\,x\right))^{3/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,dx$$

```
[In] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)
```

3.110 $\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx = -\frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{a\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

```
[Out] -a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d+a*arctanh((e*sin(d*x+c))^
(1/2)/e^(1/2))*e^(1/2)/d-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+
1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))
^(1/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3957, 2917, 2644, 335, 304, 209, 212, 2721, 2719}

$$\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx = -\frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{a\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] -((a*Sqrt[e]*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d) + (a*Sqrt[e]*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sq
rt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int(-a-a\cos(c+dx))\sec(c+dx)\sqrt{e\sin(c+dx)}\,dx\\
&= a\int\sqrt{e\sin(c+dx)}\,dx+a\int\sec(c+dx)\sqrt{e\sin(c+dx)}\,dx\\
&= \frac{a\text{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{e^2}}\,dx,x,e\sin(c+dx)\right)}{de}+\frac{\left(a\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{\sqrt{\sin(c+dx)}}\\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{(2a)\text{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{e^2}}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{de}\\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{(ae)\text{Subst}\left(\int\frac{1}{e-x^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{d}\\
&\quad-\frac{(ae)\text{Subst}\left(\int\frac{1}{e+x^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{d}\\
&= -\frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}\\
&\quad+\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx = \frac{a\left(-\arctan\left(\sqrt{\sin(c+dx)}\right)+\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)-2E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
[Out] (a*(-ArcTan[Sqrt[Sin[c + d*x]]] + ArcTanh[Sqrt[Sin[c + d*x]]] - 2*EllipticE
[(-2*c + Pi - 2*d*x)/4, 2])*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])
```

Maple [A] (verified)

Time = 8.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.37

method	result
default	$\frac{-a\sqrt{e}\,\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+a\sqrt{e}\,\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)-\frac{ae\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)}\right.\right.}{\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{ae\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-$

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] (-a*e^(1/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))+a*e^(1/2)*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))-a*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((-si
n(d*x+c)+1)^(1/2),1/2*2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 555, normalized size of antiderivative = 5.34

$$\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx = \left[\frac{8i\sqrt{2}a\sqrt{-ie}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-8i\sqrt{2}a\sqrt{ie}}{}\right.$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(8*I*sqrt(2)*a*sqrt(-I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(d*x + c) + I*sin(d*x + c))) - 8*I*sqrt(2)*a*sqrt(I*e)*weierstrassZe
ta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*a*sq
rt(-e)*arctan(1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c)
)*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x + c) - e)) + a*sqrt(-e)*log((e*cos
(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 - (cos(d*x + c)^2 -
 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) + 28*(e*cos(d*x + c)^2
- 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x
 + c)^2 - 2)*sin(d*x + c) + 8)))/d, 1/8*(8*I*sqrt(2)*a*sqrt(-I*e)*weierstra
ssZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 8*
I*sqrt(2)*a*sqrt(I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d
*x + c) - I*sin(d*x + c))) - 2*a*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin
(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x +
 c) - e)) + a*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*co
s(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*
sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4
- 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/d]
```

Sympy [F]

$$\int (a + a\sec(c + dx))\sqrt{e\sin(c + dx)}\,dx = a\left(\int \sqrt{e\sin{(c + dx)}}\,dx + \int \sqrt{e\sin{(c + dx)}}\sec{(c + dx)}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*sin(c + d*x)), x) + Integral(sqrt(e*sin(c + d*x))*sec(c
+ d*x), x))
```

Maxima [F]

$$\int (a + a\sec(c + dx))\sqrt{e\sin(c + dx)}\,dx = \int (a\sec{(dx + c)} + a)\sqrt{e\sin{(dx + c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

Giac [F]

$$\int (a + a\sec(c + dx))\sqrt{e\sin(c + dx)}\,dx = \int (a\sec{(dx + c)} + a)\sqrt{e\sin{(dx + c)}}\,dx$$

[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))\sqrt{e\sin(c + dx)}\,dx = \int \sqrt{e\,\sin{(c + d\,x)}}\,\left(a + \frac{a}{\cos{(c + d\,x)}}\right)\,dx$$

[In] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)

[Out] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)

3.111 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}$$

```
[Out] a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(1/2)+a*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))/d/e^(1/2)-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2
)/d/(e*sin(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3957, 2917, 2644, 335, 218, 212, 209, 2721, 2720}

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (a*ArcTanh[Sqrt[e*Si
n[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\\
&= a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + a\int \frac{\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\\
&= \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\sin(c+dx)\right)}{de} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{\sqrt{e\sin(c+dx)}}\\
&= \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{(2a)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de}\\
&= \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}\\
&\quad + \frac{a\text{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} + \frac{a\text{Subst}\left(\int \frac{1}{e+x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d}\\
&= \frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{a\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}\\
&\quad + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.95 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.19

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\sin(c + dx)}}\,dx$$
$$= \frac{9a\,\text{AppellF1}\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\cot^2\left(\frac{1}{2}(c\right.\right.}{5d\left(4\left(-2\,\text{AppellF1}\left(\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\cot^2\left(\frac{1}{2}(c + dx)\right), \cot^2\left(\frac{1}{2}(c + dx)\right)\right) + \text{AppellF1}\left(\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\cot^2\left(\frac{1}{2}(c\right.\right.\right.\right.}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (9*a*AppellF1[5/4, 1/2, 1, 9/4, -Cot[(c + d*x)/2]^2, Cot[(c + d*x)/2]^2]*Se
c[(c + d*x)/2]^2*(1 + Sec[c + d*x])*Sin[c + d*x]^3)/(5*d*(4*(-2*AppellF1[9/
4, 1/2, 2, 13/4, -Cot[(c + d*x)/2]^2, Cot[(c + d*x)/2]^2] + AppellF1[9/4, 3
/2, 1, 13/4, -Cot[(c + d*x)/2]^2, Cot[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^2 +
 9*AppellF1[5/4, 1/2, 1, 9/4, -Cot[(c + d*x)/2]^2, Cot[(c + d*x)/2]^2]*(-1
+ Cos[c + d*x]))*Sqrt[e*Sin[c + d*x]])
```

Maple [A] (verified)

Time = 7.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12

method	result
parts	$-\frac{a\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1}, \frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d} + \frac{a\left(\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+\text{arctanh}\left(\frac{\sqrt{e\,\text{s}}}{}\right.\right.}{\sqrt{e}\,d}$
default	$\frac{\frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{a\,\text{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{\sqrt{e}}-\frac{a\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1}, \frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

```
[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d+a*(arc
tan((e*sin(d*x+c))^(1/2)/e^(1/2))+arctanh((e*sin(d*x+c))^(1/2)/e^(1/2)))/e^
(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 550, normalized size of antiderivative = 5.34

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\sin(c + dx)}}\,dx$$
$$= \left[\frac{8\sqrt{2}a\sqrt{-i\,e}\,\mathrm{weierstrassPInverse}(4, 0, \cos(dx + c) + i\,\sin(dx + c)) + 8\sqrt{2}a\sqrt{i\,e}\,\mathrm{weierstrassPInverse}(4, 0,}{}\right.$$

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(8*sqrt(2)*a*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin
(d*x + c)) + 8*sqrt(2)*a*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) -
 I*sin(d*x + c)) - 2*a*sqrt(-e)*arctan(1/4*(cos(d*x + c)^2 - 6*sin(d*x + c)
 - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x + c) - e)
) - a*sqrt(-e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x +
 c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e
) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*co
s(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/(d*e), 1/8*(8*sqr
t(2)*a*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))
+ 8*sqrt(2)*a*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x
+ c)) + 2*a*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e
*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e)) + a*sqrt(e)
*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d
*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d
*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 +
4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/(d*e)]
```

Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\sin(c + dx)}}\,dx = a\left(\int \frac{1}{\sqrt{e\sin(c + dx)}}\,dx + \int \frac{\sec(c + dx)}{\sqrt{e\sin(c + dx)}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*sin(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*sin(c
 + d*x)), x))
```

Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\sin(c + dx)}}\,dx = \int \frac{a\sec\left(dx + c\right) + a}{\sqrt{e\sin\left(dx + c\right)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

Giac [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\sin(c + dx)}}\,dx = \int \frac{a\sec\left(dx + c\right) + a}{\sqrt{e\sin\left(dx + c\right)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\sin(c + dx)}}\,dx = \int \frac{a + \frac{a}{\cos(c+d\,x)}}{\sqrt{e\,\sin\left(c + d\,x\right)}}\,dx$$

```
[In] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(1/2), x)
```

3.112 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx=-\frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{a\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{2a}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}$$

```
[Out] -a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+a*arctanh((e*sin(d*x+c))^
(1/2)/e^(1/2))/d/e^(3/2)-2*a/d/e/(e*sin(d*x+c))^(1/2)-2*a*cos(d*x+c)/d/e/(e
*sin(d*x+c))^(1/2)+2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)
/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3957, 2917, 2644, 331, 335, 304, 209, 212, 2716, 2721, 2719}

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx=-\frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{a\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2a}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2))) + (a*ArcTanh[Sqrt[e
*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) - (2*a)/(d*e*Sqrt[e*Sin[c + d*x]]) - (
2*a*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx\\
&= a\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx + a\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx\\
&= -\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{a\int\sqrt{e\sin(c+dx)}\,dx}{e^2} + \frac{a\text{Subst}\left(\int\frac{1}{x^{3/2}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\sin(c+dx)\right)}{de}\\
&= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} + \frac{a\text{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de^3}\\
&\quad - \frac{\left(a\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{e^2\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} + \frac{(2a)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de^3}$$

$$= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de} - \frac{a\text{Subst}\left(\int \frac{1}{e+x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de}$$

$$= -\frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{a\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{a(1+\cos(c+dx))\sec\left(\frac{1}{2}(c+dx)\right)\left(\arctan\left(\sqrt{\sin(c+dx)}\right)\sec\left(\frac{1}{2}(c+dx)\right) - \text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\text{sec}}{2d(e\sin(c}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(3/2),x]

[Out] -1/2*(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]*(ArcTan[Sqrt[Sin[c + d*x]]]*Sec
[(c + d*x)/2] - ArcTanh[Sqrt[Sin[c + d*x]]]*Sec[(c + d*x)/2] - 2*EllipticE[
(-2*c + Pi - 2*d*x)/4, 2]*Sec[(c + d*x)/2] + 2*Csc[(c + d*x)/2]*Sqrt[Sin[c
+ d*x]])*Sin[c + d*x]^(3/2))/(d*(e*Sin[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 10.07 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.30

method	result
default	$\frac{-\frac{a \arctan\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{3}{2}}}-\frac{2a}{e\sqrt{e\sin(dx+c)}}+\frac{a\ \mathrm{arctanh}\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{3}{2}}}+\frac{a\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \mathrm{EllipticE}\left(\sqrt{-\sin(dx\cdots\right.\right.}{}}{d}$
parts	$\frac{a\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+\cdots\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] (-a/e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))-2*a/e/(e*sin(d*x+c))^(1/2)
+a/e^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+a/e*(2*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))-(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*
EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 670, normalized size of antiderivative = 4.32

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \left[\frac{-8i\sqrt{2}a\sqrt{-i\,e}\sin(dx+c)\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx\cdots}{}\right.$$

[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/8*(-8*I*sqrt(2)*a*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstr
assPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 8*I*sqrt(2)*a*sqrt(I*e)
*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*a*sqrt(-e)*arctan(1/4*(cos(d*x + c)^2 - 6*sin(d*x +
c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x + c) -
e))*sin(d*x + c) - a*sqrt(-e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 +
8*(7*cos(d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*
x + c))*sqrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*
x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))*sin
(d*x + c) - 16*(a*cos(d*x + c) + a)*sqrt(e*sin(d*x + c)))/(d*e^2*sin(d*x +
c)), 1/8*(-8*I*sqrt(2)*a*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weie
rstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 8*I*sqrt(2)*a*sqrt(
I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x +
c) - I*sin(d*x + c))) - 2*a*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x
+ c) - 2)*sqrt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c)

```
- e))*sin(d*x + c) + a*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2
- 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d
*x + c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*
x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))*sin
(d*x + c) - 16*(a*cos(d*x + c) + a)*sqrt(e*sin(d*x + c)))/(d*e^2*sin(d*x +
c))]
```

Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = a\left(\int \frac{1}{(e\sin{(c+dx)})^{\frac{3}{2}}}\,dx + \int \frac{\sec{(c+dx)}}{(e\sin{(c+dx)})^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
[Out] a*(Integral((e*sin(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*sin(c +
 d*x))**(3/2), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{a\sec{(dx+c)}+a}{(e\sin{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx=\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(e\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(3/2), x)
```

3.113 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx &= \frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} \\ &+ \frac{a\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{2a}{3de(e\sin(c+dx))^{3/2}} \\ &- \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}\end{aligned}$$

```
[Out] a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)+a*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))/d/e^(5/2)-2/3*a/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*cos(d*x+c)/d/e
/(e*sin(d*x+c))^(3/2)-2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2
)/d/e^2/(e*sin(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules

used = {3957, 2917, 2644, 331, 335, 218, 212, 209, 2716, 2721, 2720}

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{a\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}} - \frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}$$

```
[In] Int[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) + (a*ArcTanh[Sqrt[e*Si
n[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) - (2*a)/(3*d*e*(e*Sin[c + d*x])^(3/2)) -
(2*a*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a*EllipticF[(c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx \\
&= a\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx + a\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx \\
&= -\frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2} + \frac{a\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\sin(c+dx)\right)}{de} \\
&= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\sin(c+dx)\right)}{de^3} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e^2\sqrt{e\sin(c+dx)}} \\
&= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{(2a)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de^3} \\
&= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de^2} + \frac{a\text{Subst}\left(\int \frac{1}{e+x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de^2} \\
&= \frac{a\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{a\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{2a}{3de(e\sin(c+dx))^{3/2}} \\
&\quad - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.75

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx =$$
$$-\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-3\arctan\left(\sqrt{\sin(c+dx)}\right)-3\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)+2\,\text{EllipticF}\right.}{6de^2\sqrt{e\sin(c+dx)}}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/6*(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-3*ArcTan[Sqrt[Sin[c + d*x]]
] - 3*ArcTanh[Sqrt[Sin[c + d*x]]] + 2*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] +
 Csc[(c + d*x)/2]^2*Sqrt[Sin[c + d*x]])*Sqrt[Sin[c + d*x]])/(d*e^2*Sqrt[e*S
in[c + d*x]])
```

Maple [A] (verified)

Time = 11.42 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02

method	result
default	$\frac{-\frac{2a}{3e(e\sin(dx+c))^{\frac{3}{2}}}+\frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{5}{2}}}+\frac{a\,\text{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{5}{2}}}-\frac{a\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{5}{2}}\,\text{EllipticF}\left(\sqrt{-\sin\right.\right.}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin}}}{d}$
parts	$-\frac{a\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{5}{2}}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^3+2\sin(dx+c)\right)}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{a\left(-\frac{}{3e(e\,\mathrm{s}}\right.}{}$

```
[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*a/e/(e*sin(d*x+c))^(3/2)+a/e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2
))+a/e^(5/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))-1/3*a/e^2*((-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))-2*sin(d*x+c)^3+2*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 705, normalized size of antiderivative = 4.41

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \left[-\frac{6\,(a\cos\,(dx+c)-a)\sqrt{-e}\arctan\left(\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{e\sin(dx+c)}\sqrt{-e}}{4\left(e\cos(dx+c)^2-e\sin(dx+c)-e\right)}\right)+3\,(\right.$$

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(6*(a*cos(d*x + c) - a)*sqrt(-e)*arctan(1/4*(cos(d*x + c)^2 - 6*sin(
d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x +
 c) - e)) + 3*(a*cos(d*x + c) - a)*sqrt(-e)*log((e*cos(d*x + c)^4 - 72*e*co
s(d*x + c)^2 - 8*(7*cos(d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)
*sqrt(e*sin(d*x + c))*sqrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) +
 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x
+ c) + 8)) - 8*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-I*e)*weierstrassP
Inverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) - 8*(sqrt(2)*a*cos(d*x + c) -
sqrt(2)*a)*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c
)) - 16*sqrt(e*sin(d*x + c))*a)/(d*e^3*cos(d*x + c) - d*e^3), 1/24*(6*(a*co
s(d*x + c) - a)*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sq
rt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e)) + 3*(a*
cos(d*x + c) - a)*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(
7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x +
c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c
)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 8*(sqr
t(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 8*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(I*e
)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 16*sqrt(e*sin(
d*x + c))*a)/(d*e^3*cos(d*x + c) - d*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{(e\sin(c + dx))^{5/2}}\,dx = \int \frac{a\sec\left(dx + c\right) + a}{\left(e\sin\left(dx + c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{a + a\sec(c + dx)}{(e\sin(c + dx))^{5/2}}\,dx = \int \frac{a\sec\left(dx + c\right) + a}{\left(e\sin\left(dx + c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\sin(c + dx))^{5/2}}\,dx = \int \frac{a + \frac{a}{\cos(c+d\,x)}}{\left(e\,\sin\left(c + d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(5/2), x)
```

3.114 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 194

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = -\frac{2a^2e^{5/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{5/2}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{4a^2e(e\sin(c+dx))^{3/2}}{3d} - \frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3/2}}{d}$$

```
[Out] -2*a^2*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+2*a^2*e^(5/2)*arctanh
((e*sin(d*x+c))^(1/2)/e^(1/2))/d-4/3*a^2*e*(e*sin(d*x+c))^(3/2)/d-2/5*a^2*e
*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+a^2*e*sec(d*x+c)*(e*sin(d*x+c))^(3/2)/d+
9/5*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules

used = {3957, 2952, 2715, 2721, 2719, 2644, 327, 335, 304, 209, 212, 2646}

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = -\frac{2a^2e^{5/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{5/2}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{9a^2e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{4a^2e(e\sin(c+dx))^{3/2}}{3d} - \frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3/2}}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*e^(5/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(5/2)*Ar
cTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (9*a^2*e^2*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (4*a^2*e*(e*Sin[
c + d*x])^(3/2))/(3*d) - (2*a^2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d
) + (a^2*e*Sec[c + d*x]*(e*Sin[c + d*x])^(3/2))/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a - a\cos(c+dx))^2 \sec^2(c+dx)(e\sin(c+dx))^{5/2}\,dx \\
&= \int \Big(a^2(e\sin(c+dx))^{5/2} \\
&\qquad + 2a^2\sec(c+dx)(e\sin(c+dx))^{5/2} + a^2\sec^2(c+dx)(e\sin(c+dx))^{5/2}\Big)\,dx \\
&= a^2\int (e\sin(c+dx))^{5/2}\,dx \\
&\quad + a^2\int \sec^2(c+dx)(e\sin(c+dx))^{5/2}\,dx + \left(2a^2\right)\int \sec(c+dx)(e\sin(c+dx))^{5/2}\,dx \\
&= -\frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3/2}}{d} \\
&\quad + \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de} \\
&\quad + \frac{1}{5}\left(3a^2e^2\right)\int \sqrt{e\sin(c+dx)}\,dx - \frac{1}{2}\left(3a^2e^2\right)\int \sqrt{e\sin(c+dx)}\,dx \\
&= -\frac{4a^2e(e\sin(c+dx))^{3/2}}{3d} - \frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} \\
&\quad + \frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3/2}}{d} + \frac{(2a^2e)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{d} \\
&\quad + \frac{\left(3a^2e^2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{5\sqrt{\sin(c+dx)}} \\
&\quad - \frac{\left(3a^2e^2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{2\sqrt{\sin(c+dx)}} \\
&= -\frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{4a^2e(e\sin(c+dx))^{3/2}}{3d} \\
&\quad - \frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3/2}}{d} \\
&\quad + \frac{(4a^2e)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d}
\end{aligned}
$$

$$= -\frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{4a^2e(e\sin(c+dx))^{3/2}}{3d} - \frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3/2}}{d} + \frac{(2a^2e^3)\,\mathrm{Subst}\left(\int\frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} - \frac{(2a^2e^3)\,\mathrm{Subst}\left(\int\frac{1}{e+x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d}$$

$$= -\frac{2a^2e^{5/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{4a^2e(e\sin(c+dx))^{3/2}}{3d} - \frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3/2}}{d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 17.64 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.06

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\arcsin(\sin(c+dx))\right)(e\sin(c+dx))^{5/2}\left(-15\arctan\left(\sqrt{\right.\right.}}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*(e*Sin
[c + d*x])^(5/2)*(-15*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 15*
ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 9*Sin[c + d*x]^(3/2) - 1
0*Sqrt[Cos[c + d*x]^2]*Sin[c + d*x]^(3/2) + 9*Sqrt[Cos[c + d*x]^2]*Hypergeo
metric2F1[3/4, 3/2, 7/4, Sin[c + d*x]^2]*Sin[c + d*x]^(3/2) + 3*Sin[c + d*x
]^(7/2)))/(15*d*Sin[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 19.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.37

method	result
default	$\frac{a^2\left(-60\sqrt{e\sin(dx+c)}\,e^{\frac{5}{2}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\cos(dx+c)+60\sqrt{e\sin(dx+c)}\,e^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\cos(dx+c)+54\sqrt{-\sin(dx-}\right.}{}$
parts	$-\frac{a^2e^3\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \operatorname{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(-60*(e*sin(d*x+c))^(1/2)*e^(5/2)*
arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)+60*(e*sin(d*x+c))^(1/2)*e^(
5/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)+54*(-sin(d*x+c)+1)^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2)
,1/2*2^(1/2))*e^3-27*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e^3+12*e^3*cos(d*x+c)^
4+40*e^3*cos(d*x+c)^3-42*e^3*cos(d*x+c)^2-40*cos(d*x+c)*e^3+30*e^3)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 769, normalized size of antiderivative = 3.96

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/60*(-54*I*sqrt(2)*a^2*sqrt(-I*e)*e^2*cos(d*x + c)*weierstrassZeta(4, 0,
weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 54*I*sqrt(2)*a^
2*sqrt(I*e)*e^2*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 30*a^2*sqrt(-e)*e^2*arctan(1/4*(cos(d*x
 + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)
^2 - e*sin(d*x + c) - e))*cos(d*x + c) + 15*a^2*sqrt(-e)*e^2*cos(d*x + c)*l
og((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 - (cos(d*x
 + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) + 28*(e*cos(d*
x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4
*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 4*(6*a^2*e^2*cos(d*x + c)^2 + 20
*a^2*e^2*cos(d*x + c) - 15*a^2*e^2)*sqrt(e*sin(d*x + c))*sin(d*x + c))/(d*c
os(d*x + c)), 1/60*(-54*I*sqrt(2)*a^2*sqrt(-I*e)*e^2*cos(d*x + c)*weierstra
ssZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 54
*I*sqrt(2)*a^2*sqrt(I*e)*e^2*cos(d*x + c)*weierstrassZeta(4, 0, weierstrass
```

```
PInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 30*a^2*e^(5/2)*arctan(1/4*
(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(e)/(e*cos(d
*x + c)^2 + e*sin(d*x + c) - e))*cos(d*x + c) + 15*a^2*e^(5/2)*cos(d*x + c)
*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d
*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d
*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 +
4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 4*(6*a^2*e^2*cos(d*x + c)^2 + 2
0*a^2*e^2*cos(d*x + c) - 15*a^2*e^2)*sqrt(e*sin(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{5/2}\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{5/2}\, dx = \int (a\sec(dx + c) + a)^2 (e\sin(dx + c))^{\frac{5}{2}}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{5/2}\, dx = \int (a\sec(dx + c) + a)^2 (e\sin(dx + c))^{\frac{5}{2}}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx=\int (e\,\sin\left(c+d\,x\right))^{5/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)
```

3.115 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 192

$$\begin{aligned}\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}\,dx &= \frac{2a^2e^{3/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}\\ &+\frac{2a^2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{a^2e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}}\\ &-\frac{4a^2e\sqrt{e\sin(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\\ &+\frac{a^2e\sec(c+dx)\sqrt{e\sin(c+dx)}}{d}\end{aligned}$$

```
[Out] 2*a^2*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+2*a^2*e^(3/2)*arctanh(
(e*sin(d*x+c))^(1/2)/e^(1/2))/d+1/3*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))
*sin(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-4*a^2*e*(e*sin(d*x+c))^(1/2)/d-2/3
*a^2*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d+a^2*e*sec(d*x+c)*(e*sin(d*x+c))^(1
/2)/d
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules

used = {3957, 2952, 2715, 2721, 2720, 2644, 327, 335, 218, 212, 209, 2646}

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{2a^2e^{3/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2e^{3/2}\mathrm{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{a^2e^2\sqrt{\sin(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}} - \frac{4a^2e\sqrt{e\sin(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{a^2e\sec(c+dx)\sqrt{e\sin(c+dx)}}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*e^(3/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(3/2)*Arc
Tanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (a^2*e^2*EllipticF[(c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (4*a^2*e*Sqrt[e*Sin
[c + d*x]])/d - (2*a^2*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d) + (a^2*e*
Sec[c + d*x]*Sqrt[e*Sin[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^{3/2}\,dx\\
&= \int \Big(a^2(e\sin(c+dx))^{3/2}\\
&\qquad +2a^2\sec(c+dx)(e\sin(c+dx))^{3/2}+a^2\sec^2(c+dx)(e\sin(c+dx))^{3/2}\Big)\,dx\\
&= a^2\int (e\sin(c+dx))^{3/2}\,dx\\
&\quad +a^2\int \sec^2(c+dx)(e\sin(c+dx))^{3/2}\,dx+\left(2a^2\right)\int \sec(c+dx)(e\sin(c+dx))^{3/2}\,dx\\
&= -\frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}+\frac{a^2e\sec(c+dx)\sqrt{e\sin(c+dx)}}{d}\\
&\quad +\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{e^2}}\,dx,x,e\sin(c+dx)\right)}{de}\\
&\quad +\frac{1}{3}\left(a^2e^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx-\frac{1}{2}\left(a^2e^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx\\
&= -\frac{4a^2e\sqrt{e\sin(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\\
&\quad +\frac{a^2e\sec(c+dx)\sqrt{e\sin(c+dx)}}{d}+\frac{(2a^2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\,dx,x,e\sin(c+dx)\right)}{d}\\
&\quad +\frac{\left(a^2e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3\sqrt{e\sin(c+dx)}}-\frac{\left(a^2e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{2\sqrt{e\sin(c+dx)}}\\
&= -\frac{a^2e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}}-\frac{4a^2e\sqrt{e\sin(c+dx)}}{d}\\
&\quad -\frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}+\frac{a^2e\sec(c+dx)\sqrt{e\sin(c+dx)}}{d}\\
&\quad +\frac{(4a^2e)\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{e^2}}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{d}
\end{aligned}
$$

$$= -\frac{a^2e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{4a^2e\sqrt{e\sin(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{a^2e\sec(c+dx)\sqrt{e\sin(c+dx)}}{d} + \frac{(2a^2e^2)\,\text{Subst}\left(\int\frac{1}{e-x^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{d} + \frac{(2a^2e^2)\,\text{Subst}\left(\int\frac{1}{e+x^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{d}$$

$$= \frac{2a^2e^{3/2}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2e^{3/2}\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{a^2e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{4a^2e\sqrt{e\sin(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{a^2e\sec(c+dx)\sqrt{e\sin(c+dx)}}{d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 27.02 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.06

$$\int(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{16a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\left(6\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{\cos^2(c+dx)}+6\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a^2*e*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]]*(6*ArcTan[Sq
rt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 6*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt
[Cos[c + d*x]^2] + Sqrt[Sin[c + d*x]] - 12*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin[c
+ d*x]] - Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Sin[c + d*x
]^2]*Sqrt[Sin[c + d*x]] + 2*Sin[c + d*x]^(5/2))*Sin[ArcSin[Sin[c + d*x]]/2]
^4)/(3*d*Sin[c + d*x]^(9/2))
```

Maple [A] (verified)

Time = 13.58 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.05

method	result
default	$\frac{a^2\left(12\cos(dx+c)\sqrt{e\sin(dx+c)}\,e^{\frac{3}{2}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+12\cos(dx+c)\sqrt{e\sin(dx+c)}\,e^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+\sqrt{-\sin(dx+c)+}\right.}{6\cos(dx-}$
parts	$-\frac{a^2e^2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^3+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{a^2e^2\sqrt{}}{}$

[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/6/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(12*cos(d*x+c)*(e*sin(d*x+c))^(1/2)
*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))+12*cos(d*x+c)*(e*sin(d*x+c))^
(1/2)*e^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+(-sin(d*x+c)+1)^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*
2^(1/2))*e^2-4*e^2*cos(d*x+c)^2*sin(d*x+c)-24*e^2*sin(d*x+c)*cos(d*x+c)+6*e
^2*sin(d*x+c))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 721, normalized size of antiderivative = 3.76

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \left[-\frac{2\sqrt{2}a^2\sqrt{-i\,e}e\cos(dx+c)\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+2\sqrt{2}a}{}\right.$$

$$-\frac{2\sqrt{2}a^2\sqrt{-i\,e}e\cos(dx+c)\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+2\sqrt{2}a^2\sqrt{i\,e}e\cos(dx-}{}$$

[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/12*(2*sqrt(2)*a^2*sqrt(-I*e)*e*cos(d*x + c)*weierstrassPInverse(4, 0, c
os(d*x + c) + I*sin(d*x + c)) + 2*sqrt(2)*a^2*sqrt(I*e)*e*cos(d*x + c)*weie
rstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 6*a^2*sqrt(-e)*e*arc
tan(1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)
/(e*cos(d*x + c)^2 - e*sin(d*x + c) - e))*cos(d*x + c) - 3*a^2*sqrt(-e)*e*c
os(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(7*cos(d*x + c)

```
^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) +
 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 4*(2*a^2*e*cos(d*x
+ c)^2 + 12*a^2*e*cos(d*x + c) - 3*a^2*e)*sqrt(e*sin(d*x + c)))/(d*cos(d*x
+ c)), -1/12*(2*sqrt(2)*a^2*sqrt(-I*e)*e*cos(d*x + c)*weierstrassPInverse(4
, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*sqrt(2)*a^2*sqrt(I*e)*e*cos(d*x + c
)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 6*a^2*e^(3/2)*
arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(
e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e))*cos(d*x + c) - 3*a^2*e^(3/2)*co
s(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^
2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 2
8*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x
 + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 4*(2*a^2*e*cos(d*x +
c)^2 + 12*a^2*e*cos(d*x + c) - 3*a^2*e)*sqrt(e*sin(d*x + c)))/(d*cos(d*x +
c))]
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{3/2} \, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{3/2} \, dx = \int (a\sec(dx + c) + a)^2 (e\sin(dx + c))^{\frac{3}{2}} \, dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{3/2}\, dx = \int (a\sec(dx + c) + a)^2 (e\sin(dx + c))^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\sin(c + dx))^{3/2}\, dx = \int (e\,\sin(c + d\,x))^{3/2} \left(a + \frac{a}{\cos(c + d\,x)}\right)^2 dx$$

```
[In] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)
```

3.116 $\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = -\frac{2a^2\sqrt{e}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de}$$

```
[Out] a^2*sec(d*x+c)*(e*sin(d*x+c))^(3/2)/d/e-2*a^2*arctan((e*sin(d*x+c))^(1/2)/e
^(1/2))*e^(1/2)/d+2*a^2*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d-a^2
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules

used = {3957, 2952, 2721, 2719, 2644, 335, 304, 209, 212, 2651}

$$\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = -\frac{2a^2\sqrt{e}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de} + \frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (-2*a^2*Sqrt[e]*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*Sqrt[e]*Ar
cTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (a^2*EllipticE[(c - Pi/2 + d*x)/2,
 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) + (a^2*Sec[c + d*x]*(e*Sin
[c + d*x])^(3/2))/(d*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sqrt{e\sin(c+dx)}\,dx\\ &= \int \left(a^2\sqrt{e\sin(c+dx)}+2a^2\sec(c+dx)\sqrt{e\sin(c+dx)}+a^2\sec^2(c+dx)\sqrt{e\sin(c+dx)}\right)\,dx\end{aligned}$$

$$
\begin{aligned}
&= a^2 \int \sqrt{e \sin(c+dx)}\,dx + a^2 \int \sec^2(c+dx)\sqrt{e\sin(c+dx)}\,dx \\
&\quad + (2a^2) \int \sec(c+dx)\sqrt{e\sin(c+dx)}\,dx \\
&= \frac{a^2 \sec(c+dx)(e\sin(c+dx))^{3/2}}{de} - \frac{1}{2}a^2 \int \sqrt{e\sin(c+dx)}\,dx \\
&\quad + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de} \\
&\quad + \frac{\left(a^2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{\sqrt{\sin(c+dx)}} \\
&= \frac{2a^2 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de} \\
&\quad + \frac{(4a^2)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de} - \frac{\left(a^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{2\sqrt{\sin(c+dx)}} \\
&= \frac{a^2 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de} \\
&\quad + \frac{(2a^2e)\,\mathrm{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} \\
&\quad - \frac{(2a^2e)\,\mathrm{Subst}\left(\int \frac{1}{e+x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} \\
&= -\frac{2a^2\sqrt{e}\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} \\
&\quad + \frac{a^2 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.75 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.22

$$
\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx =
$$

$$
-\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\arcsin(\sin(c+dx))\right)\sqrt{e\sin(c+dx)}\left(3\arctan\left(\sqrt{\sin(c+dx)}\right)\right)}{}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (-2*a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*Sqrt[
e*Sin[c + d*x]]*(3*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 3*ArcT
anh[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 3*Sin[c + d*x]^(3/2) + Sqrt[
Cos[c + d*x]^2]*Hypergeometric2F1[3/4, 3/2, 7/4, Sin[c + d*x]^2]*Sin[c + d*
x]^(3/2)))/(3*d*Sqrt[Sin[c + d*x]])
```

Maple [A] (verified)

Time = 14.07 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.59

method	result
default	$-\frac{a^2\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)e-\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin}\right.}{}$
parts	$-\frac{a^2 e\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\left(2\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{a}{}$

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
*e-(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e+4*cos(d*x+c)*e^(1/2)*(e*sin(d*x+c))^(1
/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))-4*cos(d*x+c)*e^(1/2)*(e*sin(d*x+c)
)^(1/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+2*e*cos(d*x+c)^2-2*e)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 677, normalized size of antiderivative = 4.91

$$\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx$$
$$=\left[\frac{2i\sqrt{2}a^2\sqrt{-i\,e}\cos(dx+c)\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))-}{}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*I*sqrt(2)*a^2*sqrt(-I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierst
rassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 2*I*sqrt(2)*a^2*sqrt(I
*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x +
```

```
c) - I*sin(d*x + c))) - 2*a^2*sqrt(-e)*arctan(1/4*(cos(d*x + c)^2 - 6*sin(d
*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x +
c) - e))*cos(d*x + c) + a^2*sqrt(-e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 7
2*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c
) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x
+ c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*si
n(d*x + c) + 8)) + 4*sqrt(e*sin(d*x + c))*a^2*sin(d*x + c))/(d*cos(d*x + c)
), 1/4*(2*I*sqrt(2)*a^2*sqrt(-I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weier
strassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 2*I*sqrt(2)*a^2*sqrt
(I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x
+ c) - I*sin(d*x + c))) - 2*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(
d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x +
c) - e))*cos(d*x + c) + a^2*sqrt(e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72
*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c)
 - 8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x +
c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(
d*x + c) + 8)) + 4*sqrt(e*sin(d*x + c))*a^2*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F]

$$\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = a^2\left(\int\sqrt{e\sin{(c+dx)}}\,dx + \int 2\sqrt{e\sin{(c+dx)}}\sec{(c+dx)}\,dx + \int\sqrt{e\sin{(c+dx)}}\sec^2{(c+dx)}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*sin(c + d*x)), x) + Integral(2*sqrt(e*sin(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*sin(c + d*x))*sec(c + d*x)**2, x))
```

Maxima [F]

$$\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \int (a\sec{(dx+c)}+a)^2\sqrt{e\sin{(dx+c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \int (a\sec(dx+c)+a)^2\sqrt{e\sin(dx+c)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \int \sqrt{e\,\sin(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,dx$$

```
[In] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

3.117 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a^2\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{3a^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de}$$

```
[Out] 2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(1/2)+2*a^2*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))/d/e^(1/2)-3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d
*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)+a^2*sec(d*x+c)*(e*sin(d*x+c))^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3957, 2952, 2721, 2720, 2644, 335, 218, 212, 209, 2651}

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a^2\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de} + \frac{3a^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (2*a^2*ArcTanh[S
qrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (3*a^2*EllipticF[(c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]]) + (a^2*Sec[c + d*x]*S
qrt[e*Sin[c + d*x]])/(d*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\\
&= \int \left(\frac{a^2}{\sqrt{e\sin(c+dx)}}+\frac{2a^2\sec(c+dx)}{\sqrt{e\sin(c+dx)}}+\frac{a^2\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\right)dx\\
&= a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx+a^2\int \frac{\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx+\left(2a^2\right)\int \frac{\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\\
&= \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de}+\frac{1}{2}a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx\\
&\quad+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\,dx,x,e\sin(c+dx)\right)}{de}\\
&\quad+\frac{\left(a^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{\sqrt{e\sin(c+dx)}}
\end{aligned}$$

$$= \frac{2a^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de}$$
$$+ \frac{(4a^2)\operatorname{Subst}\left(\int \frac{1}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de} + \frac{\left(a^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{2\sqrt{e\sin(c+dx)}}$$

$$= \frac{3a^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de}$$
$$+ \frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} + \frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{e+x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d}$$

$$= \frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}$$
$$+ \frac{3a^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 19.81 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.18

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\arcsin(\sin(c+dx))\right)\left(2\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{\cos^2(c+dx)}+2\mathrm{ar}\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*(2*ArcTa
n[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 2*ArcTanh[Sqrt[Sin[c + d*x]]]*
Sqrt[Cos[c + d*x]^2] + Sqrt[Sin[c + d*x]] + 3*Sqrt[Cos[c + d*x]^2]*Hypergeo
metric2F1[1/4, 1/2, 5/4, Sin[c + d*x]^2]*Sqrt[Sin[c + d*x]])*Sqrt[Sin[c + d
*x]])/(d*Sqrt[e*Sin[c + d*x]])
```

Maple [A] (verified)

Time = 11.95 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.17

method	result
default	$\frac{a^{2}\left(-3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)\sqrt{e}+4\cos(dx+c)\sqrt{e\sin(dx+c)}\ \arctan\left(\frac{\sqrt{\quad}}{}\right.\right.}{2\sqrt{e}\ \cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$
parts	$-\frac{a^{2}\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{a^{2}\sqrt{\cos(dx+c)^{2}e\sin(dx+c)}\left(\sqrt{-\sin(d\right.}{2\sqrt{-e\sin(dx}}$

[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/e^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e^(1/2)+4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))+4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+2*e^(1/2)*sin(d*x+c))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.75

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$$
$$=\left[\frac{6\sqrt{2}a^{2}\sqrt{-i\,e}\cos{(dx+c)}\ \mathrm{weierstrassPInverse}(4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+6\sqrt{2}a^{2}\sqrt{i\,e}\cos{(dx+}}{}\right.$$

[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/4*(6*sqrt(2)*a^2*sqrt(-I*e)*cos(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 6*sqrt(2)*a^2*sqrt(I*e)*cos(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*a^2*sqrt(-e)*arctan(1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x + c)^2 - e*sin(d*x + c) - e))*cos(d*x + c) - a^2*sqrt(-e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 4*sqrt(e*sin(d*x + c))*a^2)/(d*e*cos(d*x + c)), 1/4*(6*sqrt(2)*a^2*sqrt(-I*e)*cos(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 6*sqrt(2)*a^2*sqrt(I*e)*cos(d*x

```
 + c)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*a^2*sqrt
(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*s
qrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e))*cos(d*x + c) + a^2*sqrt(e)*
cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c
)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(e) -
 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d
*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 4*sqrt(e*sin(d*x +
c))*a^2)/(d*e*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + \int \frac{2\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
[Out] a**2*(Integral(1/sqrt(e*sin(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
sin(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*sin(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\sin(dx+c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\sin(dx+c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{e\,\sin\left(c+d\,x\right)}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(1/2), x)
```

3.118 $\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 224

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx=-\frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{2a^2\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{4a^2}{de\sqrt{e\sin(c+dx)}}-\frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{5a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}+\frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3}$$

```
[Out] -2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+2*a^2*arctanh((e*sin(
d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+3*a^2*sec(d*x+c)*(e*sin(d*x+c))^(3/2)/d/e^
3-4*a^2/d/e/(e*sin(d*x+c))^(1/2)-2*a^2*cos(d*x+c)/d/e/(e*sin(d*x+c))^(1/2)-
2*a^2*sec(d*x+c)/d/e/(e*sin(d*x+c))^(1/2)+5*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1
/2))*(e*sin(d*x+c))^(1/2)/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules

used = {3957, 2952, 2716, 2721, 2719, 2644, 331, 335, 304, 209, 212, 2650, 2651}

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3} - \frac{5a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) + (2*a^2*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[e*Sin[c + d*
x]]) - (2*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a^2*Sec[c + d*x
])/(d*e*Sqrt[e*Sin[c + d*x]]) - (5*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqr
t[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]]) + (3*a^2*Sec[c + d*x]*(e*Sin[
c + d*x])^(3/2))/(d*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx\\
&= \int \left(\frac{a^2}{(e\sin(c+dx))^{3/2}}+\frac{2a^2\sec(c+dx)}{(e\sin(c+dx))^{3/2}}+\frac{a^2\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\right)dx\\
&= a^2\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx+a^2\int \frac{\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx+\left(2a^2\right)\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx\\
&= -\frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{a^2\int \sqrt{e\sin(c+dx)}\,dx}{e^2}\\
&\quad+\frac{(3a^2)\int \sec^2(c+dx)\sqrt{e\sin(c+dx)}\,dx}{e^2}\\
&\quad+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{e^2}\right)}\,dx,x,e\sin(c+dx)\right)}{de}\\
&= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}}-\frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}}\\
&\quad+\frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{e^2}}\,dx,x,e\sin(c+dx)\right)}{de^3}\\
&\quad-\frac{(3a^2)\int \sqrt{e\sin(c+dx)}\,dx}{2e^2}-\frac{\left(a^2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{e^2\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} + \frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3} + \frac{(4a^2)\,\mathrm{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de^3} - \frac{\left(3a^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{2e^2\sqrt{\sin(c+dx)}}$$

$$= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{5a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} + \frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3} + \frac{(2a^2)\,\mathrm{Subst}\left(\int\frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de} - \frac{(2a^2)\,\mathrm{Subst}\left(\int\frac{1}{e+x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de}$$

$$= -\frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{5a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} + \frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 19.95 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.60

$$\int\frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\cot(c+dx)\sec^4\left(\frac{1}{2}\arcsin(\sin(c+dx))\right)\sqrt{e\sin(c+dx)}\left(6\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{4},1,\right.\right.}{3}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*Cos[(c + d*x)/2]^4*Cot[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*Sqrt[
e*Sin[c + d*x]]*(6*Hypergeometric2F1[-1/4, 1, 3/4, Sin[c + d*x]^2] + 6*Hype
rgeometric2F1[-1/4, 3/2, 3/4, Sin[c + d*x]^2] + Hypergeometric2F1[3/4, 3/2,
 7/4, Sin[c + d*x]^2]*Sin[c + d*x]^2))/(3*d*e^2*Sqrt[Cos[c + d*x]^2])
```

Maple [A] (verified)

Time = 17.57 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.06

method	result
default	$\frac{a^2\left(10e^{\frac{3}{2}}\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-5e^{\frac{3}{2}}\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+\cdots}\right.}{}$
parts	$\frac{a^2\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(\cdots}\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e^(5/2)/(e*sin(d*x+c))^(1/2)/cos(d*x+c)*a^2*(10*e^(3/2)*(-sin(d*x+c)+1)
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1
/2),1/2*2^(1/2))-5*e^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-10*e^(3/2)*cos(d
*x+c)^2-8*e^(3/2)*cos(d*x+c)-4*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*(e*sin(
d*x+c))^(1/2)*cos(d*x+c)*e+4*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*(e*sin(d
*x+c))^(1/2)*cos(d*x+c)*e+2*e^(3/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 790, normalized size of antiderivative = 3.53

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(-10*I*sqrt(2)*a^2*sqrt(-I*e)*cos(d*x + c)*sin(d*x + c)*weierstrassZet
a(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 10*I*sq
rt(2)*a^2*sqrt(I*e)*cos(d*x + c)*sin(d*x + c)*weierstrassZeta(4, 0, weierst
rassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*a^2*sqrt(-e)*arctan(
1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*
cos(d*x + c)^2 - e*sin(d*x + c) - e))*cos(d*x + c)*sin(d*x + c) - a^2*sqrt(
-e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(7*cos(d*x
 + c)^2 - (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(
-e) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))*sin(d*x + c) - 4
*(5*a^2*cos(d*x + c)^2 + 4*a^2*cos(d*x + c) - a^2)*sqrt(e*sin(d*x + c)))/(d
*e^2*cos(d*x + c)*sin(d*x + c)), 1/4*(-10*I*sqrt(2)*a^2*sqrt(-I*e)*cos(d*x
+ c)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x +
```

```
c) + I*sin(d*x + c))) + 10*I*sqrt(2)*a^2*sqrt(I*e)*cos(d*x + c)*sin(d*x +
c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x
 + c))) - 2*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sq
rt(e*sin(d*x + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e))*cos(d*x
 + c)*sin(d*x + c) + a^2*sqrt(e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*
cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) -
8)*sqrt(e*sin(d*x + c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c)
+ 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x
 + c) + 8))*sin(d*x + c) - 4*(5*a^2*cos(d*x + c)^2 + 4*a^2*cos(d*x + c) - a
^2)*sqrt(e*sin(d*x + c)))/(d*e^2*cos(d*x + c)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = a^2\left(\int \frac{1}{(e\sin{(c+dx)})^{\frac{3}{2}}}\,dx + \int \frac{2\sec{(c+dx)}}{(e\sin{(c+dx)})^{\frac{3}{2}}}\,dx + \int \frac{\sec^2{(c+dx)}}{(e\sin{(c+dx)})^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*sin(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*si
n(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*sin(c + d*x))**(3/2),
x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*sin(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{(e\,\sin(c+d\,x))^{3/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(3/2), x)
```

3.119 $\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 234

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{2a^2\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{7a^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} + \frac{5a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{3de^3}$$

```
[Out] 2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)+2*a^2*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))/d/e^(5/2)-4/3*a^2/d/e/(e*sin(d*x+c))^(3/2)-2/3*a^2*co
s(d*x+c)/d/e/(e*sin(d*x+c))^(3/2)-2/3*a^2*sec(d*x+c)/d/e/(e*sin(d*x+c))^(3/
2)-7/3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d*
x+c))^(1/2)+5/3*a^2*sec(d*x+c)*(e*sin(d*x+c))^(1/2)/d/e^3
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules

used = {3957, 2952, 2716, 2721, 2720, 2644, 331, 335, 218, 212, 209, 2650, 2651}

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{2a^2\text{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{5a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{3de^3} + \frac{7a^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}} - \frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) + (2*a^2*ArcTanh[S
qrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) - (4*a^2)/(3*d*e*(e*Sin[c + d*x])
^(3/2)) - (2*a^2*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) - (2*a^2*Sec[
c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (7*a^2*EllipticF[(c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]]) + (5*a^2*Sec[c +
 d*x]*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx \\
&= \int \left(\frac{a^2}{(e\sin(c+dx))^{5/2}}+\frac{2a^2\sec(c+dx)}{(e\sin(c+dx))^{5/2}}+\frac{a^2\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\right)dx \\
&= a^2\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx+a^2\int \frac{\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx+\left(2a^2\right)\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx \\
&= -\frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}}+\frac{a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2} \\
&\quad +\frac{\left(5a^2\right)\int \frac{\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{e^2}\right)}\,dx,x,e\sin(c+dx)\right)}{de} \\
&= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} \\
&\quad +\frac{5a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{3de^3}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\,dx,x,e\sin(c+dx)\right)}{de^3} \\
&\quad +\frac{\left(5a^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{6e^2}+\frac{\left(a^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} \\
&+ \frac{2a^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} \\
&+ \frac{5a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{3de^3} + \frac{(4a^2)\operatorname{Subst}\left(\int \frac{1}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de^3} \\
&+ \frac{\left(5a^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{6e^2\sqrt{e\sin(c+dx)}} \\
&= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} \\
&+ \frac{7a^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} \\
&+ \frac{5a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{3de^3} + \frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de^2} \\
&+ \frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{e+x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de^2} \\
&= \frac{2a^2\arctan\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} \\
&- \frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} \\
&+ \frac{7a^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} + \frac{5a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{3de^3}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 18.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.72

$$
\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx =
$$

$$
-\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\left(3+4\sqrt{\cos^2(c+dx)}\csc^2(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},\sin^2(c+dx)\right)+4\sqrt{\text{co}\right.}{}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/3*(a^2*Cos[(c + d*x)/2]^4*(3 + 4*Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]^2*Hyp
ergeometric2F1[-3/4, 1, 1/4, Sin[c + d*x]^2] + 4*Sqrt[Cos[c + d*x]^2]*Csc[c
```

```
 + d*x]^2*Hypergeometric2F1[-3/4, 3/2, 1/4, Sin[c + d*x]^2] + 3*Sqrt[Cos[c
+ d*x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Sin[c + d*x]^2])*Sec[c + d*x]*Se
c[ArcSin[Sin[c + d*x]]/2]^4*Sqrt[e*Sin[c + d*x]])/(d*e^3)
```

Maple [A] (verified)

Time = 17.50 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.29

method	result
default	$\frac{a^2\left(7\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{7}{2}}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)e^{\frac{7}{2}}-14e^{\frac{7}{2}}\cos(dx+c)^4-8e^{\frac{7}{2}}\cos(dx+c)^3+12\left(\right.\right.}{}$
parts	$-\frac{a^2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{5}{2}}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^3+2\sin(dx+c)\right)}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{a^2\sqrt{\cos(}}{}$

```
[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/e^(9/2)/(e*sin(d*x+c))^(3/2)/cos(d*x+c)/(cos(d*x+c)^2-1)*a^2*(7*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))*e^(7/2)-14*e^(7/2)*cos(d*x+c)^4-8*e^(7/2)*cos(d*x+
c)^3+12*(e*sin(d*x+c))^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c
)^3*e^2+12*(e*sin(d*x+c))^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d
*x+c)^3*e^2+20*e^(7/2)*cos(d*x+c)^2+8*e^(7/2)*cos(d*x+c)-12*(e*sin(d*x+c))^
(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)*e^2-12*(e*sin(d*x+c))
^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)*e^2-6*e^(7/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 849, normalized size of antiderivative = 3.63

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*sqrt(-e)*arctan(1/4*(cos(
d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x + c))*sqrt(-e)/(e*cos(d*x +
 c)^2 - e*sin(d*x + c) - e)) + 3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*sq
rt(-e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(7*cos(d*x + c)^2 -
(cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin(d*x + c))*sqrt(-e) + 28*(
e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 14*(sqrt(2)*a^2*cos(d*x
+ c)^2 - sqrt(2)*a^2*cos(d*x + c))*sqrt(-I*e)*weierstrassPInverse(4, 0, cos
```

```
(d*x + c) + I*sin(d*x + c)) - 14*(sqrt(2)*a^2*cos(d*x + c)^2 - sqrt(2)*a^2*
cos(d*x + c))*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x
+ c)) - 4*(7*a^2*cos(d*x + c) - 3*a^2)*sqrt(e*sin(d*x + c)))/(d*e^3*cos(d*x
 + c)^2 - d*e^3*cos(d*x + c)), 1/12*(6*(a^2*cos(d*x + c)^2 - a^2*cos(d*x +
c))*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e*sin(d*x
 + c))*sqrt(e)/(e*cos(d*x + c)^2 + e*sin(d*x + c) - e)) + 3*(a^2*cos(d*x +
c)^2 - a^2*cos(d*x + c))*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^
2 - 8*(7*cos(d*x + c)^2 + (cos(d*x + c)^2 - 8)*sin(d*x + c) - 8)*sqrt(e*sin
(d*x + c))*sqrt(e) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(
d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) +
 14*(sqrt(2)*a^2*cos(d*x + c)^2 - sqrt(2)*a^2*cos(d*x + c))*sqrt(-I*e)*weie
rstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 14*(sqrt(2)*a^2*cos(
d*x + c)^2 - sqrt(2)*a^2*cos(d*x + c))*sqrt(I*e)*weierstrassPInverse(4, 0,
cos(d*x + c) - I*sin(d*x + c)) + 4*(7*a^2*cos(d*x + c) - 3*a^2)*sqrt(e*sin(
d*x + c)))/(d*e^3*cos(d*x + c)^2 - d*e^3*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^2}{(e\sin(c + dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^2}{(e\sin(c + dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*sin(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{(e\,\sin(c+d\,x))^{5/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(5/2), x)
```

3.120 $\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21ad\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{2e(e\sin(c+dx))^{5/2}}{5ad}$$

```
[Out] 2/5*e*(e*sin(d*x+c))^(5/2)/a/d+4/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin
(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(1/2)-2/21*e^3*cos(d*x+c)*(e*sin(d*x+c))^(
1/2)/a/d+2/7*e^3*cos(d*x+c)^3*(e*sin(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3957, 2918, 2644, 30, 2648, 2649, 2721, 2720}

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e^4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21ad\sqrt{e\sin(c+dx)}} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} - \frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e(e\sin(c+dx))^{5/2}}{5ad}$$

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d*Sqrt[e
*Sin[c + d*x]]) - (2*e^3*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(21*a*d) + (2*e
^3*Cos[c + d*x]^3*Sqrt[e*Sin[c + d*x]])/(7*a*d) + (2*e*(e*Sin[c + d*x])^(5/
2))/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
```

```
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{-a-a\cos(c+dx)}\,dx \\
&= \frac{e^2\int \cos(c+dx)(e\sin(c+dx))^{3/2}\,dx}{a} - \frac{e^2\int \cos^2(c+dx)(e\sin(c+dx))^{3/2}\,dx}{a} \\
&= \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{e\mathrm{Subst}\left(\int x^{3/2}\,dx, x, e\sin(c+dx)\right)}{ad} - \frac{e^4\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{7a} \\
&= -\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} \\
&\quad + \frac{2e(e\sin(c+dx))^{5/2}}{5ad} - \frac{(2e^4)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{21a} \\
&= -\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} \\
&\quad + \frac{2e(e\sin(c+dx))^{5/2}}{5ad} - \frac{\left(2e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21a\sqrt{e\sin(c+dx)}} \\
&= -\frac{4e^4\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21ad\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} \\
&\quad + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{2e(e\sin(c+dx))^{5/2}}{5ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \frac{e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(40\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+(42+25\,\text{co}\right.}{105ad(1+\sec(c+}$$

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(40*EllipticF[(-2*c + Pi - 2*d*x)/4, 2
] + (42 + 25*Cos[c + d*x] - 42*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqrt
[Sin[c + d*x]])*Sqrt[e*Sin[c + d*x]])/(105*a*d*(1 + Sec[c + d*x])*Sqrt[Sin[
c + d*x]])
```

Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

method	result
default	$\frac{\frac{2e(e\sin(dx+c))^{\frac{5}{2}}}{5a}+\frac{2e^4\left(3\sin(dx+c)^5+\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-5\sin(dx+c)^3+2\sin(dx+c\right.}{21a\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

```
[In] int((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (2/5/a*e*(e*sin(d*x+c))^(5/2)+2/21*e^4*(3*sin(d*x+c)^5+(-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))-5*sin(d*x+c)^3+2*sin(d*x+c))/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2))
/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx =$$

$$-\frac{2\left(5\sqrt{2}\sqrt{-i\,e}e^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\sqrt{i\,e}e^3\text{weierstrassPInverse}(\right.}{}$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/105*(5*sqrt(2)*sqrt(-I*e)*e^3*weierstrassPInverse(4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 5*sqrt(2)*sqrt(I*e)*e^3*weierstrassPInverse(4, 0, cos(d*x
+ c) - I*sin(d*x + c)) - (15*e^3*cos(d*x + c)^3 - 21*e^3*cos(d*x + c)^2 - 5
*e^3*cos(d*x + c) + 21*e^3)*sqrt(e*sin(d*x + c)))/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\,\sin(c+d\,x))^{7/2}}{a\,(\cos(c+d\,x)+1)}\,dx$$

```
[In] int((e*sin(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(7/2))/(a*(cos(c + d*x) + 1)), x)
```

3.121 $\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 104

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}} + \frac{2e(e\sin(c+dx))^{3/2}}{3ad} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad}$$

```
[Out] 2/3*e*(e*sin(d*x+c))^(3/2)/a/d-2/5*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/a/d+4/
5*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x+c)^(
1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3957, 2918, 2644, 30, 2649, 2721, 2719}

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}} + \frac{2e(e\sin(c+dx))^{3/2}}{3ad} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad}$$

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a*d*Sqrt[
Sin[c + d*x]]) + (2*e*(e*Sin[c + d*x])^(3/2))/(3*a*d) - (2*e*Cos[c + d*x]*(
e*Sin[c + d*x])^(3/2))/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{-a-a\cos(c+dx)}\,dx \\
&= \frac{e^2\int\cos(c+dx)\sqrt{e\sin(c+dx)}\,dx}{a}-\frac{e^2\int\cos^2(c+dx)\sqrt{e\sin(c+dx)}\,dx}{a} \\
&= -\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad} \\
&\quad+\frac{e\text{Subst}\left(\int\sqrt{x}\,dx,x,e\sin(c+dx)\right)}{ad}-\frac{(2e^2)\int\sqrt{e\sin(c+dx)}\,dx}{5a} \\
&= \frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad} \\
&\quad-\frac{\left(2e^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{5a\sqrt{\sin(c+dx)}} \\
&= -\frac{4e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}} \\
&\quad+\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.86

$$
\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \frac{2e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(10\sqrt{\csc^2(c)}\sin^2(c+dx)+6\csc(c)\csc(dx-\text{arc}\right.}{}
$$

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(10*Sqrt[Csc[c]^2]*Sin[c + d*x]^2 +
6*Csc[c]*Csc[d*x - ArcTan[Cot[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, C
os[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Sin[d*x - ArcTan[Cot[c]]]^2] + 3*Csc[c]*Se
c[c]*(Sin[c + d*x - ArcTan[Cot[c]]] + 3*Sin[c - d*x + ArcTan[Cot[c]]]) - 3*
Sqrt[Csc[c]^2]*Sin[c + d*x]*(Sin[2*(c + d*x)] + 4*Tan[c])))/(15*a*d*Sqrt[Cs
c[c]^2]*(1 + Sec[c + d*x])*Sqrt[e*Sin[c + d*x]])
```

Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.66

method	result
default	$\frac{2e^3\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(d}\right.}{15a\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^3*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elliptic
F((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*cos(d*x+c)^4-5*cos(d*x+c)^3-3*cos(d*
x+c)^2+5*cos(d*x+c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = -\frac{2\left(3i\sqrt{2}\sqrt{-i\,e}e^2\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))-3i\sqrt{2}\sqrt{i\,ee}\right.}{}$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/15*(3*I*sqrt(2)*sqrt(-I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInverse
(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*sqrt(I*e)*e^2*weierstr
assZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + (
3*e^2*cos(d*x + c) - 5*e^2)*sqrt(e*sin(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^{5/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

3.122 $\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 102

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3ad\sqrt{e\sin(c+dx)}} + \frac{2e\sqrt{e\sin(c+dx)}}{ad} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad}$$

```
[Out] 4/3*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellip
ticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))
^(1/2)+2*e*(e*sin(d*x+c))^(1/2)/a/d-2/3*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a
/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3957, 2918, 2644, 30, 2649, 2721, 2720}

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3ad\sqrt{e\sin(c+dx)}} + \frac{2e\sqrt{e\sin(c+dx)}}{ad} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad}$$

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a*d*Sqrt[e*
Sin[c + d*x]]) + (2*e*Sqrt[e*Sin[c + d*x]])/(a*d) - (2*e*Cos[c + d*x]*Sqrt[
e*Sin[c + d*x]])/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{-a-a\cos(c+dx)}\,dx \\
&= \frac{e^2\int \frac{\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a} \\
&= -\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad} + \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, e\sin(c+dx)\right)}{ad} - \frac{(2e^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a} \\
&= \frac{2e\sqrt{e\sin(c+dx)}}{ad} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad} - \frac{\left(2e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a\sqrt{e\sin(c+dx)}} \\
&= -\frac{4e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3ad\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2e\sqrt{e\sin(c+dx)}}{ad} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$
\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\frac{2\left(-2\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+(-3+\cos(c+dx))\sqrt{\sin(c+dx)}\right)(e\sin(c+dx))^{3/2}}{3ad\sin^{\frac{3}{2}}(c+dx)}
$$

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
[Out] (-2*(-2*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (-3 + Cos[c + d*x])*Sqrt[Sin[
c + d*x]])*(e*Sin[c + d*x])^(3/2))/(3*a*d*Sin[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

method	result
default	$\frac{2e^2 (\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\text{EllipticF}(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2})-\cos(dx+c)^2\sin(dx+c)+3\cos(dx+c)\sin(dx+c}{3a\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] 2/3/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^2*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-cos(d*x+c)^2*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\frac{2\left(\sqrt{2}\sqrt{-i\,e}e\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\sqrt{i\,e}e\text{weierstrassPInverse}(4,0,c\right.}{3\,ad}$$

[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -2/3*(sqrt(2)*sqrt(-I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*sqrt(I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + (e*cos(d*x + c) - 3*e)*sqrt(e*sin(d*x + c)))/(a*d)

Sympy [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\sin(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

[In] integrate((e*sin(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)

[Out] Integral((e*sin(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)

Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^{3/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

```
[In] int((e*sin(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.123 $\int \frac{\sqrt{e \sin(c+dx)}}{a+a \sec(c+dx)} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{e \sin(c+dx)}}{a+a \sec(c+dx)} \, dx = -\frac{2e}{ad\sqrt{e \sin(c+dx)}}+\frac{2e \cos(c+dx)}{ad\sqrt{e \sin(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e \sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

```
[Out] -2*e/a/d/(e*sin(d*x+c))^(1/2)+2*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(1/2)-4*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/
2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3957, 2918, 2644, 30, 2647, 2721, 2719}

$$\int \frac{\sqrt{e \sin(c+dx)}}{a+a \sec(c+dx)} \, dx = -\frac{2e}{ad\sqrt{e \sin(c+dx)}}+\frac{2e \cos(c+dx)}{ad\sqrt{e \sin(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e \sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*e)/(a*d*Sqrt[e*Sin[c + d*x]]) + (2*e*Cos[c + d*x])/(a*d*Sqrt[e*Sin[c +
d*x]]) + (4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a*d*Sqr
t[Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{-a-a\cos(c+dx)}\,dx \\
&= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a} \\
&= \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{2\int \sqrt{e\sin(c+dx)}\,dx}{a} + \frac{e\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, e\sin(c+dx)\right)}{ad} \\
&= -\frac{2e}{ad\sqrt{e\sin(c+dx)}} + \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{\left(2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{a\sqrt{\sin(c+dx)}} \\
&= -\frac{2e}{ad\sqrt{e\sin(c+dx)}} + \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.75 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.21

$$
\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx
= \frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\left(\sec\left(\frac{c}{2}\right)\sec(c)\left(3\sin\left(\frac{c}{2}\right)+\sin\left(\frac{3c}{2}\right)\right)-2\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+d\right.\right.}{}
$$

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^2*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]]*(Sec[c/2]*Sec[c]*(3
*Sin[c/2] + Sin[(3*c)/2]) - 2*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 2*Sq
rt[Csc[c]^2]*Csc[c + d*x]*Csc[d*x - ArcTan[Cot[c]]]*HypergeometricPFQ[{-1/2
, -1/4}, {3/4}, Cos[d*x - ArcTan[Cot[c]]]^2]*Sin[c]*Sqrt[Sin[d*x - ArcTan[C
ot[c]]]^2] - (Csc[c]*Csc[c + d*x]*Sec[c]*(Sin[c + d*x - ArcTan[Cot[c]]] + 3
*Sin[c - d*x + ArcTan[Cot[c]]]))/Sqrt[Csc[c]^2]))/(a*d*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.57

method	result
default	$-\frac{2e\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(c}\right.}{a\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-(
-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-si
n(d*x+c)+1)^(1/2),1/2*2^(1/2))-cos(d*x+c)^2+cos(d*x+c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{2\left(\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\sqrt{-i\,e}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(\right.\right.\right.}{}$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*sqrt(-I*e)*weierstrassZeta(4, 0,
weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + (I*sqrt(2)*cos(
d*x + c) + I*sqrt(2))*sqrt(I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4
, 0, cos(d*x + c) - I*sin(d*x + c))) + sqrt(e*sin(d*x + c))*sin(d*x + c))/(
a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{e\sin\left(c+dx\right)}}{\sec\left(c+dx\right)+1}\,dx}{a}$$

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\sin\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\sin\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\sqrt{e\,\sin\left(c+d\,x\right)}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

```
[In] int((e*sin(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.124 $\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = -\frac{2e}{3ad(e\sin(c+dx))^{3/2}}+\frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}}+\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3ad\sqrt{e\sin(c+dx)}}$$

```
[Out] -2/3*e/a/d/(e*sin(d*x+c))^(3/2)+2/3*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(3/2)-4
/3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(
cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(1/2
)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3957, 2918, 2644, 30, 2647, 2721, 2720}

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = -\frac{2e}{3ad(e\sin(c+dx))^{3/2}}+\frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}}+\frac{4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3ad\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (-2*e)/(3*a*d*(e*Sin[c + d*x])^(3/2)) + (2*e*Cos[c + d*x])/(3*a*d*(e*Sin[c
+ d*x])^(3/2)) + (4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3
*a*d*Sqrt[e*Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{e^2 \int \frac{\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a} - \frac{e^2 \int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a} \\
&= \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a} + \frac{e\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, e\sin(c+dx)\right)}{ad} \\
&= -\frac{2e}{3ad(e\sin(c+dx))^{3/2}} + \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{\left(2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a\sqrt{e\sin(c+dx)}} \\
&= -\frac{2e}{3ad(e\sin(c+dx))^{3/2}} + \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3ad\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$
\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \frac{2\cot\left(\frac{1}{2}(c+dx)\right)\left(-1+\cos(c+dx)-2\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3ad(1+\cos(c+dx))\sqrt{e\sin(c+dx)}}
$$

```
[In] Integrate[1/((a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
[Out] (2*Cot[(c + d*x)/2]*(-1 + Cos[c + d*x] - 2*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2]*Sin[c + d*x]^(3/2)))/(3*a*d*(1 + Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])
```

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{-\frac{2e}{3a(e\sin(dx+c))^{\frac{3}{2}}}-\frac{2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{5}{2}}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)+\sin(dx+c)^{3}-\sin(dx+c)\right)}{3a\sin(dx+c)^{2}\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$	121

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

[Out] (-2/3/a*e/(e*sin(d*x+c))^(3/2)-2/3*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+sin(d*x+c)^3-sin(d*x+c))/a/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{2\left(\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\sqrt{-i\,e}\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(\sqrt{2}\cos\left(dx+\right.\right.\right.}{3\left(ade\cos\left(dx+c\right)+\right.}$$

[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 2/3*((sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(e*sin(d*x + c)))/(a*d*e*cos(d*x + c) + a*d*e)

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \frac{\int \frac{1}{\sqrt{e\sin\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{e\sin\left(c+dx\right)}}\,dx}{a}$$

[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(e*sin(c + d*x))*sec(c + d*x) + sqrt(e*sin(c + d*x))), x)/a

Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\sqrt{e\sin\left(dx+c\right)}}\,dx$$

[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)

Giac [F]

$$\int \frac{1}{(a + a\sec(c + dx))\sqrt{e\sin(c + dx)}}\,dx = \int \frac{1}{(a\sec(dx + c) + a)\sqrt{e\sin(dx + c)}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(c + dx))\sqrt{e\sin(c + dx)}}\,dx = \int \frac{\cos(c + d\,x)}{a\,\sqrt{e\,\sin(c + d\,x)}\,(\cos(c + d\,x) + 1)}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.125 $\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ade^2\sqrt{\sin(c+dx)}}$$

```
[Out] -2/5*e/a/d/(e*sin(d*x+c))^(5/2)+2/5*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(5/2)-4
/5*cos(d*x+c)/a/d/e/(e*sin(d*x+c))^(1/2)+4/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)
)*(e*sin(d*x+c))^(1/2)/a/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3957, 2918, 2644, 30, 2647, 2716, 2721, 2719}

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ade^2\sqrt{\sin(c+dx)}} - \frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*e)/(5*a*d*(e*Sin[c + d*x])^(5/2)) + (2*e*Cos[c + d*x])/(5*a*d*(e*Sin[c
+ d*x])^(5/2)) - (4*Cos[c + d*x])/(5*a*d*e*Sqrt[e*Sin[c + d*x]]) - (4*Ellip
```

```
ticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a*d*e^2*Sqrt[Sin[c + d
*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx \\
&= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a} \\
&= \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} + \frac{2\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{5a} + \frac{e\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, e\sin(c+dx)\right)}{ad} \\
&= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} \\
&\quad - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}} - \frac{2\int \sqrt{e\sin(c+dx)}\,dx}{5ae^2} \\
&= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} \\
&\quad - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}} - \frac{\left(2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{5ae^2\sqrt{\sin(c+dx)}} \\
&= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} \\
&\quad - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ade^2\sqrt{\sin(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+i\sin(c+dx))\left(-6-9\cos(c+\right.}{}$$

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^2*(Cos[c + d*x] + I*Sin[c + d*x])*(-6 - 9*Cos[c + d*x] +
2*Sqrt[1 - E^((2*I)*(c + d*x))]*(1 + Cos[c + d*x])*Hypergeometric2F1[1/2, 3
/4, 7/4, E^((2*I)*(c + d*x))] + (3*I)*Sin[c + d*x]))/(15*a*d*e*Sqrt[e*Sin[c
 + d*x]])
```

Maple [A] (verified)

Time = 5.84 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.39

method	result
default	$\frac{-\frac{2e}{5a(e\sin(dx+c))^{\frac{5}{2}}}+\frac{\frac{4\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{7}{2}}\operatorname{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)}{5}-\frac{2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx}{5}}{ea\sin(dx+c)^{3}\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/5/a*e/(e*sin(d*x+c))^(5/2)+2/5/e*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))+2*sin(d*x+c)^5-3*sin(d*x+c)^3+sin(d*x+c))/a/si
n(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(\left(i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\sqrt{-i\,e}\sin(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)\right.\right.}{}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*sqrt(-I*e)*sin(d*x + c)*weierstr
assZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + (
-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*sqrt(I*e)*sin(d*x + c)*weierstrassZeta
(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + (2*cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(e*sin(d*x + c)))/((a*d*e^2*cos(d*x + c
) + a*d*e^2)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \frac{\int \frac{1}{(e\sin(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\sin(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
[Out] Integral(1/((e*sin(c + d*x))**(3/2)*sec(c + d*x) + (e*sin(c + d*x))**(3/2))
, x)/a
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)}{a\,(e\,\sin(c+d\,x))^{3/2}\,(\cos(c+d\,x)+1)}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.126 $\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx=-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}+\frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}}-\frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}}+\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21ade^2\sqrt{e\sin(c+dx)}}$$

```
[Out] -2/7*e/a/d/(e*sin(d*x+c))^(7/2)+2/7*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(7/2)-4
/21*cos(d*x+c)/a/d/e/(e*sin(d*x+c))^(3/2)-4/21*(sin(1/2*c+1/4*Pi+1/2*d*x)^2
)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/
2))*sin(d*x+c)^(1/2)/a/d/e^2/(e*sin(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3957, 2918, 2644, 30, 2647, 2716, 2721, 2720}

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx=\frac{4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21ade^2\sqrt{e\sin(c+dx)}}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}+\frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}}-\frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}}$$

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*e)/(7*a*d*(e*Sin[c + d*x])^(7/2)) + (2*e*Cos[c + d*x])/(7*a*d*(e*Sin[c
+ d*x])^(7/2)) - (4*Cos[c + d*x])/(21*a*d*e*(e*Sin[c + d*x])^(3/2)) + (4*El
```

```
lipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d*e^2*Sqrt[e*Sin[c
 + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx \\
&= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a} \\
&= \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} + \frac{2\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{7a} + \frac{e\mathrm{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, e\sin(c+dx)\right)}{ad} \\
&= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{21ae^2} \\
&= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} \\
&\quad - \frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}} + \frac{\left(2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21ae^2\sqrt{e\sin(c+dx)}} \\
&= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{4\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21ade^2\sqrt{e\sin(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(4+2\cos(c+dx)+\cos(2(c+dx))+\csc^2\left(\frac{1}{2}(c+dx)\right)\mathrm{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{7}{2}}(c+dx)\right)}{21ade(1+\cos(c+dx))(e\sin(c+dx))^{3/2}}$$

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
[Out] (-2*(4 + 2*Cos[c + d*x] + Cos[2*(c + d*x)] + Csc[(c + d*x)/2]^2*EllipticF[(
-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(21*a*d*e*(1 + Cos[c + d*x])*
(e*Sin[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

method	result
default	$\frac{-\frac{2e}{7a(e\sin(dx+c))^{\frac{7}{2}}}-\frac{2\Big(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{9}{2}}\,\mathrm{EllipticF}\Big(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\Big)-2\sin(dx+c)^{5}+5\sin(dx+c)^{3}-3\sin(dx+c)}{21e^{2}a\sin(dx+c)^{4}\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/7/a*e/(e*sin(d*x+c))^(7/2)-2/21/e^2*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*
sin(d*x+c)^5+5*sin(d*x+c)^3-3*sin(d*x+c))/a/sin(d*x+c)^4/cos(d*x+c)/(e*sin(
d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.59

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \frac{2\Big(\big(\sqrt{2}\cos{(dx+c)}^{3}+\sqrt{2}\cos{(dx+c)}^{2}-\sqrt{2}\cos{(dx+c)}-}{}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*((sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2 - sqrt(2)*cos(d*x +
c) - sqrt(2))*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + (sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2 - sqrt(2)*cos(d*x
 + c) - sqrt(2))*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d
*x + c)) + (2*cos(d*x + c)^2 + 2*cos(d*x + c) + 3)*sqrt(e*sin(d*x + c)))/(a
*d*e^3*cos(d*x + c)^3 + a*d*e^3*cos(d*x + c)^2 - a*d*e^3*cos(d*x + c) - a*d
*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(c + dx))(e\sin(c + dx))^{5/2}}\, dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a + a\sec(c + dx))(e\sin(c + dx))^{5/2}}\, dx = \int \frac{1}{(a\sec(dx + c) + a)(e\sin(dx + c))^{\frac{5}{2}}}\, dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(c + dx))(e\sin(c + dx))^{5/2}}\, dx = \int \frac{\cos(c + d\,x)}{a\,(e\,\sin(c + d\,x))^{5/2}\,(\cos(c + d\,x) + 1)}\, dx$$

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.127 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{52e^4\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d} + \frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d} + \frac{4e(e\sin(c+dx))^{5/2}}{5a^2d}$$

```
[Out] 4/5*e*(e*sin(d*x+c))^(5/2)/a^2/d-52/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*
sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)-4*e^3*(e*sin(d*x+c))^(1/2)/a^2/
d+26/21*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d+2/7*e^3*cos(d*x+c)^3*(e*s
in(d*x+c))^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3957, 2954, 2952, 2649, 2721, 2720, 2644, 14}

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{52e^4\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d} + \frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} + \frac{4e(e\sin(c+dx))^{5/2}}{5a^2d}$$

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (52*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a^2*d*Sqrt
[e*Sin[c + d*x]]) - (4*e^3*Sqrt[e*Sin[c + d*x]])/(a^2*d) + (26*e^3*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(21*a^2*d) + (2*e^3*Cos[c + d*x]^3*Sqrt[e*Sin[c
+ d*x]])/(7*a^2*d) + (4*e*(e*Sin[c + d*x])^(5/2))/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{7/2}}{(-a-a\cos(c+dx))^2}\,dx\\
&= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx}{a^4}\\
&= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}-\frac{2a^2\cos^3(c+dx)}{\sqrt{e\sin(c+dx)}}+\frac{a^2\cos^4(c+dx)}{\sqrt{e\sin(c+dx)}}\right)\,dx}{a^4}\\
&= \frac{e^4\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2}+\frac{e^4\int \frac{\cos^4(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2}-\frac{(2e^4)\int \frac{\cos^3(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2}\\
&= \frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d}\\
&\quad-\frac{(2e^3)\,\text{Subst}\left(\int \frac{1-\frac{x^2}{e^2}}{\sqrt{x}}\,dx,x,e\sin(c+dx)\right)}{a^2d}\\
&\quad+\frac{(2e^4)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^2}+\frac{(6e^4)\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{7a^2}\\
&= \frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d}\\
&\quad-\frac{(2e^3)\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{x}}-\frac{x^{3/2}}{e^2}\right)\,dx,x,e\sin(c+dx)\right)}{a^2d}\\
&\quad+\frac{(4e^4)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{7a^2}+\frac{\left(2e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} - \frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d} \\
&\quad + \frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d} \\
&\quad + \frac{4e(e\sin(c+dx))^{5/2}}{5a^2d} + \frac{\left(4e^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{7a^2\sqrt{e\sin(c+dx)}} \\
&= \frac{52e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d} + \frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} \\
&\quad + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d} + \frac{4e(e\sin(c+dx))^{5/2}}{5a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$
\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = -\frac{e^3\left(520 \operatorname{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+(756-305\cos(c+dx)+84\cos(2(c+dx))-15\cos(3(c+dx)))\right.}{210a^2d\sqrt{\sin(c+dx)}}
$$

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/210*(e^3*(520*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (756 - 305*Cos[c + d
*x] + 84*Cos[2*(c + d*x)] - 15*Cos[3*(c + d*x)])*Sqrt[Sin[c + d*x]])*Sqrt[e
*Sin[c + d*x]])/(a^2*d*Sqrt[Sin[c + d*x]])
```

Maple [A] (verified)

Time = 6.87 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

method	result
default	$-\frac{2e^4\left(-15\cos(dx+c)^4\sin(dx+c)+65\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)+42\cos(dx+c\right.}{105a^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

```
[In] int((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/a^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^4*(-15*cos(d*x+c)^4*sin(d*x+c)
+65*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF
```

```
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+42*cos(d*x+c)^3*sin(d*x+c)-65*cos(d*x+c
)^2*sin(d*x+c)+168*cos(d*x+c)*sin(d*x+c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{2\left(65\sqrt{2}\sqrt{-i\,e}e^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+65\sqrt{}\right.}{}$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2/105*(65*sqrt(2)*sqrt(-I*e)*e^3*weierstrassPInverse(4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 65*sqrt(2)*sqrt(I*e)*e^3*weierstrassPInverse(4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + (15*e^3*cos(d*x + c)^3 - 42*e^3*cos(d*x + c)^2 +
65*e^3*cos(d*x + c) - 168*e^3)*sqrt(e*sin(d*x + c)))/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^{7/2}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(7/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.128 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx &= \frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}\\ &-\frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{44e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}\\ &+\frac{4e(e\sin(c+dx))^{3/2}}{3a^2d}-\frac{12e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d}\end{aligned}$$

```
[Out] 4/3*e*(e*sin(d*x+c))^(3/2)/a^2/d-12/5*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/a^2
/d+4*e^3/a^2/d/(e*sin(d*x+c))^(1/2)-2*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(
1/2)-2*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(1/2)+44/5*e^2*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3957, 2954, 2952, 2647, 2721, 2719, 2644, 14, 2649}

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx &= \frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}\\ &-\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{44e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}\\ &+\frac{4e(e\sin(c+dx))^{3/2}}{3a^2d}-\frac{12e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d}\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*e^3)/(a^2*d*Sqrt[e*Sin[c + d*x]]) - (2*e^3*Cos[c + d*x])/(a^2*d*Sqrt[e*S
in[c + d*x]]) - (2*e^3*Cos[c + d*x]^3)/(a^2*d*Sqrt[e*Sin[c + d*x]]) - (44*e
^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^2*d*Sqrt[Sin
[c + d*x]]) + (4*e*(e*Sin[c + d*x])^(3/2))/(3*a^2*d) - (12*e*Cos[c + d*x]*(
e*Sin[c + d*x])^(3/2))/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{5/2}}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx}{a^4} \\
&= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}-\frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{3/2}}+\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{3/2}}\right)dx}{a^4} \\
&= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a^2}+\frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a^2}-\frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a^2} \\
&= -\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} \\
&\quad -\frac{(2e^2)\int \sqrt{e\sin(c+dx)}\,dx}{a^2}-\frac{(6e^2)\int \cos^2(c+dx)\sqrt{e\sin(c+dx)}\,dx}{a^2} \\
&\quad -\frac{(2e^3)\,\text{Subst}\left(\int \frac{1-\frac{x^2}{e^2}}{x^{3/2}}\,dx,x,e\sin(c+dx)\right)}{a^2d}
\end{aligned}
$$

$$= -\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{12e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d}$$
$$- \frac{(12e^2)\int\sqrt{e\sin(c+dx)}\,dx}{5a^2} - \frac{(2e^3)\,\text{Subst}\left(\int\left(\frac{1}{x^{3/2}} - \frac{\sqrt{x}}{e^2}\right)\,dx, x, e\sin(c+dx)\right)}{a^2d}$$
$$- \frac{\left(2e^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{a^2\sqrt{\sin(c+dx)}}$$

$$= \frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}$$
$$- \frac{4e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}} + \frac{4e(e\sin(c+dx))^{3/2}}{3a^2d}$$
$$- \frac{12e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} - \frac{\left(12e^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{5a^2\sqrt{\sin(c+dx)}}$$

$$= \frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}$$
$$- \frac{44e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}$$
$$+ \frac{4e(e\sin(c+dx))^{3/2}}{3a^2d} - \frac{12e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.28 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.39

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^2(c+dx)\sec^2(c+dx)\left(\frac{16\cos(dx)\sin(c)}{3d} - \frac{16\sec\left(\frac{c}{2}\right)\sec(c)\left(8\sin\left(\frac{c}{2}\right)+3\sin(\right.}{5d}\right.}{(\ldots}$$
$$- \frac{88\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c)\sec^2(c+dx)(e\sin(c+dx))^{5/2}\left(-\frac{\cot(c)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx-\arctan(\cot(c)))\right)}{\sqrt{1-\cos(dx-\arctan(\cot(c)))}\sqrt{1+\cos(dx-\arctan(\cot(c)))}\sqrt{1+\cot^2(\ldots}}\right.}{5d(a+a\sec(c+dx))^2\sin^{\ldots}}$$

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^4*Csc[c + d*x]^2*Sec[c + d*x]^2*((16*Cos[d*x]*Sin[c])/(
3*d) - (16*Sec[c/2]*Sec[c]*(8*Sin[c/2] + 3*Sin[(3*c)/2]))/(5*d) - (4*Cos[2*
d*x]*Sin[2*c])/(5*d) + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d + (1
6*Cos[c]*Sin[d*x])/(3*d) - (4*Cos[2*c]*Sin[2*d*x])/(5*d))*(e*Sin[c + d*x])^
```

```
(5/2))/(a + a*Sec[c + d*x])^2 - (88*Cos[c/2 + (d*x)/2]^4*Sec[c]*Sec[c + d*x
]^2*(e*Sin[c + d*x])^(5/2)*(-((Cot[c]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x - ArcTan[Cot[c]]]^2]*Sin[d*x - ArcTan[Cot[c]]])/(Sqrt[1 - Cos[d*x
 - ArcTan[Cot[c]]]]*Sqrt[1 + Cos[d*x - ArcTan[Cot[c]]]]*Sqrt[1 + Cot[c]^2]*
Sqrt[Cos[d*x - ArcTan[Cot[c]]]*Sqrt[1 + Cot[c]^2]*Sin[c]])) - ((2*Cos[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 + Cot[c]^2]*Sin[c]^2)/(Cos[c]^2 + Sin[c]^2) - (Cot[
c]*Sin[d*x - ArcTan[Cot[c]]])/Sqrt[1 + Cot[c]^2])/Sqrt[Cos[d*x - ArcTan[Cot
[c]]]*Sqrt[1 + Cot[c]^2]*Sin[c]]))/(5*d*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
^(5/2))
```

Maple [A] (verified)

Time = 7.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.93

method	result
default	$\frac{2e^3\left(66\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\ \text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-33\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{}}{15a^2\cos(dx+c)\sqrt{e\sin(dx+}}$

```
[In] int((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^3*(66*(-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(
1/2))-33*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elli
pticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*cos(d*x+c)^4-10*cos(d*x+c)^3-33*
cos(d*x+c)^2+40*cos(d*x+c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.91

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{2\left(\left(3\,e^2\cos\left(dx+c\right)^2-7\,e^2\cos\left(dx+c\right)-40\,e^2\right)\sqrt{e\sin\left(dx+c\right)}\sin\left(dx+c\right)+33\left(i\sqrt{2}e^2\cos\left(dx+c\right)+i\right.}{}$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*((3*e^2*cos(d*x + c)^2 - 7*e^2*cos(d*x + c) - 40*e^2)*sqrt(e*sin(d*x
+ c))*sin(d*x + c) + 33*(I*sqrt(2)*e^2*cos(d*x + c) + I*sqrt(2)*e^2)*sqrt(-
I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d
*x + c))) + 33*(-I*sqrt(2)*e^2*cos(d*x + c) - I*sqrt(2)*e^2)*sqrt(I*e)*weie
rstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))
)/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^{5/2}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.129 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 189

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx &= \frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}\\&\quad-\frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{4e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}}\\&\quad+\frac{4e\sqrt{e\sin(c+dx)}}{a^2d}-\frac{4e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d}\end{aligned}$$

```
[Out] 4/3*e^3/a^2/d/(e*sin(d*x+c))^(3/2)-2/3*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(3/2)-2/3*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(3/2)+4*e^2*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)+4*e*(e*sin(d*x
+c))^(1/2)/a^2/d-4/3*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3957, 2954, 2952, 2647, 2721, 2720, 2644, 14, 2649}

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx &= \frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}\\&\quad-\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{4e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^2d\sqrt{e\sin(c+dx)}}\\&\quad+\frac{4e\sqrt{e\sin(c+dx)}}{a^2d}-\frac{4e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d}\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*e^3)/(3*a^2*d*(e*Sin[c + d*x])^(3/2)) - (2*e^3*Cos[c + d*x])/(3*a^2*d*(e
*Sin[c + d*x])^(3/2)) - (2*e^3*Cos[c + d*x]^3)/(3*a^2*d*(e*Sin[c + d*x])^(3
/2)) - (4*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*d*S
qrt[e*Sin[c + d*x]]) + (4*e*Sqrt[e*Sin[c + d*x]])/(a^2*d) - (4*e*Cos[c + d*
x]*Sqrt[e*Sin[c + d*x]])/(3*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx}{a^4} \\
&= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}-\frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{5/2}}+\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{5/2}}\right)dx}{a^4} \\
&= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2}+\frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2}-\frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2} \\
&= -\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{(2e^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^2} \\
&\quad -\frac{(2e^2)\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2}-\frac{(2e^3)\,\text{Subst}\left(\int \frac{1-\frac{x^2}{e^2}}{x^{5/2}}\,dx,x,e\sin(c+dx)\right)}{a^2d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{4e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} \\
&\quad - \frac{(4e^2)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^2} - \frac{(2e^3)\,\text{Subst}\left(\int\left(\frac{1}{x^{5/2}} - \frac{1}{e^2\sqrt{x}}\right)\,dx, x, e\sin(c+dx)\right)}{a^2d} \\
&\quad - \frac{\left(2e^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a^2\sqrt{e\sin(c+dx)}} \\
&= \frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} \\
&\quad - \frac{4e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} + \frac{4e\sqrt{e\sin(c+dx)}}{a^2d} \\
&\quad - \frac{4e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} - \frac{\left(4e^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a^2\sqrt{e\sin(c+dx)}} \\
&= \frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} \\
&\quad - \frac{4e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{4e\sqrt{e\sin(c+dx)}}{a^2d} - \frac{4e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.63

$$
\int\frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left((15+10\cos(c+dx)-\cos(2(c+dx)))\csc(c+\right.}{3a^2d(1+\sec(c+}
$$

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*((15 + 10*Cos[c + d*x] - Cos[2*(c + d*
x)])*Csc[c + d*x]*Sec[(c + d*x)/2]^2 + (24*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(3*a^2*d*(1 + Sec[c + d*x]
)^2)
```

Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.81

method	result
default	$-\frac{2e^3\left(3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{7}{2}}\operatorname{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\cos(dx+c)^6+6\cos(dx+c)^5+4\cos(dx+c)\right.}{3a^2(e\sin(dx+c))^{\frac{3}{2}}\cos(dx+c)\left(\cos(dx+c)^2-1\right)d}$

[In] int((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -2/3/a^2/(e*sin(d*x+c))^(3/2)/cos(d*x+c)/(cos(d*x+c)^2-1)*e^3*(3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-cos(d*x+c)^6+6*cos(d*x+c)^5+4*cos(d*x+c)^4-14*cos(d*x+c)^3-3*cos(d*x+c)^2+8*cos(d*x+c))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.74

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{2\left(3\left(\sqrt{2}e\cos(dx+c)+\sqrt{2}e\right)\sqrt{-i\,e}\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+3\left(\sqrt{2}e\cos\right.\right.}{}$$

[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] -2/3*(3*(sqrt(2)*e*cos(d*x + c) + sqrt(2)*e)*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 3*(sqrt(2)*e*cos(d*x + c) + sqrt(2)*e)*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + (e*cos(d*x + c)^2 - 5*e*cos(d*x + c) - 8*e)*sqrt(e*sin(d*x + c)))/(a^2*d*cos(d*x + c) + a^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

[In] integrate((e*sin(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin\left(dx+c\right))^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin\left(dx+c\right))^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(e\,\sin\left(c+d\,x\right)\right)^{3/2}}{a^2\,\left(\cos\left(c+d\,x\right)+1\right)^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.130 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 188

$$\begin{aligned}\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx &= \frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} \\ &\quad - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} \\ &\quad - \frac{4e}{a^2d\sqrt{e\sin(c+dx)}} + \frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}} \\ &\quad + \frac{28E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}\end{aligned}$$

```
[Out] 4/5*e^3/a^2/d/(e*sin(d*x+c))^(5/2)-2/5*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(5/2)-2/5*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(5/2)-4*e/a^2/d/(e*sin(d*x+
c))^(1/2)+16/5*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(1/2)-28/5*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

= {3957, 2954, 2952, 2647, 2716, 2721, 2719, 2644, 14}

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{4e}{a^2d\sqrt{e\sin(c+dx)}} + \frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}} + \frac{28E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}$$

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*e^3)/(5*a^2*d*(e*Sin[c + d*x])^(5/2)) - (2*e^3*Cos[c + d*x])/(5*a^2*d*(e
*Sin[c + d*x])^(5/2)) - (2*e^3*Cos[c + d*x]^3)/(5*a^2*d*(e*Sin[c + d*x])^(5
/2)) - (4*e)/(a^2*d*Sqrt[e*Sin[c + d*x]]) + (16*e*Cos[c + d*x])/(5*a^2*d*Sq
rt[e*Sin[c + d*x]]) + (28*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d
*x]])/(5*a^2*d*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sqrt{e\sin(c+dx)}}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx}{a^4} \\
&= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}-\frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{7/2}}+\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{7/2}}\right)dx}{a^4} \\
&= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2}+\frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2}-\frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2}
\end{aligned}
$$

$$= -\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{(2e^2)\int\frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{5a^2} - \frac{(6e^2)\int\frac{\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{5a^2} - \frac{(2e^3)\,\mathrm{Subst}\left(\int\frac{1-\frac{x^2}{e^2}}{x^{7/2}}\,dx,x,e\sin(c+dx)\right)}{a^2d}$$

$$= -\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} + \frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}} + \frac{2\int\sqrt{e\sin(c+dx)}\,dx}{5a^2} + \frac{12\int\sqrt{e\sin(c+dx)}\,dx}{5a^2} - \frac{(2e^3)\,\mathrm{Subst}\left(\int\left(\frac{1}{x^{7/2}}-\frac{1}{e^2x^{3/2}}\right)\,dx,x,e\sin(c+dx)\right)}{a^2d}$$

$$= \frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{4e}{a^2d\sqrt{e\sin(c+dx)}} + \frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}} + \frac{\left(2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{5a^2\sqrt{\sin(c+dx)}} + \frac{\left(12\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{5a^2\sqrt{\sin(c+dx)}}$$

$$= \frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{4e}{a^2d\sqrt{e\sin(c+dx)}} + \frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}} + \frac{28E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.24

$$\int\frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{e\sin(c+dx)}\left(\sec(c)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(49\sin\left(\frac{1}{2}(c-dx)\right)+35\sin\left(\frac{1}{2}(3c+d\right.\right.\right.}{}$$

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*Sqrt[e*Sin[c + d*x]]*(Sec[c]*Sec[(c + d*
x)/2]^3*(49*Sin[(c - d*x)/2] + 35*Sin[(3*c + d*x)/2] - 23*Sin[(c + 3*d*x)/2
] + 5*Sin[(5*c + 3*d*x)/2]) - 56*Sqrt[Csc[c]^2]*Csc[c + d*x]*Csc[d*x - ArcT
```

```
an[Cot[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x - ArcTan[Cot[c]]
]^2]*Sin[c]*Sqrt[Sin[d*x - ArcTan[Cot[c]]]^2] - 28*Sqrt[Csc[c]^2]*Csc[c + d
*x]*(Sin[c + d*x - ArcTan[Cot[c]]] + 3*Sin[c - d*x + ArcTan[Cot[c]]])*Tan[c
]))/(5*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.09

method	result
default	$\frac{-\frac{2e\left(-\frac{2e^2}{5(e\sin(dx+c))^{\frac{5}{2}}}+\frac{2}{\sqrt{e\sin(dx+c)}}\right)}{a^2}-\frac{2e\left(14\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{7}{2}}\operatorname{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-7\sqrt{-\sin(dx+}}{5a^2\sin(dx+c)}}{d}$

```
[In] int((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e/a^2*(-2/5*e^2/(e*sin(d*x+c))^(5/2)+2/(e*sin(d*x+c))^(1/2))-2/5*e*(14*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-s
in(d*x+c)+1)^(1/2),1/2*2^(1/2))-7*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+9*sin(d*x
+c)^5-11*sin(d*x+c)^3+2*sin(d*x+c))/a^2/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+
c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\left(\sqrt{e\sin\left(dx+c\right)}(9\,\cos\left(dx+c\right)+8)\sin\left(dx+c\right)+7\left(-i\,\sqrt{2}\cos\left(dx+c\right)^2-2i\,\sqrt{2}\cos\left(dx+c\right)-i\,\sqrt{}\right.\right.}{}$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/5*(sqrt(e*sin(d*x + c))*(9*cos(d*x + c) + 8)*sin(d*x + c) + 7*(-I*sqrt(2
)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*sqrt(-I*e)*weierst
rassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) +
7*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*sqrt(I*
e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x
 + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{e\sin(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

[In] integrate((e*sin(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)

[Out] Integral(sqrt(e*sin(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2

Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2\,\sqrt{e\,\sin(c+d\,x)}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

[In] int((e*sin(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)

[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)

3.131 $\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 190

$$\begin{aligned}\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx &= \frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}\\&\quad-\frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}\\&\quad-\frac{4e}{3a^2d(e\sin(c+dx))^{3/2}}\\&\quad+\frac{16e\cos(c+dx)}{21a^2d(e\sin(c+dx))^{3/2}}\\&\quad+\frac{20\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}}\end{aligned}$$

```
[Out] 4/7*e^3/a^2/d/(e*sin(d*x+c))^(7/2)-2/7*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(7/2)-2/7*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(7/2)-4/3*e/a^2/d/(e*sin(d*
x+c))^(3/2)+16/21*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(3/2)-20/21*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3957, 2954, 2952, 2647, 2716, 2721, 2720, 2644, 14}

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{4e}{3a^2d(e\sin(c+dx))^{3/2}} + \frac{16e\cos(c+dx)}{21a^2d(e\sin(c+dx))^{3/2}} + \frac{20\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21a^2d\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (4*e^3)/(7*a^2*d*(e*Sin[c + d*x])^(7/2)) - (2*e^3*Cos[c + d*x])/(7*a^2*d*(e
*Sin[c + d*x])^(7/2)) - (2*e^3*Cos[c + d*x]^3)/(7*a^2*d*(e*Sin[c + d*x])^(7
/2)) - (4*e)/(3*a^2*d*(e*Sin[c + d*x])^(3/2)) + (16*e*Cos[c + d*x])/(21*a^2
*d*(e*Sin[c + d*x])^(3/2)) + (20*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[
c + d*x]])/(21*a^2*d*Sqrt[e*Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx\\ &= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{9/2}}\,dx}{a^4}\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^4 \int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{9/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{9/2}}\right) dx}{a^4} \\
&= \frac{e^4 \int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a^2} + \frac{e^4 \int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a^2} \\
&= -\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{(2e^2)\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{7a^2} \\
&\quad - \frac{(6e^2)\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{7a^2} - \frac{(2e^3)\,\text{Subst}\left(\int \frac{1-\frac{x^2}{e^2}}{x^{9/2}}\,dx, x, e\sin(c+dx)\right)}{a^2d} \\
&= -\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} \\
&\quad + \frac{16e\cos(c+dx)}{21a^2d(e\sin(c+dx))^{3/2}} - \frac{2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{21a^2} + \frac{4\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{7a^2} \\
&\quad - \frac{(2e^3)\,\text{Subst}\left(\int \left(\frac{1}{x^{9/2}} - \frac{1}{e^2x^{5/2}}\right) dx, x, e\sin(c+dx)\right)}{a^2d} \\
&= \frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} \\
&\quad - \frac{4e}{3a^2d(e\sin(c+dx))^{3/2}} + \frac{16e\cos(c+dx)}{21a^2d(e\sin(c+dx))^{3/2}} \\
&\quad - \frac{\left(2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21a^2\sqrt{e\sin(c+dx)}} + \frac{\left(4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{7a^2\sqrt{e\sin(c+dx)}} \\
&= \frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} \\
&\quad - \frac{4e}{3a^2d(e\sin(c+dx))^{3/2}} + \frac{16e\cos(c+dx)}{21a^2d(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{20\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.43

$$
\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx =
$$

$$
-\frac{\csc^3(c+dx)\left(16(8+11\cos(c+dx))\sin^4\left(\frac{1}{2}(c+dx)\right) + 40\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx), 2\right)\sin^{\frac{7}{2}}(c+dx)\right.}{42a^2d\sqrt{e\sin(c+dx)}}
$$

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] -1/42*(Csc[c + d*x]^3*(16*(8 + 11*Cos[c + d*x])*Sin[(c + d*x)/2]^4 + 40*Ell
ipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(a^2*d*Sqrt[e*Sin[c +
 d*x]])
```

Maple [A] (verified)

Time = 5.89 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.78

method	result
default	$\frac{\frac{4e^3\left(7\cos(dx+c)^2-4\right)}{21a^2(e\sin(dx+c))^{\frac{7}{2}}}-\frac{2\left(5\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{9}{2}}\,\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)+11\sin(dx+c)^5-17\sin(dx+c)^3+6\sin(\cdots\right)}{21a^2\sin(dx+c)^4\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/21/a^2*e^3/(e*sin(d*x+c))^(7/2)*(7*cos(d*x+c)^2-4)-2/21*(5*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))+11*sin(d*x+c)^5-17*sin(d*x+c)^3+6*sin(d*x+c))/a^2/sin(d*
x+c)^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \frac{2\left(5\left(\sqrt{2}\cos(dx+c)^2+2\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-i\,e}\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+\cdots\right.}{2\cdots}$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*(sqrt(2)*cos(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-I
*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*c
os(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(I*e)*weierstrassPInv
erse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(e*sin(d*x + c))*(11*cos(d*
x + c) + 8))/(a^2*d*e*cos(d*x + c)^2 + 2*a^2*d*e*cos(d*x + c) + a^2*d*e)
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \frac{\int \frac{1}{\sqrt{e\sin{(c+dx)}}\sec^2{(c+dx)}+2\sqrt{e\sin{(c+dx)}}\sec{(c+dx)}+\sqrt{e\sin{(c+dx)}}}\,dx}{a^2}$$

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*sin(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*sin(c + d*x))*s
ec(c + d*x) + sqrt(e*sin(c + d*x))), x)/a**2
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(a\sec{(dx+c)}+a)^2\sqrt{e\sin{(dx+c)}}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos{(c+d\,x)}^2}{a^2\,\sqrt{e\,\sin{(c+d\,x)}}\,{(\cos{(c+d\,x)}+1)}^2}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.132 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 224

$$
\begin{aligned}
\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx &= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} \\
&- \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} \\
&- \frac{4e}{5a^2d(e\sin(c+dx))^{5/2}} + \frac{16e\cos(c+dx)}{45a^2d(e\sin(c+dx))^{5/2}} \\
&- \frac{4\cos(c+dx)}{15a^2de\sqrt{e\sin(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2de^2\sqrt{\sin(c+dx)}}
\end{aligned}
$$

```
[Out] 4/9*e^3/a^2/d/(e*sin(d*x+c))^(9/2)-2/9*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(9/2)-2/9*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(9/2)-4/5*e/a^2/d/(e*sin(d*
x+c))^(5/2)+16/45*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(5/2)-4/15*cos(d*x+c)/a
^2/d/e/(e*sin(d*x+c))^(1/2)+4/15*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x
+c))^(1/2)/a^2/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

$= \{3957, 2954, 2952, 2647, 2716, 2721, 2719, 2644, 14\}$

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2de^2\sqrt{\sin(c+dx)}} - \frac{4e}{5a^2d(e\sin(c+dx))^{5/2}} + \frac{16e\cos(c+dx)}{45a^2d(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{15a^2de\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (4*e^3)/(9*a^2*d*(e*Sin[c + d*x])^(9/2)) - (2*e^3*Cos[c + d*x])/(9*a^2*d*(e
*Sin[c + d*x])^(9/2)) - (2*e^3*Cos[c + d*x]^3)/(9*a^2*d*(e*Sin[c + d*x])^(9
/2)) - (4*e)/(5*a^2*d*(e*Sin[c + d*x])^(5/2)) + (16*e*Cos[c + d*x])/(45*a^2
*d*(e*Sin[c + d*x])^(5/2)) - (4*Cos[c + d*x])/(15*a^2*d*e*Sqrt[e*Sin[c + d*
x]]) - (4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(15*a^2*d*
e^2*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx \\
&= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{11/2}}\,dx}{a^4} \\
&= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{11/2}}-\frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{11/2}}+\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{11/2}}\right)dx}{a^4} \\
&= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{11/2}}\,dx}{a^2}+\frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{11/2}}\,dx}{a^2}-\frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{11/2}}\,dx}{a^2}
\end{aligned}
$$

$$= -\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{(2e^2)\int \frac{1}{(e\sin(c+dx))^{7/2}}\,dx}{9a^2} - \frac{(2e^2)\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{3a^2} - \frac{(2e^3)\,\text{Subst}\left(\int \frac{1-\frac{x^2}{e^2}}{x^{11/2}}\,dx, x, e\sin(c+dx)\right)}{a^2d}$$

$$= -\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} + \frac{16e\cos(c+dx)}{45a^2d(e\sin(c+dx))^{5/2}} - \frac{2\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{15a^2} + \frac{4\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{15a^2} - \frac{(2e^3)\,\text{Subst}\left(\int \left(\frac{1}{x^{11/2}} - \frac{1}{e^2x^{7/2}}\right)\,dx, x, e\sin(c+dx)\right)}{a^2d}$$

$$= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{4e}{5a^2d(e\sin(c+dx))^{5/2}} + \frac{16e\cos(c+dx)}{45a^2d(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{15a^2de\sqrt{e\sin(c+dx)}} + \frac{2\int \sqrt{e\sin(c+dx)}\,dx}{15a^2e^2} - \frac{4\int \sqrt{e\sin(c+dx)}\,dx}{15a^2e^2}$$

$$= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{4e}{5a^2d(e\sin(c+dx))^{5/2}} + \frac{16e\cos(c+dx)}{45a^2d(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{15a^2de\sqrt{e\sin(c+dx)}} + \frac{\left(2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{15a^2e^2\sqrt{\sin(c+dx)}} - \frac{\left(4\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{15a^2e^2\sqrt{\sin(c+dx)}}$$

$$= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{4e}{5a^2d(e\sin(c+dx))^{5/2}} + \frac{16e\cos(c+dx)}{45a^2d(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{15a^2de\sqrt{e\sin(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2de^2\sqrt{\sin(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.99 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+i\sin(c+dx))\left(-31-40\cos(\right.}{}$$

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^4*(Cos[c + d*x] + I*Sin[c + d*x])*(-31 - 40*Cos[c + d*x]
- 19*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^4*Sqrt[1 - E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*
x)) + (16*I)*Sin[c + d*x] + (13*I)*Sin[2*(c + d*x)]))/(180*a^2*d*e*Sqrt[e*S
in[c + d*x]])
```

Maple [A] (verified)

Time = 6.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{4e^3\left(9\cos(dx+c)^2-4\right)}{45a^2(e\sin(dx+c))^{\frac{9}{2}}}+\frac{\frac{4\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{11}{2}}\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)}{15}-\frac{2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\mathrm{s}}{}}{e\,a^2\sin(dx+c)^5\cos(dx+c)\sqrt{}}}{d}$

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/45*e^3/a^2/(e*sin(d*x+c))^(9/2)*(9*cos(d*x+c)^2-4)+2/45/e*(6*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(11/2)*EllipticE((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(11/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+6*sin(d*x+c)^7-19*
sin(d*x+c)^5+23*sin(d*x+c)^3-10*sin(d*x+c))/a^2/sin(d*x+c)^5/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(3\left(i\sqrt{2}\cos{(dx+c)}^2+2i\sqrt{2}\cos{(dx+c)}+i\sqrt{2}\right)\sqrt{-i\,e}\sin{(dx+c)}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPIn}}{}$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45*(3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos
(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*c
os(d*x + c) - I*sqrt(2))*sqrt(I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weier
strassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + (6*cos(d*x + c)^3 +
12*cos(d*x + c)^2 + 19*cos(d*x + c) + 8)*sqrt(e*sin(d*x + c)))/((a^2*d*e^2*
cos(d*x + c)^2 + 2*a^2*d*e^2*cos(d*x + c) + a^2*d*e^2)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^2}{a^2\,(e\,\sin(c+d\,x))^{3/2}\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

3.133 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 224

$$\begin{aligned}\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx &= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}}\\ &-\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}\\ &-\frac{4e}{7a^2d(e\sin(c+dx))^{7/2}}+\frac{16e\cos(c+dx)}{77a^2d(e\sin(c+dx))^{7/2}}\\ &-\frac{4\cos(c+dx)}{231a^2de(e\sin(c+dx))^{3/2}}+\frac{4\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231a^2de^2\sqrt{e\sin(c+dx)}}\end{aligned}$$

```
[Out] 4/11*e^3/a^2/d/(e*sin(d*x+c))^(11/2)-2/11*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c
))^(11/2)-2/11*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(11/2)-4/7*e/a^2/d/(e*
sin(d*x+c))^(7/2)+16/77*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(7/2)-4/231*cos(d
*x+c)/a^2/d/e/(e*sin(d*x+c))^(3/2)-4/231*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*si
n(d*x+c)^(1/2)/a^2/d/e^2/(e*sin(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

$= \{3957, 2954, 2952, 2647, 2716, 2721, 2720, 2644, 14\}$

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} + \frac{4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{231a^2de^2\sqrt{e\sin(c+dx)}} - \frac{4e}{7a^2d(e\sin(c+dx))^{7/2}} + \frac{16e\cos(c+dx)}{77a^2d(e\sin(c+dx))^{7/2}} - \frac{4\cos(c+dx)}{231a^2de(e\sin(c+dx))^{3/2}}$$

```
[In] Int[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (4*e^3)/(11*a^2*d*(e*Sin[c + d*x])^(11/2)) - (2*e^3*Cos[c + d*x])/(11*a^2*d
*(e*Sin[c + d*x])^(11/2)) - (2*e^3*Cos[c + d*x]^3)/(11*a^2*d*(e*Sin[c + d*x
])^(11/2)) - (4*e)/(7*a^2*d*(e*Sin[c + d*x])^(7/2)) + (16*e*Cos[c + d*x])/(
77*a^2*d*(e*Sin[c + d*x])^(7/2)) - (4*Cos[c + d*x])/(231*a^2*d*e*(e*Sin[c +
 d*x])^(3/2)) + (4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(23
1*a^2*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx \\
&= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{13/2}}\,dx}{a^4} \\
&= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{13/2}}-\frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{13/2}}+\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{13/2}}\right)dx}{a^4} \\
&= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{13/2}}\,dx}{a^2}+\frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{13/2}}\,dx}{a^2}-\frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{13/2}}\,dx}{a^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{(2e^2)\int \frac{1}{(e\sin(c+dx))^{9/2}}\,dx}{11a^2} \\
&\quad - \frac{(6e^2)\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{11a^2} - \frac{(2e^3)\,\text{Subst}\left(\int \frac{1-\frac{x^2}{e^2}}{x^{13/2}}\,dx, x, e\sin(c+dx)\right)}{a^2d} \\
&= -\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} \\
&\quad + \frac{16e\cos(c+dx)}{77a^2d(e\sin(c+dx))^{7/2}} - \frac{10\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{77a^2} + \frac{12\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{77a^2} \\
&\quad - \frac{(2e^3)\,\text{Subst}\left(\int \left(\frac{1}{x^{13/2}} - \frac{1}{e^2x^{9/2}}\right)\,dx, x, e\sin(c+dx)\right)}{a^2d} \\
&= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} \\
&\quad - \frac{4e}{7a^2d(e\sin(c+dx))^{7/2}} + \frac{16e\cos(c+dx)}{77a^2d(e\sin(c+dx))^{7/2}} \\
&\quad - \frac{4\cos(c+dx)}{231a^2de(e\sin(c+dx))^{3/2}} - \frac{10\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{231a^2e^2} + \frac{4\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{77a^2e^2} \\
&= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} \\
&\quad - \frac{4e}{7a^2d(e\sin(c+dx))^{7/2}} + \frac{16e\cos(c+dx)}{77a^2d(e\sin(c+dx))^{7/2}} - \frac{4\cos(c+dx)}{231a^2de(e\sin(c+dx))^{3/2}} \\
&\quad - \frac{\left(10\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{231a^2e^2\sqrt{e\sin(c+dx)}} + \frac{\left(4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{77a^2e^2\sqrt{e\sin(c+dx)}} \\
&= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} \\
&\quad - \frac{4e}{7a^2d(e\sin(c+dx))^{7/2}} + \frac{16e\cos(c+dx)}{77a^2d(e\sin(c+dx))^{7/2}} \\
&\quad - \frac{4\cos(c+dx)}{231a^2de(e\sin(c+dx))^{3/2}} + \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{231a^2de^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx =$$

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(52+97\cos(c+dx)+4\cos(2(c+dx))+\cos(3(c+dx))+\csc^4\left(\frac{1}{2}(c+dx\right.\right.}{1848a^2de^2\sqrt{e\sin(c+dx)}}$$

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -1/1848*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]^5*(52 + 97*Cos[c + d*x] + 4*Cos[
2*(c + d*x)] + Cos[3*(c + d*x)] + Csc[(c + d*x)/2]^4*EllipticF[(-2*c + Pi -
 2*d*x)/4, 2]*Sin[c + d*x]^(11/2)))/(a^2*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Maple [A] (verified)

Time = 6.73 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.71

method	result
default	$\frac{\frac{4e^3\left(11\cos(dx+c)^2-4\right)}{77a^2(e\sin(dx+c))^{\frac{11}{2}}}-\frac{2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sin(dx+c)^{\frac{13}{2}}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)^7+47\sin(dx+c)^5-87\sin(d\right.}{231e^2a^2\sin(dx+c)^6\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/77*e^3/a^2/(e*sin(d*x+c))^(11/2)*(11*cos(d*x+c)^2-4)-2/231/e^2*((-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(13/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^7+47*sin(d*x+c)^5-87*sin(d*x+c)^3+42*
sin(d*x+c))/a^2/sin(d*x+c)^6/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \frac{2\left(\left(\sqrt{2}\cos\left(dx+c\right)^4+2\sqrt{2}\cos\left(dx+c\right)^3-2\sqrt{2}\cos\left(dx+c\right)\right.\right.}{}$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/231*((sqrt(2)*cos(d*x + c)^4 + 2*sqrt(2)*cos(d*x + c)^3 - 2*sqrt(2)*cos(d
*x + c) - sqrt(2))*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*si
```

```
n(d*x + c)) + (sqrt(2)*cos(d*x + c)^4 + 2*sqrt(2)*cos(d*x + c)^3 - 2*sqrt(2
)*cos(d*x + c) - sqrt(2))*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c)
- I*sin(d*x + c)) + (2*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + 47*cos(d*x + c)
+ 24)*sqrt(e*sin(d*x + c)))/(a^2*d*e^3*cos(d*x + c)^4 + 2*a^2*d*e^3*cos(d*x
 + c)^3 - 2*a^2*d*e^3*cos(d*x + c) - a^2*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{a^2\,\left(e\,\sin\left(c+d\,x\right)\right)^{5/2}\,\left(\cos\left(c+d\,x\right)+1\right)^2}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

3.134 $\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 247

$$\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx = \frac{a^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{3a^3\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} + \frac{a^3\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} + \frac{3a^3\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{1+m}}{de(1+m)}$$

```
[Out] 3*a^3*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+
m)/d/e/(1+m)+a^3*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(
d*x+c))^(1+m)/d/e/(1+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*
m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+3*a^3*
hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*sin(d*x+
c))^(1+m)*(cos(d*x+c)^2)^(1/2)/d/e/(1+m)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3957, 2952, 2722, 2644, 371, 2657}

$$\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx$$
$$=\frac{3a^3(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$
$$+\frac{a^3(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$
$$+\frac{a^3\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}$$
$$+\frac{3a^3\sqrt{\cos^2(c+dx)}\sec(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^
(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Si
n[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a^3*Sqrt[Cos[c +
 d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c
 + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-a-a\cos(c+dx))^3\sec^3(c+dx)(e\sin(c+dx))^m\,dx \\
&= -\int \left(-a^3(e\sin(c+dx))^m-3a^3\sec(c+dx)(e\sin(c+dx))^m \right.\\
&\qquad \left. -3a^3\sec^2(c+dx)(e\sin(c+dx))^m-a^3\sec^3(c+dx)(e\sin(c+dx))^m\right)\,dx \\
&= a^3\int (e\sin(c+dx))^m\,dx+a^3\int \sec^3(c+dx)(e\sin(c+dx))^m\,dx \\
&\quad +\left(3a^3\right)\int \sec(c+dx)(e\sin(c+dx))^m\,dx+\left(3a^3\right)\int \sec^2(c+dx)(e\sin(c+dx))^m\,dx \\
&= \frac{a^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} \\
&\quad +\frac{3a^3\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{1+}}{de(1+m)} \\
&\quad +\frac{a^3\text{Subst}\left(\int \frac{x^m}{\left(1-\frac{x^2}{e^2}\right)^2}\,dx,x,e\sin(c+dx)\right)}{de}+\frac{\left(3a^3\right)\text{Subst}\left(\int \frac{x^m}{1-\frac{x^2}{e^2}}\,dx,x,e\sin(c+dx)\right)}{de}
\end{aligned}$$

$$= \frac{a^3 \cos(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) (e \sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{3a^3 \operatorname{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) (e \sin(c+dx))^{1+m}}{de(1+m)}$$
$$+ \frac{a^3 \operatorname{Hypergeometric2F1}\left(2, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) (e \sin(c+dx))^{1+m}}{de(1+m)}$$
$$+ \frac{3a^3 \sqrt{\cos^2(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) \sec(c+dx)(e \sin(c+dx))^{1+m}}{de(1+m)}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.65

$$\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx$$
$$= \frac{a^3(e\sin(c+dx))^m \left(3 \operatorname{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) \sin(c+dx) + \operatorname{Hypergeometric2F1}\left(2\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^3*(e*Sin[c + d*x])^m*(3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c
 + d*x]^2]*Sin[c + d*x] + Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Sin[c
+ d*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*(Hypergeometric2F1[1/2, (1 +
m)/2, (3 + m)/2, Sin[c + d*x]^2] + 3*Hypergeometric2F1[3/2, (1 + m)/2, (3 +
 m)/2, Sin[c + d*x]^2])*Tan[c + d*x]))/(d*(1 + m))
```

Maple [F]

$$\int (a+a\sec(dx+c))^3 (e\sin(dx+c))^m\,dx$$

```
[In] int((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx=\int (a\sec(dx+c)+a)^3(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*(e*sin(d*x + c))^m, x)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx=a^3\left(\int (e\sin(c+dx))^m\,dx+\int 3(e\sin(c+dx))^m\sec(c+dx)\,dx+\int 3(e\sin(c+dx))^m\sec^2(c+dx)\,dx+\int (e\sin(c+dx))^m\sec^3(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**3*(e*sin(d*x+c))**m,x)
```

```
[Out] a**3*(Integral((e*sin(c + d*x))**m, x) + Integral(3*(e*sin(c + d*x))**m*sec
(c + d*x), x) + Integral(3*(e*sin(c + d*x))**m*sec(c + d*x)**2, x) + Integr
al((e*sin(c + d*x))**m*sec(c + d*x)**3, x))
```

Maxima [F]

$$\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx=\int (a\sec(dx+c)+a)^3(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx=\int (a\sec(dx+c)+a)^3(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx=\int (e\,\sin(c+d\,x))^m\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\,dx$$

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^3, x)
```

3.135 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 195

$$
\begin{aligned}
&\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx \\
&= \frac{a^2\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} \\
&\quad+\frac{2a^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} \\
&\quad+\frac{a^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{1+m}}{de(1+m)}
\end{aligned}
$$

```
[Out] 2*a^2*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+
m)/d/e/(1+m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+
c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+a^2*hypergeom([3/
2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*sin(d*x+c))^(1+m)*(co
s(d*x+c)^2)^(1/2)/d/e/(1+m)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{3957, 2952, 2722, 2644, 371, 2657\}$

$$\int (a + a\sec(c+dx))^2 (e\sin(c+dx))^m\,dx$$
$$= \frac{2a^2(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1, \frac{m+1}{2}, \frac{m+3}{2}, \sin^2(c+dx)\right)}{de(m+1)} + \frac{a^2\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}} + \frac{a^2\sqrt{\cos^2(c+dx)}\sec(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \sin^2(c+dx)\right)}{de(m+1)}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^
(1 + m))/(d*e*(1 + m)) + (a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (
1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Sin[c + d*x])^(1 + m))
/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-a-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^m\,dx\\
&= \int \left(a^2(e\sin(c+dx))^m+2a^2\sec(c+dx)(e\sin(c+dx))^m+a^2\sec^2(c+dx)(e\sin(c+dx))^m\right)\,dx\\
&= a^2\int (e\sin(c+dx))^m\,dx+a^2\int \sec^2(c+dx)(e\sin(c+dx))^m\,dx\\
&\quad+\left(2a^2\right)\int \sec(c+dx)(e\sin(c+dx))^m\,dx\\
&= \frac{a^2\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}\\
&\quad+\frac{a^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{1+m}}{de(1+m)}\\
&\quad+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^m}{1-\frac{x^2}{e^2}}\,dx,x,e\sin(c+dx)\right)}{de}\\
&= \frac{a^2\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}\\
&\quad+\frac{2a^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}\\
&\quad+\frac{a^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{1+m}}{de(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx$$
$$=\frac{a^2(e\sin(c+dx))^m\left(2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin(c+dx)+\sqrt{\cos^2(c+dx)}(\text{Hyper}\right.}{d(1\ }$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^2*(e*Sin[c + d*x])^m*(2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c
 + d*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*(Hypergeometric2F1[1/2, (1 +
 m)/2, (3 + m)/2, Sin[c + d*x]^2] + Hypergeometric2F1[3/2, (1 + m)/2, (3 +
m)/2, Sin[c + d*x]^2])*Tan[c + d*x]))/(d*(1 + m))
```

Maple [F]

$$\int \left(a+a\sec\left(dx+c\right)\right)^{2}\left(e\sin\left(dx+c\right)\right)^{m}dx$$

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx=\int (a\sec(dx+c)+a)^2(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*(e*sin(d*x + c))^m
, x)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx=a^2\left(\int (e\sin(c+dx))^m\,dx+\int 2(e\sin(c+dx))^m\sec(c+dx)\,dx+\int (e\sin(c+dx))^m\sec^2(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**m,x)
```

```
[Out] a**2*(Integral((e*sin(c + d*x))**m, x) + Integral(2*(e*sin(c + d*x))**m*sec
(c + d*x), x) + Integral((e*sin(c + d*x))**m*sec(c + d*x)**2, x))
```

Maxima [F]

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx=\int (a\sec(dx+c)+a)^2(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx=\int (a\sec(dx+c)+a)^2(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx=\int (e\,\sin(c+d\,x))^m\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,dx$$

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^2, x)
```

3.136 $\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx = \frac{a\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d
/e/(1+m)+a*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*
(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2917, 2644, 371, 2722}

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx = \frac{a(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)} + \frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (a*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m
))/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int (-a - a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^m\,dx\\ &= a\int (e\sin(c+dx))^m\,dx + a\int \sec(c+dx)(e\sin(c+dx))^m\,dx\end{aligned}$$

$$= \frac{a\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{x^m}{1-\frac{x^2}{e^2}}\,dx,x,e\sin(c+dx)\right)}{de}$$

$$= \frac{a\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx = \frac{a(e\sin(c+dx))^m\left(\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin(c+dx)+\sqrt{\cos^2(c+dx)}\,\text{Hypergeo}\right.}{d(1+m)}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a*(e*Sin[c + d*x])^m*(Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d
*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x]))/(d*(1 + m))
```

Maple [F]

$$\int (a+a\sec(dx+c))\,(e\sin(dx+c))^m\,dx$$

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx = \int (a\sec(dx+c)+a)(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [F]

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx = a\left(\int (e\sin(c+dx))^m\,dx + \int (e\sin(c+dx))^m\sec(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**m,x)
[Out] a*(Integral((e*sin(c + d*x))**m, x) + Integral((e*sin(c + d*x))**m*sec(c +
d*x), x))
```

Maxima [F]

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx = \int (a\sec(dx+c)+a)(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Giac [F]

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx = \int (a\sec(dx+c)+a)(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx = \int (e\sin(c+d\,x))^m\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x)), x)
```

3.137 $\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = -\frac{e(e\sin(c+dx))^{-1+m}}{ad(1-m)} + \frac{e\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{-1+m}}{ad(1-m)\sqrt{\cos^2(c+dx)}}$$

```
[Out] -e*(e*sin(d*x+c))^(-1+m)/a/d/(1-m)+e*cos(d*x+c)*hypergeom([-1/2, -1/2+1/2*m
],[1/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-1+m)/a/d/(1-m)/(cos(d*x+c)^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3957, 2918, 2644, 30, 2657}

$$\begin{aligned}\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx &= \frac{e\cos(c+dx)(e\sin(c+dx))^{m-1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{m+1}{2},\sin^2(c+dx)\right)}{ad(1-m)\sqrt{\cos^2(c+dx)}}\\ &\quad -\frac{e(e\sin(c+dx))^{m-1}}{ad(1-m)}\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((e*(e*Sin[c + d*x])^(-1 + m))/(a*d*(1 - m))) + (e*Cos[c + d*x]*Hypergeome
tric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(-1 +
 m))/(a*d*(1 - m)*Sqrt[Cos[c + d*x]^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^m}{-a-a\cos(c+dx)}\,dx \\
&= \frac{e^2\int \cos(c+dx)(e\sin(c+dx))^{-2+m}\,dx}{a} - \frac{e^2\int \cos^2(c+dx)(e\sin(c+dx))^{-2+m}\,dx}{a} \\
&= \frac{e\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{-1+m}}{ad(1-m)\sqrt{\cos^2(c+dx)}} \\
&\quad + \frac{e\text{Subst}\left(\int x^{-2+m}\,dx, x, e\sin(c+dx)\right)}{ad}
\end{aligned}
$$

$$= -\frac{e(e\sin(c+dx))^{-1+m}}{ad(1-m)} + \frac{e\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{-1+m}}{ad(1-m)\sqrt{\cos^2(c+dx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.38 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.83

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \frac{i2^{1-m}\left(1-e^{2i(c+dx)}\right)^{-m}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^m\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2e^{i(c+dx)}(-2+m)m\,\text{Hypergeometric2}\right.}{}$$

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

```
[Out] (I*2^(1 - m)*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^m*Cos[c/2
+ (d*x)/2]^2*(2*E^(I*(c + d*x))*(-2 + m)*m*Hypergeometric2F1[(1 - m)/2, 1 -
 m, (3 - m)/2, E^((2*I)*(c + d*x))] - 4*E^(I*(c + d*x))*(-2 + m)*m*Hypergeo
metric2F1[(1 - m)/2, 2 - m, (3 - m)/2, E^((2*I)*(c + d*x))] - (-1 + m)*(2*(
-2 + m)*Hypergeometric2F1[1 - m, -1/2*m, 1 - m/2, E^((2*I)*(c + d*x))] - 2*
E^((2*I)*(c + d*x))*m*Hypergeometric2F1[2 - m, 1 - m/2, 2 - m/2, E^((2*I)*(
c + d*x))] - (-2 + m)*(2*Hypergeometric2F1[2 - m, -1/2*m, 1 - m/2, E^((2*I)
*(c + d*x))] + Hypergeometric2F1[-m, -1/2*m, 1 - m/2, E^((2*I)*(c + d*x))])
))*Sec[c + d*x]*(e*Sin[c + d*x])^m)/(d*(1 - E^((2*I)*(c + d*x)))^m*(-2 + m)
*(-1 + m)*m*(a + a*Sec[c + d*x])*Sin[c + d*x]^m)
```

Maple [F]

$$\int \frac{(e\sin(dx+c))^m}{a+a\sec(dx+c)}dx$$

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

Fricas [F]

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin\left(dx+c\right))^m}{a\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\sin\left(c+dx\right))^m}{\sec\left(c+dx\right)+1}\,dx}{a}$$

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c)),x)
[Out] Integral((e*sin(c + d*x))**m/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin\left(dx+c\right))^m}{a\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{(e\sin\left(dx+c\right))^m}{a\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(e\,\sin\left(c+d\,x\right)\right)^m}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^m)/(a*(cos(c + d*x) + 1)), x)
```

3.138 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 207

$$\begin{aligned}\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx &= \frac{2e^3(e\sin(c+dx))^{-3+m}}{a^2d(3-m)}\\ &- \frac{e^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-3+m}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}\\ &- \frac{e^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-3+m}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}\\ &- \frac{2e(e\sin(c+dx))^{-1+m}}{a^2d(1-m)}\end{aligned}$$

```
[Out] 2*e^3*(e*sin(d*x+c))^(-3+m)/a^2/d/(3-m)-2*e*(e*sin(d*x+c))^(-1+m)/a^2/d/(1-
m)-e^3*cos(d*x+c)*hypergeom([-3/2, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)*(
e*sin(d*x+c))^(-3+m)/a^2/d/(3-m)/(cos(d*x+c)^2)^(1/2)-e^3*cos(d*x+c)*hyperg
eom([-1/2, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-3+m)/a^2
/d/(3-m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{3957, 2954, 2952, 2657, 2644, 14\}$

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$$
$$= -\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},\frac{m-1}{2},\sin^2(c+dx)\right)}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}$$
$$-\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-3}{2},\frac{m-1}{2},\sin^2(c+dx)\right)}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}$$
$$+\frac{2e^3(e\sin(c+dx))^{m-3}}{a^2d(3-m)}-\frac{2e(e\sin(c+dx))^{m-1}}{a^2d(1-m)}$$

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*e^3*(e*Sin[c + d*x])^(-3 + m))/(a^2*d*(3 - m)) - (e^3*Cos[c + d*x]*Hyper
geometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])
^(-3 + m))/(a^2*d*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (e^3*Cos[c + d*x]*Hyperge
ometric2F1[-1/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(
-3 + m))/(a^2*d*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (2*e*(e*Sin[c + d*x])^(-1 +
 m))/(a^2*d*(1 - m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
```

```
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^2}\,dx$$

$$= \frac{e^4\int \cos^2(c+dx)(-a+a\cos(c+dx))^2(e\sin(c+dx))^{-4+m}\,dx}{a^4}$$

$$= \frac{e^4\int \left(a^2\cos^2(c+dx)(e\sin(c+dx))^{-4+m}-2a^2\cos^3(c+dx)(e\sin(c+dx))^{-4+m}+a^2\cos^4(c+dx)\right.}{a^4}$$

$$= \frac{e^4\int \cos^2(c+dx)(e\sin(c+dx))^{-4+m}\,dx}{a^2}+\frac{e^4\int \cos^4(c+dx)(e\sin(c+dx))^{-4+m}\,dx}{a^2} - \frac{(2e^4)\int \cos^3(c+dx)(e\sin(c+dx))^{-4+m}\,dx}{a^2}$$

$$= -\frac{e^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-3}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}} - \frac{e^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-3}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}} - \frac{(2e^3)\,\text{Subst}\left(\int x^{-4+m}\left(1-\frac{x^2}{e^2}\right)\,dx,x,e\sin(c+dx)\right)}{a^2d}$$

$$= -\frac{e^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-3}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}} - \frac{e^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-3}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}} - \frac{(2e^3)\,\text{Subst}\left(\int \left(x^{-4+m}-\frac{x^{-2+m}}{e^2}\right)\,dx,x,e\sin(c+dx)\right)}{a^2d}$$

$$= \frac{2e^3(e\sin(c+dx))^{-3+m}}{a^2d(3-m)}$$
$$- \frac{e^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-3+}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}$$
$$- \frac{e^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-3+}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}$$
$$- \frac{2e(e\sin(c+dx))^{-1+m}}{a^2d(1-m)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.96

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{i2^{4-m}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^m\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\left(-1+e^{2i(c+dx)}\right)\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},1-\frac{m}{2},e^{2i(c+dx)}\right)}{4m}+\right.}{}$$

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(4 - m)*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^m*Cos[(
c + d*x)/2]^4*(((-1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1, (2 + m)/2,
1 - m/2, E^((2*I)*(c + d*x))])/(4*m) + (E^(I*(c + d*x))*((6 - 5*m + m^2)*Hy
pergeometric2F1[(1 - m)/2, 2 - m, (3 - m)/2, E^((2*I)*(c + d*x))] + E^(I*(c
 + d*x))*(-1 + m)*(E^(I*(c + d*x))*(-2 + m)*Hypergeometric2F1[2 - m, (3 - m
)/2, (5 - m)/2, E^((2*I)*(c + d*x))] - 2*(-3 + m)*Hypergeometric2F1[2 - m,
1 - m/2, 2 - m/2, E^((2*I)*(c + d*x))])))/((1 - E^((2*I)*(c + d*x)))^m*(-3
+ m)*(-2 + m)*(-1 + m)) + (E^((2*I)*(c + d*x))*((4*E^(I*(c + d*x))*Hypergeo
metric2F1[(3 - m)/2, 4 - m, (5 - m)/2, E^((2*I)*(c + d*x))])/(-3 + m) + (4*
E^((3*I)*(c + d*x))*Hypergeometric2F1[4 - m, (5 - m)/2, (7 - m)/2, E^((2*I)
*(c + d*x))])/(-5 + m) - Hypergeometric2F1[4 - m, 1 - m/2, 2 - m/2, E^((2*I
)*(c + d*x))]/(-2 + m) - (6*E^((2*I)*(c + d*x))*Hypergeometric2F1[4 - m, 2
- m/2, 3 - m/2, E^((2*I)*(c + d*x))])/(-4 + m) - (E^((4*I)*(c + d*x))*Hyper
geometric2F1[4 - m, 3 - m/2, 4 - m/2, E^((2*I)*(c + d*x))])/(-6 + m)))/(1 -
 E^((2*I)*(c + d*x)))^m)*Sec[c + d*x]^2*(e*Sin[c + d*x])^m)/(a^2*d*(1 + Sec
[c + d*x])^2*Sin[c + d*x]^m)
```

Maple [F]

$$\int \frac{(e\sin(dx+c))^m}{(a+a\sec(dx+c))^2}dx$$

[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x)

[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x)

Fricas [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((e*sin(d*x + c))^m/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)

Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{(e\sin(c+dx))^m}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**2,x)

[Out] Integral((e*sin(c + d*x))**m/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2

Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^m}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^m)/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.139 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 236

$$\begin{aligned}\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx &= -\frac{4e^5(e\sin(c+dx))^{-5+m}}{a^3d(5-m)}\\ &+\frac{e^5\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-5+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-5+m}}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}\\ &+\frac{3e^5\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-5+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right)(e\sin(c+dx))^{-5+m}}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}\\ &+\frac{7e^3(e\sin(c+dx))^{-3+m}}{a^3d(3-m)}-\frac{3e(e\sin(c+dx))^{-1+m}}{a^3d(1-m)}\end{aligned}$$

```
[Out] -4*e^5*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m)+7*e^3*(e*sin(d*x+c))^(-3+m)/a^3/d/
(3-m)-3*e*(e*sin(d*x+c))^(-1+m)/a^3/d/(1-m)+e^5*cos(d*x+c)*hypergeom([-5/2,
 -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m)/(
cos(d*x+c)^2)^(1/2)+3*e^5*cos(d*x+c)*hypergeom([-3/2, -5/2+1/2*m],[-3/2+1/2
*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used

$= \{3957, 2954, 2952, 2644, 14, 2657, 276\}$

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{e^5\cos(c+dx)(e\sin(c+dx))^{m-5}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m-5}{2},\frac{m-3}{2},\sin^2(c+dx)\right)}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{3e^5\cos(c+dx)(e\sin(c+dx))^{m-5}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-5}{2},\frac{m-3}{2},\sin^2(c+dx)\right)}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}$$
$$- \frac{4e^5(e\sin(c+dx))^{m-5}}{a^3d(5-m)} + \frac{7e^3(e\sin(c+dx))^{m-3}}{a^3d(3-m)} - \frac{3e(e\sin(c+dx))^{m-1}}{a^3d(1-m)}$$

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-4*e^5*(e*Sin[c + d*x])^(-5 + m))/(a^3*d*(5 - m)) + (e^5*Cos[c + d*x]*Hype
rgeometric2F1[-5/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(-5 + m))/(a^3*d*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (3*e^5*Cos[c + d*x]*Hype
rgeometric2F1[-3/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(-5 + m))/(a^3*d*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (7*e^3*(e*Sin[c + d*x])^
(-3 + m))/(a^3*d*(3 - m)) - (3*e*(e*Sin[c + d*x])^(-1 + m))/(a^3*d*(1 - m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
```

```
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{e^6\int \cos^3(c+dx)(-a+a\cos(c+dx))^3(e\sin(c+dx))^{-6+m}\,dx}{a^6} \\
&= -\frac{e^6\int \left(-a^3\cos^3(c+dx)(e\sin(c+dx))^{-6+m}+3a^3\cos^4(c+dx)(e\sin(c+dx))^{-6+m}-3a^3\cos^5(c\cdots\right.}{a^6} \\
&= \frac{e^6\int \cos^3(c+dx)(e\sin(c+dx))^{-6+m}\,dx}{a^3}-\frac{e^6\int \cos^6(c+dx)(e\sin(c+dx))^{-6+m}\,dx}{a^3} \\
&\quad -\frac{(3e^6)\int \cos^4(c+dx)(e\sin(c+dx))^{-6+m}\,dx}{a^3}+\frac{(3e^6)\int \cos^5(c+dx)(e\sin(c+dx))^{-6+m}\,dx}{a^3}
\end{aligned}
$$

$$= \frac{e^5 \cos(c+dx) \text{Hypergeometric2F1}\left(-\frac{5}{2}, \frac{1}{2}(-5+m), \frac{1}{2}(-3+m), \sin^2(c+dx)\right) (e\sin(c+dx))^{-5+m}}{a^3 d(5-m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{3e^5 \cos(c+dx) \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{2}(-5+m), \frac{1}{2}(-3+m), \sin^2(c+dx)\right) (e\sin(c+dx))^{-5}}{a^3 d(5-m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{e^5 \text{Subst}\left(\int x^{-6+m}\left(1-\frac{x^2}{e^2}\right) dx, x, e\sin(c+dx)\right)}{a^3 d}$$
$$+ \frac{(3e^5)\,\text{Subst}\left(\int x^{-6+m}\left(1-\frac{x^2}{e^2}\right)^2 dx, x, e\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{e^5 \cos(c+dx) \text{Hypergeometric2F1}\left(-\frac{5}{2}, \frac{1}{2}(-5+m), \frac{1}{2}(-3+m), \sin^2(c+dx)\right) (e\sin(c+dx))^{-5+m}}{a^3 d(5-m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{3e^5 \cos(c+dx) \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{2}(-5+m), \frac{1}{2}(-3+m), \sin^2(c+dx)\right) (e\sin(c+dx))^{-5}}{a^3 d(5-m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{e^5 \text{Subst}\left(\int \left(x^{-6+m}-\frac{x^{-4+m}}{e^2}\right) dx, x, e\sin(c+dx)\right)}{a^3 d}$$
$$+ \frac{(3e^5)\,\text{Subst}\left(\int \left(x^{-6+m}-\frac{2x^{-4+m}}{e^2}+\frac{x^{-2+m}}{e^4}\right) dx, x, e\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{4e^5(e\sin(c+dx))^{-5+m}}{a^3 d(5-m)}$$
$$+ \frac{e^5 \cos(c+dx) \text{Hypergeometric2F1}\left(-\frac{5}{2}, \frac{1}{2}(-5+m), \frac{1}{2}(-3+m), \sin^2(c+dx)\right) (e\sin(c+dx))^{-5+}}{a^3 d(5-m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{3e^5 \cos(c+dx) \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{2}(-5+m), \frac{1}{2}(-3+m), \sin^2(c+dx)\right) (e\sin(c+dx))^{-5}}{a^3 d(5-m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{7e^3(e\sin(c+dx))^{-3+m}}{a^3 d(3-m)} - \frac{3e(e\sin(c+dx))^{-1+m}}{a^3 d(1-m)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.09 (sec) , antiderivative size = 995, normalized size of antiderivative = 4.22

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{i2^{4-m}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^m \cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\left(-1+e^{2i(c+dx)}\right)\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},1-\frac{m}{2},e^{2i(c+dx)}\right)}{2m}+\right.}{}$$

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((-I)*2^(4 - m)*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^m*Cos[(
c + d*x)/2]^6*(((-1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1, (2 + m)/2,
1 - m/2, E^((2*I)*(c + d*x))])/(2*m) + (3*E^(I*(c + d*x))*((6 - 5*m + m^2)*
Hypergeometric2F1[(1 - m)/2, 2 - m, (3 - m)/2, E^((2*I)*(c + d*x))] + E^(I*
(c + d*x))*(-1 + m)*(E^(I*(c + d*x))*(-2 + m)*Hypergeometric2F1[2 - m, (3 -
 m)/2, (5 - m)/2, E^((2*I)*(c + d*x))] - 2*(-3 + m)*Hypergeometric2F1[2 - m
, 1 - m/2, 2 - m/2, E^((2*I)*(c + d*x))])))/((1 - E^((2*I)*(c + d*x)))^m*(-
3 + m)*(-2 + m)*(-1 + m)) + (6*E^((2*I)*(c + d*x))*((4*E^(I*(c + d*x))*Hype
rgeometric2F1[(3 - m)/2, 4 - m, (5 - m)/2, E^((2*I)*(c + d*x))])/(-3 + m) +
 (4*E^((3*I)*(c + d*x))*Hypergeometric2F1[4 - m, (5 - m)/2, (7 - m)/2, E^((
2*I)*(c + d*x))])/(-5 + m) - Hypergeometric2F1[4 - m, 1 - m/2, 2 - m/2, E^(
(2*I)*(c + d*x))]/(-2 + m) - (6*E^((2*I)*(c + d*x))*Hypergeometric2F1[4 - m
, 2 - m/2, 3 - m/2, E^((2*I)*(c + d*x))])/(-4 + m) - (E^((4*I)*(c + d*x))*H
ypergeometric2F1[4 - m, 3 - m/2, 4 - m/2, E^((2*I)*(c + d*x))])/(-6 + m)))/
(1 - E^((2*I)*(c + d*x)))^m - (4*E^((3*I)*(c + d*x))*(-(Hypergeometric2F1[(
3 - m)/2, 6 - m, (5 - m)/2, E^((2*I)*(c + d*x))]/(-3 + m)) - (15*E^((2*I)*(
c + d*x))*Hypergeometric2F1[(5 - m)/2, 6 - m, (7 - m)/2, E^((2*I)*(c + d*x)
)])/(-5 + m) - (15*E^((4*I)*(c + d*x))*Hypergeometric2F1[6 - m, (7 - m)/2,
(9 - m)/2, E^((2*I)*(c + d*x))])/(-7 + m) - (E^((6*I)*(c + d*x))*Hypergeome
tric2F1[6 - m, (9 - m)/2, (11 - m)/2, E^((2*I)*(c + d*x))])/(-9 + m) + (6*E
^(I*(c + d*x))*Hypergeometric2F1[6 - m, 2 - m/2, 3 - m/2, E^((2*I)*(c + d*x
))])/(-4 + m) + (20*E^((3*I)*(c + d*x))*Hypergeometric2F1[6 - m, 3 - m/2, 4
 - m/2, E^((2*I)*(c + d*x))])/(-6 + m) + (6*E^((5*I)*(c + d*x))*Hypergeomet
ric2F1[6 - m, 4 - m/2, 5 - m/2, E^((2*I)*(c + d*x))])/(-8 + m)))/(1 - E^((2
*I)*(c + d*x)))^m)*Sec[c + d*x]^3*(e*Sin[c + d*x])^m)/(a^3*d*(1 + Sec[c + d
*x])^3*Sin[c + d*x]^m)
```

Maple [F]

$$\int \frac{(e \sin (dx+c))^m}{(a+a \sec (dx+c))^3} dx$$

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

Fricas [F]

$$\int \frac{(e \sin(c+dx))^m}{(a+a \sec(c+dx))^3} \, dx = \int \frac{(e \sin (dx+c))^m}{(a \sec (dx+c)+a)^3} \, dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{(e\sin(c+dx))^m}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**3,x)
[Out] Integral((e*sin(c + d*x))**m/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos(c+d\,x)^3\,(e\,\sin(c+d\,x))^m}{a^3\,(\cos(c+d\,x)+1)^3}\,dx$$

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^3,x)
[Out] int((cos(c + d*x)^3*(e*sin(c + d*x))^m)/(a^3*(cos(c + d*x) + 1)^3), x)
```

3.140 $\int (a+a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int (a+a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx=\frac{2ae\operatorname{AppellF1}\left(-\frac{1}{2},\frac{1-m}{2},\frac{1}{2}(-2-m),\frac{1}{2},\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c}{d}$$

```
[Out] 2*a*e*AppellF1(-1/2,-1-1/2*m,1/2-1/2*m,1/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d
*x+c))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)*(a+a*sec(d*x+c))^(1/2)/d/((1+cos(d
*x+c))^(1/2*m))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3961, 2965, 140, 138}

$$\int (a+a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx=\frac{2ae\sqrt{a\sec(c+dx)+a}(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}\operatorname{AppellF1}(}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] (2*a*e*AppellF1[-1/2, (1 - m)/2, (-2 - m)/2, 1/2, Cos[c + d*x], -Cos[c + d*
x]]*(1 - Cos[c + d*x])^((1 - m)/2)*Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x]
)^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^{3/2}(e\sin(c+dx))^m}{(-\cos(c+dx))^{3/2}}\,dx}{\sqrt{-a-a\cos(c+dx)}}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}\sqrt{a+a\sec(c+dx)}(e\sin\right.}{d}$$

$$= \frac{\left(ae\sqrt{-\cos(c+dx)}(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}\right.}{d}$$

$$= \frac{\left(ae(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}\sqrt{-\cos(c+dx)}(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-\right.}{}$$

$$= \frac{2ae\,\text{AppellF1}\left(-\frac{1}{2},\frac{1-m}{2},\frac{1}{2}(-2-m),\frac{1}{2},\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}(1+\cos(}{d}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1243 vs. $2(106) = 212$.

Time = 9.56 (sec) , antiderivative size = 1243, normalized size of antiderivative = 11.73

$$\int (a+a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \frac{}{d(1+m)\left(6\,\text{AppellF1}\left(\frac{1+m}{2},\frac{3}{2},m,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right.}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d
*x]))^(3/2)*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*(6*AppellF1[(1
+ m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*m*A
ppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] - 2*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, 1/2,
 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 +
 m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 4*m*Ap
pellF1[(3 + m)/2, 3/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] + 6*AppellF1[(3 + m)/2, 5/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2] + 6*AppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*m*AppellF1[(1 + m)/2, 3/2, m, (3
+ m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*AppellF1[
(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Cos[c + d*x] + 2*m*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + AppellF1[(3 + m)/2, 1/2, 1 + m
, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*m*Ap
pellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*Cos[c + d*x] - AppellF1[(3 + m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 4*m*AppellF1[(3 + m)/2, 3/2, 1 +
m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] - 6*App
ellF1[(3 + m)/2, 5/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Cos[c + d*x] + (3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c
```

```
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x]) + (3 + m)*AppellF1[(1
 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + C
os[c + d*x])))
```

Maple [F]

$$\int \left(a + a\sec\left(dx + c\right)\right)^{\frac{3}{2}} \left(e\sin\left(dx + c\right)\right)^{m} dx$$

[In] int((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)

[Out] int((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)

Fricas [F]

$$\int (a + a\sec(c + dx))^{3/2}(e\sin(c + dx))^m\,dx = \int \left(a\sec\left(dx + c\right) + a\right)^{\frac{3}{2}} \left(e\sin\left(dx + c\right)\right)^{m}\,dx$$

[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")

[Out] integral((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2}(e\sin(c + dx))^m\,dx = \text{Timed out}$$

[In] integrate((a+a*sec(d*x+c))**(3/2)*(e*sin(d*x+c))**m,x)

[Out] Timed out

Maxima [F]

$$\int (a + a\sec(c + dx))^{3/2}(e\sin(c + dx))^m\,dx = \int \left(a\sec\left(dx + c\right) + a\right)^{\frac{3}{2}} \left(e\sin\left(dx + c\right)\right)^{m}\,dx$$

[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)

Giac [F]

$$\int (a+a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \int (e\,\sin(c+d\,x))^m\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(3/2),x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(3/2), x)
```

3.141 $\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 107

$$\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx = -\frac{2e\,\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},-\frac{m}{2},\frac{3}{2},\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+dx)(1+\cos(c+d}{d}$$

```
[Out] -2*e*AppellF1(1/2,-1/2*m,1/2-1/2*m,3/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c
))^(1/2-1/2*m)*cos(d*x+c)*(e*sin(d*x+c))^(-1+m)*(a+a*sec(d*x+c))^(1/2)/d/((
1+cos(d*x+c))^(1/2*m))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3961, 2965, 140, 138}

$$\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx = -\frac{2e\cos(c+dx)\sqrt{a\sec(c+dx)+a}(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},-\frac{m}{2},\frac{3}{2},c}{d}$$

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

```
[Out] (-2*e*AppellF1[1/2, (1 - m)/2, -1/2*m, 3/2, Cos[c + d*x], -Cos[c + d*x]]*(1
 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c
 + d*x])^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\right)\int\frac{\sqrt{-a-a\cos(c+dx)}(e\sin(c+dx))^m}{\sqrt{-\cos(c+dx)}}\,dx}{\sqrt{-a-a\cos(c+dx)}}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}\sqrt{a+a\sec(c+dx)}(e\,\mathrm{s}\right.}{d}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx))^{1}\right.}{}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}\sqrt{-\cos(c+dx)}(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}\right.}{}$$

$$= -\frac{2e\,\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},-\frac{m}{2},\frac{3}{2},\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+dx)(1+\text{co}}{d}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 433 vs. $2(107) = 214$.

Time = 2.37 (sec) , antiderivative size = 433, normalized size of antiderivative = 4.05

$$\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx$$

$$= \frac{}{d(1+m)\left(\left(2(1+m)\,\text{AppellF1}\left(\frac{3+m}{2},-\frac{1}{2},2+m,\frac{5+m}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+(1+2m)\,\text{A}\right.\right.}$$

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*Sqrt[a*(1 + Sec[c + d*x])]
*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*((2*(1 + m)*AppellF1[(3 +
m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (1
 + 2*m)*AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] - AppellF1[(3 + m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3 + m)*AppellF1[(1 + m)/2,
-1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c
 + d*x]) + (3 + m)*AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F]

$$\int \sqrt{a+a\sec(dx+c)}\,(e\sin(dx+c))^m\,dx$$

```
[In] int((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x)
```

Fricas [F]

$$\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx=\int \sqrt{a\sec\left(dx+c\right)+a}(e\sin\left(dx+c\right))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [F]

$$\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx=\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}(e\sin\left(c+dx\right))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*(e*sin(d*x+c))**m,x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sin(c + d*x))**m, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx=\int \sqrt{a\sec\left(dx+c\right)+a}(e\sin\left(dx+c\right))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Giac [F]

$$\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx=\int \sqrt{a\sec\left(dx+c\right)+a}(e\sin\left(dx+c\right))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int (e\,\sin\left(c+d\,x\right))^m\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(1/2),x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(1/2), x)
```

3.142 $\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2e\,\mathrm{AppellF1}\left(\frac{3}{2},\frac{1-m}{2},\frac{2-m}{2},\frac{5}{2},\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+dx)(1+\cos(c+}{3d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2/3*e*AppellF1(3/2,1-1/2*m,1/2-1/2*m,5/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d*
x+c))^(1/2-1/2*m)*cos(d*x+c)*(1+cos(d*x+c))^(1-1/2*m)*(e*sin(d*x+c))^(-1+m)
/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3961, 2965, 140, 138}

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}\,\mathrm{AppellF1}\left(\frac{3}{2},\frac{1-m}{2},\frac{2-m}{2},\frac{5}{2},\cos(c+dx),-\cos(}{3d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[(e*Sin[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*e*AppellF1[3/2, (1 - m)/2, (2 - m)/2, 5/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1 - m/2)*(
e*Sin[c + d*x])^(-1 + m))/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-a-a\cos(c+dx)}\int\frac{\sqrt{-\cos(c+dx)}(e\sin(c+dx))^m}{\sqrt{-a-a\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(e(-a-a\cos(c+dx))^{\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+dx))^{-1+m}\right)\text{Subst}\left(\int\sqrt{-x}(-a-\right.}{d\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(e(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+dx))^{-}\right.}{d\sqrt{-\cos(c+dx)}\sqrt{a+a\,\mathrm{sec}}}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+d\right.}{d\sqrt{-\cos(c+d}}$$

$$= -\frac{2e\,\mathrm{AppellF1}\left(\frac{3}{2},\frac{1-m}{2},\frac{2-m}{2},\frac{5}{2},\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+dx)(1+}{3d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 277 vs. $2(115) = 230$.

Time = 2.85 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.41

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{4(3+m)\,\mathrm{Appel}}{d(1+m)\left(\left(2(1+m)\,\mathrm{AppellF1}\left(\frac{3+m}{2},-\frac{1}{2},2+m,\frac{5+m}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+\mathrm{AppellF1}\right.\right.}$$

```
[In] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^3*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m
)/(d*(1 + m)*((2*(1 + m)*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m
)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3 + m
)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x]))*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [F]

$$\int \frac{(e\sin(dx+c))^m}{\sqrt{a+a\sec(dx+c)}}dx$$

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sin(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sin(c+dx))^m}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral((e*sin(c + d*x))**m/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sin(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sin(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\,\sin(c+d\,x))^m}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(1/2),x)
[Out] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(1/2), x)
```

3.143 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2e\,\mathrm{AppellF1}\left(\frac{5}{2},\frac{1-m}{2},\frac{4-m}{2},\frac{7}{2},\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos^2(c+dx)(1+\cos(c+d}{5ad\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2/5*e*AppellF1(5/2,2-1/2*m,1/2-1/2*m,7/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d*
x+c))^(1/2-1/2*m)*cos(d*x+c)^2*(1+cos(d*x+c))^(1-1/2*m)*(e*sin(d*x+c))^(-1+
m)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3961, 2965, 140, 138}

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2e\cos^2(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}\,\mathrm{AppellF1}\left(\frac{5}{2},\frac{1-m}{2},\frac{4-m}{2},\frac{7}{2},\cos(c+dx),-\cos(c}{5ad\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*AppellF1[5/2, (1 - m)/2, (4 - m)/2, 7/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]^2*(1 + Cos[c + d*x])^(1 - m/2)
*(e*Sin[c + d*x])^(-1 + m))/(5*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{-a-a\cos(c+dx)}\int\frac{(-\cos(c+dx))^{3/2}(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^{3/2}}\,dx}{\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{\left(e(-a-a\cos(c+dx))^{\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+dx))^{-1+m}\right)\text{Subst}\left(\int(-x)^{3/2}(-a \cdots\right.}{d\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= \frac{\left(e(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+dx))^{-1+ \cdots}\right.}{ad\sqrt{-\cos(c+dx)}\sqrt{a+a\sec \cdots}}\\
&= \frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx)) \cdots\right.}{ad\sqrt{-\cos(c+dx \cdots}}
\end{aligned}
$$

$$= -\frac{2e\,\text{AppellF1}\left(\frac{5}{2},\frac{1-m}{2},\frac{4-m}{2},\frac{7}{2},\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos^2(c+dx)(1+}{5ad\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 484 vs. $2(120) = 240$.

Time = 2.32 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.03

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{}{d(1+m)\left(-4(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2},1+m,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\text{t}\right.\right.}$$

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - 2*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*Sin[(c + d*x)/2]*(e*Sin
[c + d*x])^m)/(d*(1 + m)*(-4*(3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 +
m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2 + (2*m*Ap
pellF1[(3 + m)/2, -1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] - 4*(1 + m)*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 + m)/2, 1/2, m, (5 + m)/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[(3 + m)/2, 1/2, 1 + m, (5
 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3
 + m)*AppellF1[(1 + m)/2, -1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x]))*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [F]

$$\int \frac{(e\sin(dx+c))^m}{(a+a\sec(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)

Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(c+dx))^m}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**(3/2),x)

[Out] Integral((e*sin(c + d*x))**m/(a*(sec(c + d*x) + 1))**(3/2), x)

Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> an error occurred running a Giac command: INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{%%%{%%{poly1[1,0]:[1,0,-2]%%},[5,12]%%%}+%%%{%%{[6,0]:[1,0,-2]%%},[5,10]%%%}+%%%{%%{[15,0]

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\,\sin\left(c+d\,x\right))^m}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(3/2), x)
```

3.144 $\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx = -\frac{e\,\text{AppellF1}\left(1-n,\frac{1-m}{2},\frac{1}{2}(1-m-2n),2-n,\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+}{d(1-n)}$$

[Out] -e*AppellF1(1-n,1/2-1/2*m-n,1/2-1/2*m,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(1/2-1/2*m)*cos(d*x+c)*(1+cos(d*x+c))^(1/2-1/2*m-n)*(a+a*sec(d*x+c))^n*(e*sin(d*x+c))^(-1+m)/d/(1-n)

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3961, 2965, 140, 138}

$$\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx = -\frac{e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(a\sec(c+dx)+a)^n(e\sin(c+dx))^{m-1}(\cos(c+dx)+1)^{\frac{1}{2}(-m-2n+1)}\,\text{Ap}}{d(1-n)}$$

[In] Int[(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]

[Out] -((e*AppellF1[1 - n, (1 - m)/2, (1 - m - 2*n)/2, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(1 + Cos[c + d*x])^((1 - m - 2*n)/2)*(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^(-1 + m))/(d*(1 - n)))

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\text{integral} = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int(-\cos(c+dx))^{-n}(-a - a\cos(c+dx))^n(e\sin(c+dx))^m\,dx$$

$$= -\frac{\left(e(-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1-m}{2}-n}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(a+a\sec(c+dx))^n(e\,\mathrm{s}\right.}{d}$$

$$= -\frac{\left(e(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}-n}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx\right.}{}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}-n}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\right.}{}$$

$$= -\frac{e\,\text{AppellF1}\left(1-n,\frac{1-m}{2},\frac{1}{2}(1-m-2n),2-n,\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}}{d(1-n)}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 276 vs. $2(130) = 260$.

Time = 2.06 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.12

$$\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

$$= \frac{4(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},n,1+m,\frac{3+m}{2}\right.}{d(1+m)\left((3+m)\,\text{AppellF1}\left(\frac{1+m}{2},n,1+m,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))\right.}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d*x]))^n*Sin[(c + d*x)
/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m, (
3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x]) - 4*(
(1 + m)*AppellF1[(3 + m)/2, n, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2])*Sin[(c + d*x)/2]^2))
```

Maple [F]

$$\int (a+a\sec(dx+c))^n\,(e\sin(dx+c))^m\,dx$$

```
[In] int((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx = \int (a\sec(dx+c)+a)^n(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx = \int (a(\sec(c+dx)+1))^n\,(e\sin(c+dx))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**n*(e*sin(d*x+c))**m,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*(e*sin(c + d*x))**m, x)
```

Maxima [F]

$$\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx = \int (a\sec(dx+c)+a)^n(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx = \int (a\sec(dx+c)+a)^n(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx = \int (e\,\sin(c+d\,x))^m\left(a+\frac{a}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^n,x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^n, x)
```

3.145 $\int (a+a\sec(c+dx))^n \sin^7(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 180

$$\int (a+a\sec(c+dx))^n \sin^7(c+dx)\,dx = -\frac{(3-n)(8-n)(16-n)\,\text{Hypergeometric2F1}(6,4+n,5+n,1+\sec(c+dx))(a+a\sec(c+dx))^{4+n}}{42a^4d(1-n)(4+n)} - \frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a+a\sec(c+dx))^{4+n}}{a^4d(1-n)} + \frac{\cos^7(c+dx)(a+a\sec(c+dx))^{4+n}\left(6(8-n)-(108-25n+n^2)\sec(c+dx)\right)}{42a^4d(1-n)}$$

```
[Out] -1/42*(3-n)*(8-n)*(16-n)*hypergeom([6, 4+n],[5+n],1+sec(d*x+c))*(a+a*sec(d*
x+c))^(4+n)/a^4/d/(-n^2-3*n+4)-cos(d*x+c)^7*(1-sec(d*x+c))^2*(a+a*sec(d*x+c
))^(4+n)/a^4/d/(1-n)+1/42*cos(d*x+c)^7*(a+a*sec(d*x+c))^(4+n)*(48-6*n-(n^2-
25*n+108)*sec(d*x+c))/a^4/d/(1-n)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3958, 102, 150, 67}

$$\int (a+a\sec(c+dx))^n \sin^7(c+dx)\,dx = -\frac{(3-n)(8-n)(16-n)(a\sec(c+dx)+a)^{n+4}\,\text{Hypergeometric2F1}(6,n+4,n+5,\sec(c+dx)+1)}{42a^4d(1-n)(n+4)} + \frac{\cos^7(c+dx)\left(6(8-n)-(n^2-25n+108)\sec(c+dx)\right)(a\sec(c+dx)+a)^{n+4}}{42a^4d(1-n)} - \frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{n+4}}{a^4d(1-n)}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^7,x]
```

```
[Out] -1/42*((3 - n)*(8 - n)*(16 - n)*Hypergeometric2F1[6, 4 + n, 5 + n, 1 + Sec[
c + d*x]]*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(1 - n)*(4 + n)) - (Cos[c +
d*x]^7*(1 - Sec[c + d*x])^2*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(1 - n)) +
 (Cos[c + d*x]^7*(a + a*Sec[c + d*x])^(4 + n)*(6*(8 - n) - (108 - 25*n + n^
2)*Sec[c + d*x]))/(42*a^4*d*(1 - n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(-a-ax)^3(a-ax)^{3+n}}{x^8}\,dx, x, -\sec(c+dx)\right)}{a^6d}$$

$$= -\frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a+a\sec(c+dx))^{4+n}}{a^4d(1-n)} - \frac{\text{Subst}\left(\int \frac{(-a-ax)(a-ax)^{3+n}(a^3(8-n)+a^3(4-n)x)}{x^8}\,dx, x, -\sec(c+dx)\right)}{a^7d(1-n)}$$

$$= -\frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a+a\sec(c+dx))^{4+n}}{a^4d(1-n)} + \frac{\cos^7(c+dx)(a+a\sec(c+dx))^{4+n}\left(6(8-n)-(108-25n+n^2)\sec(c+dx)\right)}{42a^4d(1-n)} + \frac{((3-n)(8-n)(16-n))\text{Subst}\left(\int \frac{(a-ax)^{3+n}}{x^6}\,dx, x, -\sec(c+dx)\right)}{42a^3d(1-n)}$$

$$= -\frac{(3-n)(8-n)(16-n)\,\text{Hypergeometric2F1}(6,4+n,5+n,1+\sec(c+dx))(a+a\sec(c+dx))^{4}}{42a^4d(1-n)(4+n)} - \frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a+a\sec(c+dx))^{4+n}}{a^4d(1-n)} + \frac{\cos^7(c+dx)(a+a\sec(c+dx))^{4+n}\left(6(8-n)-(108-25n+n^2)\sec(c+dx)\right)}{42a^4d(1-n)}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

$$\int (a+a\sec(c+dx))^n\sin^7(c+dx)\,dx = \frac{((4+n)\cos^5(c+dx)\left((24-25n+n^2)\cos(c+dx)+3(13+n+(-1+n)\cos(2(c+dx)))\right)-(-384+200}{42d(-1}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^7,x]
```

```
[Out] (((4 + n)*Cos[c + d*x]^5*((24 - 25*n + n^2)*Cos[c + d*x] + 3*(13 + n + (-1
+ n)*Cos[2*(c + d*x)])) - (-384 + 200*n - 27*n^2 + n^3)*Hypergeometric2F1[6
, 4 + n, 5 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*x])^4*(a*(1 + Sec[c + d*x
]))^n)/(42*d*(-1 + n)*(4 + n))
```

Maple [F]

$$\int (a + a\sec(dx + c))^n \sin(dx + c)^7\, dx$$

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x)
```

Fricas [F]

$$\int (a + a\sec(c + dx))^n \sin^7(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^7\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*(a*sec
(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^7(c + dx)\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**7,x)
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin^7(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^7\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^7, x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^n\sin^7(c+dx)\,dx=\int (a\sec\left(dx+c\right)+a)^n\sin\left(dx+c\right)^7\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^n\sin^7(c+dx)\,dx=\int \sin\left(c+d\,x\right)^7\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^n\,dx$$

```
[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^n, x)
```

3.146 $\int (a+a\sec(c+dx))^n \sin^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int (a+a\sec(c+dx))^n \sin^5(c+dx)\,dx = \frac{(12-n)\cos^4(c+dx)(a+a\sec(c+dx))^{3+n}}{20a^3d} - \frac{\cos^5(c+dx)(a+a\sec(c+dx))^{3+n}}{5a^3d} + \frac{(32-13n+n^2)\,\text{Hypergeometric2F1}(4,3+n,4+n,1+\sec(c+dx))(a+a\sec(c+dx))^{3+n}}{20a^3d(3+n)}$$

```
[Out] 1/20*(12-n)*cos(d*x+c)^4*(a+a*sec(d*x+c))^(3+n)/a^3/d-1/5*cos(d*x+c)^5*(a+a
*sec(d*x+c))^(3+n)/a^3/d+1/20*(n^2-13*n+32)*hypergeom([4, 3+n],[4+n],1+sec(
d*x+c))*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3958, 91, 79, 67}

$$\int (a+a\sec(c+dx))^n \sin^5(c+dx)\,dx = \frac{(n^2-13n+32)\,(a\sec(c+dx)+a)^{n+3}\,\text{Hypergeometric2F1}(4,n+3,n+4,\sec(c+dx)+1)}{20a^3d(n+3)} - \frac{\cos^5(c+dx)(a\sec(c+dx)+a)^{n+3}}{5a^3d} + \frac{(12-n)\cos^4(c+dx)(a\sec(c+dx)+a)^{n+3}}{20a^3d}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] ((12 - n)*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3 + n))/(20*a^3*d) - (Cos[c
+ d*x]^5*(a + a*Sec[c + d*x])^(3 + n))/(5*a^3*d) + ((32 - 13*n + n^2)*Hyper
```

```
geometric2F1[4, 3 + n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n
))/(20*a^3*d*(3 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(-a-ax)^2(a-ax)^{2+n}}{x^6}\,dx, x, -\sec(c+dx)\right)}{a^4 d}$$

$$= -\frac{\cos^5(c+dx)(a+a\sec(c+dx))^{3+n}}{5a^3 d} - \frac{\text{Subst}\left(\int \frac{(a-ax)^{2+n}(a^3(12-n)+5a^3x)}{x^5}\,dx, x, -\sec(c+dx)\right)}{5a^5 d}$$

$$= \frac{(12-n)\cos^4(c+dx)(a+a\sec(c+dx))^{3+n}}{20a^3d} - \frac{\cos^5(c+dx)(a+a\sec(c+dx))^{3+n}}{5a^3d} - \frac{(32-13n+n^2)\,\mathrm{Subst}\left(\int \frac{(a-ax)^{2+n}}{x^4}\,dx, x, -\sec(c+dx)\right)}{20a^2d}$$

$$= \frac{(12-n)\cos^4(c+dx)(a+a\sec(c+dx))^{3+n}}{20a^3d} - \frac{\cos^5(c+dx)(a+a\sec(c+dx))^{3+n}}{5a^3d} + \frac{(32-13n+n^2)\,\mathrm{Hypergeometric2F1}(4,3+n,4+n,1+\sec(c+dx))(a+a\sec(c+dx))^{3+n}}{20a^3d(3+n)}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int (a+a\sec(c+dx))^n \sin^5(c+dx)\,dx = -\frac{((3+n)\cos^4(c+dx)(-12+n+4\cos(c+dx)) - (32-13n+n^2)\,\mathrm{Hypergeometric2F1}(4,3+n,4+n}{20d(3+n)}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
[Out] -1/20*(((3 + n)*Cos[c + d*x]^4*(-12 + n + 4*Cos[c + d*x]) - (32 - 13*n + n^
2)*Hypergeometric2F1[4, 3 + n, 4 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*x])
^3*(a*(1 + Sec[c + d*x]))^n)/(d*(3 + n))
```

Maple [F]

$$\int (a+a\sec(dx+c))^n \sin(dx+c)^5\,dx$$

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^n \sin^5(c+dx)\,dx = \int (a\sec(dx+c)+a)^n \sin(dx+c)^5\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="fricas")
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sec(d*x + c) + a)^n*sin
(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^5(c + dx)\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin^5(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^5\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^n \sin^5(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^5\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^5(c + dx)\, dx = \int \sin(c + d\,x)^5 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^n, x)
```

3.147 $\int (a+a\sec(c+dx))^n \sin^3(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int (a+a\sec(c+dx))^n \sin^3(c+dx)\,dx = \frac{\cos^3(c+dx)(a+a\sec(c+dx))^{2+n}}{3a^2d} - \frac{(4-n)\,\text{Hypergeometric2F1}(3,2+n,3+n,1+\sec(c+dx))(a+a\sec(c+dx))^{2+n}}{3a^2d(2+n)}$$

```
[Out] 1/3*cos(d*x+c)^3*(a+a*sec(d*x+c))^(2+n)/a^2/d-1/3*(4-n)*hypergeom([3, 2+n],
[3+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3958, 79, 67}

$$\int (a+a\sec(c+dx))^n \sin^3(c+dx)\,dx = \frac{\cos^3(c+dx)(a\sec(c+dx)+a)^{n+2}}{3a^2d} - \frac{(4-n)(a\sec(c+dx)+a)^{n+2}\,\text{Hypergeometric2F1}(3,n+2,n+3,\sec(c+dx)+1)}{3a^2d(n+2)}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(2 + n))/(3*a^2*d) - ((4 - n)*Hypergeo
metric2F1[3, 2 + n, 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/
(3*a^2*d*(2 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(-a-ax)(a-ax)^{1+n}}{x^4}\,dx, x, -\sec(c+dx)\right)}{a^2d} \\
&= \frac{\cos^3(c+dx)(a+a\sec(c+dx))^{2+n}}{3a^2d} + \frac{(4-n)\text{Subst}\left(\int \frac{(a-ax)^{1+n}}{x^3}\,dx, x, -\sec(c+dx)\right)}{3ad} \\
&= \frac{\cos^3(c+dx)(a+a\sec(c+dx))^{2+n}}{3a^2d} \\
&\quad - \frac{(4-n)\,\text{Hypergeometric2F1}(3, 2+n, 3+n, 1+\sec(c+dx))(a+a\sec(c+dx))^{2+n}}{3a^2d(2+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int (a+a\sec(c+dx))^n\sin^3(c+dx)\,dx$$
$$=\frac{((2+n)\cos^3(c+dx)+(-4+n)\,\text{Hypergeometric2F1}(3,2+n,3+n,1+\sec(c+dx)))\,(1+\sec(c+dx))}{3d(2+n)}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
[Out] (((2 + n)*Cos[c + d*x]^3 + (-4 + n)*Hypergeometric2F1[3, 2 + n, 3 + n, 1 +
Sec[c + d*x]])*(1 + Sec[c + d*x])^2*(a*(1 + Sec[c + d*x]))^n)/(3*d*(2 + n))
```

Maple [F]

$$\int (a+a\sec(dx+c))^n\sin(dx+c)^3\,dx$$

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^n\sin^3(c+dx)\,dx=\int (a\sec(dx+c)+a)^n\sin(dx+c)^3\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^n\sin^3(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sec(c+dx))^n \sin^3(c+dx)\,dx = \int (a\sec(dx+c)+a)^n \sin(dx+c)^3\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+a\sec(c+dx))^n \sin^3(c+dx)\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^n \sin^3(c+dx)\,dx = \int \sin(c+d\,x)^3 \left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

```
[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

3.148 $\int (a+a\sec(c+dx))^n \sin(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 42

$$\int (a+a\sec(c+dx))^n \sin(c+dx)\,dx = \frac{\text{Hypergeometric2F1}(2,1+n,2+n,1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}$$

```
[Out] hypergeom([2, 1+n],[2+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3958, 67}

$$\int (a+a\sec(c+dx))^n \sin(c+dx)\,dx = \frac{(a\sec(c+dx)+a)^{n+1}\,\text{Hypergeometric2F1}(2,n+1,n+2,\sec(c+dx)+1)}{ad(n+1)}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^
(1 + n))/(a*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
```

```
|| GtQ[-d/(b*c), 0])
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^n}{x^2}\,dx, x, -\sec(c+dx)\right)}{d}\\ &= \frac{\text{Hypergeometric2F1}(2,1+n,2+n,1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (a+a\sec(c+dx))^n \sin(c+dx)\,dx = \frac{\text{Hypergeometric2F1}(2,1+n,2+n,1+\sec(c+dx))(a(1+\sec(c+dx)))^{1+n}}{ad(1+n)}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x],x]
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*x])
)^(1 + n))/(a*d*(1 + n))
```

Maple [F]

$$\int (a+a\sec(dx+c))^n \sin(dx+c)\,dx$$

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c),x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c),x)
```

Fricas [F]

$$\int (a + a\sec(c + dx))^n \sin(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)\ dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F]

$$\int (a + a\sec(c + dx))^n \sin(c + dx)\, dx = \int (a(\sec(c + dx) + 1))^n \sin(c + dx)\, dx$$

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*sin(c + d*x), x)
```

Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)\ dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^n \sin(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)\ dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52

$$\int (a+a\sec(c+dx))^n\sin(c+dx)\,dx$$
$$=\frac{\cos\left(c+d\,x\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^n\,{}_2\mathrm{F}_1\left(1-n,-n;\ 2-n;\ -\cos\left(c+d\,x\right)\right)}{d\,\left(\cos\left(c+d\,x\right)+1\right)^n\,\left(n-1\right)}$$

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x))^n,x)
```

```
[Out] (cos(c + d*x)*(a + a/cos(c + d*x))^n*hypergeom([1 - n, -n], 2 - n, -cos(c +
 d*x)))/(d*(cos(c + d*x) + 1)^n*(n - 1))
```

3.149 $\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn}$$

```
[Out] -1/2*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3958, 70}

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{(a\sec(c+dx)+a)^n\,\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d
*x])^n)/(d*n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a^2\text{Subst}\left(\int \frac{(a-ax)^{-1+n}}{-a-ax}\,dx, x, -\sec(c+dx)\right)}{d} \\ &= -\frac{\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 92 vs. $2(40) = 80$.

Time = 0.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx$$
$$= \frac{2^{-1+n}\,\text{Hypergeometric2F1}\left(1,1-n,2-n,\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{-1+}}{d(-1+n)}$$

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (2^(-1 + n)*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec[(c + d*x)/2
]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*(a*(1 + Sec[c + d*x]))^n)/(
d*(-1 + n)*(1 + Sec[c + d*x])^n)
```

Maple [F]

$$\int \csc(dx+c)\,(a+a\sec(dx+c))^n\,dx$$

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\csc(dx+c)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Sympy [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a(\sec(c+dx)+1))^n\csc(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**n,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*csc(c + d*x), x)
```

Maxima [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\csc(dx+c)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Giac [F]

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\csc(dx+c)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)}\,dx$$

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x),x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x), x)
```

3.150 $\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a(2-n)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)}+\frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{(2+n)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{8dn}$$

```
[Out] -1/4*a*(2-n)*(a+a*sec(d*x+c))^(-1+n)/d/(1-n)+1/2*a*(a+a*sec(d*x+c))^(-1+n)/
d/(1-sec(d*x+c))-1/8*(2+n)*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+a*
sec(d*x+c))^n/d/n
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3958, 91, 80, 70}

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{(n+2)(a\sec(c+dx)+a)^n\,\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{8dn} - \frac{a(2-n)(a\sec(c+dx)+a)^{n-1}}{4d(1-n)}+\frac{a(a\sec(c+dx)+a)^{n-1}}{2d(1-\sec(c+dx))}$$

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/4*(a*(2 - n)*(a + a*Sec[c + d*x])^(-1 + n))/(d*(1 - n)) + (a*(a + a*Sec[
c + d*x])^(-1 + n))/(2*d*(1 - Sec[c + d*x])) - ((2 + n)*Hypergeometric2F1[1
, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])^n)/(8*d*n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^4\text{Subst}\left(\int \frac{x^2(a-ax)^{-2+n}}{(-a-ax)^2}\,dx, x, -\sec(c+dx)\right)}{d} \\
&= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} + \frac{\text{Subst}\left(\int \frac{(a-ax)^{-2+n}(-a^3n+2a^3x)}{-a-ax}\,dx, x, -\sec(c+dx)\right)}{2d}
\end{aligned}$$

$$= -\frac{a(2-n)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{(a^2(2+n))\,\text{Subst}\left(\int \frac{(a-ax)^{-1+n}}{-a-ax}\,dx, x, -\sec(c+dx)\right)}{4d}$$

$$= -\frac{a(2-n)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{(2+n)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{8dn}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.60

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = \frac{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^{-n}(a(1+\sec(c+dx)))^n\left(2^{1+n}\,\text{Hypergeometric2F1}\left(1,1-n\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (Cos[c + d*x]*Sec[(c + d*x)/2]^2*(a*(1 + Sec[c + d*x]))^n*(2^(1 + n)*Hyperg
eometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x
)/2]^2*Sec[c + d*x])^n + 2^n*Hypergeometric2F1[2, 1 - n, 2 - n, Cos[c + d*x
]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n + (1 + Sec[c + d*
x])^n))/(8*d*(-1 + n)*(1 + Sec[c + d*x])^n)
```

Maple [F]

$$\int \csc(dx+c)^3\,(a+a\sec(dx+c))^n\,dx$$

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^3\,dx$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Sympy [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a(\sec{(c+dx)}+1))^n \csc^3{(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**n,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*csc(c + d*x)**3, x)
```

Maxima [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec{(dx+c)}+a)^n \csc{(dx+c)}^3\,dx$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Giac [F]

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec{(dx+c)}+a)^n \csc{(dx+c)}^3\,dx$$

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin{(c+d\,x)}^3}\,dx$$

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^3,x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^3, x)
```

3.151 $\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 240

$$\begin{aligned}
&\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx\\
&=\frac{a^2(12+9n+n^2)\,\text{Hypergeometric2F1}\left(1,-2+n,-1+n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^{-2+n}}{16d(2-n)}\\
&\quad+\frac{a^2(3+n)\sec^2(c+dx)(a+a\sec(c+dx))^{-2+n}}{4d(1-n)(1-\sec(c+dx))^2}-\frac{a^2\sec^3(c+dx)(a+a\sec(c+dx))^{-2+n}}{d(1-n)(1-\sec(c+dx))^2}\\
&\quad-\frac{a^2(a+a\sec(c+dx))^{-2+n}\left(12+4n-7n^2-n^3-2(1-n)(6+n)\sec(c+dx)\right)}{8d\left(2-3n+n^2\right)(1-\sec(c+dx))}
\end{aligned}$$

```
[Out] 1/16*a^2*(n^2+9*n+12)*hypergeom([1, -2+n],[-1+n],1/2+1/2*sec(d*x+c))*(a+a*s
ec(d*x+c))^(-2+n)/d/(2-n)+1/4*a^2*(3+n)*sec(d*x+c)^2*(a+a*sec(d*x+c))^(-2+n
)/d/(1-n)/(1-sec(d*x+c))^2-a^2*sec(d*x+c)^3*(a+a*sec(d*x+c))^(-2+n)/d/(1-n)
/(1-sec(d*x+c))^2-1/8*a^2*(a+a*sec(d*x+c))^(-2+n)*(12+4*n-7*n^2-n^3-2*(1-n)
*(6+n)*sec(d*x+c))/d/(n^2-3*n+2)/(1-sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used

$= \{3958, 102, 154, 151, 70\}$

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx$$
$$= \frac{a^2(n^2+9n+12)\,(a\sec(c+dx)+a)^{n-2}\,\text{Hypergeometric2F1}\left(1,n-2,n-1,\frac{1}{2}(\sec(c+dx)+1)\right)}{16d(2-n)}$$
$$- \frac{a^2(-2(1-n)(n+6)\sec(c+dx)-n^3-7n^2+4n+12)\,(a\sec(c+dx)+a)^{n-2}}{8d\,(n^2-3n+2)\,(1-\sec(c+dx))}$$
$$- \frac{a^2\sec^3(c+dx)(a\sec(c+dx)+a)^{n-2}}{d(1-n)(1-\sec(c+dx))^2} + \frac{a^2(n+3)\sec^2(c+dx)(a\sec(c+dx)+a)^{n-2}}{4d(1-n)(1-\sec(c+dx))^2}$$

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (a^2*(12 + 9*n + n^2)*Hypergeometric2F1[1, -2 + n, -1 + n, (1 + Sec[c + d*x
])/2]*(a + a*Sec[c + d*x])^(-2 + n))/(16*d*(2 - n)) + (a^2*(3 + n)*Sec[c +
d*x]^2*(a + a*Sec[c + d*x])^(-2 + n))/(4*d*(1 - n)*(1 - Sec[c + d*x])^2) -
(a^2*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(-2 + n))/(d*(1 - n)*(1 - Sec[c +
d*x])^2) - (a^2*(a + a*Sec[c + d*x])^(-2 + n)*(12 + 4*n - 7*n^2 - n^3 - 2*(
1 - n)*(6 + n)*Sec[c + d*x]))/(8*d*(2 - 3*n + n^2)*(1 - Sec[c + d*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
```

```
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^6\text{Subst}\left(\int \frac{x^4(a-ax)^{-3+n}}{(-a-ax)^3}\,dx, x, -\sec(c+dx)\right)}{d}\\
&= -\frac{a^2\sec^3(c+dx)(a+a\sec(c+dx))^{-2+n}}{d(1-n)(1-\sec(c+dx))^2}\\
&\quad+\frac{a^4\text{Subst}\left(\int \frac{x^2(a-ax)^{-3+n}(3a^2-a^2nx)}{(-a-ax)^3}\,dx, x, -\sec(c+dx)\right)}{d(1-n)}\\
&= \frac{a^2(3+n)\sec^2(c+dx)(a+a\sec(c+dx))^{-2+n}}{4d(1-n)(1-\sec(c+dx))^2}\\
&\quad-\frac{a^2\sec^3(c+dx)(a+a\sec(c+dx))^{-2+n}}{d(1-n)(1-\sec(c+dx))^2}\\
&\quad+\frac{a\text{Subst}\left(\int \frac{x(a-ax)^{-3+n}(-2a^4(3+n)-a^4(1-n)(6+n)x)}{(-a-ax)^2}\,dx, x, -\sec(c+dx)\right)}{4d(1-n)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a^2(3+n)\sec^2(c+dx)(a+a\sec(c+dx))^{-2+n}}{4d(1-n)(1-\sec(c+dx))^2} \\
&\quad - \frac{a^2\sec^3(c+dx)(a+a\sec(c+dx))^{-2+n}}{d(1-n)(1-\sec(c+dx))^2} \\
&\quad - \frac{a^2(a+a\sec(c+dx))^{-2+n}\left(12+4n-7n^2-n^3-2(1-n)(6+n)\sec(c+dx)\right)}{8d\left(2-3n+n^2\right)(1-\sec(c+dx))} \\
&\quad - \frac{(a^4(12+9n+n^2))\,\text{Subst}\left(\int \frac{(a-ax)^{-3+n}}{-a-ax}\,dx, x, -\sec(c+dx)\right)}{8d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a^2(12+9n+n^2)\,\text{Hypergeometric2F1}\left(1,-2+n,-1+n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^{-}}{16d(2-n)} \\
&\quad + \frac{a^2(3+n)\sec^2(c+dx)(a+a\sec(c+dx))^{-2+n}}{4d(1-n)(1-\sec(c+dx))^2} \\
&\quad - \frac{a^2\sec^3(c+dx)(a+a\sec(c+dx))^{-2+n}}{d(1-n)(1-\sec(c+dx))^2} \\
&\quad - \frac{a^2(a+a\sec(c+dx))^{-2+n}\left(12+4n-7n^2-n^3-2(1-n)(6+n)\sec(c+dx)\right)}{8d\left(2-3n+n^2\right)(1-\sec(c+dx))}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 492 vs. $2(240) = 480$.

Time = 3.89 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.05

$$
\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx =
$$

$$
-\frac{\cos(c+dx)(1+\sec(c+dx))^{-n}(a(1+\sec(c+dx)))^n\,\big(2^{1+n}\cot^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\,\big(\cos^2\left(\frac{1}{2}(c+dx\right.}{}
$$

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/64*(Cos[c + d*x]*(a*(1 + Sec[c + d*x]))^n*(2^(1 + n)*Cot[(c + d*x)/2]^4*
Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) - 3*2^n*n*Cot[(c +
d*x)/2]^4*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) + 2^n*n^2
*Cot[(c + d*x)/2]^4*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)
 - 3*2^(2 + n)*(-2 + n)*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec
[(c + d*x)/2]^2]*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) -
2^n*(-18 + 7*n + n^2)*Hypergeometric2F1[2, 1 - n, 2 - n, Cos[c + d*x]*Sec[(
c + d*x)/2]^2]*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) + 32
*Sec[c + d*x]*(1 + Sec[c + d*x])^n - 12*n*Sec[c + d*x]*(1 + Sec[c + d*x])^n
 - 12*Sec[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sec[c + d*x])^n + 2*n*Sec[(c + d
*x)/2]^2*Sec[c + d*x]*(1 + Sec[c + d*x])^n - 2*Sec[(c + d*x)/2]^4*Sec[c + d
*x]*(1 + Sec[c + d*x])^n + 2*n*Sec[(c + d*x)/2]^4*Sec[c + d*x]*(1 + Sec[c +
 d*x])^n))/(d*(-2 + n)*(-1 + n)*(1 + Sec[c + d*x])^n)
```

Maple [F]

$$\int \csc\left(dx+c\right)^{5}\left(a+a\sec\left(dx+c\right)\right)^{n}dx$$

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x)
[Out] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx=\int \left(a\sec\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right)^{5}\,dx$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx=\text{Timed out}$$

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx=\int \left(a\sec\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right)^{5}\,dx$$

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

Giac [F]

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx=\int (a\sec(dx+c)+a)^n\csc(dx+c)^5\,dx$$

[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)

Mupad [F(-1)]

Timed out.

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^5}\,dx$$

[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^5,x)

[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^5, x)

3.152 $\int (a+a\sec(c+dx))^n \sin^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 230

$$\int (a+a\sec(c+dx))^n \sin^4(c+dx)\,dx = -\frac{\text{AppellF1}\left(1-n,-\frac{1}{2},\frac{1}{2}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)(1+\cos(c+dx))^{\frac{1}{2}-n}(n-n\cos(c+dx)}{d(1-n)\sqrt{1-\cos(c+dx)}}$$
$$-\frac{\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d}$$
$$+\frac{2^{\frac{1}{2}+n}\text{AppellF1}\left(\frac{1}{2},-4+n,\frac{1}{2}-n,\frac{3}{2},1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)\cos^n(c+dx)(1+\cos(c+dx)}{d}$$

```
[Out] -cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d+2^(1/2+n)*AppellF1(1/2,-4+n,1/2
-n,3/2,1-cos(d*x+c),1/2-1/2*cos(d*x+c))*cos(d*x+c)^n*(1+cos(d*x+c))^(-1/2-n
)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d-AppellF1(1-n,1/2-n,-1/2,2-n,-cos(d*x+c),c
os(d*x+c))*(1+cos(d*x+c))^(1/2-n)*(n-n*cos(d*x+c))*cot(d*x+c)*(a+a*sec(d*x+
c))^n/d/(1-n)/(1-cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{3961, 2960, 2866, 2865, 2864, 138, 3125, 3087, 140\}$

$$\int (a + a\sec(c + dx))^n \sin^4(c + dx)\, dx$$
$$= \frac{2^{n+\frac{1}{2}} \sin(c + dx) \cos^n(c + dx)(\cos(c + dx) + 1)^{-n-\frac{1}{2}}(a\sec(c + dx) + a)^n \operatorname{AppellF1}\left(\frac{1}{2}, n - 4, \frac{1}{2} - n, \frac{3}{2}, 1 - c\right.}{d}$$
$$- \frac{\cot(c + dx)(n - n\cos(c + dx))(\cos(c + dx) + 1)^{\frac{1}{2}-n}(a\sec(c + dx) + a)^n \operatorname{AppellF1}\left(1 - n, -\frac{1}{2}, \frac{1}{2} - n, 2 -\right.}{d(1 - n)\sqrt{1 - \cos(c + dx)}}$$
$$- \frac{\sin(c + dx)\cos(c + dx)(a\sec(c + dx) + a)^n}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

```
[Out] -((AppellF1[1 - n, -1/2, 1/2 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 +
Cos[c + d*x])^(1/2 - n)*(n - n*Cos[c + d*x])*Cot[c + d*x]*(a + a*Sec[c + d*
x])^n)/(d*(1 - n)*Sqrt[1 - Cos[c + d*x]])) - (Cos[c + d*x]*(a + a*Sec[c + d
*x])^n*Sin[c + d*x])/d + (2^(1/2 + n)*AppellF1[1/2, -4 + n, 1/2 - n, 3/2, 1
 - Cos[c + d*x], (1 - Cos[c + d*x])/2]*Cos[c + d*x]^n*(1 + Cos[c + d*x])^(-
1/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/d
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
```

```
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !In
tegerQ[m] && GtQ[a, 0] && !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rule 3087

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3125

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1
)/(d*f*(m + n + 2))), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x
])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1)
) + C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
```

```
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\text{integral} = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int(-\cos(c+dx))^{-n}(-a-a\cos(c+dx))^n\sin^4(c+dx)\,dx$$

$$= \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int(-\cos(c+dx))^{4-n}(-a-a\cos(c+dx))^n\,dx + \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int(-\cos(c+dx))^{-n}(-a-a\cos(c+dx))^n\left(1-2\cos^2(c+dx)\right)\,dx$$

$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d} + \left((-\cos(c+dx))^n(1+\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int(-\cos(c+dx))^{4-n}(1+\cos(c+dx))^n\,dx + \frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int(-\cos(c+dx))^{-n}(-a-a\cos(c}{2a}$$

$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d} + \left(\cos^n(c+dx)(1+\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int\cos^{4-n}(c+dx)(1+\cos(c+dx))^n\,dx - \frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1}{2}-n}\sqrt{2an-2an\cos(c+dx)}\csc(c+dx)(a+a\sec(c+dx}{2ad}$$

$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d} - \frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{2}-n}\sqrt{2an-2an\cos(c+dx)}\csc(c+dx)(a+a\sec(c+dx))^n\right.}{2ad} + \frac{\left(\cos^n(c+dx)(1+\cos(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx))^n\sin(c+dx)\right)\text{Subst}\left(\int\frac{(1-x)^{4-n}(2-x)^{-\frac{1}{2}+n}}{\sqrt{x}}\right.}{d\sqrt{1-\cos(c+dx)}}$$

$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d} + \frac{2^{\frac{1}{2}+n}\,\text{AppellF1}\left(\frac{1}{2},-4+n,\frac{1}{2}-n,\frac{3}{2},1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)\cos^n(c+dx)(1+\cos(c}{d} - \frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{2}-n}(2an-2an\cos(c+dx))\csc(c+dx)(a+a\sec(c+dx))^n\right)}{2ad\sqrt{1-\cos(c+dx)}}$$

$$= -\frac{\text{AppellF1}\left(1-n,-\frac{1}{2},\frac{1}{2}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)(1+\cos(c+dx))^{\frac{1}{2}-n}(n-n\cos}{d(1-n)\sqrt{1-\cos(c+dx)}}$$
$$-\frac{\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d}$$
$$+\frac{2^{\frac{1}{2}+n}\text{AppellF1}\left(\frac{1}{2},-4+n,\frac{1}{2}-n,\frac{3}{2},1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)\cos^n(c+dx)(1+\cos(}{d}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 21.75 (sec) , antiderivative size = 7069, normalized size of antiderivative = 30.73

$$\int (a+a\sec(c+dx))^n\sin^4(c+dx)\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
[Out] Result too large to show
```

Maple [F]

$$\int (a+a\sec(dx+c))^n\sin(dx+c)^4\,dx$$

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^n\sin^4(c+dx)\,dx = \int (a\sec(dx+c)+a)^n\sin(dx+c)^4\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="fricas")
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sec(d*x + c) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^n \sin^4(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sec(c+dx))^n \sin^4(c+dx)\,dx = \int (a\sec(dx+c)+a)^n \sin(dx+c)^4\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^n \sin^4(c+dx)\,dx = \int (a\sec(dx+c)+a)^n \sin(dx+c)^4\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^n \sin^4(c+dx)\,dx = \int \sin(c+d\,x)^4 \left(a+\frac{a}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

3.153 $\int (a+a\sec(c+dx))^n \sin^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int (a+a\sec(c+dx))^n \sin^2(c+dx)\,dx =$$
$$-\frac{\operatorname{AppellF1}\left(1-n,-\frac{1}{2},-\frac{1}{2}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{\frac{1}{2}-}}{d(1-n)}$$

```
[Out] -AppellF1(1-n,-1/2-n,-1/2,2-n,-cos(d*x+c),cos(d*x+c))*(1+cos(d*x+c))^(1/2-n
)*cot(d*x+c)*(a+a*sec(d*x+c))^n*(1-cos(d*x+c))^(1/2)/d/(1-n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3961, 2953, 3087, 140, 138}

$$\int (a+a\sec(c+dx))^n \sin^2(c+dx)\,dx =$$
$$-\frac{\sqrt{1-\cos(c+dx)}\cot(c+dx)(\cos(c+dx)+1)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n \operatorname{AppellF1}\left(1-n,-\frac{1}{2},-n-\frac{1}{2},2\right.}{d(1-n)}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] -((AppellF1[1 - n, -1/2, -1/2 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*Sqrt
[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(1/2 - n)*Cot[c + d*x]*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
```

```
m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3087

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\text{integral} = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int(-\cos(c+dx))^{-n}(-a-a\cos(c+dx))^n\sin^2(c+dx)\,dx$$

$$= \frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int(-\cos(c+dx))^{-n}(-a-a\cos(c+$$

$$a^2$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1}{2}-n}\sqrt{-a+a\cos(c+dx)}\csc(c+dx)(a+a\sec(c+dx))^n$$

$$a^2 d$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{2}-n}(-a-a\cos(c+dx))\sqrt{-a+a\cos(c+dx)}\csc(c+dx)\right.}{a^2d}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{2}-n}(-a-a\cos(c+dx))(-a+a\cos(c+dx))\csc(c+dx)\right.}{a^2d\sqrt{1-\cos(c+dx}}$$

$$= -\frac{\text{AppellF1}\left(1-n,-\frac{1}{2},-\frac{1}{2}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)\sqrt{1-\cos(c+dx)}(1+\cos(c+}{d(1-n)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 17.34 (sec) , antiderivative size = 4297, normalized size of antiderivative = 45.23

$$\int (a+a\sec(c+dx))^n\sin^2(c+dx)\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] (2^(3 + n)*Cos[(c + d*x)/2]^5*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + S
ec[c + d*x]))^n*Sin[(c + d*x)/2]*(Cos[2*(c + d*x)]*(-1/4*(1 + Sec[c + d*x])
^n - ((1 + Sec[c + d*x])^n*Sin[c + d*x]^2)/2 - ((1 + Sec[c + d*x])^n*Sin[c
+ d*x]^4)/4) + (I/4)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)] + (I/2)*(1 + Sec
[c + d*x])^n*Sin[c + d*x]^2*Sin[2*(c + d*x)] + (I/4)*(1 + Sec[c + d*x])^n*S
in[c + d*x]^4*Sin[2*(c + d*x)] + Cos[c + d*x]^4*(-1/4*(Cos[2*(c + d*x)]*(1
+ Sec[c + d*x])^n) + (I/4)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)]) + Cos[c +
 d*x]^3*((-I)*Cos[2*(c + d*x)]*(1 + Sec[c + d*x])^n*Sin[c + d*x] - (1 + Sec
[c + d*x])^n*Sin[c + d*x]*Sin[2*(c + d*x)]) + Cos[c + d*x]^2*(Cos[2*(c + d*
x)]*((1 + Sec[c + d*x])^n/2 + (3*(1 + Sec[c + d*x])^n*Sin[c + d*x]^2)/2) -
(I/2)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)] - ((3*I)/2)*(1 + Sec[c + d*x])^
n*Sin[c + d*x]^2*Sin[2*(c + d*x)]) + Cos[c + d*x]*(Cos[2*(c + d*x)]*(I*(1 +
 Sec[c + d*x])^n*Sin[c + d*x] + I*(1 + Sec[c + d*x])^n*Sin[c + d*x]^3) + (1
 + Sec[c + d*x])^n*Sin[c + d*x]*Sin[2*(c + d*x)] + (1 + Sec[c + d*x])^n*Sin
[c + d*x]^3*Sin[2*(c + d*x)]))*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(3*AppellF1[1/2, n, 2, 3/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - AppellF1[1/2,
n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/(AppellF1[1/2, n, 3, 3/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3)))/(d*(1
```

```
+ Sec[c + d*x])^n*(2^(2 + n)*Cos[(c + d*x)/2]^6*(Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^n*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Sec[(c + d*x)/2]^2)/(3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3)) - 5*2^(2 + n)*Cos[(c + d*x)/2
]^4*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*Sin[(c + d*x)/2]^2*((3*AppellF1[1/2
, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(
3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2
*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*Appe
llF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
 d*x)/2]^2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (
2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n
*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan
[(c + d*x)/2]^2)/3)) + 2^(3 + n)*Cos[(c + d*x)/2]^5*(Cos[(c + d*x)/2]^2*Sec
[c + d*x])^n*Sin[(c + d*x)/2]*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*AppellF1[1
/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/
2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1
+ n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)
+ (3*Sec[(c + d*x)/2]^2*((-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (n*AppellF1[3/
2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2])/3))/(3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - (-(AppellF1[3/2, n, 4, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (n
*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2])/3)/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3) - (3*AppellF1[1/2, n, 2,
 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(2*(-2*Ap
pellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF
1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2] + 3*((-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (n*Appell
F1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/3) + 2*Tan[(c + d*x)/2]^2*(-2*((-9*AppellF1[5/2,
```

```
n, 4, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2])/5 + (3*n*AppellF1[5/2, 1 + n, 3, 7/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + n*((-6*AppellF1
[5/2, 1 + n, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 + n, 2, 7/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)))
)/(3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*
(-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*A
ppellF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(
c + d*x)/2]^2)^2 + (AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*(-(AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (n*AppellF1[3/2, 1 + n, 3, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/3 + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*Tan[(c + d*x)/2]^2*(-3*((
-12*AppellF1[5/2, n, 5, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (3*n*AppellF1[5/2, 1 + n, 4, 7/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)
 + n*((-9*AppellF1[5/2, 1 + n, 4, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 +
n, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2])/5)))/3))/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3)^2) + 2^(3 + n)*n*Cos[(c + d*x)/2]^5*(
Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*Sin[(c + d*x)/2]*((3*AppellF1[1/2
, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(
3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2
*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*Appe
llF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
 d*x)/2]^2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (
2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n
*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan
[(c + d*x)/2]^2)/3))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + C
os[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))
```

Maple [F]

$$\int (a + a\sec(dx + c))^n \sin(dx + c)^2 \, dx$$

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

Fricas [F]

$$\int (a + a\sec(c + dx))^n \sin^2(c + dx) \, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^2 \, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sec(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int (a + a\sec(c + dx))^n \sin^2(c + dx) \, dx = \int (a(\sec(c + dx) + 1))^n \sin^2(c + dx) \, dx$$

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**2,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*sin(c + d*x)**2, x)
```

Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin^2(c + dx) \, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^2 \, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^n \sin^2(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^2\, dx$$

[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^2(c + dx)\, dx = \int \sin(c + d\,x)^2 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^n,x)

[Out] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^n, x)

3.154 $\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + \frac{2^{-\frac{1}{2}+n}n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)(1+\sec(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx))^n}{d}$$

```
[Out] -cot(d*x+c)*(a+a*sec(d*x+c))^n/d+2^(-1/2+n)*n*hypergeom([1/2, 3/2-n],[3/2],
1/2-1/2*sec(d*x+c))*(1+sec(d*x+c))^(-1/2-n)*(a+a*sec(d*x+c))^n*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3960, 3913, 3912, 71}

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx$$
$$= \frac{2^{n-\frac{1}{2}}n\tan(c+dx)(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}-n,\frac{3}{2},\frac{1}{2}(1-\sec(c\right.}{d} - \frac{\cot(c+dx)(a\sec(c+dx)+a)^n}{d}$$

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((Cot[c + d*x]*(a + a*Sec[c + d*x])^n)/d) + (2^(-1/2 + n)*n*Hypergeometric
2F1[1/2, 3/2 - n, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*
(a + a*Sec[c + d*x])^n*Tan[c + d*x])/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + (an)\int \sec(c+dx)(a+a\sec(c+dx))^{-1+n}\,dx \\
&= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} \\
&\quad + \left(n(1+\sec(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int \sec(c+dx)(1+\sec(c+dx))^{-1+n}\,dx \\
&= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} \\
&\quad - \frac{\left(n(1+\sec(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx))^n\tan(c+dx)\right)\operatorname{Subst}\left(\int \frac{(1+x)^{-\frac{3}{2}+n}}{\sqrt{1-x}}\,dx, x, \sec(c+dx\right.}{d\sqrt{1-\sec(c+dx)}}
\end{aligned}$$

$$= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + \frac{2^{-\frac{1}{2}+n} n \,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{2}-n, \frac{3}{2}, \frac{1}{2}(1-\sec(c+dx))\right)(1+\sec(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+}{d}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.45

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n \, dx = -\frac{2^{-1+n}\left(\cot^2\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(-\frac{1}{2}, n, \frac{1}{2}, \tan^2\left(\frac{1}{2}(c+dx)\right)\right) - \text{Hypergeometric2F1}\left(\frac{1}{2}, n, \frac{3}{2},\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((2^(-1 + n)*(Cot[(c + d*x)/2]^2*Hypergeometric2F1[-1/2, n, 1/2, Tan[(c +
d*x)/2]^2] - Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2])*(Cos[c + d
*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c
 + d*x]))^n*Tan[(c + d*x)/2])/(d*(1 + Sec[c + d*x])^n))
```

Maple [F]

$$\int \csc(dx+c)^2 (a+a\sec(dx+c))^n \, dx$$

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^2 \, dx$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Sympy [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a(\sec(c+dx)+1))^n \csc^2(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**n,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*csc(c + d*x)**2, x)
```

Maxima [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^2\,dx$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Giac [F]

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \csc(dx+c)^2\,dx$$

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^2}\,dx$$

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^2,x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^2, x)
```

3.155 $\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 349

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\,(1-4n^2)\,(1-\cos(c+dx))^2} - \frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2} - \frac{a^3(4-n)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d\,(3-8n+4n^2)\,(a-a\cos(c+dx))^2(a+a\cos(c+dx))} + \frac{n(7-3n-n^2)\cos(c+dx)\left(\frac{1+\cos(c+dx)}{1-\cos(c+dx)}\right)^{-\frac{1}{2}-n}\text{Hypergeometric2F1}\left(-\frac{1}{2}-n,1-n,2-n,-\frac{2\cos(c+dx)}{1-\cos(c+dx)}\right)(}{d(1-2n)(3-2n)(1-n)(1+2n)(1-\cos(c+dx))^2}$$

```
[Out] (n^2-n+2)*cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/(-4*n^2+1)/(1-
cos(d*x+c))^2-a^4*cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/(a-a*c
os(d*x+c))^2/(a+a*cos(d*x+c))^2-a^3*(4-n)*cos(d*x+c)*(a+a*sec(d*x+c))^n*sin
(d*x+c)/d/(4*n^2-8*n+3)/(a-a*cos(d*x+c))^2/(a+a*cos(d*x+c))+n*(-n^2-3*n+7)*
cos(d*x+c)*((1+cos(d*x+c))/(1-cos(d*x+c)))^(-1/2-n)*hypergeom([1-n, -1/2-n]
,[2-n],-2*cos(d*x+c)/(1-cos(d*x+c)))*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(-8*n^
4+20*n^3-10*n^2-5*n+3)/(1-cos(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3961, 2962, 136, 160, 12, 134}

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a^4\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d(3-2n)(a-a\cos(c+dx))^2(a\cos(c+dx)+a)^2}$$
$$-\frac{a^3(4-n)\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d\,(4n^2-8n+3)\,(a-a\cos(c+dx))^2(a\cos(c+dx)+a)}$$
$$+\frac{n(-n^2-3n+7)\sin(c+dx)\cos(c+dx)\left(\frac{\cos(c+dx)+1}{1-\cos(c+dx)}\right)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\,\text{Hypergeometric2F1}\Big(-}{d(1-2n)(3-2n)(1-n)(2n+1)(1-\cos(c+dx))^2}$$
$$+\frac{(n^2-n+2)\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d(3-2n)\,(1-4n^2)\,(1-\cos(c+dx))^2}$$

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] ((2 - n + n^2)*Cos[c + d*x]*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*
n)*(1 - 4*n^2)*(1 - Cos[c + d*x])^2) - (a^4*Cos[c + d*x]*(a + a*Sec[c + d*x
])^n*Sin[c + d*x])/(d*(3 - 2*n)*(a - a*Cos[c + d*x])^2*(a + a*Cos[c + d*x])
^2) - (a^3*(4 - n)*Cos[c + d*x]*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3
- 8*n + 4*n^2)*(a - a*Cos[c + d*x])^2*(a + a*Cos[c + d*x])) + (n*(7 - 3*n -
 n^2)*Cos[c + d*x]*((1 + Cos[c + d*x])/(1 - Cos[c + d*x]))^(-1/2 - n)*Hyper
geometric2F1[-1/2 - n, 1 - n, 2 - n, (-2*Cos[c + d*x])/(1 - Cos[c + d*x])]*
(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(3 - 2*n)*(1 - n)*(1 + 2*
n)*(1 - Cos[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))]])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 136

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimp
lerQ[p, 1]))
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 2962

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[Cos[e + f*x]/(a^(
p - 2)*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(d*x
)^n*(a + b*x)^(m + p/2 - 1/2)*(a - b*x)^(p/2 - 1/2), x], x, Sin[e + f*x]],
x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[p/2]
 &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\text{integral} = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int(-\cos(c+dx))^{-n}(-a - a\cos(c+dx))^n\csc^4(c+dx)\,dx$$

$$= -\frac{\left(a^6(-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx))^n\sin(c+dx)\right)\text{Subst}\left(\int\frac{(-x)^{-n}(\cdots}{(-\cdots}\right.}{d\sqrt{-a+a\cos(c+dx)}}$$

$$= -\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2}$$

$$-\frac{\left(a^3(-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx))^n\sin(c+dx)\right)\text{Subst}\left(\int\frac{(-x)^{-}}{}\right.}{d(3-2n)\sqrt{-a+a\cos(c+dx)}}$$

$$= -\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2}$$

$$-\frac{a^3(4-n)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))}$$

$$-\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx))^n\sin(c+dx)\right)\text{Subst}\left(\int\frac{(-x)^{-n}(-}{}\right.}{d(1-2n)(3-2n)\sqrt{-a+a\cos(c+dx)}}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2}$$

$$-\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2}$$

$$-\frac{a^3(4-n)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))}$$

$$-\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx))^n\sin(c+dx)\right)\text{Subst}\left(\int\frac{a^6n(7-3n}{}\right.}{a^3d(1-2n)(3-2n)(1+2n)\sqrt{-a+a\cos(c+dx)}}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2}$$

$$-\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2}$$

$$-\frac{a^3(4-n)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))}$$

$$-\frac{\left(a^3n(7-3n-n^2)(-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx))^n\sin(c+dx)\right)}{d(1-2n)(3-2n)(1+2n)\sqrt{-a+a\cos(c+dx)}}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2}$$

$$-\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2}$$

$$-\frac{a^3(4-n)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))}$$

$$+\frac{n(7-3n-n^2)\cos(c+dx)\left(\frac{1+\cos(c+dx)}{1-\cos(c+dx)}\right)^{-\frac{1}{2}-n}\text{Hypergeometric2F1}\left(-\frac{1}{2}-n,1-n,2-n,-\frac{2\cos}{1-c}\right.}{d(1-2n)(3-2n)(1-n)(1+2n)(1-\cos(c+dx))^2}$$

Mathematica [A] (verified)

Time = 4.57 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx$$
$$=\frac{(a(1+\sec(c+dx)))^n\left(-2\cot^2\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},n,\frac{1}{2},\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(\cos(c+dx)\,\mathrm{s}\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^n*((-2*Cot[(c + d*x)/2]^2*Hypergeometric2F1[-1/2, n
, 1/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(3*2^n*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^n + 2*(1 + Sec[c + d*x])^n + n*(1 + Sec[c + d*
x])^n))/(1 + Sec[c + d*x])^n + (-(Cos[c + d*x]*(4*n*Cos[c + d*x] + (-3 + n)
*(3 + Cos[2*(c + d*x)]))*Csc[(c + d*x)/2]^4*Sec[(c + d*x)/2]^2) + (24*Hyper
geometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2
]^2)^n*(-3*2^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n - 2*(1 + Sec[c + d*x])^n
 + n*(2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n + (1 + Sec[c + d*x])^n)
))/(1 + Sec[c + d*x])^n)/(4*(-3 + 2*n)))*Tan[(c + d*x)/2])/(24*d)
```

Maple [F]

$$\int \csc\left(dx+c\right)^4\left(a+a\sec\left(dx+c\right)\right)^n dx$$

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx=\int \left(a\sec\left(dx+c\right)+a\right)^n\csc\left(dx+c\right)^4\,dx$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\csc(dx+c)^4\,dx$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Giac [F]

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\csc(dx+c)^4\,dx$$

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^4}\,dx$$

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^4,x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^4, x)
```

3.156 $\int (a+a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int (a+a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx =$$
$$-\frac{\text{AppellF1}\left(1-n,-\frac{1}{4},-\frac{1}{4}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)\cos(c+dx)(1+\cos(c+dx))^{-\frac{1}{4}-n}(a+}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

```
[Out] -AppellF1(1-n,-1/4-n,-1/4,2-n,-cos(d*x+c),cos(d*x+c))*cos(d*x+c)*(1+cos(d*x
+c))^(-1/4-n)*(a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2)/d/(1-n)/(1-cos(d*x+c))^(1
/4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3961, 2965, 140, 138}

$$\int (a+a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx =$$
$$-\frac{\sqrt{\sin(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{4}}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(1-n,-\frac{1}{4},-n-\frac{1}{4},2-n\right.}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

```
[Out] -((AppellF1[1 - n, -1/4, -1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*Cos[
c + d*x]*(1 + Cos[c + d*x])^(-1/4 - n)*(a + a*Sec[c + d*x])^n*Sqrt[Sin[c +
d*x]])/(d*(1 - n)*(1 - Cos[c + d*x])^(1/4)))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\text{integral} = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int(-\cos(c+dx))^{-n}(-a - a\cos(c+dx))^n\sin^{\frac{3}{2}}(c+dx)\,dx$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\right)\mathrm{Subst}\left(\int(-x)^{-}\right.}{d\sqrt[4]{-a+a\cos(c+dx)}}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\right)\mathrm{Subst}\left(\int(-x)^{-n}(1\right.}{d\sqrt[4]{-a+a\cos(c+dx)}}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\right)\mathrm{Subst}\left(\int\sqrt[4]{1-x}(-\right.}{d\sqrt[4]{1-\cos(c+dx)}}$$

$$= -\frac{\text{AppellF1}\left(1-n,-\frac{1}{4},-\frac{1}{4}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)\cos(c+dx)(1+\cos(c+dx))^{-\frac{1}{4}}}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 382 vs. $2(105) = 210$.

Time = 16.60 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.64

$$\int (a+a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

$$= \frac{}{d\left(2\left(3\,\text{AppellF1}\left(\frac{5}{4},n,\frac{5}{2},\frac{9}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-5\,\text{AppellF1}\left(\frac{5}{4},n,\frac{7}{2},\frac{9}{4},\tan^2\left(\frac{1}{2}(c+dx)\right.\right.\right.\right.}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

```
[Out] (10*(AppellF1[1/4, n, 3/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
AppellF1[1/4, n, 5/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(1 + C
os[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x]^(5/2))/(d*(2*(3*AppellF1
[5/4, n, 5/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 5*AppellF1[5/
4, n, 7/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1[5/4
, 1 + n, 3/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*n*AppellF1[
5/4, 1 + n, 5/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c
 + d*x]) + 5*AppellF1[1/4, n, 3/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*(1 + Cos[c + d*x]) - 5*AppellF1[1/4, n, 5/2, 5/4, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F]

$$\int (a+a\sec(dx+c))^n \sin(dx+c)^{\frac{3}{2}}\,dx$$

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int (a + a\sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx)\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \sin(dx + c)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sin^{\frac{3}{2}}(c + dx)\,dx = \int \sin(c + d\,x)^{3/2}\left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

```
[In] int(sin(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n, x)
```

3.157 $\int (a+a\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int (a+a\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx = -\frac{\operatorname{AppellF1}\left(1-n,\frac{1}{4},\frac{1}{4}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)\sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(1+\cos(c-$$
$$d(1-n)\sqrt{\sin(c+dx)}$$

```
[Out] -AppellF1(1-n,1/4-n,1/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(1/4)*co
s(d*x+c)*(1+cos(d*x+c))^(1/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3961, 2965, 140, 138}

$$\int (a+a\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx = -\frac{\sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)+a)^n \operatorname{AppellF1}\left(1-n,\frac{1}{4},\frac{1}{4}-n,2-\right.}{d(1-n)\sqrt{\sin(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

```
[Out] -((AppellF1[1 - n, 1/4, 1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(1/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sqrt[Sin[c + d*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\text{integral} = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int(-\cos(c+dx))^{-n}(-a - a\cos(c+dx))^n\sqrt{\sin(c+dx)}\,dx$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1}{4}-n}\sqrt[4]{-a+a\cos(c+dx)}(a+a\sec(c+dx))^n\right)\text{Subst}\left(\int\right.}{d\sqrt{\sin(c+dx)}}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}\sqrt[4]{-a+a\cos(c+dx)}(a+a\sec(c+dx))^n\right)\text{Subst}\left(\int\frac{(-x)}{\sqrt[4]{}}\right.}{d\sqrt{\sin(c+dx)}}$$

$$= -\frac{\left(\sqrt[4]{1-\cos(c+dx)}(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}(a+a\sec(c+dx))^n\right)\operatorname{Subst}\left(\int \frac{(-x)^{-n}}{\sqrt[4]{}}\right.}{d\sqrt{\sin(c+dx)}}$$

$$= -\frac{\operatorname{AppellF1}\left(1-n,\frac{1}{4},\frac{1}{4}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)\sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(1+}{d(1-n)\sqrt{\sin(c+dx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 214 vs. $2(105) = 210$.

Time = 3.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.04

$$\int (a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$$

$$= \frac{14\operatorname{AppellF1}\left(\frac{3}{4},n,\frac{3}{2},\frac{7}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),\right.}{d\left(6\left(3\operatorname{AppellF1}\left(\frac{7}{4},n,\frac{5}{2},\frac{11}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2n\operatorname{AppellF1}\left(\frac{7}{4},1+n,\frac{3}{2},\frac{11}{4},\tan^2\left(\frac{1}{2}\right.\right.\right.}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

```
[Out] (14*AppellF1[3/4, n, 3/2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1
+ Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x]^(3/2))/(d*(6*(3*Appel
lF1[7/4, n, 5/2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*Appel
lF1[7/4, 1 + n, 3/2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 +
Cos[c + d*x]) + 21*AppellF1[3/4, n, 3/2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F]

$$\int (a+a\sec(dx+c))^n\sqrt{\sin(dx+c)}dx$$

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int (a\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int (a(\sec(c+dx)+1))^n\sqrt{\sin(c+dx)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**(1/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*sqrt(sin(c + d*x)), x)
```

Maxima [F]

$$\int (a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int (a\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int (a\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int \sqrt{\sin\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

```
[In] int(sin(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n, x)
```

3.158 $\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = -\frac{\text{AppellF1}\left(1-n,\frac{3}{4},\frac{3}{4}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{3/4}\cos(c+dx)(1+\cos(c}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

```
[Out] -AppellF1(1-n,3/4-n,3/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(3/4)*co
s(d*x+c)*(1+cos(d*x+c))^(3/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3961, 2965, 140, 138}

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = -\frac{(1-\cos(c+dx))^{3/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{3}{4}-n}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(1-n,\frac{3}{4},\frac{3}{4}-n,2-\right.}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

```
[In] Int[(a + a*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

```
[Out] -((AppellF1[1 - n, 3/4, 3/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(3/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(3/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sin[c + d*x]^(3/2)))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\text{integral} = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int \frac{(-\cos(c+dx))^{-n}(-a-a\cos(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{3}{4}-n}(-a+a\cos(c+dx))^{3/4}(a+a\sec(c+dx))^n\right)\text{Subst}}{d\sin^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{3}{4}-n}(-a+a\cos(c+dx))^{3/4}(a+a\sec(c+dx))^n\right)\text{Subst}\Big(\int}{d\sin^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{\left((1-\cos(c+dx))^{3/4}(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{3}{4}-n}(a+a\sec(c+dx))^n\right)\text{Subst}\left(\int \frac{(-x)^{-}}{(}\right.}{d\sin^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{\text{AppellF1}\left(1-n,\frac{3}{4},\frac{3}{4}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{3/4}\cos(c+dx)(1}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 212 vs. $2(105) = 210$.

Time $= 3.53$ (sec) , antiderivative size $= 212$, normalized size of antiderivative $= 2.02$

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

$$= \frac{10\,\text{AppellF1}\left(\frac{1}{4},n,\frac{1}{2},\frac{5}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-\text{ta}\right.}{d\left(2\left(\text{AppellF1}\left(\frac{5}{4},n,\frac{3}{2},\frac{9}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2n\,\text{AppellF1}\left(\frac{5}{4},1+n,\frac{1}{2},\frac{9}{4},\tan^2\left(\frac{1}{2}(c+\right.\right.\right.}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

```
[Out] (10*AppellF1[1/4, n, 1/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1
+ Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sqrt[Sin[c + d*x]])/(d*(2*(AppellF
1[5/4, n, 3/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1
[5/4, 1 + n, 1/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[
c + d*x]) + 5*AppellF1[1/4, n, 1/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F]

$$\int \frac{(a+a\sec(dx+c))^n}{\sqrt{\sin(dx+c)}}dx$$

```
[In] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx=\int \frac{(a(\sec(c+dx)+1))^n}{\sqrt{\sin(c+dx)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**n/sin(d*x+c)**(1/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n/sqrt(sin(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sqrt{\sin\left(c+d\,x\right)}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(1/2),x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(1/2), x)
```

3.159 $\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\text{AppellF1}\left(1-n,\frac{5}{4},\frac{5}{4}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{5/4}\cos(c+dx)(1+\cos(}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

```
[Out] -AppellF1(1-n,5/4-n,5/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(5/4)*co
s(d*x+c)*(1+cos(d*x+c))^(5/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3961, 2965, 140, 138}

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = -\frac{(1-\cos(c+dx))^{5/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{5}{4}-n}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(1-n,\frac{5}{4},\frac{5}{4}-n,2\right.}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

```
[In] Int[(a + a*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] -((AppellF1[1 - n, 5/4, 5/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(5/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(5/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sin[c + d*x]^(5/2)))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\text{integral} = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int \frac{(-\cos(c+dx))^{-n}(-a-a\cos(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{5}{4}-n}(-a+a\cos(c+dx))^{5/4}(a+a\sec(c+dx))^n\right)\text{Subst}\Big(}{d\sin^{\frac{5}{2}}(c+dx)}$$

$$= -\Bigg(-\frac{\Big((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}(-a-a\cos(c+dx))(-a+a\cos(c+dx))^{5/4}(a+a\,\mathrm{se}}{ad\sin^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{\left(\sqrt[4]{1-\cos(c+dx)}(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}(-a-a\cos(c+dx))(-a+a\cos(c+\right.}{a^2 d\sin^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{\text{AppellF1}\left(1-n,\frac{5}{4},\frac{5}{4}-n,2-n,\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{5/4}\cos(c+dx)(1}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 212 vs. $2(105) = 210$.

Time = 4.38 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.02

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{6\,\text{AppellF1}\left(-\frac{1}{4},n,-\frac{1}{2},\frac{3}{4},\right.}{d\left(-2\left(\text{AppellF1}\left(\frac{3}{4},n,\frac{1}{2},\frac{7}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2n\,\text{AppellF1}\left(\frac{3}{4},1+n,-\frac{1}{2},\frac{7}{4},\tan^2\right.\right.\right.}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] (-6*AppellF1[-1/4, n, -1/2, 3/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
1 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n)/(d*(-2*(AppellF1[3/4, n, 1/2, 7
/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*n*AppellF1[3/4, 1 + n, -1/
2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 3*A
ppellF1[-1/4, n, -1/2, 3/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + C
os[c + d*x]))*Sqrt[Sin[c + d*x]])
```

Maple [F]

$$\int \frac{(a+a\sec(dx+c))^n}{\sin(dx+c)^{\frac{3}{2}}}dx$$

```
[In] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral(-(a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c))/(cos(d*x + c)^2 - 1), x
)
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a(\sec(c+dx)+1))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**n/sin(d*x+c)**(3/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n/sin(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(3/2),x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(3/2), x)
```

3.160 $\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 119

$$\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx = -\frac{a\cos(c+dx)}{d}+\frac{3b\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{d}-\frac{3b\cos^4(c+dx)}{4d}-\frac{3a\cos^5(c+dx)}{5d}+\frac{b\cos^6(c+dx)}{6d}+\frac{a\cos^7(c+dx)}{7d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+3/2*b*cos(d*x+c)^2/d+a*cos(d*x+c)^3/d-3/4*b*cos(d*x+c)^4/d-
3/5*a*cos(d*x+c)^5/d+1/6*b*cos(d*x+c)^6/d+1/7*a*cos(d*x+c)^7/d-b*ln(cos(d*x
+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2916, 12, 780}

$$\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx = \frac{a\cos^7(c+dx)}{7d}-\frac{3a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{d}-\frac{a\cos(c+dx)}{d}+\frac{b\cos^6(c+dx)}{6d}-\frac{3b\cos^4(c+dx)}{4d}+\frac{3b\cos^2(c+dx)}{2d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (3*b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/d -
 (3*b*Cos[c + d*x]^4)/(4*d) - (3*a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]^
6)/(6*d) + (a*Cos[c + d*x]^7)/(7*d) - (b*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b - a\cos(c+dx))\sin^6(c+dx)\tan(c+dx)\,dx \\
&= \frac{\operatorname{Subst}\left(\int \frac{a(-b+x)\left(a^2-x^2\right)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^7 d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{(-b+x)\left(a^2-x^2\right)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^6 d} \\
&= \frac{\operatorname{Subst}\left(\int \left(a^6 - \frac{a^6 b}{x} + 3a^4bx - 3a^4x^2 - 3a^2bx^3 + 3a^2x^4 + bx^5 - x^6\right)\,dx, x, -a\cos(c+dx)\right)}{a^6 d} \\
&= -\frac{a\cos(c+dx)}{d} + \frac{3b\cos^2(c+dx)}{2d} + \frac{a\cos^3(c+dx)}{d} - \frac{3b\cos^4(c+dx)}{4d} \\
&\quad - \frac{3a\cos^5(c+dx)}{5d} + \frac{b\cos^6(c+dx)}{6d} + \frac{a\cos^7(c+dx)}{7d} - \frac{b\log(\cos(c+dx))}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx$$
$$= -\frac{35a\cos(c+dx)}{64d} + \frac{7a\cos(3(c+dx))}{64d} - \frac{7a\cos(5(c+dx))}{320d} + \frac{a\cos(7(c+dx))}{448d} - \frac{b\left(-\frac{3}{2}\cos^2(c+dx)+\frac{3}{4}\cos^4(c+dx)-\frac{1}{6}\cos^6(c+dx)+\log(\cos(c+dx))\right)}{d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] (-35*a*Cos[c + d*x])/(64*d) + (7*a*Cos[3*(c + d*x)])/(64*d) - (7*a*Cos[5*(c
 + d*x)])/(320*d) + (a*Cos[7*(c + d*x)])/(448*d) - (b*((-3*Cos[c + d*x]^2)/
2 + (3*Cos[c + d*x]^4)/4 - Cos[c + d*x]^6/6 + Log[Cos[c + d*x]]))/d
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}+b\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}+b\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parts	$-\frac{a\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7d}+\frac{b\left(-\frac{\sin(dx+c)^6}{6}-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c$
parallelrisch	$\frac{-3675a\cos(dx+c)+15a\cos(7dx+7c)-147a\cos(5dx+5c)+735a\cos(3dx+3c)+35b\cos(6dx+6c)-420b\cos(4dx+4c)+3045}{6720d}$
risch	$ibx+\frac{29b\,e^{2i(dx+c)}}{128d}+\frac{29b\,e^{-2i(dx+c)}}{128d}+\frac{2ibc}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{35a\cos(dx+c)}{64d}+\frac{a\cos(7dx+7c)}{448d}+\frac{b\cos(6}{19}$
norman	$\frac{-\frac{32a}{35d}-\frac{128b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-\frac{14b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{(96a+70b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{5d}-\frac{(96a+128b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3d}-\frac{(32a+}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7}$

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*a*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c
)+b*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx$$
$$= \frac{60\,a\cos\left(dx+c\right)^7+70\,b\cos\left(dx+c\right)^6-252\,a\cos\left(dx+c\right)^5-315\,b\cos\left(dx+c\right)^4+420\,a\cos\left(dx+c\right)^3+6}{420\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*b*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*b*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*b*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*b*log(-cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx$$
$$= \frac{60\,a\cos\left(dx+c\right)^7+70\,b\cos\left(dx+c\right)^6-252\,a\cos\left(dx+c\right)^5-315\,b\cos\left(dx+c\right)^4+420\,a\cos\left(dx+c\right)^3+6}{420\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*b*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*b*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*b*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*b*log(cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(109) = 218$.

Time = 0.34 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.66

$$\int (a + b\sec(c+dx))\sin^7(c+dx)\,dx$$
$$= \frac{420\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 420\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{384\,a+1089\,b-\frac{2688\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8463\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+}{}}{}$$

[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="giac")

[Out] 1/420*(420*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (384*a + 1089*b - 2688*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8463*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8064*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 28749*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 13440*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 56035*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 28749*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8463*b*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)/d

Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int (a + b\sec(c+dx))\sin^7(c+dx)\,dx =$$
$$-\frac{a\,\cos\left(c+d\,x\right)-a\cos\left(c+d\,x\right)^3+\frac{3\,a\cos(c+d\,x)^5}{5}-\frac{a\cos(c+d\,x)^7}{7}-\frac{3\,b\cos(c+d\,x)^2}{2}+\frac{3\,b\cos(c+d\,x)^4}{4}-\frac{b\cos(c+d\,x)^6}{6}+}{d}$$

[In] int(sin(c + d*x)^7*(a + b/cos(c + d*x)),x)

[Out] -(a*cos(c + d*x) - a*cos(c + d*x)^3 + (3*a*cos(c + d*x)^5)/5 - (a*cos(c + d*x)^7)/7 - (3*b*cos(c + d*x)^2)/2 + (3*b*cos(c + d*x)^4)/4 - (b*cos(c + d*x)^6)/6 + b*log(cos(c + d*x)))/d

3.161 $\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx = -\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{b\cos^4(c+dx)}{4d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+b*cos(d*x+c)^2/d+2/3*a*cos(d*x+c)^3/d-1/4*b*cos(d*x+c)^4/d-
1/5*a*cos(d*x+c)^5/d-b*ln(cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2916, 12, 780}

$$\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx = -\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}+\frac{b\cos^2(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (b*Cos[c + d*x]^2)/d + (2*a*Cos[c + d*x]^3)/(3*d) -
 (b*Cos[c + d*x]^4)/(4*d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Log[Cos[c + d*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b - a\cos(c+dx))\sin^4(c+dx)\tan(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{a(-b+x)\left(a^2-x^2\right)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^5 d} \\
&= \frac{\text{Subst}\left(\int \frac{(-b+x)\left(a^2-x^2\right)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^4 d} \\
&= \frac{\text{Subst}\left(\int \left(a^4 - \frac{a^4 b}{x} + 2a^2 bx - 2a^2x^2 - bx^3 + x^4\right)\,dx, x, -a\cos(c+dx)\right)}{a^4 d} \\
&= -\frac{a\cos(c+dx)}{d} + \frac{b\cos^2(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} \\
&\quad - \frac{b\cos^4(c+dx)}{4d} - \frac{a\cos^5(c+dx)}{5d} - \frac{b\log(\cos(c+dx))}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx$$
$$= -\frac{5a\cos(c+dx)}{8d}+\frac{5a\cos(3(c+dx))}{48d}-\frac{a\cos(5(c+dx))}{80d} - \frac{b\left(-\cos^2(c+dx)+\frac{1}{4}\cos^4(c+dx)+\log(\cos(c+dx))\right)}{d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^5,x]
[Out] (-5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*x)])/(48*d) - (a*Cos[5*(c + d
*x)])/(80*d) - (b*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c + d*x]]))
/d
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+b\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+b\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parts	$-\frac{a\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{b\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parallelrisch	$\frac{-300a\cos(dx+c)-6a\cos(5dx+5c)+50a\cos(3dx+3c)-15b\cos(4dx+4c)+180\cos(2dx+2c)b+480b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-}{480d}$
risch	$ibx+\frac{3b\,e^{2i(dx+c)}}{16d}+\frac{3b\,e^{-2i(dx+c)}}{16d}+\frac{2ibc}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{5a\cos(dx+c)}{8d}-\frac{a\cos(5dx+5c)}{80d}-\frac{b\cos(4d}{32}$
norman	$\frac{-\frac{16a}{15d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{10b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{(16a+6b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}-\frac{2(16a+15b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}+\frac{b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}$

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/5*a*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+b*(-1/4*sin(d*x+
c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx = -\frac{12\,a\cos\left(dx+c\right)^5+15\,b\cos\left(dx+c\right)^4-40\,a\cos\left(dx+c\right)^3-60\,b\cos\left(dx+c\right)^2+60\,a\cos\left(dx+c\right)+60\,b}{60\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*b*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*b*log(-cos(d*x + c)))/d
```

Sympy [F]

$$\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx = \int (a+b\sec{(c+dx)})\sin^5{(c+dx)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**5,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx = -\frac{12\,a\cos\left(dx+c\right)^5+15\,b\cos\left(dx+c\right)^4-40\,a\cos\left(dx+c\right)^3-60\,b\cos\left(dx+c\right)^2+60\,a\cos\left(dx+c\right)+60\,b}{60\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*b*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*b*log(cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(81) = 162$.

Time = 0.33 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.85

$$\int (a + b\sec(c + dx))\sin^5(c + dx)\,dx$$
$$= \frac{60\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{64\,a+137\,b-\frac{320\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{805\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{640\,a(}{(\cos}}{}}{60\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*b*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (64*a + 137*b - 320*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 805*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 640*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*b*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 - 1970*b*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 805*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*b*(cos(d*x
 + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int (a + b\sec(c + dx))\sin^5(c + dx)\,dx =$$
$$-\frac{a\,\cos\left(c+d\,x\right)-\frac{2\,a\,{\cos\left(c+d\,x\right)}^3}{3}+\frac{a\,{\cos\left(c+d\,x\right)}^5}{5}-b\,{\cos\left(c+d\,x\right)}^2+\frac{b\,{\cos\left(c+d\,x\right)}^4}{4}+b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - (2*a*cos(c + d*x)^3)/3 + (a*cos(c + d*x)^5)/5 - b*cos(c
+ d*x)^2 + (b*cos(c + d*x)^4)/4 + b*log(cos(c + d*x)))/d
```

3.162 $\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx = -\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{3d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+1/2*b*cos(d*x+c)^2/d+1/3*a*cos(d*x+c)^3/d-b*ln(cos(d*x+c))/
d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2916, 12, 780}

$$\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx = \frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{2d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/(3*d)
 - (b*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-b - a\cos(c+dx))\sin^2(c+dx)\tan(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{a(-b+x)(a^2-x^2)}{x}\,dx, x, -a\cos(c+dx)\right)}{a^3 d} \\
&= \frac{\text{Subst}\left(\int \frac{(-b+x)(a^2-x^2)}{x}\,dx, x, -a\cos(c+dx)\right)}{a^2 d} \\
&= \frac{\text{Subst}\left(\int \left(a^2 - \frac{a^2 b}{x} + bx - x^2\right)\,dx, x, -a\cos(c+dx)\right)}{a^2 d} \\
&= -\frac{a\cos(c+dx)}{d} + \frac{b\cos^2(c+dx)}{2d} + \frac{a\cos^3(c+dx)}{3d} - \frac{b\log(\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int (a + b\sec(c+dx))\sin^3(c+dx)\,dx = -\frac{3a\cos(c+dx)}{4d} + \frac{a\cos(3(c+dx))}{12d} - \frac{b\left(-\frac{1}{2}\cos^2(c+dx) + \log(\cos(c+dx))\right)}{d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] (-3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (b*(-1/2*Cos[c +
d*x]^2 + Log[Cos[c + d*x]]))/d
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+b\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+b\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
parts	$-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{b\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}$
risch	$ibx+\frac{b\,\mathrm{e}^{2i(dx+c)}}{8d}+\frac{b\,\mathrm{e}^{-2i(dx+c)}}{8d}+\frac{2ibc}{d}-\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}-\frac{3a\cos(dx+c)}{4d}+\frac{a\cos(3dx+3c)}{12d}$
parallelrisch	$\frac{12b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-12b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-12b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-9a\cos(dx+c)+3\cos(2dx+2c)b+a\cos(3dx+3}{12d}$
norman	$\frac{-\frac{4a}{3d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{(4a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{d}$

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a*(2+sin(d*x+c)^2)*cos(d*x+c)+b*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c))
))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx = \frac{2\,a\cos\left(dx+c\right)^3+3\,b\cos\left(dx+c\right)^2-6\,a\cos\left(dx+c\right)-6\,b\log\left(-\cos\left(dx+c\right)\right)}{6\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*b*log(-
cos(d*x + c)))/d
```

Sympy [F]

$$\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx=\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx$$

[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**3,x)

[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx = \frac{2\,a\cos(dx+c)^3+3\,b\cos(dx+c)^2-6\,a\cos(dx+c)-6\,b\log(\cos(dx+c))}{6\,d}$$

[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="maxima")

[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*b*log(cos(d*x + c)))/d

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx = -\frac{b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{2\,ad^2\cos(dx+c)^3+3\,bd^2\cos(dx+c)^2-6\,ad^2\cos(dx+c)}{6\,d^3}$$

[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="giac")

[Out] -b*log(abs(cos(d*x + c))/abs(d))/d + 1/6*(2*a*d^2*cos(d*x + c)^3 + 3*b*d^2*cos(d*x + c)^2 - 6*a*d^2*cos(d*x + c))/d^3

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int (a + b\sec(c + dx))\sin^3(c + dx)\,dx$$
$$= -\frac{a\,\cos\left(c+d\,x\right)-\frac{a\cos\left(c+d\,x\right)^3}{3}-\frac{b\cos\left(c+d\,x\right)^2}{2}+b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x)),x)
[Out] -(a*cos(c + d*x) - (a*cos(c + d*x)^3)/3 - (b*cos(c + d*x)^2)/2 + b*log(cos(
c + d*x)))/d
```

3.163 $\int (a+b\sec(c+dx))\sin(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 26

$$\int (a+b\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(c+dx)}{d} - \frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d-b*ln(cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3957, 2800, 45}

$$\int (a+b\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(c+dx)}{d} - \frac{b\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-b - a\cos(c+dx))\tan(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{-b+x}{x}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{b}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= -\frac{a\cos(c+dx)}{d} - \frac{b\log(\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int (a+b\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(c)\cos(dx)}{d} - \frac{b\log(\cos(c+dx))}{d} + \frac{a\sin(c)\sin(dx)}{d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x],x]
[Out] -((a*Cos[c]*Cos[d*x])/d) - (b*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{-\frac{a}{\sec(dx+c)}+b\ln(\sec(dx+c))}{d}$	26
default	$\frac{-\frac{a}{\sec(dx+c)}+b\ln(\sec(dx+c))}{d}$	26
parts	$-\frac{a\cos(dx+c)}{d}+\frac{b\ln(\sec(dx+c))}{d}$	26
risch	$ibx+\frac{2ibc}{d}-\frac{b\ln(\mathrm{e}^{2i(dx+c)}+1)}{d}-\frac{a\cos(dx+c)}{d}$	45
parallelrisch	$\frac{-a\cos(dx+c)-b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)+a}{d}$	60
norman	$\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}+\frac{b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	89

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(-a/sec(d*x+c)+b*ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int (a+b\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(dx+c)+b\log(-\cos(dx+c))}{d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="fricas")
[Out] -(a*cos(d*x + c) + b*log(-cos(d*x + c)))/d
```

Sympy [F]

$$\int (a+b\sec(c+dx))\sin(c+dx)\,dx = \int (a+b\sec(c+dx))\sin(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x)
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int (a+b\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(dx+c)+b\log(\cos(dx+c))}{d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="maxima")
[Out] -(a*cos(d*x + c) + b*log(cos(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (a+b\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\cos(dx+c)}{d} - \frac{b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="giac")
[Out] -a*cos(d*x + c)/d - b*log(abs(cos(d*x + c))/abs(d))/d
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int (a+b\sec(c+dx))\sin(c+dx)\,dx = -\frac{a\,\cos(c+d\,x)+b\,\ln(\cos(c+d\,x))}{d}$$

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x)),x)
[Out] -(a*cos(c + d*x) + b*log(cos(c + d*x)))/d
```

3.164 $\int \csc(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\,\text{arctanh}(\cos(c+dx))}{d} + \frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+b*ln(tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3957, 2913, 2700, 29, 3855}

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = \frac{b\log(\tan(c+dx))}{d} - \frac{a\,\text{arctanh}(\cos(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (b*Log[Tan[c + d*x]])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))\csc(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc(c+dx)\,dx + b\int \csc(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{arctanh}(\cos(c+dx))}{d} + \frac{b\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(c+dx)\right)}{d} \\
&= -\frac{a\text{arctanh}(\cos(c+dx))}{d} + \frac{b\log(\tan(c+dx))}{d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{b\log(\cos(c+dx))}{d} + \frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{b\log(\sin(c+dx))}{d}$$

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[c/2 + (d*x)/2]])/d) - (b*Log[Cos[c + d*x]])/d + (a*Log[Sin[c/2
 + (d*x)/2]])/d + (b*Log[Sin[c + d*x]])/d
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{a\ln(-\cot(dx+c)+\csc(dx+c))+b\ln(\tan(dx+c))}{d}$	33
default	$\frac{a\ln(-\cot(dx+c)+\csc(dx+c))+b\ln(\tan(dx+c))}{d}$	33
parallelrisch	$\frac{-b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)(a+b)}{d}$	50
norman	$\frac{(a+b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	55
risch	$-\frac{a\ln(e^{i(dx+c)}+1)}{d}+\frac{\ln(e^{i(dx+c)}+1)b}{d}+\frac{a\ln(e^{i(dx+c)}-1)}{d}+\frac{\ln(e^{i(dx+c)}-1)b}{d}-\frac{b\ln(e^{2i(dx+c)}+1)}{d}$	89

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*ln(-cot(d*x+c)+csc(d*x+c))+b*ln(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = -\frac{2\,b\log\left(-\cos\left(dx+c\right)\right)+(a-b)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-(a+b)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,d}$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(2*b*log(-cos(d*x + c)) + (a - b)*log(1/2*cos(d*x + c) + 1/2) - (a + b
)*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec\left(c+dx\right))\csc\left(c+dx\right)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x)
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx$$
$$= -\frac{(a-b)\log(\cos(dx+c)+1)-(a+b)\log(\cos(dx+c)-1)+2\,b\log(\cos(dx+c))}{2\,d}$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/2*((a - b)*log(cos(d*x + c) + 1) - (a + b)*log(cos(d*x + c) - 1) + 2*b*l
og(cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(26) = 52.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx = \frac{(a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{2\,d}$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] 1/2*((a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*b*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \csc(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{\frac{a\ln(\cos(c+d\,x)-1)}{2}-b\ln(\cos(c+d\,x))-\frac{a\ln(\cos(c+d\,x)+1)}{2}+\frac{b\ln(\cos(c+d\,x)-1)}{2}+\frac{b\ln(\cos(c+d\,x)+1)}{2}}{d}$$

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x),x)
[Out] ((a*log(cos(c + d*x) - 1))/2 - b*log(cos(c + d*x)) - (a*log(cos(c + d*x) +
1))/2 + (b*log(cos(c + d*x) - 1))/2 + (b*log(cos(c + d*x) + 1))/2)/d
```

3.165 $\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\text{arctanh}(\cos(c+dx))}{2d} - \frac{b\cot^2(c+dx)}{2d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} + \frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -1/2*a*arctanh(cos(d*x+c))/d-1/2*b*cot(d*x+c)^2/d-1/2*a*cot(d*x+c)*csc(d*x+
c)/d+b*ln(tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3957, 2913, 2700, 14, 3853, 3855}

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\text{arctanh}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} - \frac{b\cot^2(c+dx)}{2d} + \frac{b\log(\tan(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/2*(a*ArcTanh[Cos[c + d*x]])/d - (b*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*
x]*Csc[c + d*x])/(2*d) + (b*Log[Tan[c + d*x]])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int (-b - a\cos(c+dx))\csc^3(c+dx)\sec(c+dx)\,dx$$

$$= a\int \csc^3(c+dx)\,dx + b\int \csc^3(c+dx)\sec(c+dx)\,dx$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)}{2d} + \frac{1}{2}a\int \csc(c+dx)\,dx + \frac{b\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$=-\frac{a\text{arctanh}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{b\text{Subst}\left(\int\left(\frac{1}{x^3}+\frac{1}{x}\right)\,dx,x,\tan(c+dx)\right)}{d}$$

$$=-\frac{a\text{arctanh}(\cos(c+dx))}{2d}-\frac{b\cot^2(c+dx)}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.88

$$\begin{aligned}\int\csc^3(c+dx)(a+b\sec(c+dx))\,dx=&-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\csc^2(c+dx)}{2d}\\&-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}\\&-\frac{b\log(\cos(c+dx))}{d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}\\&+\frac{b\log(\sin(c+dx))}{d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}\end{aligned}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (b*Csc[c + d*x]^2)/(2*d) - (a*Log[Cos[(c +
d*x)/2]])/(2*d) - (b*Log[Cos[c + d*x]])/d + (a*Log[Sin[(c + d*x)/2]])/(2*d)
 + (b*Log[Sin[c + d*x]])/d + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{-8b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-8b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(4a+8b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a}{8d}$
norman	$\frac{-\frac{a+b}{8d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{(a+2b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$
risch	$\frac{a\,e^{3i(dx+c)}+2b\,e^{2i(dx+c)}+e^{i(dx+c)}a}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}+1\right)b}{d}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{2d}+\frac{\ln(e^{i(dx+c)}-1}{d}$

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2*cot(d*x+c)*csc(d*x+c)+1/2*ln(-cot(d*x+c)+csc(d*x+c)))+b*(-1/2/
sin(d*x+c)^2+ln(tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{2\,a\cos\left(dx+c\right) - 4\,\left(b\cos\left(dx+c\right)^2 - b\right)\log\left(-\cos\left(dx+c\right)\right) - \left(\left(a-2\,b\right)\cos\left(dx+c\right)^2 - a + 2\,b\right)\log\left(\frac{1}{2}\cos\right.}{4\,\left(d\cos\left(dx+c\right)^2 - d\right)}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*cos(d*x + c) - 4*(b*cos(d*x + c)^2 - b)*log(-cos(d*x + c)) - ((a -
 2*b)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c) + 1/2) + ((a + 2*b)*co
s(d*x + c)^2 - a - 2*b)*log(-1/2*cos(d*x + c) + 1/2) + 2*b)/(d*cos(d*x + c)
^2 - d)
```

Sympy [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx = \int \left(a+b\sec\left(c+dx\right)\right)\csc^3\left(c+dx\right)dx$$

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\left(a-2\,b\right)\log\left(\cos\left(dx+c\right)+1\right) - \left(a+2\,b\right)\log\left(\cos\left(dx+c\right)-1\right) + 4\,b\log\left(\cos\left(dx+c\right)\right) - \frac{2\,\left(a\cos\left(dx+c\right)+b\right)}{\cos\left(dx+c\right)^2-1}}{4\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*((a - 2*b)*log(cos(d*x + c) + 1) - (a + 2*b)*log(cos(d*x + c) - 1) + 4
*b*log(cos(d*x + c)) - 2*(a*cos(d*x + c) + b)/(cos(d*x + c)^2 - 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(58) = 116$.

Time = 0.30 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.64

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{2\,(a+2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 8\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{\left(a+b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{8\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*b*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + b - 2*a*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(
cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{\frac{\frac{b}{2}+\frac{a\,\cos(c+d\,x)}{2}}{\cos(c+d\,x)^2-1} + \ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{a}{4}+\frac{b}{2}\right) - \ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{a}{4}-\frac{b}{2}\right) - b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^3,x)
```

```
[Out] ((b/2 + (a*cos(c + d*x))/2)/(cos(c + d*x)^2 - 1) + log(cos(c + d*x) - 1)*(a
/4 + b/2) - log(cos(c + d*x) + 1)*(a/4 - b/2) - b*log(cos(c + d*x)))/d
```

3.166 $\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx = -\frac{3a\text{arctanh}(\cos(c+dx))}{8d} - \frac{b\cot^2(c+dx)}{d} - \frac{b\cot^4(c+dx)}{4d} - \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -3/8*a*arctanh(cos(d*x+c))/d-b*cot(d*x+c)^2/d-1/4*b*cot(d*x+c)^4/d-3/8*a*co
t(d*x+c)*csc(d*x+c)/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d+b*ln(tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3957, 2913, 2700, 272, 45, 3853, 3855}

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx = -\frac{3a\text{arctanh}(\cos(c+dx))}{8d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\cot^4(c+dx)}{4d} - \frac{b\cot^2(c+dx)}{d} + \frac{b\log(\tan(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) - (b*Cot[c + d*x]^2)/d - (b*Cot[c + d*x]
^4)/(4*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c +
 d*x]^3)/(4*d) + (b*Log[Tan[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))\csc^5(c+dx)\sec(c+dx)\,dx\\
&= a\int \csc^5(c+dx)\,dx+b\int \csc^5(c+dx)\sec(c+dx)\,dx\\
&= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}(3a)\int \csc^3(c+dx)\,dx\\
&\quad+\frac{b\text{Subst}\left(\int \frac{(1+x^2)^2}{x^5}\,dx,x,\tan(c+dx)\right)}{d}\\
&= -\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}\\
&\quad+\frac{1}{8}(3a)\int \csc(c+dx)\,dx+\frac{b\text{Subst}\left(\int \frac{(1+x)^2}{x^3}\,dx,x,\tan^2(c+dx)\right)}{2d}\\
&= -\frac{3a\text{arctanh}(\cos(c+dx))}{8d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}\\
&\quad-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{b\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{2}{x^2}+\frac{1}{x}\right)\,dx,x,\tan^2(c+dx)\right)}{2d}\\
&= -\frac{3a\text{arctanh}(\cos(c+dx))}{8d}-\frac{b\cot^2(c+dx)}{d}-\frac{b\cot^4(c+dx)}{4d}\\
&\quad-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{b\log(\tan(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.76

$$\begin{aligned}
\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}\\
&\quad-\frac{b\csc^2(c+dx)}{2d}-\frac{b\csc^4(c+dx)}{4d}\\
&\quad-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{b\log(\cos(c+dx))}{d}\\
&\quad+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{b\log(\sin(c+dx))}{d}\\
&\quad+\frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}
\end{aligned}$$

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (b*Csc[c
 + d*x]^2)/(2*d) - (b*Csc[c + d*x]^4)/(4*d) - (3*a*Log[Cos[(c + d*x)/2]])/(
8*d) - (b*Log[Cos[c + d*x]])/d + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (b*Log
[Sin[c + d*x]])/d + (3*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4
)/(64*d)
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{a\left(\left(-\frac{\csc(dx+c)^3}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c}{d}$
default	$\frac{a\left(\left(-\frac{\csc(dx+c)^3}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c}{d}$
parallelrisch	$\frac{-64b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-64b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(24a+64b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-8a-12b)}{64d}$
norman	$\frac{-\frac{a+b}{64d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{64d}+\frac{(2a-3b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16d}-\frac{(2a+3b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{16d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}$
risch	$\frac{3a\,e^{7i(dx+c)}+8b\,e^{6i(dx+c)}-11a\,e^{5i(dx+c)}-32b\,e^{4i(dx+c)}-11a\,e^{3i(dx+c)}+8b\,e^{2i(dx+c)}+3\,e^{i(dx+c)}a}{4d\left(e^{2i(dx+c)}-1\right)^4}+\frac{3a\ln\left(e^{i(dx+c)}-1\right.}{8d}$

```
[In] int(csc(d*x+c)^5*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(-cot(d*x+c)+cs
c(d*x+c)))+b*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(92) = 184$.

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.01

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx = \frac{6\,a\cos\left(dx+c\right)^3+8\,b\cos\left(dx+c\right)^2-10\,a\cos\left(dx+c\right)-16\,\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+b\right)\log\left(-\right.}{\ }$$

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 - 10*a*cos(d*x + c) - 16*(b*c
os(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-cos(d*x + c)) - ((3*a - 8*b)*c
os(d*x + c)^4 - 2*(3*a - 8*b)*cos(d*x + c)^2 + 3*a - 8*b)*log(1/2*cos(d*x +
 c) + 1/2) + ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a + 8*b)*cos(d*x + c)^2 + 3
```

```
*a + 8*b)*log(-1/2*cos(d*x + c) + 1/2) - 12*b)/(d*cos(d*x + c)^4 - 2*d*cos(
d*x + c)^2 + d)
```

Sympy [F]

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\csc^5(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**5*(a+b*sec(d*x+c)),x)
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx =$$
$$-\frac{(3a-8b)\log(\cos(dx+c)+1)-(3a+8b)\log(\cos(dx+c)-1)+16b\log(\cos(dx+c))-\frac{2(3a\cos(dx\cdots}{c\cdots}}{16d}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/16*((3*a - 8*b)*log(cos(d*x + c) + 1) - (3*a + 8*b)*log(cos(d*x + c) - 1
) + 16*b*log(cos(d*x + c)) - 2*(3*a*cos(d*x + c)^3 + 4*b*cos(d*x + c)^2 - 5
*a*cos(d*x + c) - 6*b)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(92) = 184$.

Time = 0.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.66

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{4(3a+8b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-64b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{\left(a+b-\frac{8a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{18a(\cos(dx+\cdots}{(\cos(dx+c\cdots}\cdots}{(\cos(dx+c)-\cdots}}{64d}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] 1/64*(4*(3*a + 8*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 64*
b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a + b - 8*a*(cos(
```

```
d*x + c) - 1)/(cos(d*x + c) + 1) - 12*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 18*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 48*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 8*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.91 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx = \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3\,a}{16}+\frac{b}{2}\right)}{d} - \frac{-\frac{3\,a\cos\left(c+d\,x\right)^3}{8}-\frac{b\cos\left(c+d\,x\right)^2}{2}+\frac{5\,a\,\cos\left(c+d\,x\right)}{8}+\frac{3\,b}{4}}{d\,\left(\cos\left(c+d\,x\right)^4-2\cos\left(c+d\,x\right)^2+1\right)} - \frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{3\,a}{16}-\frac{b}{2}\right)}{d} - \frac{b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^5,x)
```

```
[Out] (log(cos(c + d*x) - 1)*((3*a)/16 + b/2))/d - ((3*b)/4 + (5*a*cos(c + d*x))/
8 - (3*a*cos(c + d*x)^3)/8 - (b*cos(c + d*x)^2)/2)/(d*(cos(c + d*x)^4 - 2*c
os(c + d*x)^2 + 1)) - (log(cos(c + d*x) + 1)*((3*a)/16 - b/2))/d - (b*log(c
os(c + d*x)))/d
```

3.167 $\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx = -\frac{5a\text{arctanh}(\cos(c+dx))}{16d} - \frac{3b\cot^2(c+dx)}{2d} - \frac{3b\cot^4(c+dx)}{4d} - \frac{b\cot^6(c+dx)}{6d} - \frac{5a\cot(c+dx)\csc(c+dx)}{16d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -5/16*a*arctanh(cos(d*x+c))/d-3/2*b*cot(d*x+c)^2/d-3/4*b*cot(d*x+c)^4/d-1/6
*b*cot(d*x+c)^6/d-5/16*a*cot(d*x+c)*csc(d*x+c)/d-5/24*a*cot(d*x+c)*csc(d*x+
c)^3/d-1/6*a*cot(d*x+c)*csc(d*x+c)^5/d+b*ln(tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used

$= \{3957, 2913, 2700, 272, 45, 3853, 3855\}$

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx = -\frac{5a\text{arctanh}(\cos(c+dx))}{16d} - \frac{a\cot(c+dx)\csc^5(c+dx)}{6d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{5a\cot(c+dx)\csc(c+dx)}{16d} - \frac{b\cot^6(c+dx)}{6d} - \frac{3b\cot^4(c+dx)}{4d} - \frac{3b\cot^2(c+dx)}{2d} + \frac{b\log(\tan(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (3*b*Cot[c + d*x]^2)/(2*d) - (3*b*Cot
[c + d*x]^4)/(4*d) - (b*Cot[c + d*x]^6)/(6*d) - (5*a*Cot[c + d*x]*Csc[c + d
*x])/(16*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a*Cot[c + d*x]*Cs
c[c + d*x]^5)/(6*d) + (b*Log[Tan[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
```

```
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b - a\cos(c+dx))\csc^7(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^7(c+dx)\,dx + b\int \csc^7(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \csc^5(c+dx)\,dx \\
&\quad + \frac{b\text{Subst}\left(\int \frac{(1+x^2)^3}{x^7}\,dx, x, \tan(c+dx)\right)}{d} \\
&= -\frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a\cot(c+dx)\csc^5(c+dx)}{6d} \\
&\quad + \frac{1}{8}(5a)\int \csc^3(c+dx)\,dx + \frac{b\text{Subst}\left(\int \frac{(1+x)^3}{x^4}\,dx, x, \tan^2(c+dx)\right)}{2d} \\
&= -\frac{5a\cot(c+dx)\csc(c+dx)}{16d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a\cot(c+dx)\csc^5(c+dx)}{6d} \\
&\quad + \frac{1}{16}(5a)\int \csc(c+dx)\,dx + \frac{b\text{Subst}\left(\int \left(\frac{1}{x^4} + \frac{3}{x^3} + \frac{3}{x^2} + \frac{1}{x}\right)\,dx, x, \tan^2(c+dx)\right)}{2d}
\end{aligned}
$$

$$= -\frac{5a\text{arctanh}(\cos(c+dx))}{16d} - \frac{3b\cot^2(c+dx)}{2d} - \frac{3b\cot^4(c+dx)}{4d} - \frac{b\cot^6(c+dx)}{6d} - \frac{5a\cot(c+dx)\csc(c+dx)}{16d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{b\log(\tan(c+dx))}{d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.66

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx = -\frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{b\csc^2(c+dx)}{2d} - \frac{b\csc^4(c+dx)}{4d} - \frac{b\csc^6(c+dx)}{6d} - \frac{5a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{b\log(\cos(c+dx))}{d} + \frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{b\log(\sin(c+dx))}{d} + \frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

```
[In] Integrate[Csc[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-5*a*Csc[(c + d*x)/2]^2)/(64*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Csc[(
c + d*x)/2]^6)/(384*d) - (b*Csc[c + d*x]^2)/(2*d) - (b*Csc[c + d*x]^4)/(4*d
) - (b*Csc[c + d*x]^6)/(6*d) - (5*a*Log[Cos[(c + d*x)/2]])/(16*d) - (b*Log[
Cos[c + d*x]])/d + (5*a*Log[Sin[(c + d*x)/2]])/(16*d) + (b*Log[Sin[c + d*x]
])/d + (5*a*Sec[(c + d*x)/2]^2)/(64*d) + (a*Sec[(c + d*x)/2]^4)/(64*d) + (a
*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{a\left(\left(-\frac{\csc(dx+c)^5}{6}-\frac{5\csc(dx+c)^3}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx+c)+\frac{5\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right)+b\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}-\cdots\right.}{d}$
default	$\frac{a\left(\left(-\frac{\csc(dx+c)^5}{6}-\frac{5\csc(dx+c)^3}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx+c)+\frac{5\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right)+b\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}-\cdots\right.}{d}$
parallelrisch	$\frac{-384b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-384b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(120a+384b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-9a-12\cdots}{384\cdots}$
norman	$\frac{-\frac{a+b}{384d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{384d}+\frac{(3a-4b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{128d}-\frac{(3a+4b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{128d}+\frac{(15a-29b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{128d}-\frac{(15a+29b)\tan\left(\frac{dx}{2}+\frac{c}{2}\cdots\right.}{128d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}$
risch	$\frac{15a\,e^{11i(dx+c)}+48b\,e^{10i(dx+c)}-85a\,e^{9i(dx+c)}-288b\,e^{8i(dx+c)}+198a\,e^{7i(dx+c)}+736b\,e^{6i(dx+c)}+198a\,e^{5i(dx+c)}-288b\,e^{4i(d\cdots}}{24d\left(e^{2i(dx+c)}-1\right)^6}$

```
[In] int(csc(d*x+c)^7*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*((-1/6*csc(d*x+c)^5-5/24*csc(d*x+c)^3-5/16*csc(d*x+c))*cot(d*x+c)+5/
16*ln(-cot(d*x+c)+csc(d*x+c)))+b*(-1/6/sin(d*x+c)^6-1/4/sin(d*x+c)^4-1/2/si
n(d*x+c)^2+ln(tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(126) = 252$.

Time = 0.27 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.03

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx = \frac{30\,a\cos\left(dx+c\right)^5+48\,b\cos\left(dx+c\right)^4-80\,a\cos\left(dx+c\right)^3-120\,b\cos\left(dx+c\right)^2+66\,a\cos\left(dx+c\right)-96\,(b\cdots}{}$$

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(30*a*cos(d*x + c)^5 + 48*b*cos(d*x + c)^4 - 80*a*cos(d*x + c)^3 - 120
*b*cos(d*x + c)^2 + 66*a*cos(d*x + c) - 96*(b*cos(d*x + c)^6 - 3*b*cos(d*x
+ c)^4 + 3*b*cos(d*x + c)^2 - b)*log(-cos(d*x + c)) - 3*((5*a - 16*b)*cos(d
*x + c)^6 - 3*(5*a - 16*b)*cos(d*x + c)^4 + 3*(5*a - 16*b)*cos(d*x + c)^2 -
 5*a + 16*b)*log(1/2*cos(d*x + c) + 1/2) + 3*((5*a + 16*b)*cos(d*x + c)^6 -
 3*(5*a + 16*b)*cos(d*x + c)^4 + 3*(5*a + 16*b)*cos(d*x + c)^2 - 5*a - 16*b
)*log(-1/2*cos(d*x + c) + 1/2) + 88*b)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)
^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**7*(a+b*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx =$$
$$-\frac{3\,(5\,a-16\,b)\log\left(\cos\left(dx+c\right)+1\right)-3\,(5\,a+16\,b)\log\left(\cos\left(dx+c\right)-1\right)+96\,b\log\left(\cos\left(dx+c\right)\right)-\frac{2\,(1}{}}{96\,d}$$

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/96*(3*(5*a - 16*b)*log(cos(d*x + c) + 1) - 3*(5*a + 16*b)*log(cos(d*x +
c) - 1) + 96*b*log(cos(d*x + c)) - 2*(15*a*cos(d*x + c)^5 + 24*b*cos(d*x +
c)^4 - 40*a*cos(d*x + c)^3 - 60*b*cos(d*x + c)^2 + 33*a*cos(d*x + c) + 44*b
)/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(126) = 252.

Time = 0.31 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.55

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{12\,(5\,a+16\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(a+b-\frac{9\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(}{(\mathrm{co}}\right.}{}}{}$$

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] 1/384*(12*(5*a + 16*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) -
384*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + b - 9*a*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 87*b*(cos(d*x + c
```

```
) - 1)^2/(cos(d*x + c) + 1)^2 - 110*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
1)^3 - 352*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^
3/(cos(d*x + c) - 1)^3 - 45*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 87*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 12*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a*(cos(d*x +
c) - 1)^3/(cos(d*x + c) + 1)^3 + b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^
3)/d
```

Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{\frac{5\,a\cos(c+d\,x)^5}{16}+\frac{b\cos(c+d\,x)^4}{2}-\frac{5\,a\cos(c+d\,x)^3}{6}-\frac{5\,b\cos(c+d\,x)^2}{4}+\frac{11\,a\,\cos(c+d\,x)}{16}+\frac{11\,b}{12}}{d\,\left(\cos\left(c+d\,x\right)^6-3\cos\left(c+d\,x\right)^4+3\cos\left(c+d\,x\right)^2-1\right)}+\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{5\,a}{32}+\frac{b}{2}\right)}{d}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{5\,a}{32}-\frac{b}{2}\right)}{d}-\frac{b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^7,x)
```

```
[Out] ((11*b)/12 + (11*a*cos(c + d*x))/16 - (5*a*cos(c + d*x)^3)/6 + (5*a*cos(c +
 d*x)^5)/16 - (5*b*cos(c + d*x)^2)/4 + (b*cos(c + d*x)^4)/2)/(d*(3*cos(c +
d*x)^2 - 3*cos(c + d*x)^4 + cos(c + d*x)^6 - 1)) + (log(cos(c + d*x) - 1)*(
(5*a)/32 + b/2))/d - (log(cos(c + d*x) + 1)*((5*a)/32 - b/2))/d - (b*log(co
s(c + d*x)))/d
```

3.168 $\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 127

$$\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx = \frac{5ax}{16}+\frac{b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{5a\cos(c+dx)\sin(c+dx)}{16d}-\frac{b\sin^3(c+dx)}{3d}-\frac{5a\cos(c+dx)\sin^3(c+dx)}{24d}-\frac{b\sin^5(c+dx)}{5d}-\frac{a\cos(c+dx)\sin^5(c+dx)}{6d}$$

```
[Out] 5/16*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-5/16*a*cos(d*x+c)*sin(d*x+c
)/d-1/3*b*sin(d*x+c)^3/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*b*sin(d*x+c)^
5/d-1/6*a*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3957, 2917, 2672, 308, 212, 2715, 8}

$$\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx = -\frac{a\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}+\frac{b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*x)/16 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (5*a*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) - (b*Sin[c + d*x]^3)/(3*d) - (5*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) - (b*Sin[c + d*x]^5)/(5*d) - (a*Cos[c + d*x]*Sin[c +
 d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b - a\cos(c+dx))\sin^5(c+dx)\tan(c+dx)\,dx \\
&= a\int \sin^6(c+dx)\,dx + b\int \sin^5(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \sin^4(c+dx)\,dx + \frac{b\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= -\frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d} \\
&\quad + \frac{1}{8}(5a)\int \sin^2(c+dx)\,dx + \frac{b\text{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)\,dx, x, \sin(c+dx)\right)}{d} \\
&= -\frac{b\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} - \frac{b\sin^3(c+dx)}{3d} \\
&\quad - \frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{b\sin^5(c+dx)}{5d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d} \\
&\quad + \frac{1}{16}(5a)\int 1\,dx + \frac{b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= \frac{5ax}{16} + \frac{b\text{arctanh}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad - \frac{b\sin^3(c+dx)}{3d} - \frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{b\sin^5(c+dx)}{5d} \\
&\quad - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$
\begin{aligned}
\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx = {} & \frac{5a(c+dx)}{16d} + \frac{b\text{arctanh}(\sin(c+dx))}{d} \\
& - \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d} \\
& - \frac{b\sin^5(c+dx)}{5d} - \frac{15a\sin(2(c+dx))}{64d} \\
& + \frac{3a\sin(4(c+dx))}{64d} - \frac{a\sin(6(c+dx))}{192d}
\end{aligned}
$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

[Out] (5*a*(c + d*x))/(16*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d) - (15*a*Sin[2*(c + d*x)])/(64*d) + (3*a*Sin[4*(c + d*x)])/(64*d) - (a*Sin[6*(c + d*x)])/(192*d)

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+b\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx-\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+b\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx-\right.}{d}$
parts	$\frac{a\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(\right.}{d}$
parallelrisch	$\frac{300axd-1320\sin(dx+c)b-5a\sin(6dx+6c)+45a\sin(4dx+4c)-225a\sin(2dx+2c)-12\sin(5dx+5c)b+140\sin(3dx+3c)b-96}{960d}$
risch	$\frac{5ax}{16}+\frac{11ib\,e^{i(dx+c)}}{16d}-\frac{11ib\,e^{-i(dx+c)}}{16d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\sin(6dx+6c)}{192d}-\frac{b\sin(5dx+5c)}{80d}$
norman	$\frac{\frac{5ax}{16}+\frac{15ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8}+\frac{75ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16}+\frac{25ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4}+\frac{75ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{16}+\frac{15ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{8}+\frac{5ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16}}{}$

[In] int((a+b*sec(d*x+c))*sin(d*x+c)^6,x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/16*d*x+5/16*c)+b*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx$$
$$=\frac{75\,adx+120\,b\log\left(\sin\left(dx+c\right)+1\right)-120\,b\log\left(-\sin\left(dx+c\right)+1\right)-\left(40\,a\cos\left(dx+c\right)^5+48\,b\cos\left(dx+c\right.\right.}{240\,d}$$

[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="fricas")

```
[Out] 1/240*(75*a*d*x + 120*b*log(sin(d*x + c) + 1) - 120*b*log(-sin(d*x + c) + 1
) - (40*a*cos(d*x + c)^5 + 48*b*cos(d*x + c)^4 - 130*a*cos(d*x + c)^3 - 176
*b*cos(d*x + c)^2 + 165*a*cos(d*x + c) + 368*b)*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx=\int (a+b\sec{(c+dx)})\sin^6{(c+dx)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**6,x)
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx = \frac{5\left(4\sin{(2\,dx+2\,c)}^3+60\,dx+60\,c+9\sin{(4\,dx+4\,c)}-48\sin{(2\,dx+2\,c)}\right)a-32\left(6\sin{(dx+c)}^5+10\right.}{960\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="maxima")
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a - 32*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d
*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*b)/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.80

$$\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx = \frac{75\,(dx+c)a+240\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-240\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(75\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}\right.}{}}{}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(75*(d*x + c)*a + 240*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(75*a*tan(1/2*d*x + 1/2*c)^11 - 240*b*
tan(1/2*d*x + 1/2*c)^11 + 425*a*tan(1/2*d*x + 1/2*c)^9 - 1520*b*tan(1/2*d*x
 + 1/2*c)^9 + 990*a*tan(1/2*d*x + 1/2*c)^7 - 4128*b*tan(1/2*d*x + 1/2*c)^7
- 990*a*tan(1/2*d*x + 1/2*c)^5 - 4128*b*tan(1/2*d*x + 1/2*c)^5 - 425*a*tan(
1/2*d*x + 1/2*c)^3 - 1520*b*tan(1/2*d*x + 1/2*c)^3 - 75*a*tan(1/2*d*x + 1/2
*c) - 240*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 15.25 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.61

$$\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx = \frac{5\,a\,\mathrm{atan}\left(\frac{125\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,\left(\frac{125\,a^3}{64}+20\,a\,b^2\right)}+\frac{20\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\frac{125\,a^3}{64}+20\,a\,b^2}\right)}{8\,d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{64\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\frac{25\,a^2\,b}{4}+64\,b^3}+\frac{25\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(\frac{25\,a^2\,b}{4}+64\,b^3\right)}\right)}{d} - \frac{\left(2\,b-\frac{5\,a}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{38\,b}{3}-\frac{85\,a}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(\frac{172\,b}{5}-\frac{33\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+\left(\frac{33\,a}{4}+\frac{172\,b}{5}\right)\,\tan}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\right.}$$

```
[In] int(sin(c + d*x)^6*(a + b/cos(c + d*x)),x)
```

```
[Out] (5*a*atan((125*a^3*tan(c/2 + (d*x)/2))/(64*(20*a*b^2 + (125*a^3)/64)) + (20
*a*b^2*tan(c/2 + (d*x)/2))/(20*a*b^2 + (125*a^3)/64)))/(8*d) + (2*b*atanh((
64*b^3*tan(c/2 + (d*x)/2))/((25*a^2*b)/4 + 64*b^3) + (25*a^2*b*tan(c/2 + (d
*x)/2))/(4*((25*a^2*b)/4 + 64*b^3))))/d - (tan(c/2 + (d*x)/2)*((5*a)/8 + 2*
b) - tan(c/2 + (d*x)/2)^11*((5*a)/8 - 2*b) + tan(c/2 + (d*x)/2)^3*((85*a)/2
4 + (38*b)/3) - tan(c/2 + (d*x)/2)^9*((85*a)/24 - (38*b)/3) + tan(c/2 + (d*
x)/2)^5*((33*a)/4 + (172*b)/5) - tan(c/2 + (d*x)/2)^7*((33*a)/4 - (172*b)/5
))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x
)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x
)/2)^12 + 1))
```

3.169 $\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx = \frac{3ax}{8}+\frac{b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d}-\frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{b\sin^3(c+dx)}{3d}-\frac{a\cos(c+dx)\sin^3(c+dx)}{4d}$$

```
[Out] 3/8*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-3/8*a*cos(d*x+c)*sin(d*x+c)/
d-1/3*b*sin(d*x+c)^3/d-1/4*a*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3957, 2917, 2672, 308, 212, 2715, 8}

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx = -\frac{a\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{3a\sin(c+dx)\cos(c+dx)}{8d} + \frac{3ax}{8}+\frac{b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*x)/8 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (3*a*Cos[c +
 d*x]*Sin[c + d*x])/(8*d) - (b*Sin[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Sin[
c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))\sin^3(c+dx)\tan(c+dx)\,dx\\
&= a\int \sin^4(c+dx)\,dx + b\int \sin^3(c+dx)\tan(c+dx)\,dx\\
&= -\frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{4}(3a)\int \sin^2(c+dx)\,dx + \frac{b\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}\\
&= -\frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4d}\\
&\quad + \frac{1}{8}(3a)\int 1\,dx + \frac{b\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{3ax}{8} - \frac{b\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{b\sin^3(c+dx)}{3d}\\
&\quad - \frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{3ax}{8} + \frac{b\text{arctanh}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d}\\
&\quad - \frac{b\sin^3(c+dx)}{3d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx = \frac{3a(c+dx)}{8d} + \frac{b\text{arctanh}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d} - \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^4,x]
[Out] (3*a*(c + d*x))/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d -
(b*Sin[c + d*x]^3)/(3*d) - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)]
)/(32*d)
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
parts	$\frac{a\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{36axd-120\sin(dx+c)b+3a\sin(4dx+4c)-24a\sin(2dx+2c)+96b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-96b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\sin(3dx}{96d}$
risch	$\frac{3ax}{8}+\frac{5ib\,e^{i(dx+c)}}{8d}-\frac{5ib\,e^{-i(dx+c)}}{8d}+\frac{b\ln(e^{i(dx+c)}+i)}{d}-\frac{b\ln(e^{i(dx+c)}-i)}{d}+\frac{a\sin(4dx+4c)}{32d}+\frac{b\sin(3dx+3c)}{12d}-$
norman	$\frac{\frac{3ax}{8}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{9ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}+\frac{(3a-8b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}-\frac{(3a+8b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{4d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+b*(-1/
3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx$$
$$=\frac{9\,adx+12\,b\log\left(\sin\left(dx+c\right)+1\right)-12\,b\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,a\cos\left(dx+c\right)^3+8\,b\cos\left(dx+c\right)^2-1\right.}{24\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + 12*b*log(sin(d*x + c) + 1) - 12*b*log(-sin(d*x + c) + 1) +
(6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 32*b)*sin(d*
x + c))/d
```

Sympy [F]

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx = \int (a+b\sec{(c+dx)})\sin^4{(c+dx)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx$$
$$= \frac{3\,(12\,dx+12\,c+\sin{(4\,dx+4\,c)}-8\,\sin{(2\,dx+2\,c)})a-16\,\left(2\,\sin{(dx+c)}^3-3\,\log{(\sin{(dx+c)}+1)}+3\right.}{96\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a - 16*(2*s
in(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d
*x + c))*b)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(81) = 162.

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.93

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx$$
$$= \frac{9\,(dx+c)a+24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,b\,\mathrm{ta}\right.}{}}{2}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 24*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*b*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2
*d*x + 1/2*c)^7 + 33*a*tan(1/2*d*x + 1/2*c)^5 - 104*b*tan(1/2*d*x + 1/2*c)^
5 - 33*a*tan(1/2*d*x + 1/2*c)^3 - 104*b*tan(1/2*d*x + 1/2*c)^3 - 9*a*tan(1/
2*d*x + 1/2*c) - 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)
/d
```

Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.00

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx$$

$$=\frac{3\,a\,\mathrm{atan}\left(\frac{27\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\left(\frac{27\,a^3}{8}+24\,a\,b^2\right)}+\frac{24\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\frac{27\,a^3}{8}+24\,a\,b^2}\right)}{4\,d}+\frac{2\,b\,\mathrm{atanh}\left(\frac{64\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\,a^2\,b+64\,b^3}+\frac{9\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\,a^2\,b+64\,b^3}\right)}{d}$$

$$-\frac{\left(2\,b-\frac{3\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{26\,b}{3}-\frac{11\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{11\,a}{4}+\frac{26\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{3\,a}{4}+2\,b\right)\,\tan\left(\frac{c}{2}+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

```
[Out] (3*a*atan((27*a^3*tan(c/2 + (d*x)/2))/(8*(24*a*b^2 + (27*a^3)/8)) + (24*a*b
^2*tan(c/2 + (d*x)/2))/(24*a*b^2 + (27*a^3)/8)))/(4*d) + (2*b*atanh((64*b^3
*tan(c/2 + (d*x)/2))/(9*a^2*b + 64*b^3) + (9*a^2*b*tan(c/2 + (d*x)/2))/(9*a
^2*b + 64*b^3)))/d - (tan(c/2 + (d*x)/2)*((3*a)/4 + 2*b) - tan(c/2 + (d*x)/
2)^7*((3*a)/4 - 2*b) + tan(c/2 + (d*x)/2)^3*((11*a)/4 + (26*b)/3) - tan(c/2
 + (d*x)/2)^5*((11*a)/4 - (26*b)/3))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2
 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.170 $\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx = \frac{ax}{2}+\frac{b\,\text{arctanh}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d}-\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-1/2*a*cos(d*x+c)*sin(d*x+c)/
d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3957, 2917, 2672, 327, 212, 2715, 8}

$$\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx = -\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2} +\frac{b\,\text{arctanh}(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*x)/2 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b - a\cos(c+dx))\sin(c+dx)\tan(c+dx)\,dx \\
&= a\int \sin^2(c+dx)\,dx + b\int \sin(c+dx)\tan(c+dx)\,dx
\end{aligned}
$$

$$= -\frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx + \frac{b\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{ax}{2} - \frac{b\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{b\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{ax}{2} + \frac{b\mathrm{arctanh}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx = \frac{a(c+dx)}{2d} + \frac{b\mathrm{arctanh}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{a\sin(2(c+dx))}{4d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*(c + d*x))/(2*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (a
*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
parts	$\frac{a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
parallelrisch	$\frac{2axd-4\sin(dx+c)b-a\sin(2dx+2c)+4b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-4b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4d}$
risch	$\frac{ax}{2}+\frac{ib\,\mathrm{e}^{i(dx+c)}}{2d}-\frac{ib\,\mathrm{e}^{-i(dx+c)}}{2d}+\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}-\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{(a-2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{ax}{2}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}-\frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-$ b ln(ta

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b*(-sin(d*x+c)+ln(sec(d*x
+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx$$
$$=\frac{adx+b\log\left(\sin\left(dx+c\right)+1\right)-b\log\left(-\sin\left(dx+c\right)+1\right)-\left(a\cos\left(dx+c\right)+2\,b\right)\sin\left(dx+c\right)}{2\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1) - (a*cos(d*
x + c) + 2*b)*sin(d*x + c))/d
```

Sympy [F]

$$\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx=\int \left(a+b\sec\left(c+dx\right)\right)\sin^2\left(c+dx\right)dx$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx$$
$$=\frac{\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)a+2\,b(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))}{4\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a + 2*b*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(47) = 94$.

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.24

$$\int (a + b\sec(c + dx))\sin^2(c + dx)\,dx$$
$$= \frac{(dx + c)a + 2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 2\,b\tan\left(\frac{1}{2}\,dx-\right.\right.}{\left(\tan\left(\right.\right.}}{2\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1
/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int (a + b\sec(c + dx))\sin^2(c + dx)\,dx = \frac{a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} - \frac{b\,\sin\left(c+d\,x\right)}{d}$$

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

```
[Out] (a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b*atanh(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*sin(2*c + 2*d*x))/(4*d) - (b*sin(c + d*x
))/d
```

3.171 $\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-b*csc(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3957, 2917, 2701, 327, 213, 3852, 8}

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx=-\frac{a\cot(c+dx)}{d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (b*Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^2(c+dx)\,dx + b\int \csc^2(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int 1\,dx, x, \cot(c+dx)\right)}{d} - \frac{b\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{b\csc(c+dx)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&= \frac{b\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{b\csc(c+dx)}{d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\cot(c+dx)}{d} - \frac{b\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(c+dx)\right)}{d}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x]),x]
[Out] -((a*Cot[c + d*x])/d) - (b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin
[c + d*x]^2])/d
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{-a\cot(dx+c)+b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$	42
default	$\frac{-a\cot(dx+c)+b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$	42
parallelrisch	$\frac{-2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(-a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{2d}$	69
risch	$-\frac{2i\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}$	71
norman	$\frac{-\frac{a+b}{2d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	79

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-a*cot(d*x+c)+b*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{b\log(\sin(dx+c)+1)\sin(dx+c) - b\log(-\sin(dx+c)+1)\sin(dx+c) - 2\,a\cos(dx+c) - 2\,b}{2\,d\sin(dx+c)}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(b*log(sin(d*x + c) + 1)*sin(d*x + c) - b*log(-sin(d*x + c) + 1)*sin(d*
x + c) - 2*a*cos(d*x + c) - 2*b)/(d*sin(d*x + c))
```

Sympy [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\csc^2(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c)),x)
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{b\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + \frac{2\,a}{\tan(dx+c)}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) +
2*a/tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(37) = 74$.

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.08

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-}{2\,d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c) - (a + b)/tan(1
/2*d*x + 1/2*c))/d
```

Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{\frac{a}{2}+\frac{b}{2}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{d}$$

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^2,x)
```

```
[Out] (2*b*atanh(tan(c/2 + (d*x)/2)))/d - (a/2 + b/2)/(d*tan(c/2 + (d*x)/2)) + (t
an(c/2 + (d*x)/2)*(a/2 - b/2))/d
```

3.172 $\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\cot^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d-b*csc(d*x+c)/d-
1/3*b*csc(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3957, 2917, 2701, 308, 213, 3852}

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b - a\cos(c+dx))\csc^4(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^4(c+dx)\,dx + b\int \csc^4(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, \cot(c+dx)\right)}{d} - \frac{b\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}
\end{aligned}
$$

$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right) dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{3d} - \frac{b\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \csc(c+dx)\right)}{d}$$

$$= \frac{b\,\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{3d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = -\frac{2a\cot(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d} - \frac{b\csc^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(c+dx)\right)}{3d}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x]),x]
[Out] (-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (b*Csc[
c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{-24b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+24b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(-a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+(-9a-15b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}{24d}$
risch	$-\frac{2i\left(3b\,e^{5i(dx+c)}-10b\,e^{3i(dx+c)}-6a\,e^{2i(dx+c)}+3b\,e^{i(dx+c)}+2a\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}$
norman	$\frac{-\frac{a+b}{24d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{24d}+\frac{(3a-5b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8d}-\frac{(3a+5b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}$

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)
+ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.81

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = -\frac{4\,a\cos\left(dx+c\right)^3+6\,b\cos\left(dx+c\right)^2-3\,\left(b\cos\left(dx+c\right)^2-b\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\,\left(b\cos\right.}{6\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+\right.}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^3 + 6*b*cos(d*x + c)^2 - 3*(b*cos(d*x + c)^2 - b)*lo
g(sin(d*x + c) + 1)*sin(d*x + c) + 3*(b*cos(d*x + c)^2 - b)*log(-sin(d*x +
c) + 1)*sin(d*x + c) - 6*a*cos(d*x + c) - 8*b)/((d*cos(d*x + c)^2 - d)*sin(
d*x + c))
```

Sympy [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\csc^4(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = -\frac{b\left(\frac{2\left(3\,\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\left(3\,\tan(dx+c)^2+1\right)a}{\tan(dx+c)^3}}{6\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*a/tan(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(65) = 130$.

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.93

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\right.\right.}{24\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 + 24*b*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 24*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 9*a*ta
n(1/2*d*x + 1/2*c) - 15*b*tan(1/2*d*x + 1/2*c) - (9*a*tan(1/2*d*x + 1/2*c)^
2 + 15*b*tan(1/2*d*x + 1/2*c)^2 + a + b)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.85 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{a}{24}-\frac{b}{24}\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left((3\,a+5\,b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a}{3}+\frac{b}{3}\right)}{8\,d} + \frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,a}{8}-\frac{5\,b}{8}\right)}{d}$$

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(a/24 - b/24))/d - (cot(c/2 + (d*x)/2)^3*(a/3 + b/3 +
 tan(c/2 + (d*x)/2)^2*(3*a + 5*b)))/(8*d) + (2*b*atanh(tan(c/2 + (d*x)/2)))
/d + (tan(c/2 + (d*x)/2)*((3*a)/8 - (5*b)/8))/d
```

3.173 $\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx = \frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot^5(c+dx)}{5d}-\frac{b\csc(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc^5(c+dx)}{5d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-2/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+c
)^5/d-b*csc(d*x+c)/d-1/3*b*csc(d*x+c)^3/d-1/5*b*csc(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3957, 2917, 2701, 308, 213, 3852}

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx = -\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (2*a*Cot[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]^5)/(5*d) - (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*
d) - (b*Csc[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))\csc^6(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^6(c+dx)\,dx + b\int \csc^6(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, \cot(c+dx)\right)}{d} - \frac{b\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}
\end{aligned}
$$

$$= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{b\text{Subst}\left(\int\left(1+x^2+x^4+\frac{1}{-1+x^2}\right)dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{3d} - \frac{b\csc^5(c+dx)}{5d} - \frac{b\text{Subst}\left(\int\frac{1}{-1+x^2}dx, x, \csc(c+dx)\right)}{d}$$

$$= \frac{b\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{3d} - \frac{b\csc^5(c+dx)}{5d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx = -\frac{8a\cot(c+dx)}{15d} - \frac{4a\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} - \frac{b\csc^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\sin^2(c+dx)\right)}{5d}$$

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*
Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (b*Csc[c + d*x]^5*Hypergeometric2F1[-5
/2, 1, -3/2, Sin[c + d*x]^2])/(5*d)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c)+b\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c)+b\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{-960b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+960b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\csc\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(a\cos(5dx+5c)+10a\cos(dx+c)-5a\cos(3dx+3c)\right.}{960d}$
risch	$-\frac{2i\left(15b\,e^{9i(dx+c)}-80b\,e^{7i(dx+c)}+178b\,e^{5i(dx+c)}+80a\,e^{4i(dx+c)}-80b\,e^{3i(dx+c)}-40a\,e^{2i(dx+c)}+15b\,e^{i(dx+c)}+8a\right)}{15d\left(e^{2i(dx+c)}-1\right)^5}-\frac{b}{}$
norman	$\frac{-\frac{a+b}{160d}+\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{160d}+\frac{(5a-11b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16d}+\frac{(5a-7b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{96d}-\frac{(5a+7b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{96d}-\frac{(5a+11b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}$

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+b*(-1/5/sin(d*
x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.72

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx =$$
$$-\frac{16\,a\cos\left(dx+c\right)^5+30\,b\cos\left(dx+c\right)^4-40\,a\cos\left(dx+c\right)^3-70\,b\cos\left(dx+c\right)^2-15\,\left(b\cos\left(dx+c\right)^4-2\right.}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(16*a*cos(d*x + c)^5 + 30*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 70
*b*cos(d*x + c)^2 - 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(sin(
d*x + c) + 1)*sin(d*x + c) + 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)
*log(-sin(d*x + c) + 1)*sin(d*x + c) + 30*a*cos(d*x + c) + 46*b)/((d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F]

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\csc^6(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx =$$
$$-\frac{b\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)+\frac{2\left(15\tan(dx+c)^4+1\right.}{\tan(dx}}{30\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 2*(15*tan(d*x + c)^4 +
10*tan(d*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(93) = 186$.

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.92

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+480\,b\log\left(\left|\mathrm{ta}\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 + 25*a*tan(1
/2*d*x + 1/2*c)^3 - 35*b*tan(1/2*d*x + 1/2*c)^3 + 480*b*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - 480*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 150*a*tan(1/2*
d*x + 1/2*c) - 330*b*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 +
 330*b*tan(1/2*d*x + 1/2*c)^4 + 25*a*tan(1/2*d*x + 1/2*c)^2 + 35*b*tan(1/2*
d*x + 1/2*c)^2 + 3*a + 3*b)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.41

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{5\,a}{96}-\frac{7\,b}{96}\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left((10\,a+22\,b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{5\,a}{3}+\frac{7\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a}{5}+\frac{b}{5}\right)}{32\,d} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{a}{160}-\frac{b}{160}\right)}{d} + \frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a}{16}-\frac{11\,b}{16}\right)}{d}$$

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((5*a)/96 - (7*b)/96))/d - (cot(c/2 + (d*x)/2)^5*(a/5
 + b/5 + tan(c/2 + (d*x)/2)^2*((5*a)/3 + (7*b)/3) + tan(c/2 + (d*x)/2)^4*(1
0*a + 22*b)))/(32*d) + (tan(c/2 + (d*x)/2)^5*(a/160 - b/160))/d + (2*b*atan
h(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)*((5*a)/16 - (11*b)/16))/d
```

3.174 $\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\begin{aligned}\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx = &-\frac{(a^2-2b^2)\cos(c+dx)}{d}+\frac{2ab\cos^2(c+dx)}{d}\\&+\frac{(2a^2-b^2)\cos^3(c+dx)}{3d}\\&-\frac{ab\cos^4(c+dx)}{2d}-\frac{a^2\cos^5(c+dx)}{5d}\\&-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}\end{aligned}$$

```
[Out] -(a^2-2*b^2)*cos(d*x+c)/d+2*a*b*cos(d*x+c)^2/d+1/3*(2*a^2-b^2)*cos(d*x+c)^3
/d-1/2*a*b*cos(d*x+c)^4/d-1/5*a^2*cos(d*x+c)^5/d-2*a*b*ln(cos(d*x+c))/d+b^2
*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2916, 12, 962}

$$\begin{aligned}\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx = &\frac{(2a^2-b^2)\cos^3(c+dx)}{3d}\\&-\frac{(a^2-2b^2)\cos(c+dx)}{d}-\frac{a^2\cos^5(c+dx)}{5d}\\&-\frac{ab\cos^4(c+dx)}{2d}+\frac{2ab\cos^2(c+dx)}{d}\\&-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}\end{aligned}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] -(((a^2 - 2*b^2)*Cos[c + d*x])/d) + (2*a*b*Cos[c + d*x]^2)/d + ((2*a^2 - b^
2)*Cos[c + d*x]^3)/(3*d) - (a*b*Cos[c + d*x]^4)/(2*d) - (a^2*Cos[c + d*x]^5
)/(5*d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-b-a\cos(c+dx))^2\sin^3(c+dx)\tan^2(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^2(-b+x)^2\left(a^2-x^2\right)^2}{x^2}\,dx,x,-a\cos(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \frac{(-b+x)^2\left(a^2-x^2\right)^2}{x^2}\,dx,x,-a\cos(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(a^4\left(1-\frac{2b^2}{a^2}\right)+\frac{a^4b^2}{x^2}-\frac{2a^4b}{x}+4a^2bx-\left(2a^2-b^2\right)x^2-2bx^3+x^4\right)\,dx,x,-a\cos(c+dx)\right)}{a^3d}
\end{aligned}
$$

$$= -\frac{(a^2-2b^2)\cos(c+dx)}{d} + \frac{2ab\cos^2(c+dx)}{d} + \frac{(2a^2-b^2)\cos^3(c+dx)}{3d} - \frac{ab\cos^4(c+dx)}{2d} - \frac{a^2\cos^5(c+dx)}{5d} - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx =$$

$$-\frac{30(5a^2-14b^2)\cos(c+dx) - 180ab\cos(2(c+dx)) - 25a^2\cos(3(c+dx)) + 20b^2\cos(3(c+dx)) + 15ab\,\mathrm{c}}{240d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] -1/240*(30*(5*a^2 - 14*b^2)*Cos[c + d*x] - 180*a*b*Cos[2*(c + d*x)] - 25*a^
2*Cos[3*(c + d*x)] + 20*b^2*Cos[3*(c + d*x)] + 15*a*b*Cos[4*(c + d*x)] + 3*
a^2*Cos[5*(c + d*x)] + 480*a*b*Log[Cos[c + d*x]] - 240*b^2*Sec[c + d*x])/d
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+2ab\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(\right.\right.}{d}$
default	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+2ab\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(\right.\right.}{d}$
parts	$-\frac{a^2\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{b^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)}{d}+\frac{2ab}{}$
parallelrisch	$\frac{-960ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+960ab\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)-960ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-256}{}$
norman	$\frac{\frac{16a^2-80b^2}{15d}-\frac{32(a^2+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3d}-\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{16ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}+\frac{4(16a^2+15ab-80b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{15d}+\frac{(16a^2+48}{}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$
risch	$2iabx+\frac{3ab\,e^{2i(dx+c)}}{8d}-\frac{5a^2e^{i(dx+c)}}{16d}+\frac{7\,e^{i(dx+c)}b^2}{8d}-\frac{5\,e^{-i(dx+c)}a^2}{16d}+\frac{7\,e^{-i(dx+c)}b^2}{8d}+\frac{3ab\,e^{-2i(dx+c)}}{8d}+\frac{4i}{}$

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a^2*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+2*a*b*(-1/4*si
n(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+b^2*(sin(d*x+c)^6/cos(d*x+c)+(8
```

```
/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

$$\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx =$$
$$-\frac{48\,a^2\cos{(dx+c)}^6+120\,ab\cos{(dx+c)}^5-480\,ab\cos{(dx+c)}^3-80\,(2\,a^2-b^2)\cos{(dx+c)}^4+480\,ab\,c}{240\,d\cos{(dx-}}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^6 + 120*a*b*cos(d*x + c)^5 - 480*a*b*cos(d*x +
c)^3 - 80*(2*a^2 - b^2)*cos(d*x + c)^4 + 480*a*b*cos(d*x + c)*log(-cos(d*x
+ c)) + 195*a*b*cos(d*x + c) + 240*(a^2 - 2*b^2)*cos(d*x + c)^2 - 240*b^2)/
(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx = \int \left(a+b\sec{(c+dx)}\right)^2\sin^5{(c+dx)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**5,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx =$$
$$-\frac{6\,a^2\cos{(dx+c)}^5+15\,ab\cos{(dx+c)}^4-60\,ab\cos{(dx+c)}^2-10\,(2\,a^2-b^2)\cos{(dx+c)}^3+60\,ab\log{(c}}{30\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/30*(6*a^2*cos(d*x + c)^5 + 15*a*b*cos(d*x + c)^4 - 60*a*b*cos(d*x + c)^2
 - 10*(2*a^2 - b^2)*cos(d*x + c)^3 + 60*a*b*log(cos(d*x + c)) + 30*(a^2 - 2
*b^2)*cos(d*x + c) - 30*b^2/cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(118) = 236$.

Time = 0.39 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.37

$$\int (a + b\sec(c + dx))^2 \sin^5(c + dx)\, dx$$
$$= \frac{60\, ab \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 60\, ab \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{60\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1} + \frac{32\, a^2+137\, ab-100\, b^2-}{}}{}$$

[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="giac")

[Out] 1/30*(60*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 60*(a*b + b^2 + a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1) + (32*a^2 + 137*a*b - 100*b^2 - 160*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 805*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 440*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 640*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1970*a*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 360*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 60*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*a*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d

Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int (a + b\sec(c + dx))^2 \sin^5(c + dx)\, dx =$$
$$-\frac{\cos(c+d\,x)\,\left(a^2-2\,b^2\right)-{\cos(c+d\,x)}^3\,\left(\frac{2\,a^2}{3}-\frac{b^2}{3}\right)+\frac{a^2\,{\cos(c+d\,x)}^5}{5}-\frac{b^2}{\cos(c+d\,x)}-2\,a\,b\,{\cos(c+d\,x)}^2+\frac{a\,b\,\mathrm{co}}{}}{d}$$

[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^2,x)

[Out] -(cos(c + d*x)*(a^2 - 2*b^2) - cos(c + d*x)^3*((2*a^2)/3 - b^2/3) + (a^2*cos(c + d*x)^5)/5 - b^2/cos(c + d*x) - 2*a*b*cos(c + d*x)^2 + (a*b*cos(c + d*x)^4)/2 + 2*a*b*log(cos(c + d*x)))/d

3.175 $\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx = -\frac{(a^2-b^2)\cos(c+dx)}{d} + \frac{ab\cos^2(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}$$

```
[Out] -(a^2-b^2)*cos(d*x+c)/d+a*b*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-2*a*b*ln(
cos(d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2916, 12, 908}

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx = -\frac{(a^2-b^2)\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} + \frac{ab\cos^2(c+dx)}{d} - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] -(((a^2 - b^2)*Cos[c + d*x])/d) + (a*b*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x
]^3)/(3*d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-b-a\cos(c+dx))^2\sin(c+dx)\tan^2(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^2(-b+x)^2(a^2-x^2)}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \frac{(-b+x)^2(a^2-x^2)}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{a^2b^2}{x^2}-\frac{2a^2b}{x}+2bx-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{ad}\\
&= -\frac{(a^2-b^2)\cos(c+dx)}{d}+\frac{ab\cos^2(c+dx)}{d}\\
&\quad+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx = \frac{(-9a^2+12b^2)\cos(c+dx)+6ab\cos(2(c+dx))+a^2\cos(3(c+dx))-24ab\log(\cos(c+dx))+12b^2\sec(c+dx)}{12d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] ((-9*a^2 + 12*b^2)*Cos[c + d*x] + 6*a*b*Cos[2*(c + d*x)] + a^2*Cos[3*(c + d
*x)] - 24*a*b*Log[Cos[c + d*x]] + 12*b^2*Sec[c + d*x])/(12*d)
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{-\frac{a^2(2+\sin(dx+c)^2)\cos(dx+c)}{3}+2ab\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+(2+\sin(dx+c)^2)\cos(dx+c)\right)}{d}$
default	$\frac{-\frac{a^2(2+\sin(dx+c)^2)\cos(dx+c)}{3}+2ab\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+(2+\sin(dx+c)^2)\cos(dx+c)\right)}{d}$
parts	$-\frac{a^2(2+\sin(dx+c)^2)\cos(dx+c)}{3d}+\frac{b^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+(2+\sin(dx+c)^2)\cos(dx+c)\right)}{d}+\frac{2ab\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+$
parallelrisch	$\frac{48ab\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)-48ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-48ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+(-8a^2}{24d\cos(dx+c)}$
norman	$\frac{\frac{4a^2-12b^2}{3d}-\frac{4(a^2+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}+\frac{2(4a^2+6ab-12b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-$
risch	$2iabx+\frac{ab\,e^{2i(dx+c)}}{4d}-\frac{3a^2e^{i(dx+c)}}{8d}+\frac{e^{i(dx+c)}b^2}{2d}-\frac{3\,e^{-i(dx+c)}a^2}{8d}+\frac{e^{-i(dx+c)}b^2}{2d}+\frac{ab\,e^{-2i(dx+c)}}{4d}+\frac{4iabc}{d}$

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2*(2+sin(d*x+c)^2)*cos(d*x+c)+2*a*b*(-1/2*sin(d*x+c)^2-ln(cos(d
*x+c)))+b^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx$$
$$= \frac{2\,a^2\cos\left(dx+c\right)^4+6\,ab\cos\left(dx+c\right)^3-12\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-3\,ab\cos\left(dx+c\right)-6\,(a^2-}{6\,d\cos\left(dx+c\right)}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="fricas")
[Out] 1/6*(2*a^2*cos(d*x + c)^4 + 6*a*b*cos(d*x + c)^3 - 12*a*b*cos(d*x + c)*log(
-cos(d*x + c)) - 3*a*b*cos(d*x + c) - 6*(a^2 - b^2)*cos(d*x + c)^2 + 6*b^2)
/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^2\sin^3\left(c+dx\right)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**3,x)
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx$$
$$= \frac{a^2\cos\left(dx+c\right)^3+3\,ab\cos\left(dx+c\right)^2-6\,ab\log\left(\cos\left(dx+c\right)\right)-3\,(a^2-b^2)\cos\left(dx+c\right)+\frac{3\,b^2}{\cos(dx+c)}}{3\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="maxima")
[Out] 1/3*(a^2*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^2 - 6*a*b*log(cos(d*x + c)) -
3*(a^2 - b^2)*cos(d*x + c) + 3*b^2/cos(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx$$
$$=-\frac{2\,ab\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{b^2}{d\cos(dx+c)}$$
$$+\frac{a^2d^5\cos(dx+c)^3+3\,abd^5\cos(dx+c)^2-3\,a^2d^5\cos(dx+c)+3\,b^2d^5\cos(dx+c)}{3\,d^6}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(cos(d*x + c))/abs(d))/d + b^2/(d*cos(d*x + c)) + 1/3*(a^2*d^
5*cos(d*x + c)^3 + 3*a*b*d^5*cos(d*x + c)^2 - 3*a^2*d^5*cos(d*x + c) + 3*b^
2*d^5*cos(d*x + c))/d^6
```

Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx$$
$$=\frac{\frac{a^2\cos(c+d\,x)^3}{3}-\cos(c+d\,x)\,\left(a^2-b^2\right)+\frac{b^2}{\cos(c+d\,x)}+a\,b\cos(c+d\,x)^2-2\,a\,b\,\ln\left(\cos(c+d\,x)\right)}{d}$$

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((a^2*cos(c + d*x)^3)/3 - cos(c + d*x)*(a^2 - b^2) + b^2/cos(c + d*x) + a*b
*cos(c + d*x)^2 - 2*a*b*log(cos(c + d*x)))/d
```

3.176 $\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 42

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx=-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] -a^2*cos(d*x+c)/d-2*a*b*ln(cos(d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2912, 12, 45}

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx=-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x],x]
```

```
[Out] -((a^2*Cos[c + d*x])/d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/
d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-b - a\cos(c+dx))^2 \sec(c+dx)\tan(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{a\text{Subst}\left(\int \frac{(-b+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a\text{Subst}\left(\int \left(1+\frac{b^2}{x^2}-\frac{2b}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= -\frac{a^2\cos(c+dx)}{d} - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx = \frac{-a^2\cos(c+dx)+b(-2a\log(\cos(c+dx))+b\sec(c+dx))}{d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x],x]
[Out] (-(a^2*Cos[c + d*x]) + b*(-2*a*Log[Cos[c + d*x]] + b*Sec[c + d*x]))/d
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\sec(dx+c)b^2-\frac{a^2}{\sec(dx+c)}+2ab\ln(\sec(dx+c))}{d}$
default	$\frac{\sec(dx+c)b^2-\frac{a^2}{\sec(dx+c)}+2ab\ln(\sec(dx+c))}{d}$
parts	$-\frac{a^2\cos(dx+c)}{d}+\frac{b^2\sec(dx+c)}{d}+\frac{2ab\ln(\sec(dx+c))}{d}$
risch	$2iabx-\frac{a^2 e^{i(dx+c)}}{2d}-\frac{e^{-i(dx+c)}a^2}{2d}+\frac{4iabc}{d}+\frac{2b^2 e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}-\frac{2ab\ln(e^{2i(dx+c)}+1)}{d}$
parallelrisch	$\frac{4ab\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)-4ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-4ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-\cos(2dx+2c}{2d\cos(dx+c)}$
norman	$\frac{\frac{2a^2-2b^2}{d}-\frac{2(a^2+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{2ab\ln\left(1+\tan\left(\right.\right.}{d}$

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(sec(d*x+c)*b^2-a^2/sec(d*x+c)+2*a*b*ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx$$
$$=-\frac{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)\log(-\cos(dx+c))-b^2}{d\cos(dx+c)}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="fricas")
[Out] -(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)*log(-cos(d*x + c)) - b^2)/(d*cos(
d*x + c))
```

Sympy [F]

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx=\int (a+b\sec{(c+dx)})^2\sin{(c+dx)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c),x)
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx=-\frac{a^2\cos{(dx+c)}+2\,ab\log{(\cos{(dx+c)})}-\frac{b^2}{\cos(dx+c)}}{d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="maxima")
[Out] -(a^2*cos(d*x + c) + 2*a*b*log(cos(d*x + c)) - b^2/cos(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx=-\frac{a^2\cos{(dx+c)}}{d}-\frac{2\,ab\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{b^2}{d\cos{(dx+c)}}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="giac")
[Out] -a^2*cos(d*x + c)/d - 2*a*b*log(abs(cos(d*x + c))/abs(d))/d + b^2/(d*cos(d*
x + c))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx=-\frac{a^2\,\cos{(c+d\,x)}-\frac{b^2}{\cos(c+d\,x)}+2\,a\,b\,\ln{(\cos{(c+d\,x)})}}{d}$$

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x))^2,x)
[Out] -(a^2*cos(c + d*x) - b^2/cos(c + d*x) + 2*a*b*log(cos(c + d*x)))/d
```

3.177 $\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx=\frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}-\frac{(a-b)^2\log(1+\cos(c+dx))}{2d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] 1/2*(a+b)^2*ln(1-cos(d*x+c))/d-2*a*b*ln(cos(d*x+c))/d-1/2*(a-b)^2*ln(1+cos(
d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2916, 12, 1816}

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx=-\frac{(a-b)^2\log(\cos(c+dx)+1)}{2d}+\frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a + b)^2*Log[1 - Cos[c + d*x]])/(2*d) - (2*a*b*Log[Cos[c + d*x]])/d - ((a
 - b)^2*Log[1 + Cos[c + d*x]])/(2*d) + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-b-a\cos(c+dx))^2\csc(c+dx)\sec^2(c+dx)\,dx\\
&= \frac{a\text{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^3\text{Subst}\left(\int \frac{(-b+x)^2}{x^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^3\text{Subst}\left(\int \left(\frac{(a-b)^2}{2a^3(a-x)}+\frac{b^2}{a^2x^2}-\frac{2b}{a^2x}+\frac{(a+b)^2}{2a^3(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}\\
&\quad -\frac{(a-b)^2\log(1+\cos(c+dx))}{2d}+\frac{b^2\sec(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{-(a-b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2ab\log(\cos(c+dx))+a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-((a - b)^2*Log[Cos[(c + d*x)/2]]) - 2*a*b*Log[Cos[c + d*x]] + a^2*Log[Sin
[(c + d*x)/2]] + 2*a*b*Log[Sin[(c + d*x)/2]] + b^2*Log[Sin[(c + d*x)/2]] +
b^2*Sec[c + d*x])/d
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{a^2\ln(-\cot(dx+c)+\csc(dx+c))+2ab\ln(\tan(dx+c))+b^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)}{d}$
default	$\frac{a^2\ln(-\cot(dx+c)+\csc(dx+c))+2ab\ln(\tan(dx+c))+b^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)}{d}$
norman	$-\frac{2b^2}{d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{(a^2+2ab+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$
parallelrisch	$\frac{-2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\cos(dx+c)(a+b)^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\cos(dx-}{d\cos(dx+c)}$
risch	$\frac{2b^2e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}+\frac{a^2\ln(e^{i(dx+c)}-1)}{d}+\frac{2\ln(e^{i(dx+c)}-1)ab}{d}+\frac{\ln(e^{i(dx+c)}-1)b^2}{d}-\frac{a^2\ln(e^{i(dx+c)}+1)}{d}+\frac{2\ln(e^{i(dx}{}$

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*ln(-cot(d*x+c)+csc(d*x+c))+2*a*b*ln(tan(d*x+c))+b^2*(1/cos(d*x+c)+
ln(-cot(d*x+c)+csc(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx =$$
$$-\frac{4\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2+2\,a\right.}{2\,d\cos\left(dx+c\right)}$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b*cos(d*x + c)*log(-cos(d*x + c)) + (a^2 - 2*a*b + b^2)*cos(d*x +
 c)*log(1/2*cos(d*x + c) + 1/2) - (a^2 + 2*a*b + b^2)*cos(d*x + c)*log(-1/2
*cos(d*x + c) + 1/2) - 2*b^2)/(d*cos(d*x + c))
```

Sympy [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^2\csc\left(c+dx\right)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx =$$
$$-\frac{4\,ab\log\left(\cos\left(dx+c\right)\right)+\left(a^2-2\,ab+b^2\right)\log\left(\cos\left(dx+c\right)+1\right)-\left(a^2+2\,ab+b^2\right)\log\left(\cos\left(dx+c\right)-1\right)}{2\,d}$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*b*log(cos(d*x + c)) + (a^2 - 2*a*b + b^2)*log(cos(d*x + c) + 1) -
 (a^2 + 2*a*b + b^2)*log(cos(d*x + c) - 1) - 2*b^2/cos(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.68

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{4\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-(a^2+2\,ab+b^2)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-\frac{4\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{2\,d}$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 + 2
*a*b + b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*(a*b + b^
2 + a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\frac{\ln(\cos(c+d\,x)-1)\,(a+b)^2}{2}-\frac{\ln(\cos(c+d\,x)+1)\,(a-b)^2}{2}+\frac{b^2}{\cos(c+d\,x)}-2\,a\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x),x)
```

```
[Out] ((log(cos(c + d*x) - 1)*(a + b)^2)/2 - (log(cos(c + d*x) + 1)*(a - b)^2)/2
+ b^2/cos(c + d*x) - 2*a*b*log(cos(c + d*x)))/d
```

3.178 $\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{(2ab+(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2d} + \frac{(a+b)(a+3b)\log(1-\cos(c+dx))}{4d} - \frac{2ab\log(\cos(c+dx))}{d} - \frac{(a-3b)(a-b)\log(1+\cos(c+dx))}{4d} + \frac{b^2\sec(c+dx)}{d}$$

```
[Out] -1/2*(2*a*b+(a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/d+1/4*(a+b)*(a+3*b)*ln(1-cos
(d*x+c))/d-2*a*b*ln(cos(d*x+c))/d-1/4*(a-3*b)*(a-b)*ln(1+cos(d*x+c))/d+b^2*
sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used

$= \{3957, 2916, 12, 1819, 1816\}$

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{\csc^2(c+dx)\left((a^2+b^2)\cos(c+dx)+2ab\right)}{2d} - \frac{2ab\log(\cos(c+dx))}{d} + \frac{(a+b)(a+3b)\log(1-\cos(c+dx))}{4d} - \frac{(a-3b)(a-b)\log(\cos(c+dx)+1)}{4d} + \frac{b^2\sec(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*((2*a*b + (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/d + ((a + b)*(a +
3*b)*Log[1 - Cos[c + d*x]])/(4*d) - (2*a*b*Log[Cos[c + d*x]])/d - ((a - 3*b
)*(a - b)*Log[1 + Cos[c + d*x]])/(4*d) + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-b-a\cos(c+dx))^2\csc^3(c+dx)\sec^2(c+dx)\,dx \\
&= \frac{a^3\text{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^5\text{Subst}\left(\int \frac{(-b+x)^2}{x^2(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= -\frac{a\left(2b+\frac{(a^2+b^2)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} - \frac{a^3\text{Subst}\left(\int \frac{-2b^2+4bx-\frac{(a^2+b^2)x^2}{a^2}}{x^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{2d} \\
&= -\frac{a\left(2b+\frac{(a^2+b^2)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} \\
&\quad - \frac{a^3\text{Subst}\left(\int \left(\frac{(a-3b)(-a+b)}{2a^3(a-x)} - \frac{2b^2}{a^2x^2} + \frac{4b}{a^2x} + \frac{(-a-3b)(a+b)}{2a^3(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{2d} \\
&= -\frac{a\left(2b+\frac{(a^2+b^2)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} + \frac{(a+b)(a+3b)\log(1-\cos(c+dx))}{4d} \\
&\quad - \frac{2ab\log(\cos(c+dx))}{d} - \frac{(a-3b)(a-b)\log(1+\cos(c+dx))}{4d} + \frac{b^2\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(114) = 228.

Time = 1.38 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.89

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{\csc^4(c+dx)\left(2a^2-2b^2+2(a^2+3b^2)\cos(2(c+dx))-a^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4ab\,\text{co}\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*(Csc[c + d*x]^4*(2*a^2 - 2*b^2 + 2*(a^2 + 3*b^2)*Cos[2*(c + d*x)] - a^
2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 4*a*b*Cos[3*(c + d*x)]*Log[Cos[(
c + d*x)/2]] - 3*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 4*a*b*Cos[3*(
c + d*x)]*Log[Cos[c + d*x]] + a^2*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] +
4*a*b*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 3*b^2*Cos[3*(c + d*x)]*Log[S
in[(c + d*x)/2]] + Cos[c + d*x]*(8*a*b + (a^2 - 4*a*b + 3*b^2)*Log[Cos[(c +
 d*x)/2]] + 4*a*b*Log[Cos[c + d*x]] - a^2*Log[Sin[(c + d*x)/2]] - 4*a*b*Log
[Sin[(c + d*x)/2]] - 3*b^2*Log[Sin[(c + d*x)/2]])))/(d*(Csc[(c + d*x)/2]^2
- Sec[(c + d*x)/2]^2))
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+2ab\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+2ab\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}\right.}{d}$
norman	$\frac{\frac{a^2+2ab+b^2}{8d}+\frac{(a^2-2ab+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8d}-\frac{(a^2+9b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{(a^2+4ab+3b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{d}$
parallelrisch	$\frac{16ba\left(1-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+16ba\left(1-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\right.}{8d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}$
risch	$\frac{a^2 e^{5i(dx+c)}+3b^2 e^{5i(dx+c)}+4ab\,e^{4i(dx+c)}+2a^2 e^{3i(dx+c)}-2b^2 e^{3i(dx+c)}+4ab\,e^{2i(dx+c)}+a^2 e^{i(dx+c)}+3b^2 e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}+\frac{a^2\ln(}{}$

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2*cot(d*x+c)*csc(d*x+c)+1/2*ln(-cot(d*x+c)+csc(d*x+c)))+2*a*b*
(-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/co
s(d*x+c)+3/2*ln(-cot(d*x+c)+csc(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.80

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{4\,ab\cos\left(dx+c\right)+2\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^2-4\,b^2-8\left(ab\cos\left(dx+c\right)^3-ab\cos\left(dx+c\right)\right)\log\left(-\cos\left(dx+\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*b*cos(d*x + c) + 2*(a^2 + 3*b^2)*cos(d*x + c)^2 - 4*b^2 - 8*(a*b*c
os(d*x + c)^3 - a*b*cos(d*x + c))*log(-cos(d*x + c)) - ((a^2 - 4*a*b + 3*b^
2)*cos(d*x + c)^3 - (a^2 - 4*a*b + 3*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c
) + 1/2) + ((a^2 + 4*a*b + 3*b^2)*cos(d*x + c)^3 - (a^2 + 4*a*b + 3*b^2)*co
s(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - d*cos(d*x + c
))
```

Sympy [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx=\int (a+b\sec{(c+dx)})^2\csc^3{(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx=$$

$$-\frac{8\,ab\log{(\cos{(dx+c)})}+(a^2-4\,ab+3\,b^2)\log{(\cos{(dx+c)}+1)}-(a^2+4\,ab+3\,b^2)\log{(\cos{(dx+c)}-}{4\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/4*(8*a*b*log(cos(d*x + c)) + (a^2 - 4*a*b + 3*b^2)*log(cos(d*x + c) + 1)
 - (a^2 + 4*a*b + 3*b^2)*log(cos(d*x + c) - 1) - 2*(2*a*b*cos(d*x + c) + (a
^2 + 3*b^2)*cos(d*x + c)^2 - 2*b^2)/(cos(d*x + c)^3 - cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(108)=216$.

Time = 0.33 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.75

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx=$$

$$-\frac{16\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,(a^2+4\,ab+3\,b^2)\,\mathrm{lo}}{8\,}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(16*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + a^2*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*(a^2 + 4*a*b + 3*b^2)
*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - (a^2 + 2*a*b + b^2 + 6
*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 14*b^2*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(a+b\right)\,\left(a+3\,b\right)}{4\,d} - \frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(a-b\right)\,\left(a-3\,b\right)}{4\,d} - \frac{2\,a\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d} - \frac{{\cos\left(c+d\,x\right)}^2\,\left(\frac{a^2}{2}+\frac{3\,b^2}{2}\right)-b^2+a\,b\,\cos\left(c+d\,x\right)}{d\,\left(\cos\left(c+d\,x\right)-{\cos\left(c+d\,x\right)}^3\right)}$$

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^3,x)
```

```
[Out] (log(cos(c + d*x) - 1)*(a + b)*(a + 3*b))/(4*d) - (log(cos(c + d*x) + 1)*(a
 - b)*(a - 3*b))/(4*d) - (2*a*b*log(cos(c + d*x)))/d - (cos(c + d*x)^2*(a^2
/2 + (3*b^2)/2) - b^2 + a*b*cos(c + d*x))/(d*(cos(c + d*x) - cos(c + d*x)^3
))
```

3.179 $\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 175

$$\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx = \frac{5}{16}\left(a^2-6b^2\right)x+\frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{(11a^2-18b^2)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(13a^2-6b^2)\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{2ab\sin^3(c+dx)}{3d} - \frac{2ab\sin^5(c+dx)}{5d} + \frac{b^2\tan(c+dx)}{d}$$

```
[Out] 5/16*(a^2-6*b^2)*x+2*a*b*arctanh(sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/16*(11*
a^2-18*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*(13*a^2-6*b^2)*cos(d*x+c)^3*sin(d*
x+c)/d-1/6*a^2*cos(d*x+c)^5*sin(d*x+c)/d-2/3*a*b*sin(d*x+c)^3/d-2/5*a*b*sin
(d*x+c)^5/d+b^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {3957, 2990, 2672, 308, 212, 466, 1828, 1171, 396, 209}

$$\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx = \frac{(13a^2-6b^2)\sin(c+dx)\cos^3(c+dx)}{24d} - \frac{(11a^2-18b^2)\sin(c+dx)\cos(c+dx)}{16d} + \frac{5}{16}x\left(a^2-6b^2\right) - \frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d} + \frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2ab\sin^5(c+dx)}{5d} - \frac{2ab\sin^3(c+dx)}{3d} - \frac{2ab\sin(c+dx)}{d} + \frac{b^2\tan(c+dx)}{d}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] (5*(a^2 - 6*b^2)*x)/16 + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d
*x])/d - ((11*a^2 - 18*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((13*a^2 -
6*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*
x])/(6*d) - (2*a*b*Sin[c + d*x]^3)/(3*d) - (2*a*b*Sin[c + d*x]^5)/(5*d) + (
b^2*Tan[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int (-b-a\cos(c+dx))^2\sin^4(c+dx)\tan^2(c+dx)\,dx$$

$$= (2ab)\int \sin^5(c+dx)\tan(c+dx)\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\sin^4(c+dx)\tan^2(c+dx)\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^6(a^2+b^2+b^2x^2)}{(1+x^2)^4}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(2ab)\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{\text{Subst}\left(\int \frac{-a^2+6a^2x^2-6a^2x^4-6b^2x^6}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{6d} + \frac{(2ab)\text{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{2ab\sin(c+dx)}{d} + \frac{(13a^2-6b^2)\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{2ab\sin^3(c+dx)}{3d} - \frac{2ab\sin^5(c+dx)}{5d} + \frac{\text{Subst}\left(\int \frac{-3(3a^2-2b^2)+24(a^2-b^2)x^2+24b^2x^4}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{24d} + \frac{(2ab)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{2ab\,\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{(11a^2-18b^2)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(13a^2-6b^2)\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{2ab\sin^3(c+dx)}{3d} - \frac{2ab\sin^5(c+dx)}{5d} - \frac{\text{Subst}\left(\int \frac{-3(5a^2-14b^2)-48b^2x^2}{1+x^2}\,dx, x, \tan(c+dx)\right)}{48d}$$

$$= \frac{2ab\,\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{(11a^2-18b^2)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(13a^2-6b^2)\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{2ab\sin^3(c+dx)}{3d} - \frac{2ab\sin^5(c+dx)}{5d} + \frac{b^2\tan(c+dx)}{d} + \frac{(5(a^2-6b^2))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{16d}$$

$$
\begin{aligned}
&= \frac{5}{16}\left(a^2-6b^2\right)x+\frac{2ab\,\mathrm{arctanh}(\sin(c+dx))}{d} \\
&\quad -\frac{2ab\sin(c+dx)}{d}-\frac{(11a^2-18b^2)\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad +\frac{(13a^2-6b^2)\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} \\
&\quad -\frac{2ab\sin^3(c+dx)}{3d}-\frac{2ab\sin^5(c+dx)}{5d}+\frac{b^2\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.10

$$
\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx
$$

$$
= \frac{60\left(5\left(a^2-6b^2\right)(c+dx)-32ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+32ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.\right.}{}
$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] (60*(5*(a^2 - 6*b^2)*(c + d*x) - 32*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 32*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 2128*a*b*Sin[c +
 d*x] + (-185*a^2 + 1410*b^2 - 5*(29*a^2 - 84*b^2)*Cos[2*(c + d*x)] + 232*a
*b*Cos[3*(c + d*x)] + 35*a^2*Cos[4*(c + d*x)] - 30*b^2*Cos[4*(c + d*x)] - 2
4*a*b*Cos[5*(c + d*x)] - 5*a^2*Cos[6*(c + d*x)])*Tan[c + d*x])/(960*d)
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{\left(\sin(dx+c)^{5}+\frac{5\sin(dx+c)^{3}}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+2ab\left(-\frac{\sin(dx+c)^{5}}{5}-\frac{\sin(dx+c)^{3}}{3}-\sin(dx+c)+\ln(\sec(\right.}{d}$
default	$\frac{a^{2}\left(-\frac{\left(\sin(dx+c)^{5}+\frac{5\sin(dx+c)^{3}}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+2ab\left(-\frac{\sin(dx+c)^{5}}{5}-\frac{\sin(dx+c)^{3}}{3}-\sin(dx+c)+\ln(\sec(\right.}{d}$
parts	$\frac{a^{2}\left(-\frac{\left(\sin(dx+c)^{5}+\frac{5\sin(dx+c)^{3}}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b^{2}\left(\frac{\sin(dx+c)^{7}}{\cos(dx+c)}+\left(\sin(dx+c)^{5}+\frac{5\sin(dx+c)^{3}}{4}+\frac{15}{}\right.\right.}{d}$
parallelrisch	$\frac{-3840ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+3840ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\left(-180a^{2}+450b^{2}\right)\sin(3dx+3c)+\left(40a^{2}\right.}{}$
risch	$\frac{5a^{2}x}{16}-\frac{15b^{2}x}{8}+\frac{i\,\mathrm{e}^{-2i(dx+c)}b^{2}}{4d}-\frac{15ia^{2}\mathrm{e}^{-2i(dx+c)}}{128d}+\frac{11iab\,\mathrm{e}^{i(dx+c)}}{8d}-\frac{i\mathrm{e}^{2i(dx+c)}b^{2}}{4d}+\frac{15ia^{2}\mathrm{e}^{2i(dx+c)}}{128d}-\frac{11iab}{}$
norman	$\frac{\left(-\frac{5a^{2}}{16}+\frac{15b^{2}}{8}\right)x+\left(-\frac{45a^{2}}{16}+\frac{135b^{2}}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\left(-\frac{25a^{2}}{16}+\frac{75b^{2}}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\left(-\frac{25a^{2}}{16}+\frac{75b^{2}}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\left(\right.}{}$

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5
/16*d*x+5/16*c)+2*a*b*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec
(d*x+c)+tan(d*x+c)))+b^2*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x
+c)^3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01

$$\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx$$
$$=\frac{75\left(a^{2}-6\,b^{2}\right)dx\cos\left(dx+c\right)+240\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-240\,ab\cos\left(dx+c\right)\log\left(-\sin\left(dx\right.\right.}{}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*(a^2 - 6*b^2)*d*x*cos(d*x + c) + 240*a*b*cos(d*x + c)*log(sin(d*x
 + c) + 1) - 240*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (40*a^2*cos(d*x
+ c)^6 + 96*a*b*cos(d*x + c)^5 - 352*a*b*cos(d*x + c)^3 - 10*(13*a^2 - 6*b^
```

```
2)*cos(d*x + c)^4 + 736*a*b*cos(d*x + c) + 15*(11*a^2 - 18*b^2)*cos(d*x + c
)^2 - 240*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**6,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

$$\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx = \frac{5\left(4\sin\left(2\,dx+2\,c\right)^3+60\,dx+60\,c+9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^2-64\left(6\,\sin\left(dx+c\right)^5+10\right.}{}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="maxima")
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a^2 - 64*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a*b - 120*(15*
d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*
x + c)^2 + 1) - 8*tan(d*x + c))*b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(163) = 326.

Time = 0.38 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.17

$$\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx = \frac{480\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-480\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+75\left(a^2-6\,b^2\right)(dx+c)-\frac{480\,b^2\tan}{\tan\left(\frac{1}{2}\,d\right.}}{}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(480*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 480*a*b*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) + 75*(a^2 - 6*b^2)*(d*x + c) - 480*b^2*tan(1/2*d*x + 1/
2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(75*a^2*tan(1/2*d*x + 1/2*c)^11 - 480
*a*b*tan(1/2*d*x + 1/2*c)^11 - 210*b^2*tan(1/2*d*x + 1/2*c)^11 + 425*a^2*ta
n(1/2*d*x + 1/2*c)^9 - 3040*a*b*tan(1/2*d*x + 1/2*c)^9 - 870*b^2*tan(1/2*d*
x + 1/2*c)^9 + 990*a^2*tan(1/2*d*x + 1/2*c)^7 - 8256*a*b*tan(1/2*d*x + 1/2*
c)^7 - 660*b^2*tan(1/2*d*x + 1/2*c)^7 - 990*a^2*tan(1/2*d*x + 1/2*c)^5 - 82
56*a*b*tan(1/2*d*x + 1/2*c)^5 + 660*b^2*tan(1/2*d*x + 1/2*c)^5 - 425*a^2*ta
n(1/2*d*x + 1/2*c)^3 - 3040*a*b*tan(1/2*d*x + 1/2*c)^3 + 870*b^2*tan(1/2*d*
x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c) - 480*a*b*tan(1/2*d*x + 1/2*c) +
 210*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 16.74 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.32

$$\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx$$
$$= \frac{\frac{5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)a^2}{8}+4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,a\,b-\frac{15\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)b^2}{4}}{d}$$
$$-\frac{\frac{15\,a^2\,\sin(c+d\,x)}{128}-\frac{5\,b^2\,\sin(c+d\,x)}{4}+\frac{3\,a^2\,\sin(3\,c+3\,d\,x)}{32}-\frac{a^2\,\sin(5\,c+5\,d\,x)}{48}+\frac{a^2\,\sin(7\,c+7\,d\,x)}{384}-\frac{15\,b^2\,\sin(3\,c+3\,d\,x)}{64}+\frac{b^2\,\sin(5\,c+5}{64}}{d\,\cos\left(c+d\,x\right)}$$

```
[In] int(sin(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((5*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (15*b^2*atan(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + 4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2)))/d - ((15*a^2*sin(c + d*x))/128 - (5*b^2*sin(c + d*x))/4 + (
3*a^2*sin(3*c + 3*d*x))/32 - (a^2*sin(5*c + 5*d*x))/48 + (a^2*sin(7*c + 7*d
*x))/384 - (15*b^2*sin(3*c + 3*d*x))/64 + (b^2*sin(5*c + 5*d*x))/64 + (59*a
*b*sin(2*c + 2*d*x))/48 - (2*a*b*sin(4*c + 4*d*x))/15 + (a*b*sin(6*c + 6*d*
x))/80)/(d*cos(c + d*x))
```

3.180 $\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx =& \frac{3}{8}\left(a^2-4b^2\right)x+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}\\ &-\frac{b(28a^2+b^2)\sin(c+dx)}{6ad}\\ &-\frac{(39a^2+2b^2)\cos(c+dx)\sin(c+dx)}{24d}\\ &-\frac{(12a^2+b^2)\,(b+a\cos(c+dx))^2\sin(c+dx)}{12abd}\\ &+\frac{(b+a\cos(c+dx))^3\sin(c+dx)}{4ad}\\ &+\frac{(b+a\cos(c+dx))^3\tan(c+dx)}{bd}\end{aligned}$$

```
[Out] 3/8*(a^2-4*b^2)*x+2*a*b*arctanh(sin(d*x+c))/d-1/6*b*(28*a^2+b^2)*sin(d*x+c)
/a/d-1/24*(39*a^2+2*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/12*(12*a^2+b^2)*(b+a*cos
(d*x+c))^2*sin(d*x+c)/a/b/d+1/4*(b+a*cos(d*x+c))^3*sin(d*x+c)/a/d+(b+a*cos(
d*x+c))^3*tan(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{3957, 2973, 3128, 3112, 3102, 2814, 3855\}$

$$\begin{aligned}\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx = &-\frac{b(28a^2+b^2)\sin(c+dx)}{6ad}\\ &-\frac{(12a^2+b^2)\sin(c+dx)(a\cos(c+dx)+b)^2}{12abd}\\ &-\frac{(39a^2+2b^2)\sin(c+dx)\cos(c+dx)}{24d}\\ &+\frac{3}{8}x\left(a^2-4b^2\right)+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}\\ &+\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}\\ &+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}\end{aligned}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] (3*(a^2 - 4*b^2)*x)/8 + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (b*(28*a^2 + b^2)
*Sin[c + d*x])/(6*a*d) - ((39*a^2 + 2*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*d
) - ((12*a^2 + b^2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(12*a*b*d) + ((b +
 a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d) + ((b + a*Cos[c + d*x])^3*Tan[c +
d*x])/(b*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2973

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d
*Sin[e + f*x])^(n + 2)/(b*d^2*f*(m + n + 4))), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int (-b - a\cos(c+dx))^2 \sin^2(c+dx) \tan^2(c+dx)\, dx$$

$$= \frac{(b+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{(b+a\cos(c+dx))^3\tan(c+dx)}{bd} - \frac{\int(-b-a\cos(c+dx))^2\left(-8a^2+5ab\cos(c+dx)+(12a^2+b^2)\cos^2(c+dx)\right)\sec(c+dx)\,dx}{4ab}$$

$$= -\frac{(12a^2+b^2)\,(b+a\cos(c+dx))^2\sin(c+dx)}{12abd} + \frac{(b+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{(b+a\cos(c+dx))^3\tan(c+dx)}{bd} - \frac{\int(-b-a\cos(c+dx))\left(24a^2b-17ab^2\cos(c+dx)-b(39a^2+2b^2)\cos^2(c+dx)\right)\sec(c+dx)\,dx}{12ab}$$

$$= -\frac{(39a^2+2b^2)\cos(c+dx)\sin(c+dx)}{24d} - \frac{(12a^2+b^2)\,(b+a\cos(c+dx))^2\sin(c+dx)}{12abd} + \frac{(b+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{(b+a\cos(c+dx))^3\tan(c+dx)}{bd} - \frac{\int\left(-48a^2b^2-9ab(a^2-4b^2)\cos(c+dx)+4b^2(28a^2+b^2)\cos^2(c+dx)\right)\sec(c+dx)\,dx}{24ab}$$

$$= -\frac{b(28a^2+b^2)\sin(c+dx)}{6ad} - \frac{(39a^2+2b^2)\cos(c+dx)\sin(c+dx)}{24d} - \frac{(12a^2+b^2)\,(b+a\cos(c+dx))^2\sin(c+dx)}{12abd} + \frac{(b+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{(b+a\cos(c+dx))^3\tan(c+dx)}{bd} - \frac{\int\left(-48a^2b^2-9ab(a^2-4b^2)\cos(c+dx)\right)\sec(c+dx)\,dx}{24ab}$$

$$= \frac{3}{8}\left(a^2-4b^2\right)x - \frac{b(28a^2+b^2)\sin(c+dx)}{6ad} - \frac{(39a^2+2b^2)\cos(c+dx)\sin(c+dx)}{24d} - \frac{(12a^2+b^2)\,(b+a\cos(c+dx))^2\sin(c+dx)}{12abd} + \frac{(b+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{(b+a\cos(c+dx))^3\tan(c+dx)}{bd} + (2ab)\int\sec(c+dx)\,dx$$

$$= \frac{3}{8}\left(a^2-4b^2\right)x + \frac{2ab\,\mathrm{arctanh}(\sin(c+dx))}{d} - \frac{b(28a^2+b^2)\sin(c+dx)}{6ad} - \frac{(39a^2+2b^2)\cos(c+dx)\sin(c+dx)}{24d} - \frac{(12a^2+b^2)\,(b+a\cos(c+dx))^2\sin(c+dx)}{12abd} + \frac{(b+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{(b+a\cos(c+dx))^3\tan(c+dx)}{bd}$$

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx$$
$$=\frac{12\left(3(a^2-4b^2)(c+dx)-16ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+16ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.\right.}{}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] (12*(3*(a^2 - 4*b^2)*(c + d*x) - 16*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 16*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 208*a*b*Sin[c +
d*x] + (-6*(3*a^2 - 4*b^2)*Cos[2*(c + d*x)] + 16*a*b*Cos[3*(c + d*x)] + 3*(
-7*a^2 + 40*b^2 + a^2*Cos[4*(c + d*x)]))*Tan[c + d*x])/(96*d)
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2}{d}$
default	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2}{d}$
parts	$\frac{a^2\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^2\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\right.}{d}$
parallelrisch	$\frac{72a^2xd\cos(dx+c)-288b^2dx\cos(dx+c)+384ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-384ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-}{19}$
risch	$\frac{3a^2x}{8}-\frac{3b^2x}{2}+\frac{ia^2e^{2i(dx+c)}}{8d}-\frac{ie^{2i(dx+c)}b^2}{8d}+\frac{5iab\,e^{i(dx+c)}}{4d}-\frac{5iab\,e^{-i(dx+c)}}{4d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{ie^{-2i(dx+c}}{8d}$
norman	$\frac{\left(-\frac{3a^2}{8}+\frac{3b^2}{2}\right)x+\left(-\frac{9a^2}{8}+\frac{9b^2}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(-\frac{3a^2}{4}+3b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{3a^2}{4}-3b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(\frac{3a^2}{8}-\frac{3b^2}{2}\right.}{}$

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2*a*
b*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(sin(d*x+c)^
5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.80

$$\int (a + b\sec(c + dx))^2 \sin^4(c + dx)\, dx$$
$$= \frac{9\,(a^2 - 4\,b^2)dx\cos(dx + c) + 24\,ab\cos(dx + c)\log(\sin(dx + c) + 1) - 24\,ab\cos(dx + c)\log(-\sin(dx + c}{24}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(9*(a^2 - 4*b^2)*d*x*cos(d*x + c) + 24*a*b*cos(d*x + c)*log(sin(d*x +
c) + 1) - 24*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) + (6*a^2*cos(d*x + c)^
4 + 16*a*b*cos(d*x + c)^3 - 64*a*b*cos(d*x + c) - 3*(5*a^2 - 4*b^2)*cos(d*x
 + c)^2 + 24*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a + b\sec(c + dx))^2 \sin^4(c + dx)\, dx = \int (a + b\sec(c + dx))^2 \sin^4(c + dx)\, dx$$

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int (a + b\sec(c + dx))^2 \sin^4(c + dx)\, dx$$
$$= \frac{3\,(12\,dx + 12\,c + \sin(4\,dx + 4\,c) - 8\,\sin(2\,dx + 2\,c))a^2 - 32\,\left(2\,\sin(dx + c)^3 - 3\,\log(\sin(dx + c) + 1) + 3}{96\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^2 - 32*(2
*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin
(d*x + c))*a*b - 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*ta
n(d*x + c))*b^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.60

$$\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx$$
$$= \frac{48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+9\,(a^2-4\,b^2)(dx+c)-\frac{48\,b^2\tan\left(\frac{1}{2}\,dx\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}}{}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(48*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*a*b*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) + 9*(a^2 - 4*b^2)*(d*x + c) - 48*b^2*tan(1/2*d*x + 1/2*c)/
(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan
(1/2*d*x + 1/2*c)^7 - 12*b^2*tan(1/2*d*x + 1/2*c)^7 + 33*a^2*tan(1/2*d*x +
1/2*c)^5 - 208*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*b^2*tan(1/2*d*x + 1/2*c)^5 -
 33*a^2*tan(1/2*d*x + 1/2*c)^3 - 208*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 9*a^2*tan(1/2*d*x + 1/2*c) - 48*a*b*tan(1/2*d*x + 1/
2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 14.82 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.16

$$\begin{aligned}\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx &= \frac{3\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}-\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}\\ &\quad+\frac{a^2\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}\\ &\quad-\frac{8\,a\,b\,\sin\left(c+d\,x\right)}{3\,d}+\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}\\ &\quad-\frac{5\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d}\\ &\quad+\frac{b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}\\ &\quad+\frac{2\,a\,b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}\end{aligned}$$

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d) - (3*b^2*atan(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^2*cos(c + d*x)^3*sin(c + d*x))/
```

```
(4*d) + (b^2*sin(c + d*x))/(d*cos(c + d*x)) - (8*a*b*sin(c + d*x))/(3*d) +
(4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (5*a^2*cos(c + d*x
)*sin(c + d*x))/(8*d) + (b^2*cos(c + d*x)*sin(c + d*x))/(2*d) + (2*a*b*cos(
c + d*x)^2*sin(c + d*x))/(3*d)
```

3.181 $\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx = \frac{a^2x}{2}-b^2x+\frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] 1/2*a^2*x-b^2*x+2*a*b*arctanh(sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/2*a^2*cos(
d*x+c)*sin(d*x+c)/d+b^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3957, 2801, 2715, 8, 2672, 327, 212, 3554}

$$\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx = -\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}+\frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] (a^2*x)/2 - b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/
d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-b - a\cos(c+dx))^2 \tan^2(c+dx)\,dx \\
&= \int \left(a^2\sin^2(c+dx) + 2ab\sin(c+dx)\tan(c+dx) + b^2\tan^2(c+dx)\right)\,dx \\
&= a^2\int \sin^2(c+dx)\,dx + (2ab)\int \sin(c+dx)\tan(c+dx)\,dx + b^2\int \tan^2(c+dx)\,dx \\
&= -\frac{a^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{b^2\tan(c+dx)}{d} + \frac{1}{2}a^2\int 1\,dx \\
&\quad - b^2\int 1\,dx + \frac{(2ab)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= \frac{a^2x}{2} - b^2x - \frac{2ab\sin(c+dx)}{d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} \\
&\quad + \frac{b^2\tan(c+dx)}{d} + \frac{(2ab)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= \frac{a^2x}{2} - b^2x + \frac{2ab\,\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} \\
&\quad - \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{b^2\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.57

$$\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx =$$

$$-\frac{-2a^2c + 4b^2c - 2a^2dx + 4b^2dx + 8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - 8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right.\right.}{4d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] -1/4*(-2*a^2*c + 4*b^2*c - 2*a^2*d*x + 4*b^2*d*x + 8*a*b*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] - 8*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8
*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)] - 4*b^2*Tan[c + d*x])/d
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^{2}(\tan(dx+c)-dx-c)}{d}$
default	$\frac{a^{2}\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^{2}(\tan(dx+c)-dx-c)}{d}$
parts	$\frac{a^{2}\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^{2}(\tan(dx+c)-dx-c)}{d}+\frac{2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
parallelrisch	$\frac{-16ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+16ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-8ab\sin(2dx+2c)-a^{2}\sin(3dx+3c)+4dx(a^{2}-}{8d\cos(dx+c)}$
risch	$\frac{a^{2}x}{2}-b^{2}x+\frac{ia^{2}\mathrm{e}^{2i(dx+c)}}{8d}+\frac{iab\,\mathrm{e}^{i(dx+c)}}{d}-\frac{iab\,\mathrm{e}^{-i(dx+c)}}{d}-\frac{ia^{2}\mathrm{e}^{-2i(dx+c)}}{8d}+\frac{2ib^{2}}{d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{2ab\ln(\mathrm{e}^{i(dx+}}{d}$
norman	$\frac{\left(-\frac{a^{2}}{2}+b^{2}\right)x+\left(-\frac{a^{2}}{2}+b^{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\left(\frac{a^{2}}{2}-b^{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\left(\frac{a^{2}}{2}-b^{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\frac{\left(a^{2}-4ab-2b^{2}\right)\tan\left(\frac{dx}{2}+\right.}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(-sin(d*x+c)+ln(s
ec(d*x+c)+tan(d*x+c)))+b^2*(tan(d*x+c)-d*x-c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx$$
$$=\frac{(a^2-2\,b^2)dx\cos{(dx+c)}+2\,ab\cos{(dx+c)}\log{(\sin{(dx+c)}+1)}-2\,ab\cos{(dx+c)}\log{(-\sin{(dx+c)}+}}{2\,d\cos{(dx+c)}}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 - 2*b^2)*d*x*cos(d*x + c) + 2*a*b*cos(d*x + c)*log(sin(d*x + c) +
 1) - 2*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 4*a
*b*cos(d*x + c) - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx = \int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**2,x)
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx$$
$$= \frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2-4\,(dx+c-\tan(dx+c))b^2+4\,ab(\log(\sin(dx+c)+1)-\log(\sin(dx$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="maxima")
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^2 - 4*(d*x + c - tan(d*x + c))*b^2
+ 4*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(73) = 146.

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.06

$$\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx$$
$$= \frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(a^2-2\,b^2)(dx+c)-\frac{4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{2\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="giac")
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (a^2 - 2*b^2)*(d*x + c) - 4*b^2*tan(1/2*d*x + 1/2*c)/(tan(1
/2*d*x + 1/2*c)^2 - 1) + 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x
+ 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c) - 4*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.86

$$\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx = \frac{a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)} - \frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d} + \frac{4\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*b^2*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^2*sin(c + d*x))/(d*cos(c + d*x)) - (2
*a*b*sin(c + d*x))/d + (4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d - (a^2*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.182 $\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{(a^2+b^2)\cot(c+dx)}{d} - \frac{2ab\csc(c+dx)}{d} + \frac{b^2\tan(c+dx)}{d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-(a^2+b^2)*cot(d*x+c)/d-2*a*b*csc(d*x+c)/d+b^2*t
an(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2990, 2701, 327, 213, 14}

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{(a^2+b^2)\cot(c+dx)}{d} + \frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2ab\csc(c+dx)}{d} + \frac{b^2\tan(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + b^2)*Cot[c + d*x])/d - (2*a*b*Csc
[c + d*x])/d + (b^2*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-b - a\cos(c+dx))^2 \csc^2(c+dx)\sec^2(c+dx)\,dx \\
&= (2ab)\int \csc^2(c+dx)\sec(c+dx)\,dx + \int \left(b^2 + a^2\cos^2(c+dx)\right)\csc^2(c+dx)\sec^2(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{a^2+b^2+b^2x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}
\end{aligned}
$$

$$= -\frac{2ab\csc(c+dx)}{d} + \frac{\text{Subst}\left(\int \left(b^2 + \frac{a^2+b^2}{x^2}\right) dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \csc(c+dx)\right)}{d}$$

$$= \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{(a^2+b^2)\cot(c+dx)}{d} - \frac{2ab\csc(c+dx)}{d} + \frac{b^2\tan(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 138 vs. $2(59) = 118$.

Time = 1.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{\csc^3\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(4ab\cos(c+dx) + (a^2+2b^2)\cos(2(c+dx)) + a\left(a+2b\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.\right.}{4d\left(-1+\cot^2\left(\frac{1}{2}(c+dx)\right)\right.}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*(Csc[(c + d*x)/2]^3*Sec[(c + d*x)/2]*(4*a*b*Cos[c + d*x] + (a^2 + 2*b^
2)*Cos[2*(c + d*x)] + a*(a + 2*b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[2*(c + d*x)])))/(d*(-1 + Co
t[(c + d*x)/2]^2))
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-a^2\cot(dx+c)+2ab\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
default	$\frac{-a^2\cot(dx+c)+2ab\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
parallelrisch	$\frac{-8ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+8ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-\left((a^2+2b^2)\cos(2dx+2c)+a(a-4b)\right)\sec\left(\frac{dx}{2}\right.}{4d\cos(dx+c)}$
risch	$-\frac{2i\left(2ab\,e^{3i(dx+c)}+a^2e^{2i(dx+c)}+2ab\,e^{i(dx+c)}+a^2+2b^2\right)}{d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)}-\frac{2ab\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{2ab\ln\left(e^{i(dx+c)}+i\right)}{d}$
norman	$\frac{\frac{a^2+2ab+b^2}{2d}-\frac{(a^2+3b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{(a^2-2ab+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{d}$

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*cot(d*x+c)+2*a*b*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(1
/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.76

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{ab\cos(dx+c)\log(\sin(dx+c)+1)\sin(dx+c)-ab\cos(dx+c)\log(-\sin(dx+c)+1)\sin(dx+c)-2}{d\cos(dx+c)\sin(dx+c)}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a*b*cos(d*x + c)*log(sin(d*x + c) + 1)*sin(d*x + c) - a*b*cos(d*x + c)*log
(-sin(d*x + c) + 1)*sin(d*x + c) - 2*a*b*cos(d*x + c) - (a^2 + 2*b^2)*cos(d
*x + c)^2 + b^2)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx=\int (a+b\sec(c+dx))^2\csc^2(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx=$$
$$-\frac{ab\left(\frac{2}{\sin(dx+c)}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+b^2\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)+\frac{a^2}{\tan(dx+c)}}{d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + b^
2*(1/tan(d*x + c) - tan(d*x + c)) + a^2/tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(59) = 118$.

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.83

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab\tan\left(\frac{1}{2}\,dx+\right.}{2\,d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + a^2*tan(1/2*d*x + 1/2*c) - 2*a*b*tan(1/2*d*x + 1/2*c) + b^2
*tan(1/2*d*x + 1/2*c) - (a^2*tan(1/2*d*x + 1/2*c)^2 + 2*a*b*tan(1/2*d*x + 1
/2*c)^2 + 5*b^2*tan(1/2*d*x + 1/2*c)^2 - a^2 - 2*a*b - b^2)/(tan(1/2*d*x +
1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.83

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a-b\right)}^2}{2\,d} - \frac{2\,a\,b+a^2+b^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2+2\,a\,b+5\,b^2\right)}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)} + \frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(a - b)^2)/(2*d) - (2*a*b + a^2 + b^2 - tan(c/2 + (d*x)
/2)^2*(2*a*b + a^2 + 5*b^2))/(d*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2
)^3)) + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

3.183 $\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{(a^2+2b^2)\cot(c+dx)}{d} - \frac{(a^2+b^2)\cot^3(c+dx)}{3d} - \frac{2ab\csc(c+dx)}{d} - \frac{2ab\csc^3(c+dx)}{3d} + \frac{b^2\tan(c+dx)}{d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-(a^2+2*b^2)*cot(d*x+c)/d-1/3*(a^2+b^2)*cot(d*x+
c)^3/d-2*a*b*csc(d*x+c)/d-2/3*a*b*csc(d*x+c)^3/d+b^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2990, 2701, 308, 213, 459}

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{(a^2+b^2)\cot^3(c+dx)}{3d} - \frac{(a^2+2b^2)\cot(c+dx)}{d} + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{2ab\csc(c+dx)}{d} + \frac{b^2\tan(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + 2*b^2)*Cot[c + d*x])/d - ((a^2 +
b^2)*Cot[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3
)/(3*d) + (b^2*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int (-b - a\cos(c+dx))^2 \csc^4(c+dx)\sec^2(c+dx)\,dx$$

$$= (2ab)\int \csc^4(c+dx)\sec(c+dx)\,dx + \int \left(b^2 + a^2\cos^2(c+dx)\right)\csc^4(c+dx)\sec^2(c+dx)\,dx$$

$$= \frac{\mathrm{Subst}\left(\int \frac{(1+x^2)(a^2+b^2+b^2x^2)}{x^4}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\mathrm{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \left(b^2+\frac{a^2+b^2}{x^4}+\frac{a^2+2b^2}{x^2}\right)\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\mathrm{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{(a^2+2b^2)\cot(c+dx)}{d} - \frac{(a^2+b^2)\cot^3(c+dx)}{3d} - \frac{2ab\csc(c+dx)}{d} - \frac{2ab\csc^3(c+dx)}{3d} + \frac{b^2\tan(c+dx)}{d} - \frac{(2ab)\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}$$

$$= \frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{(a^2+2b^2)\cot(c+dx)}{d} - \frac{(a^2+b^2)\cot^3(c+dx)}{3d} - \frac{2ab\csc(c+dx)}{d} - \frac{2ab\csc^3(c+dx)}{3d} + \frac{b^2\tan(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 259 vs. $2(100) = 200$.

Time = 1.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.59

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-3a^2-14ab\cos(c+dx)-2(a^2+4b^2)\cos(2(c+dx))+6ab\cos(3(c+d\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*(-3*a^2 - 14*a*b*Cos[c + d*x] - 2*(a
^2 + 4*b^2)*Cos[2*(c + d*x)] + 6*a*b*Cos[3*(c + d*x)] + a^2*Cos[4*(c + d*x)
] + 4*b^2*Cos[4*(c + d*x)] - 6*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*Sin[2*(c + d*x)] + 6*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[2*(c
 + d*x)] + 3*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]
- 3*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]))/(96*d*(
-1 + Cot[(c + d*x)/2]^2))
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{a^2\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+2ab\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx}\right.}{d}$
default	$\frac{a^2\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+2ab\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx}\right.}{d}$
parallelrisch	$\frac{-96ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+96ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left((a^2+4b^2)\cos(2dx+2c)+\right.}{48d\cos(dx+c)}$
risch	$-\frac{4i\left(3ab\,e^{7i(dx+c)}-7ab\,e^{5i(dx+c)}-3a^2e^{4i(dx+c)}-7ab\,e^{3i(dx+c)}-2a^2e^{2i(dx+c)}-8b^2e^{2i(dx+c)}+3ab\,e^{i(dx+c)}+a^2+4b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)^3}-$
norman	$\frac{\frac{a^2+2ab+b^2}{24d}-\frac{3(a^2+5b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4d}+\frac{(a^2-2ab+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{24d}+\frac{(2a^2-7ab+5b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{6d}+\frac{(2a^2+7ab+5b^2)\tan\left(\frac{dx}{2}\right.}{6d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+2*a*b*(-1/3/sin(d*x+c)^3-1/sin(
d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin
(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.78

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx =$$
$$-\frac{6\,ab\cos(dx+c)^3+2\,(a^2+4\,b^2)\cos(dx+c)^4-8\,ab\cos(dx+c)-3\,(a^2+4\,b^2)\cos(dx+c)^2-3\,(ab\,\mathrm{co}}{3\,(}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a*b*cos(d*x + c)^3 + 2*(a^2 + 4*b^2)*cos(d*x + c)^4 - 8*a*b*cos(d*x
 + c) - 3*(a^2 + 4*b^2)*cos(d*x + c)^2 - 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*
x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 3*(a*b*cos(d*x + c)^3 - a*b*co
s(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 3*b^2)/((d*cos(d*x + c)^3
 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec{(c+dx)})^2\csc^4{(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx =$$
$$-\frac{ab\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+b^2\left(\frac{6\tan(dx+c)^2+1}{\tan(dx+c)^3}-3\tan\left(dx\right.\right.}{3\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) + b^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 -
 3*tan(d*x + c)) + (3*tan(d*x + c)^2 + 1)*a^2/tan(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(96) = 192$.

Time = 0.34 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.26

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-48}{}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1
/2*d*x + 1/2*c)^3 + 48*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*a*b*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 9*a^2*tan(1/2*d*x + 1/2*c) - 30*a*b*tan(1/
2*d*x + 1/2*c) + 21*b^2*tan(1/2*d*x + 1/2*c) - 48*b^2*tan(1/2*d*x + 1/2*c)/
(tan(1/2*d*x + 1/2*c)^2 - 1) - (9*a^2*tan(1/2*d*x + 1/2*c)^2 + 30*a*b*tan(1
/2*d*x + 1/2*c)^2 + 21*b^2*tan(1/2*d*x + 1/2*c)^2 + a^2 + 2*a*b + b^2)/tan(
1/2*d*x + 1/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.68 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.82

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,{\left(a-b\right)}^2}{24\,d} - \frac{\frac{2\,a\,b}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(3\,a^2+10\,a\,b+23\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{8\,a^2}{3}+\frac{28\,a\,b}{3}+\frac{20\,b^2}{3}\right)+\frac{a^2}{3}+\frac{b^2}{3}}{d\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^2}{8}-\frac{3\,a\,b}{4}+\frac{5\,b^2}{8}+\frac{{\left(a-b\right)}^2}{4}\right)}{d} + \frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(a - b)^2)/(24*d) - ((2*a*b)/3 - tan(c/2 + (d*x)/2)^4
*(10*a*b + 3*a^2 + 23*b^2) + tan(c/2 + (d*x)/2)^2*((28*a*b)/3 + (8*a^2)/3 +
 (20*b^2)/3) + a^2/3 + b^2/3)/(d*(8*tan(c/2 + (d*x)/2)^3 - 8*tan(c/2 + (d*x
)/2)^5)) + (tan(c/2 + (d*x)/2)*(a^2/8 - (3*a*b)/4 + (5*b^2)/8 + (a - b)^2/4
))/d + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

3.184 $\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{(a^2+3b^2)\cot(c+dx)}{d} - \frac{(2a^2+3b^2)\cot^3(c+dx)}{3d} - \frac{(a^2+b^2)\cot^5(c+dx)}{5d} - \frac{2ab\csc(c+dx)}{d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{2ab\csc^5(c+dx)}{5d} + \frac{b^2\tan(c+dx)}{d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-(a^2+3*b^2)*cot(d*x+c)/d-1/3*(2*a^2+3*b^2)*cot(
d*x+c)^3/d-1/5*(a^2+b^2)*cot(d*x+c)^5/d-2*a*b*csc(d*x+c)/d-2/3*a*b*csc(d*x+
c)^3/d-2/5*a*b*csc(d*x+c)^5/d+b^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2990, 2701, 308, 213, 459}

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{(a^2+b^2)\cot^5(c+dx)}{5d} - \frac{(2a^2+3b^2)\cot^3(c+dx)}{3d} - \frac{(a^2+3b^2)\cot(c+dx)}{d} + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\csc^5(c+dx)}{5d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{2ab\csc(c+dx)}{d} + \frac{b^2\tan(c+dx)}{d}$$

[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]

[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + 3*b^2)*Cot[c + d*x])/d - ((2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - ((a^2 + b^2)*Cot[c + d*x]^5)/(5*d) - (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x])/d

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-b-a\cos(c+dx))^2\csc^6(c+dx)\sec^2(c+dx)\,dx\\
&= (2ab)\int \csc^6(c+dx)\sec(c+dx)\,dx+\int \left(b^2+a^2\cos^2(c+dx)\right)\csc^6(c+dx)\sec^2(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a^2+b^2+b^2x^2)}{x^6}\,dx,x,\tan(c+dx)\right)}{d}-\frac{(2ab)\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2+b^2}{x^6}+\frac{2a^2+3b^2}{x^4}+\frac{a^2+3b^2}{x^2}\right)\,dx,x,\tan(c+dx)\right)}{d}\\
&\quad -\frac{(2ab)\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&= -\frac{(a^2+3b^2)\cot(c+dx)}{d}-\frac{(2a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+b^2)\cot^5(c+dx)}{5d}\\
&\quad -\frac{2ab\csc(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc^5(c+dx)}{5d}\\
&\quad +\frac{b^2\tan(c+dx)}{d}-\frac{(2ab)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\
&= \frac{2ab\,\text{arctanh}(\sin(c+dx))}{d}-\frac{(a^2+3b^2)\cot(c+dx)}{d}\\
&\quad -\frac{(2a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+b^2)\cot^5(c+dx)}{5d}-\frac{2ab\csc(c+dx)}{d}\\
&\quad -\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc^5(c+dx)}{5d}+\frac{b^2\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 368 vs. $2(143) = 286$.

Time = 1.44 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.57

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(40a^2+196ab\cos(c+dx)+20(a^2+6b^2)\cos(2(c+dx))-130ab\cos(3\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/7680*(Csc[(c + d*x)/2]^7*Sec[(c + d*x)/2]^5*(40*a^2 + 196*a*b*Cos[c + d*
x] + 20*(a^2 + 6*b^2)*Cos[2*(c + d*x)] - 130*a*b*Cos[3*(c + d*x)] - 16*a^2*
Cos[4*(c + d*x)] - 96*b^2*Cos[4*(c + d*x)] + 30*a*b*Cos[5*(c + d*x)] + 4*a^
2*Cos[6*(c + d*x)] + 24*b^2*Cos[6*(c + d*x)] + 75*a*b*Log[Cos[(c + d*x)/2]
```

```
- Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] - 75*a*b*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]]*Sin[2*(c + d*x)] - 60*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]]*Sin[4*(c + d*x)] + 60*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[
4*(c + d*x)] + 15*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[6*(c + d
*x)] - 15*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[6*(c + d*x)]))/(
d*(-1 + Cot[(c + d*x)/2]^2))
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^2(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15})\cot(dx+c)+2ab(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+}{d}$
default	$\frac{a^2(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15})\cot(dx+c)+2ab(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+}{d}$
parallelrisch	$\frac{-3840ab\ln(\tan(\frac{dx}{2}+\frac{c}{2})-1)\cos(dx+c)+3840ab\ln(\tan(\frac{dx}{2}+\frac{c}{2})+1)\cos(dx+c)-\sec(\frac{dx}{2}+\frac{c}{2})^5\csc(\frac{dx}{2}+\frac{c}{2})^5(5(a^2+6}{1920}$
risch	$-\frac{4i(15ab\,e^{11i(dx+c)}-65ab\,e^{9i(dx+c)}+98ab\,e^{7i(dx+c)}+40a^2e^{6i(dx+c)}+98ab\,e^{5i(dx+c)}+20a^2e^{4i(dx+c)}+120b^2e^{4i(dx+c)}-}{15d(e^{2i(dx+c)}+1)(e^{2i(dx+c)}-1)^5}$
norman	$\frac{\frac{a^2+2ab+b^2}{160d}-\frac{5(a^2+7b^2)\tan(\frac{dx}{2}+\frac{c}{2})^6}{8d}+\frac{(a^2-2ab+b^2)\tan(\frac{dx}{2}+\frac{c}{2})^{12}}{160d}+\frac{(11a^2-32ab+21b^2)\tan(\frac{dx}{2}+\frac{c}{2})^{10}}{240d}+\frac{(11a^2+32ab+21b^2}{240}}{\tan(\frac{dx}{2}+\frac{c}{2})^5(-1+\tan(\frac{dx}{2}+\frac{c}{2})^2}$

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+2*a*b*(-1/5/
sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(
-1/5/sin(d*x+c)^5/cos(d*x+c)-2/5/sin(d*x+c)^3/cos(d*x+c)+8/5/sin(d*x+c)/cos
(d*x+c)-16/5*cot(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.69

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{30\,ab\cos(dx+c)^5+8\,(a^2+6\,b^2)\cos(dx+c)^6-70\,ab\cos(dx+c)^3-20\,(a^2+6\,b^2)\cos(dx+c)^4+46}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(30*a*b*cos(d*x + c)^5 + 8*(a^2 + 6*b^2)*cos(d*x + c)^6 - 70*a*b*cos(
d*x + c)^3 - 20*(a^2 + 6*b^2)*cos(d*x + c)^4 + 46*a*b*cos(d*x + c) + 15*(a^
2 + 6*b^2)*cos(d*x + c)^2 - 15*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^3 +
 a*b*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*(a*b*cos(d*x + c
)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d
*x + c) - 15*b^2)/((d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^3 + d*cos(d*x + c))
*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{ab\left(\frac{2\left(15\,\sin(dx+c)^4+5\,\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)+3\,b^2\left(\frac{15\,\tan(dx+}{\mathrm{t}}\right.}{15\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(a*b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3*b^2*((15*tan(d*x +
c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x + c)) + (15*tan(d*x
 + c)^4 + 10*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(135) = 270$.

Time = 0.36 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.28

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-70\,ab\tan}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2*
tan(1/2*d*x + 1/2*c)^5 + 25*a^2*tan(1/2*d*x + 1/2*c)^3 - 70*a*b*tan(1/2*d*x
 + 1/2*c)^3 + 45*b^2*tan(1/2*d*x + 1/2*c)^3 + 960*a*b*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - 960*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 150*a^2*tan(1/
2*d*x + 1/2*c) - 660*a*b*tan(1/2*d*x + 1/2*c) + 570*b^2*tan(1/2*d*x + 1/2*c
) - 960*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (150*a^2*ta
n(1/2*d*x + 1/2*c)^4 + 660*a*b*tan(1/2*d*x + 1/2*c)^4 + 570*b^2*tan(1/2*d*x
 + 1/2*c)^4 + 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 70*a*b*tan(1/2*d*x + 1/2*c)^2
 + 45*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan(1/2*d*x + 1/2
*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.73

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(a-b)^2}{160\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a^2}{32}-\frac{5\,a\,b}{48}+\frac{7\,b^2}{96}+\frac{(a-b)^2}{48}\right)}{d}$$
$$-\frac{\frac{2\,a\,b}{5}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{22\,a^2}{15}+\frac{64\,a\,b}{15}+\frac{14\,b^2}{5}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(10\,a^2+44\,a\,b+102\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{25}{3}\right.}{d\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\right)}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{7\,a^2}{32}-\frac{19\,a\,b}{16}+\frac{35\,b^2}{32}+\frac{3\,(a-b)^2}{32}\right)}{d}+\frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a - b)^2)/(160*d) + (tan(c/2 + (d*x)/2)^3*(a^2/32 -
(5*a*b)/48 + (7*b^2)/96 + (a - b)^2/48))/d - ((2*a*b)/5 + tan(c/2 + (d*x)/2
)^2*((64*a*b)/15 + (22*a^2)/15 + (14*b^2)/5) - tan(c/2 + (d*x)/2)^6*(44*a*b
 + 10*a^2 + 102*b^2) + tan(c/2 + (d*x)/2)^4*((118*a*b)/3 + (25*a^2)/3 + 35*
```

```
b^2) + a^2/5 + b^2/5)/(d*(32*tan(c/2 + (d*x)/2)^5 - 32*tan(c/2 + (d*x)/2)^7
)) + (tan(c/2 + (d*x)/2)*((7*a^2)/32 - (19*a*b)/16 + (35*b^2)/32 + (3*(a -
b)^2)/32))/d + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

3.185 $\int (a+b\sec(c+dx))^3 \sin^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 170

$$\begin{aligned}\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx = &-\frac{a(a^2-6b^2)\cos(c+dx)}{d}+\frac{b(6a^2-b^2)\cos^2(c+dx)}{2d}\\&+\frac{a(2a^2-3b^2)\cos^3(c+dx)}{3d}-\frac{3a^2b\cos^4(c+dx)}{4d}\\&-\frac{a^3\cos^5(c+dx)}{5d}-\frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d}\\&+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}\end{aligned}$$

```
[Out] -a*(a^2-6*b^2)*cos(d*x+c)/d+1/2*b*(6*a^2-b^2)*cos(d*x+c)^2/d+1/3*a*(2*a^2-3
*b^2)*cos(d*x+c)^3/d-3/4*a^2*b*cos(d*x+c)^4/d-1/5*a^3*cos(d*x+c)^5/d-b*(3*a
^2-2*b^2)*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2916, 12, 962}

$$\begin{aligned}\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx = &-\frac{a^3\cos^5(c+dx)}{5d}+\frac{a(2a^2-3b^2)\cos^3(c+dx)}{3d}\\&+\frac{b(6a^2-b^2)\cos^2(c+dx)}{2d}\\&-\frac{a(a^2-6b^2)\cos(c+dx)}{d}\\&-\frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d}-\frac{3a^2b\cos^4(c+dx)}{4d}\\&+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}\end{aligned}$$

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] -((a*(a^2 - 6*b^2)*Cos[c + d*x])/d) + (b*(6*a^2 - b^2)*Cos[c + d*x]^2)/(2*d
) + (a*(2*a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*d) - (3*a^2*b*Cos[c + d*x]^4)/(4*
d) - (a^3*Cos[c + d*x]^5)/(5*d) - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d +
 (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))^3\sin^2(c+dx)\tan^3(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^3(-b+x)^3\left(a^2-x^2\right)^2}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \frac{(-b+x)^3\left(a^2-x^2\right)^2}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^2d}\\
&= \frac{\text{Subst}\left(\int \left(a^4\left(1-\frac{6b^2}{a^2}\right)-\frac{a^4b^3}{x^3}+\frac{3a^4b^2}{x^2}+\frac{-3a^4b+2a^2b^3}{x}-b\left(-6a^2+b^2\right)x-\left(2a^2-3b^2\right)x^2-3bx^3+x^4\right)\right.}{a^2d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a(a^2-6b^2)\cos(c+dx)}{d} + \frac{b(6a^2-b^2)\cos^2(c+dx)}{2d} \\
&\quad + \frac{a(2a^2-3b^2)\cos^3(c+dx)}{3d} - \frac{3a^2b\cos^4(c+dx)}{4d} - \frac{a^3\cos^5(c+dx)}{5d} \\
&\quad - \frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$
\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx
$$
$$
= \frac{-60a(5a^2-42b^2)\cos(c+dx)+60(9a^2b-2b^3)\cos(2(c+dx))+50a^3\cos(3(c+dx))-120ab^2\cos(3(c+}{}
$$

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] (-60*a*(5*a^2 - 42*b^2)*Cos[c + d*x] + 60*(9*a^2*b - 2*b^3)*Cos[2*(c + d*x)
] + 50*a^3*Cos[3*(c + d*x)] - 120*a*b^2*Cos[3*(c + d*x)] - 45*a^2*b*Cos[4*(
c + d*x)] - 6*a^3*Cos[5*(c + d*x)] - 1440*a^2*b*Log[Cos[c + d*x]] + 960*b^3
*Log[Cos[c + d*x]] + 1440*a*b^2*Sec[c + d*x] + 240*b^3*Sec[c + d*x]^2)/(480
*d)
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{a^3\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+3a^2b\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}\right.\right.}{d}$
default	$\frac{-\frac{a^3\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}+3a^2b\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}\right.\right.}{d}$
parts	$-\frac{a^3\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{b^3\left(\frac{\sin(dx+c)^6}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^4}{2}+\sin(dx+c)^2+2\ln(\cos(dx+c))\right)}{d}+$
parallelrisch	$\frac{2880\left(a^2-\frac{2b^2}{3}\right)(1+\cos(2dx+2c))b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-2880\left(a^2-\frac{2b^2}{3}\right)(1+\cos(2dx+2c))b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2880\left(a\right.}{}$
norman	$\frac{\frac{\left(16a^3+24a^2b-48a\,b^2+16b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{16a^3-240a\,b^2}{15d}-\frac{\left(6a^2b-4b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-\frac{\left(18a^2b-12b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{(16a^3}{}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}$
risch	$\frac{6ib\,a^2c}{d}-\frac{4ib^3c}{d}+\frac{5\,\mathrm{e}^{3i(dx+c)}a^3}{96d}-\frac{\mathrm{e}^{3i(dx+c)}a\,b^2}{8d}+\frac{9\,\mathrm{e}^{2i(dx+c)}a^2b}{16d}-\frac{\mathrm{e}^{2i(dx+c)}b^3}{8d}-\frac{5a^3\mathrm{e}^{i(dx+c)}}{16d}+\frac{21\,\mathrm{e}^{i(dx+c)}}{8d}$

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a^3*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+3*a^2*b*(-1/4*
sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+3*a*b^2*(sin(d*x+c)^6/cos(d*x
+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+b^3*(1/2*sin(d*x+c)^6/c
os(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.03

$$\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx =$$
$$-\frac{96\,a^3\cos(dx+c)^7+360\,a^2b\cos(dx+c)^6-160\,(2\,a^3-3\,ab^2)\cos(dx+c)^5-240\,(6\,a^2b-b^3)\cos(dx+c)}{}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^3*cos(d*x + c)^7 + 360*a^2*b*cos(d*x + c)^6 - 160*(2*a^3 - 3*a
*b^2)*cos(d*x + c)^5 - 240*(6*a^2*b - b^3)*cos(d*x + c)^4 - 1440*a*b^2*cos(
d*x + c) + 480*(a^3 - 6*a*b^2)*cos(d*x + c)^3 + 480*(3*a^2*b - 2*b^3)*cos(d
*x + c)^2*log(-cos(d*x + c)) - 240*b^3 + 15*(39*a^2*b - 8*b^3)*cos(d*x + c)
^2)/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.84

$$\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx =$$
$$-\frac{12\,a^3\cos(dx+c)^5+45\,a^2b\cos(dx+c)^4-20\,(2\,a^3-3\,ab^2)\cos(dx+c)^3-30\,(6\,a^2b-b^3)\cos(dx+c)^2}{60\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a^3*cos(d*x + c)^5 + 45*a^2*b*cos(d*x + c)^4 - 20*(2*a^3 - 3*a*b^
2)*cos(d*x + c)^3 - 30*(6*a^2*b - b^3)*cos(d*x + c)^2 + 60*(a^3 - 6*a*b^2)*
cos(d*x + c) + 60*(3*a^2*b - 2*b^3)*log(cos(d*x + c)) - 30*(6*a*b^2*cos(d*x
 + c) + b^3)/cos(d*x + c)^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(160) = 320$.

Time = 0.44 (sec) , antiderivative size = 695, normalized size of antiderivative = 4.09

$$\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*(3*a^2*b - 2*b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)) - 60*(3*a^2*b - 2*b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 1)) + 30*(9*a^2*b + 12*a*b^2 - 6*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*b^3*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 1)^2 + (64*a^3 + 411*a^2*b - 600*a*b^2 - 274*b^
3 - 320*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2415*a^2*b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 2640*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 1490*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 640*a^3*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 5910*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - 3840*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3100*b^3*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5910*a^2*b*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 + 2160*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 + 3100*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2415*a^2*b*(cos(d*x
 + c) - 1)^4/(cos(d*x + c) + 1)^4 - 360*a*b^2*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4 - 1490*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 411*a^2
*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 274*b^3*(cos(d*x + c) - 1)^5
/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx =$$

$$-\frac{\cos\left(c+d\,x\right)^3\,\left(a\,b^2-\frac{2\,a^3}{3}\right)-\cos\left(c+d\,x\right)^2\,\left(3\,a^2\,b-\frac{b^3}{2}\right)+\ln\left(\cos\left(c+d\,x\right)\right)\,\left(3\,a^2\,b-2\,b^3\right)-\frac{\frac{b^3}{2}+3\,a\,\cos(c+}{\cos(c+d\,x)}}{d}$$

```
[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^3,x)
```

```
[Out] -(cos(c + d*x)^3*(a*b^2 - (2*a^3)/3) - cos(c + d*x)^2*(3*a^2*b - b^3/2) + l
og(cos(c + d*x))*(3*a^2*b - 2*b^3) - (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c +
 d*x)^2 - cos(c + d*x)*(6*a*b^2 - a^3) + (a^3*cos(c + d*x)^5)/5 + (3*a^2*b*
cos(c + d*x)^4)/4)/d
```

3.186 $\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx = -\frac{a(a^2-3b^2)\cos(c+dx)}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

```
[Out] -a*(a^2-3*b^2)*cos(d*x+c)/d+3/2*a^2*b*cos(d*x+c)^2/d+1/3*a^3*cos(d*x+c)^3/d
-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3957, 2800, 908}

$$\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx = \frac{a^3\cos^3(c+dx)}{3d}-\frac{a(a^2-3b^2)\cos(c+dx)}{d}-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] -((a*(a^2 - 3*b^2)*Cos[c + d*x])/d) + (3*a^2*b*Cos[c + d*x]^2)/(2*d) + (a^3
*Cos[c + d*x]^3)/(3*d) - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*S
ec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int (-b - a\cos(c+dx))^3 \tan^3(c+dx)\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{(-b+x)^3(a^2-x^2)}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{3b^2}{a^2}\right) - \frac{a^2b^3}{x^3} + \frac{3a^2b^2}{x^2} + \frac{-3a^2b+b^3}{x} + 3bx - x^2\right)\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= -\frac{a(a^2-3b^2)\cos(c+dx)}{d} + \frac{3a^2b\cos^2(c+dx)}{2d} + \frac{a^3\cos^3(c+dx)}{3d} - \frac{b(3a^2-b^2)\log(\cos(c+dx))}{d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec^2(c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.88

$$\int (a + b\sec(c+dx))^3 \sin^3(c+dx)\,dx = \frac{-9a(a^2-4b^2)\cos(c+dx) + 9a^2b\cos(2(c+dx)) + a^3\cos(3(c+dx)) - 36a^2b\log(\cos(c+dx)) + 12b^3\log(c}{12d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] (-9*a*(a^2 - 4*b^2)*Cos[c + d*x] + 9*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
+ d*x)] - 36*a^2*b*Log[Cos[c + d*x]] + 12*b^3*Log[Cos[c + d*x]] + 36*a*b^2*
Sec[c + d*x] + 6*b^3*Sec[c + d*x]^2)/(12*d)
```

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{-\frac{a^{3}\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{3}+3a^{2}b\left(-\frac{\sin(dx+c)^{2}}{2}-\ln(\cos(dx+c))\right)+3a\,b^{2}\left(\frac{\sin(dx+c)^{4}}{\cos(dx+c)}+\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)\right)+}{d}$
default	$\frac{-\frac{a^{3}\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{3}+3a^{2}b\left(-\frac{\sin(dx+c)^{2}}{2}-\ln(\cos(dx+c))\right)+3a\,b^{2}\left(\frac{\sin(dx+c)^{4}}{\cos(dx+c)}+\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)\right)+}{d}$
parts	$-\frac{a^{3}\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{3d}+\frac{b^{3}\left(\frac{\tan(dx+c)^{2}}{2}+\ln(\cos(dx+c))\right)}{d}+\frac{3a\,b^{2}\left(\frac{\sin(dx+c)^{4}}{\cos(dx+c)}+\left(2+\sin(dx+c)^{2}\right)\cos(dx-\right.}{d}$
parallelrisch	$\frac{72(1+\cos(2dx+2c))b\left(a^{2}-\frac{b^{2}}{3}\right)\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)-72(1+\cos(2dx+2c))b\left(a^{2}-\frac{b^{2}}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-72(1+\cos(2dx}{}$
risch	$3ia^{2}bx+\frac{6ib\,a^{2}c}{d}+\frac{e^{3i(dx+c)}a^{3}}{24d}+\frac{3\,e^{2i(dx+c)}a^{2}b}{8d}-\frac{3a^{3}e^{i(dx+c)}}{8d}+\frac{3\,e^{i(dx+c)}a\,b^{2}}{2d}-\frac{3a^{3}e^{-i(dx+c)}}{8d}+\frac{3\,e^{-i(}}{}$
norman	$\frac{-\frac{4a^{3}-36a\,b^{2}}{3d}-\frac{\left(6a^{2}b-2b^{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{d}-\frac{2\left(2a^{3}-3a^{2}b+6a\,b^{2}-3b^{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{d}-\frac{\left(4a^{3}+18a^{2}b-36a\,b^{2}-6b^{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3d}+}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a^2*b*(-1/2*sin(d*x+c)^2-ln(cos
(d*x+c)))+3*a*b^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b^3
*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx$$
$$=\frac{4\,a^3\cos{(dx+c)}^5+18\,a^2b\cos{(dx+c)}^4-9\,a^2b\cos{(dx+c)}^2+36\,ab^2\cos{(dx+c)}-12\,(a^3-3\,ab^2)\cos{(d}}{12\,d\cos{(dx+c)}^2}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^5 + 18*a^2*b*cos(d*x + c)^4 - 9*a^2*b*cos(d*x + c)
^2 + 36*a*b^2*cos(d*x + c) - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 12*(3*a^2*
b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + 6*b^3)/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx=\int (a+b\sec{(c+dx)})^3\sin^3{(c+dx)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**3,x)
[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx$$
$$=\frac{2\,a^3\cos{(dx+c)}^3+9\,a^2b\cos{(dx+c)}^2-6\,(a^3-3\,ab^2)\cos{(dx+c)}-6\,(3\,a^2b-b^3)\log{(\cos{(dx+c)})}+\frac{3\,(6}{}}{6\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="maxima")
[Out] 1/6*(2*a^3*cos(d*x + c)^3 + 9*a^2*b*cos(d*x + c)^2 - 6*(a^3 - 3*a*b^2)*cos(
d*x + c) - 6*(3*a^2*b - b^3)*log(cos(d*x + c)) + 3*(6*a*b^2*cos(d*x + c) +
b^3)/cos(d*x + c)^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx$$
$$=-\frac{(3\,a^2b-b^3)\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,ab^2\cos{(dx+c)}+b^3}{2\,d\cos{(dx+c)}^2}$$
$$+\frac{2\,a^3d^8\cos{(dx+c)}^3+9\,a^2bd^8\cos{(dx+c)}^2-6\,a^3d^8\cos{(dx+c)}+18\,ab^2d^8\cos{(dx+c)}}{6\,d^9}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="giac")
[Out] -(3*a^2*b - b^3)*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a*b^2*cos(d*x + c
) + b^3)/(d*cos(d*x + c)^2) + 1/6*(2*a^3*d^8*cos(d*x + c)^3 + 9*a^2*b*d^8*c
os(d*x + c)^2 - 6*a^3*d^8*cos(d*x + c) + 18*a*b^2*d^8*cos(d*x + c))/d^9
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx$$
$$=\frac{\frac{\frac{b^3}{2}+3\,a\,\cos(c+d\,x)\,b^2}{\cos(c+d\,x)^2}-\ln\left(\cos\left(c+d\,x\right)\right)\,\left(3\,a^2\,b-b^3\right)+\cos\left(c+d\,x\right)\,\left(3\,a\,b^2-a^3\right)+\frac{a^3\cos(c+d\,x)^3}{3}+\frac{3\,a^2\,b\cos(c+d\,x}{2}}{d}$$

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 - log(cos(c + d*x))*(3*a^2*b
 - b^3) + cos(c + d*x)*(3*a*b^2 - a^3) + (a^3*cos(c + d*x)^3)/3 + (3*a^2*b*
cos(c + d*x)^2)/2)/d
```

3.187 $\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx=-\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

```
[Out] -a^3*cos(d*x+c)/d-3*a^2*b*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec
(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2912, 12, 45}

$$\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx=-\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] -((a^3*Cos[c + d*x])/d) - (3*a^2*b*Log[Cos[c + d*x]])/d + (3*a*b^2*Sec[c +
d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^3(-b+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{ad}\\
&= \frac{a^2\text{Subst}\left(\int \frac{(-b+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^2\text{Subst}\left(\int \left(1-\frac{b^3}{x^3}+\frac{3b^2}{x^2}-\frac{3b}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= -\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$
\begin{aligned}
&\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx\\
&= \frac{-2a^3\cos(c+dx)+b(-6a^2\log(\cos(c+dx))+6ab\sec(c+dx)+b^2\sec^2(c+dx))}{2d}
\end{aligned}
$$

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] (-2*a^3*Cos[c + d*x] + b*(-6*a^2*Log[Cos[c + d*x]] + 6*a*b*Sec[c + d*x] + b
^2*Sec[c + d*x]^2))/(2*d)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{b^3\sec(dx+c)^2}{2}+3\sec(dx+c)a\,b^2+3a^2b\ln(\sec(dx+c))-\frac{a^3}{\sec(dx+c)}}{d}$
default	$\frac{\frac{b^3\sec(dx+c)^2}{2}+3\sec(dx+c)a\,b^2+3a^2b\ln(\sec(dx+c))-\frac{a^3}{\sec(dx+c)}}{d}$
parts	$-\frac{a^3\cos(dx+c)}{d}+\frac{b^3\sec(dx+c)^2}{2d}+\frac{3a^2b\ln(\sec(dx+c))}{d}+\frac{3a\,b^2\sec(dx+c)}{d}$
risch	$3ia^2bx-\frac{a^3\mathrm{e}^{i(dx+c)}}{2d}-\frac{a^3\mathrm{e}^{-i(dx+c)}}{2d}+\frac{6ib\,a^2c}{d}+\frac{2b^2\left(3a\,\mathrm{e}^{3i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}a\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{3b\ln\left(\mathrm{e}^{2i(dx+c)}+\right.}{d}$
norman	$\frac{\frac{\left(4a^3+2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{2a^3-6a\,b^2}{d}-\frac{\left(2a^3+6a\,b^2-2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{3a^2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{3a^2b\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{d}$
parallelrisch	$\frac{6a^2b(1+\cos(2dx+2c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-6a^2b(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-6a^2b(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{2d(1+\cos(2dx+2c}$

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b^3*sec(d*x+c)^2+3*sec(d*x+c)*a*b^2+3*a^2*b*ln(sec(d*x+c))-a^3/sec
(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx = -\frac{2\,a^3\cos\left(dx+c\right)^3+6\,a^2b\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-6\,ab^2\cos\left(dx+c\right)-b^3}{2\,d\cos\left(dx+c\right)^2}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 6*a^2*b*cos(d*x + c)^2*log(-cos(d*x + c)) - 6*
a*b^2*cos(d*x + c) - b^3)/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx = \int (a+b\sec(c+dx))^3\sin(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c),x)
[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx = -\frac{2\,a^3\cos(dx+c)+6\,a^2b\log(\cos(dx+c))-\frac{6\,ab^2}{\cos(dx+c)}-\frac{b^3}{\cos(dx+c)^2}}{2\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="maxima")
[Out] -1/2*(2*a^3*cos(d*x + c) + 6*a^2*b*log(cos(d*x + c)) - 6*a*b^2/cos(d*x + c)
 - b^3/cos(d*x + c)^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx = -\frac{a^3\cos(dx+c)}{d}-\frac{3\,a^2b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,ab^2\cos(dx+c)+b^3}{2\,d\cos(dx+c)^2}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="giac")
[Out] -a^3*cos(d*x + c)/d - 3*a^2*b*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a*b^
2*cos(d*x + c) + b^3)/(d*cos(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx$$
$$= -\frac{a^3\,\cos\left(c+d\,x\right)-\frac{\frac{b^3}{2}+3\,a\,\cos\left(c+d\,x\right)\,b^2}{{\cos\left(c+d\,x\right)}^2}+3\,a^2\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*cos(c + d*x) - (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 + 3*a^2*
b*log(cos(c + d*x)))/d
```

3.188 $\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{(a+b)^3\log(1-\cos(c+dx))}{2d} - \frac{b(3a^2+b^2)\log(\cos(c+dx))}{d} - \frac{(a-b)^3\log(1+\cos(c+dx))}{2d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec^2(c+dx)}{2d}$$

```
[Out] 1/2*(a+b)^3*ln(1-cos(d*x+c))/d-b*(3*a^2+b^2)*ln(cos(d*x+c))/d-1/2*(a-b)^3*l
n(1+cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2916, 12, 1816}

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx = -\frac{b(3a^2+b^2)\log(\cos(c+dx))}{d} + \frac{3ab^2\sec(c+dx)}{d} - \frac{(a-b)^3\log(\cos(c+dx)+1)}{2d} + \frac{(a+b)^3\log(1-\cos(c+dx))}{2d} + \frac{b^3\sec^2(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a + b)^3*Log[1 - Cos[c + d*x]])/(2*d) - (b*(3*a^2 + b^2)*Log[Cos[c + d*x]
])/d - ((a - b)^3*Log[1 + Cos[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d +
 (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))^3\csc(c+dx)\sec^3(c+dx)\,dx\\
&= \frac{a\text{Subst}\left(\int \frac{a^3(-b+x)^3}{x^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^4\text{Subst}\left(\int \frac{(-b+x)^3}{x^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^4\text{Subst}\left(\int \left(\frac{(a-b)^3}{2a^4(a-x)}-\frac{b^3}{a^2x^3}+\frac{3b^2}{a^2x^2}+\frac{b(-3a^2-b^2)}{a^4x}+\frac{(a+b)^3}{2a^4(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{(a+b)^3\log(1-\cos(c+dx))}{2d}-\frac{b(3a^2+b^2)\log(\cos(c+dx))}{d}\\
&\quad -\frac{(a-b)^3\log(1+\cos(c+dx))}{2d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{-2(a-b)^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2b(3a^2+b^2)\log(\cos(c+dx))+2(a+b)^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6ab^2}{2d}$$

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*(a - b)^3*Log[Cos[(c + d*x)/2]] - 2*b*(3*a^2 + b^2)*Log[Cos[c + d*x]] +
 2*(a + b)^3*Log[Sin[(c + d*x)/2]] + 6*a*b^2*Sec[c + d*x] + b^3*Sec[c + d*x
]^2)/(2*d)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{a^3\ln(-\cot(dx+c)+\csc(dx+c))+3a^2b\ln(\tan(dx+c))+3ab^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+b^3\left(\frac{1}{2\cos(dx+c)^2}\right.}{d}$
default	$\frac{a^3\ln(-\cot(dx+c)+\csc(dx+c))+3a^2b\ln(\tan(dx+c))+3ab^2\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+b^3\left(\frac{1}{2\cos(dx+c)^2}\right.}{d}$
norman	$\frac{\frac{6ab^2}{d}-\frac{2(3ab^2-b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{(a^3+3a^2b+3ab^2+b^3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b(3a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b(}{}$
parallelrisch	$\frac{-3\left(a^2+\frac{b^2}{3}\right)(1+\cos(2dx+2c))b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-3\left(a^2+\frac{b^2}{3}\right)(1+\cos(2dx+2c))b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+(a+b)^3(1+\cos}{d(1+\cos(2dx+2c))}$
risch	$\frac{2b^2\left(3a\,e^{3i(dx+c)}+b\,e^{2i(dx+c)}+3\,e^{i(dx+c)}a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{a^3\ln\left(e^{i(dx+c)}-1\right)}{d}+\frac{3\ln\left(e^{i(dx+c)}-1\right)a^2b}{d}+\frac{3\ln\left(e^{i(dx+c)}-1\right)ab^2}{d}+$

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*ln(-cot(d*x+c)+csc(d*x+c))+3*a^2*b*ln(tan(d*x+c))+3*a*b^2*(1/cos(d
*x+c)+ln(-cot(d*x+c)+csc(d*x+c)))+b^3*(1/2/cos(d*x+c)^2+ln(tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.36

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{6\,ab^2\cos\left(dx+c\right)-2\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\cos\left(dx+c\right)^2}{2\,d\cos\left(dx+\right.}$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*cos(d*x + c) - 2*(3*a^2*b + b^3)*cos(d*x + c)^2*log(-cos(d*x +
 c)) - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(d*x + c)^2*log(1/2*cos(d*x + c)
+ 1/2) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^2*log(-1/2*cos(d*x +
c) + 1/2) + b^3)/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx=\int\left(a+b\sec\left(c+dx\right)\right)^3\csc\left(c+dx\right)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx=$$
$$-\frac{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log\left(\cos\left(dx+c\right)+1\right)-\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(\cos\left(dx+c\right)-1\right)+2\left(3\,a^2\right.}{2\,d}$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*log(cos(d*x + c) + 1) - (a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*log(cos(d*x + c) - 1) + 2*(3*a^2*b + b^3)*log(cos(d*x +
c)) - (6*a*b^2*cos(d*x + c) + b^3)/cos(d*x + c)^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(96) = 192$.

Time = 0.34 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.45

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{(a^3+3a^2b+3ab^2+b^3)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2(3a^2b+b^3)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{9a^2b+12ab^2+3b^3+\frac{18a}{}}{}}{2d}$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x
 + c) + 1)) - 2*(3*a^2*b + b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) - 1)) + (9*a^2*b + 12*a*b^2 + 3*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^3*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 + 3*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\frac{\ln(\cos(c+d\,x)-1)\,(a+b)^3}{2}-\frac{\ln(\cos(c+d\,x)+1)\,(a-b)^3}{2}+\frac{\frac{b^3}{2}+3\,a\,\cos(c+d\,x)\,b^2}{\cos(c+d\,x)^2}-\ln\left(\cos\left(c+d\,x\right)\right)\,\left(3\,a^2\,b+b^3\right)}{d}$$

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x),x)
```

```
[Out] ((log(cos(c + d*x) - 1)*(a + b)^3)/2 - (log(cos(c + d*x) + 1)*(a - b)^3)/2
+ (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 - log(cos(c + d*x))*(3*a^2*
b + b^3))/d
```

3.189 $\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 162

$$\begin{aligned}
&\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx \\
&= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d} \\
&\quad+\frac{(a+b)^2(a+4b)\log(1-\cos(c+dx))}{4d}-\frac{b(3a^2+2b^2)\log(\cos(c+dx))}{d} \\
&\quad-\frac{(a-4b)(a-b)^2\log(1+\cos(c+dx))}{4d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}
\end{aligned}$$

```
[Out] -1/2*a^2*(b*(3+b^2/a^2)+a*(1+3*b^2/a^2)*cos(d*x+c))*csc(d*x+c)^2/d+1/4*(a+b
)^2*(a+4*b)*ln(1-cos(d*x+c))/d-b*(3*a^2+2*b^2)*ln(cos(d*x+c))/d-1/4*(a-4*b)
*(a-b)^2*ln(1+cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2916, 12, 1819, 1816}

$$\begin{aligned}
&\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx \\
&= -\frac{b(3a^2+2b^2)\log(\cos(c+dx))}{d}-\frac{a^2\csc^2(c+dx)\left(a\left(\frac{3b^2}{a^2}+1\right)\cos(c+dx)+b\left(\frac{b^2}{a^2}+3\right)\right)}{2d} \\
&\quad+\frac{3ab^2\sec(c+dx)}{d}+\frac{(a+b)^2(a+4b)\log(1-\cos(c+dx))}{4d} \\
&\quad-\frac{(a-4b)(a-b)^2\log(\cos(c+dx)+1)}{4d}+\frac{b^3\sec^2(c+dx)}{2d}
\end{aligned}$$

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*(a^2*(b*(3 + b^2/a^2) + a*(1 + (3*b^2)/a^2)*Cos[c + d*x])*Csc[c + d*x]
^2)/d + ((a + b)^2*(a + 4*b)*Log[1 - Cos[c + d*x]])/(4*d) - (b*(3*a^2 + 2*b
^2)*Log[Cos[c + d*x]])/d - ((a - 4*b)*(a - b)^2*Log[1 + Cos[c + d*x]])/(4*d
) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int (-b - a\cos(c+dx))^3 \csc^3(c+dx)\sec^3(c+dx)\,dx$$

$$= \frac{a^3\text{Subst}\left(\int \frac{a^3(-b+x)^3}{x^3(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^6\text{Subst}\left(\int \frac{(-b+x)^3}{x^3(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d} - \frac{a^4\text{Subst}\left(\int \frac{2b^3-6b^2x+2b\left(3+\frac{b^2}{a^2}\right)x^2-\left(1+\frac{3b^2}{a^2}\right)x^3}{x^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{2d}$$

$$= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d} - \frac{a^4\text{Subst}\left(\int \left(-\frac{(a-4b)(a-b)^2}{2a^4(a-x)}+\frac{2b^3}{a^2x^3}-\frac{6b^2}{a^2x^2}+\frac{2(3a^2b+2b^3)}{a^4x}-\frac{(a+b)^2(a+4b)}{2a^4(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{2d}$$

$$= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d} + \frac{(a+b)^2(a+4b)\log(1-\cos(c+dx))}{4d} - \frac{b(3a^2+2b^2)\log(\cos(c+dx))}{d} - \frac{(a-4b)(a-b)^2\log(1+\cos(c+dx))}{4d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec^2(c+dx)}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 669 vs. 2(162) = 324.

Time = 7.22 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.13

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3}{d(b+a\cos(c+dx))^3}$$
$$+ \frac{(-a^3-3a^2b-3ab^2-b^3)\cos^3(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\sec(c+dx))^3}{8d(b+a\cos(c+dx))^3}$$
$$+ \frac{(-a^3+6a^2b-9ab^2+4b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3}$$
$$+ \frac{(-3a^2b-2b^3)\cos^3(c+dx)\log(\cos(c+dx))(a+b\sec(c+dx))^3}{d(b+a\cos(c+dx))^3}$$
$$+ \frac{(a^3+6a^2b+9ab^2+4b^3)\cos^3(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3}$$
$$+ \frac{(a^3-3a^2b+3ab^2-b^3)\cos^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\sec(c+dx))^3}{8d(b+a\cos(c+dx))^3}$$
$$+ \frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$
$$+ \frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$
$$+ \frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$
$$- \frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(d*(b + a*Cos[c + d*x])^3)
+ ((-a^3 - 3*a^2*b - 3*a*b^2 - b^3)*Cos[c + d*x]^3*Csc[(c + d*x)/2]^2*(a +
b*Sec[c + d*x])^3)/(8*d*(b + a*Cos[c + d*x])^3) + ((-a^3 + 6*a^2*b - 9*a*b^
2 + 4*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*
d*(b + a*Cos[c + d*x])^3) + ((-3*a^2*b - 2*b^3)*Cos[c + d*x]^3*Log[Cos[c +
d*x]]*(a + b*Sec[c + d*x])^3)/(d*(b + a*Cos[c + d*x])^3) + ((a^3 + 6*a^2*b
+ 9*a*b^2 + 4*b^3)*Cos[c + d*x]^3*Log[Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x]
)^3)/(2*d*(b + a*Cos[c + d*x])^3) + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*Cos[c
+ d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Sec[c + d*x])^3)/(8*d*(b + a*Cos[c + d*x
])^3) + (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x
])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a
+ b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])) + (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(
4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (3*a*
```

```
b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c
 + d*x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+3a\,b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx\cdot\right.}{d}$
default	$\frac{a^3\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+3a\,b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx\cdot\right.}{d}$
norman	$\frac{-\frac{a^3+3a^2b+3a\,b^2+b^3}{8d}+\frac{\left(a^3+15a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d}+\frac{\left(a^3-3a^2b+3a\,b^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8d}-\frac{\left(2a^3-3a^2b+30a\,b^2-9b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$
parallelrisch	$\frac{-192(1+\cos(2dx+2c))\left(a^2+\frac{2b^2}{3}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-192(1+\cos(2dx+2c))\left(a^2+\frac{2b^2}{3}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+32(4b+a}{}$
risch	$\frac{a^3e^{7i(dx+c)}+9a\,b^2e^{7i(dx+c)}+6a^2b\,e^{6i(dx+c)}+4b^3e^{6i(dx+c)}+3a^3e^{5i(dx+c)}+3a\,b^2e^{5i(dx+c)}+12a^2b\,e^{4i(dx+c)}+3a^3e^{3i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^2\left(e^{2i(dx+c)}-1\right)^2}$

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2*cot(d*x+c)*csc(d*x+c)+1/2*ln(-cot(d*x+c)+csc(d*x+c)))+3*a^2*
b*(-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+3*a*b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+
3/2/cos(d*x+c)+3/2*ln(-cot(d*x+c)+csc(d*x+c)))+b^3*(1/2/sin(d*x+c)^2/cos(d*
x+c)^2-1/sin(d*x+c)^2+2*ln(tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.79

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx =$$
$$-\frac{12\,ab^2\cos\left(dx+c\right)-2\left(a^3+9\,ab^2\right)\cos\left(dx+c\right)^3+2\,b^3-2\left(3\,a^2b+2\,b^3\right)\cos\left(dx+c\right)^2+4\left(\left(3\,a^2b+2\,b^3\right)\right.}{}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(12*a*b^2*cos(d*x + c) - 2*(a^3 + 9*a*b^2)*cos(d*x + c)^3 + 2*b^3 - 2*
(3*a^2*b + 2*b^3)*cos(d*x + c)^2 + 4*((3*a^2*b + 2*b^3)*cos(d*x + c)^4 - (3
*a^2*b + 2*b^3)*cos(d*x + c)^2)*log(-cos(d*x + c)) + ((a^3 - 6*a^2*b + 9*a*
b^2 - 4*b^3)*cos(d*x + c)^4 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(d*x + c
)^2)*log(1/2*cos(d*x + c) + 1/2) - ((a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(d
```

```
*x + c)^4 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(d*x + c)^2)*log(-1/2*cos(
d*x + c) + 1/2))/(d*cos(d*x + c)^4 - d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec(c+dx))^3\csc^3(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{(a^3-6a^2b+9ab^2-4b^3)\log(\cos(dx+c)+1)-(a^3+6a^2b+9ab^2+4b^3)\log(\cos(dx+c)-1)+4}{4d}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] -1/4*((a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*log(cos(d*x + c) + 1) - (a^3 + 6*a^
2*b + 9*a*b^2 + 4*b^3)*log(cos(d*x + c) - 1) + 4*(3*a^2*b + 2*b^3)*log(cos(
d*x + c)) + 2*(6*a*b^2*cos(d*x + c) - (a^3 + 9*a*b^2)*cos(d*x + c)^3 + b^3
- (3*a^2*b + 2*b^3)*cos(d*x + c)^2)/(cos(d*x + c)^4 - cos(d*x + c)^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(154) = 308.

Time = 0.38 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.98

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-2(a^3+6a^2b+9ab^2+4b^3)\log\left(\frac{|-}{|}\right.}{}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^2*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 3*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)
*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + 8*(3*a^2*b + 2*b^3)*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^3 + 3*a^2*b + 3*a*b
^2 + b^3 - 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^2*b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 18*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 8*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d
*x + c) - 1) - 4*(9*a^2*b + 12*a*b^2 + 6*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b^3
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(a+b\right)^2\,\left(a+4\,b\right)}{4\,d}-\frac{\ln\left(\cos\left(c+d\,x\right)\right)\,\left(3\,a^2\,b+2\,b^3\right)}{d}\\ &\quad-\frac{\cos\left(c+d\,x\right)^3\,\left(\frac{a^3}{2}+\frac{9\,a\,b^2}{2}\right)-\frac{b^3}{2}+\cos\left(c+d\,x\right)^2\,\left(\frac{3\,a^2\,b}{2}+b^3\right)-3\,a\,b^2\,\cos\left(c+d\,x\right)}{d\,\left(\cos\left(c+d\,x\right)^2-\cos\left(c+d\,x\right)^4\right)}\\ &\quad-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(a-b\right)^2\,\left(a-4\,b\right)}{4\,d}\end{aligned}$$

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^3,x)
```

```
[Out] (log(cos(c + d*x) - 1)*(a + b)^2*(a + 4*b))/(4*d) - (log(cos(c + d*x))*(3*a
^2*b + 2*b^3))/d - (cos(c + d*x)^3*((9*a*b^2)/2 + a^3/2) - b^3/2 + cos(c +
d*x)^2*((3*a^2*b)/2 + b^3) - 3*a*b^2*cos(c + d*x))/(d*(cos(c + d*x)^2 - cos
(c + d*x)^4)) - (log(cos(c + d*x) + 1)*(a - b)^2*(a - 4*b))/(4*d)
```

3.190 $\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 299

$$\begin{aligned}\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx &= \frac{5a^3x}{16}-\frac{45}{8}ab^2x+\frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d}\\&-\frac{5b^3\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{3a^2b\sin(c+dx)}{d}\\&+\frac{5b^3\sin(c+dx)}{2d}-\frac{5a^3\cos(c+dx)\sin(c+dx)}{16d}\\&-\frac{a^2b\sin^3(c+dx)}{d}+\frac{5b^3\sin^3(c+dx)}{6d}\\&-\frac{5a^3\cos(c+dx)\sin^3(c+dx)}{24d}-\frac{3a^2b\sin^5(c+dx)}{5d}\\&-\frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d}+\frac{45ab^2\tan(c+dx)}{8d}\\&-\frac{15ab^2\sin^2(c+dx)\tan(c+dx)}{8d}\\&-\frac{3ab^2\sin^4(c+dx)\tan(c+dx)}{4d}\\&+\frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}\end{aligned}$$

```
[Out] 5/16*a^3*x-45/8*a*b^2*x+3*a^2*b*arctanh(sin(d*x+c))/d-5/2*b^3*arctanh(sin(d
*x+c))/d-3*a^2*b*sin(d*x+c)/d+5/2*b^3*sin(d*x+c)/d-5/16*a^3*cos(d*x+c)*sin(
d*x+c)/d-a^2*b*sin(d*x+c)^3/d+5/6*b^3*sin(d*x+c)^3/d-5/24*a^3*cos(d*x+c)*si
n(d*x+c)^3/d-3/5*a^2*b*sin(d*x+c)^5/d-1/6*a^3*cos(d*x+c)*sin(d*x+c)^5/d+45/
8*a*b^2*tan(d*x+c)/d-15/8*a*b^2*sin(d*x+c)^2*tan(d*x+c)/d-3/4*a*b^2*sin(d*x
+c)^4*tan(d*x+c)/d+1/2*b^3*sin(d*x+c)^3*tan(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3957, 2991, 2715, 8, 2672, 308, 212, 2671, 294, 327, 209}

$$\begin{aligned}\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx = &-\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}\\ &-\frac{5a^3\sin^3(c+dx)\cos(c+dx)}{24d}\\ &-\frac{5a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^3x}{16}\\ &+\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}-\frac{3a^2b\sin^5(c+dx)}{5d}\\ &-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}\\ &+\frac{45ab^2\tan(c+dx)}{8d}-\frac{3ab^2\sin^4(c+dx)\tan(c+dx)}{4d}\\ &-\frac{15ab^2\sin^2(c+dx)\tan(c+dx)}{8d}-\frac{45}{8}ab^2x\\ &-\frac{5b^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{5b^3\sin^3(c+dx)}{6d}\\ &+\frac{5b^3\sin(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}\end{aligned}$$

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] (5*a^3*x)/16 - (45*a*b^2*x)/8 + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*
ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*b*Sin[c + d*x])/d + (5*b^3*Sin[c + d*
x])/(2*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (a^2*b*Sin[c + d*x]^
3)/d + (5*b^3*Sin[c + d*x]^3)/(6*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(
24*d) - (3*a^2*b*Sin[c + d*x]^5)/(5*d) - (a^3*Cos[c + d*x]*Sin[c + d*x]^5)/
(6*d) + (45*a*b^2*Tan[c + d*x])/(8*d) - (15*a*b^2*Sin[c + d*x]^2*Tan[c + d*
x])/(8*d) - (3*a*b^2*Sin[c + d*x]^4*Tan[c + d*x])/(4*d) + (b^3*Sin[c + d*x]
^3*Tan[c + d*x]^2)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))^3\sin^3(c+dx)\tan^3(c+dx)\,dx\\
&= -\int \left(-a^3\sin^6(c+dx)-3a^2b\sin^5(c+dx)\tan(c+dx)\right.\\
&\qquad\left.-3ab^2\sin^4(c+dx)\tan^2(c+dx)-b^3\sin^3(c+dx)\tan^3(c+dx)\right)\,dx\\
&= a^3\int\sin^6(c+dx)\,dx+\left(3a^2b\right)\int\sin^5(c+dx)\tan(c+dx)\,dx\\
&\quad+\left(3ab^2\right)\int\sin^4(c+dx)\tan^2(c+dx)\,dx+b^3\int\sin^3(c+dx)\tan^3(c+dx)\,dx\\
&= -\frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d}+\frac{1}{6}\left(5a^3\right)\int\sin^4(c+dx)\,dx\\
&\quad+\frac{(3a^2b)\,\text{Subst}\left(\int\frac{x^6}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\
&\quad+\frac{(3ab^2)\,\text{Subst}\left(\int\frac{x^6}{(1+x^2)^3}\,dx,x,\tan(c+dx)\right)}{d}\\
&\quad+\frac{b^3\text{Subst}\left(\int\frac{x^6}{(1-x^2)^2}\,dx,x,\sin(c+dx)\right)}{d}
\end{aligned}
$$

$$= -\frac{5a^3\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d} - \frac{3ab^2\sin^4(c+dx)\tan(c+dx)}{4d} + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d} + \frac{1}{8}(5a^3)\int\sin^2(c+dx)\,dx + \frac{(3a^2b)\,\text{Subst}\left(\int\left(-1-x^2-x^4+\frac{1}{1-x^2}\right)\,dx,x,\sin(c+dx)\right)}{d} + \frac{(15ab^2)\,\text{Subst}\left(\int\frac{x^4}{(1+x^2)^2}\,dx,x,\tan(c+dx)\right)}{4d} - \frac{(5b^3)\,\text{Subst}\left(\int\frac{x^4}{1-x^2}\,dx,x,\sin(c+dx)\right)}{2d}$$

$$= -\frac{3a^2b\sin(c+dx)}{d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{16d} - \frac{a^2b\sin^3(c+dx)}{d} - \frac{5a^3\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{3a^2b\sin^5(c+dx)}{5d} - \frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d} - \frac{15ab^2\sin^2(c+dx)\tan(c+dx)}{8d} - \frac{3ab^2\sin^4(c+dx)\tan(c+dx)}{4d} + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d} + \frac{1}{16}(5a^3)\int 1\,dx + \frac{(3a^2b)\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d} + \frac{(45ab^2)\,\text{Subst}\left(\int\frac{x^2}{1+x^2}\,dx,x,\tan(c+dx)\right)}{8d} - \frac{(5b^3)\,\text{Subst}\left(\int\left(-1-x^2+\frac{1}{1-x^2}\right)\,dx,x,\sin(c+dx)\right)}{2d}$$

$$= \frac{5a^3x}{16} + \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} - \frac{3a^2b\sin(c+dx)}{d} + \frac{5b^3\sin(c+dx)}{2d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{16d} - \frac{a^2b\sin^3(c+dx)}{d} + \frac{5b^3\sin^3(c+dx)}{6d} - \frac{5a^3\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{3a^2b\sin^5(c+dx)}{5d} - \frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{45ab^2\tan(c+dx)}{8d} - \frac{15ab^2\sin^2(c+dx)\tan(c+dx)}{8d} - \frac{3ab^2\sin^4(c+dx)\tan(c+dx)}{4d} + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d} - \frac{(45ab^2)\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\tan(c+dx)\right)}{8d} - \frac{(5b^3)\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{2d}$$

$$
\begin{aligned}
&= \frac{5a^3x}{16} - \frac{45}{8}ab^2x + \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{5b^3\operatorname{arctanh}(\sin(c+dx))}{2d} \\
&\quad - \frac{3a^2b\sin(c+dx)}{d} + \frac{5b^3\sin(c+dx)}{2d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{16d} \\
&\quad - \frac{a^2b\sin^3(c+dx)}{d} + \frac{5b^3\sin^3(c+dx)}{6d} - \frac{5a^3\cos(c+dx)\sin^3(c+dx)}{24d} \\
&\quad - \frac{3a^2b\sin^5(c+dx)}{5d} - \frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d} \\
&\quad + \frac{45ab^2\tan(c+dx)}{8d} - \frac{15ab^2\sin^2(c+dx)\tan(c+dx)}{8d} \\
&\quad - \frac{3ab^2\sin^4(c+dx)\tan(c+dx)}{4d} + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 818 vs. $2(299) = 598$.

Time = 7.09 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.74

$$\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx$$
$$= \frac{5a(a^2-18b^2)(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^3}{16d(b+a\cos(c+dx))^3}$$
$$+\frac{(-6a^2b+5b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3}$$
$$+\frac{(6a^2b-5b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3}$$
$$+\frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$
$$+\frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$
$$-\frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$
$$+\frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$
$$+\frac{3b(-11a^2+6b^2)\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{8d(b+a\cos(c+dx))^3}$$
$$-\frac{3a(5a^2-32b^2)\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(2(c+dx))}{64d(b+a\cos(c+dx))^3}$$
$$-\frac{b(-21a^2+4b^2)\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(3(c+dx))}{48d(b+a\cos(c+dx))^3}$$
$$+\frac{3a(a^2-2b^2)\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(4(c+dx))}{64d(b+a\cos(c+dx))^3}$$
$$-\frac{3a^2b\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(5(c+dx))}{80d(b+a\cos(c+dx))^3}$$
$$-\frac{a^3\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(6(c+dx))}{192d(b+a\cos(c+dx))^3}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*(a^2 - 18*b^2)*(c + d*x)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(16*d*
(b + a*Cos[c + d*x])^3) + ((-6*a^2*b + 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x]
)^3) + ((6*a^2*b - 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])^3) + (b^3*Cos[c +
d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*S
in[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x
```

```
)/2])) - (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*
x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a
 + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])) + (3*b*(-11*a^2 + 6*b^2)*Cos[c + d*x]^3*(a + b
*Sec[c + d*x])^3*Sin[c + d*x])/(8*d*(b + a*Cos[c + d*x])^3) - (3*a*(5*a^2 -
 32*b^2)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[2*(c + d*x)])/(64*d*(b +
 a*Cos[c + d*x])^3) - (b*(-21*a^2 + 4*b^2)*Cos[c + d*x]^3*(a + b*Sec[c + d*
x])^3*Sin[3*(c + d*x)])/(48*d*(b + a*Cos[c + d*x])^3) + (3*a*(a^2 - 2*b^2)*
Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[4*(c + d*x)])/(64*d*(b + a*Cos[c
+ d*x])^3) - (3*a^2*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[5*(c + d*x)
])/(80*d*(b + a*Cos[c + d*x])^3) - (a^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])
^3*Sin[6*(c + d*x)])/(192*d*(b + a*Cos[c + d*x])^3)
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+3a^2b\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec\right.}{}$
default	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+3a^2b\left(-\frac{\sin(dx+c)^5}{5}-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec\right.}{}$
parts	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b^3\left(\frac{\sin(dx+c)^7}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{2}+\frac{5\sin(dx+c)^3}{6}+\frac{5\sin\right.}{d}$
parallelrisch	$\frac{-5760(1+\cos(2dx+2c))\left(a^2-\frac{5b^2}{6}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+5760(1+\cos(2dx+2c))\left(a^2-\frac{5b^2}{6}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+600aa}{}$
risch	$\frac{5a^3x}{16}-\frac{45a\,b^2x}{8}-\frac{15i\mathrm{e}^{-2i(dx+c)}a^3}{128d}-\frac{9i\mathrm{e}^{i(dx+c)}b^3}{8d}+\frac{7i\mathrm{e}^{-3i(dx+c)}a^2b}{32d}+\frac{15i\mathrm{e}^{2i(dx+c)}a^3}{128d}-\frac{7i\mathrm{e}^{3i(dx+c)}a^2b}{32d}+3$
norman	$\frac{(\frac{5}{16}a^3-\frac{45}{8}a\,b^2)x+(-\frac{25}{8}a^3+\frac{225}{4}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-\frac{5}{4}a^3+\frac{45}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-\frac{5}{4}a^3+\frac{45}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{}$

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5
/16*d*x+5/16*c)+3*a^2*b*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(s
ec(d*x+c)+tan(d*x+c)))+3*a*b^2*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*s
in(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+b^3*(1/2*sin(d*x+c
)^7/cos(d*x+c)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5/2*sin(d*x+c)-5/2*ln(se
c(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.81

$$\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx$$
$$=\frac{75\,(a^3-18\,ab^2)dx\cos{(dx+c)}^2+60\,(6\,a^2b-5\,b^3)\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-60\,(6\,a^2b-5\,b^3)\,c}{}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*(a^3 - 18*a*b^2)*d*x*cos(d*x + c)^2 + 60*(6*a^2*b - 5*b^3)*cos(d*
x + c)^2*log(sin(d*x + c) + 1) - 60*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) - (40*a^3*cos(d*x + c)^7 + 144*a^2*b*cos(d*x + c)^6 - 10*(
13*a^3 - 18*a*b^2)*cos(d*x + c)^5 - 16*(33*a^2*b - 5*b^3)*cos(d*x + c)^4 -
720*a*b^2*cos(d*x + c) + 15*(11*a^3 - 54*a*b^2)*cos(d*x + c)^3 - 120*b^3 +
16*(69*a^2*b - 35*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.81

$$\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx$$
$$=\frac{5\left(4\sin{(2\,dx+2\,c)}^3+60\,dx+60\,c+9\sin{(4\,dx+4\,c)}-48\sin{(2\,dx+2\,c)}\right)a^3-96\left(6\sin{(dx+c)}^5+10\right.}{}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a^3 - 96*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^2*b - 360*(1
```

```
5*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(
d*x + c)^2 + 1) - 8*tan(d*x + c))*a*b^2 + 80*(4*sin(d*x + c)^3 - 6*sin(d*x
+ c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c)
- 1) + 24*sin(d*x + c))*b^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(273) = 546$.

Time = 0.44 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.88

$$\int (a + b\sec(c + dx))^3 \sin^6(c + dx)\,dx$$

$$= \frac{75\,(a^3 - 18\,ab^2)(dx + c) + 120\,(6\,a^2b - 5\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 120\,(6\,a^2b - 5\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(75*(a^3 - 18*a*b^2)*(d*x + c) + 120*(6*a^2*b - 5*b^3)*log(abs(tan(1/
2*d*x + 1/2*c) + 1)) - 120*(6*a^2*b - 5*b^3)*log(abs(tan(1/2*d*x + 1/2*c) -
 1)) - 240*(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6
*a*b^2*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^2 + 2*(75*a^3*tan(1/2*d*x + 1/2*c)^11 - 720*a^2*b*tan(1/2*d*x + 1
/2*c)^11 - 630*a*b^2*tan(1/2*d*x + 1/2*c)^11 + 480*b^3*tan(1/2*d*x + 1/2*c)
^11 + 425*a^3*tan(1/2*d*x + 1/2*c)^9 - 4560*a^2*b*tan(1/2*d*x + 1/2*c)^9 -
2610*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 2720*b^3*tan(1/2*d*x + 1/2*c)^9 + 990*a
^3*tan(1/2*d*x + 1/2*c)^7 - 12384*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 1980*a*b^2
*tan(1/2*d*x + 1/2*c)^7 + 5760*b^3*tan(1/2*d*x + 1/2*c)^7 - 990*a^3*tan(1/2
*d*x + 1/2*c)^5 - 12384*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 1980*a*b^2*tan(1/2*d
*x + 1/2*c)^5 + 5760*b^3*tan(1/2*d*x + 1/2*c)^5 - 425*a^3*tan(1/2*d*x + 1/2
*c)^3 - 4560*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 2610*a*b^2*tan(1/2*d*x + 1/2*c)
^3 + 2720*b^3*tan(1/2*d*x + 1/2*c)^3 - 75*a^3*tan(1/2*d*x + 1/2*c) - 720*a^
2*b*tan(1/2*d*x + 1/2*c) + 630*a*b^2*tan(1/2*d*x + 1/2*c) + 480*b^3*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.25

$$\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx = \frac{7\,b^3\,\sin\left(c+d\,x\right)}{3\,d}+\frac{5\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,d}-\frac{5\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{13\,a^3\,{\cos\left(c+d\,x\right)}^3\,\sin\left(c+d\,x\right)}{24\,d}-\frac{a^3\,{\cos\left(c+d\,x\right)}^5\,\sin\left(c+d\,x\right)}{6\,d}+\frac{b^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}-\frac{b^3\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{3\,d}-\frac{11\,a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{16\,d}-\frac{23\,a^2\,b\,\sin\left(c+d\,x\right)}{5\,d}-\frac{45\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}+\frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{27\,a\,b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{11\,a^2\,b\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{5\,d}-\frac{3\,a\,b^2\,{\cos\left(c+d\,x\right)}^3\,\sin\left(c+d\,x\right)}{4\,d}-\frac{3\,a^2\,b\,{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{5\,d}$$

```
[In] int(sin(c + d*x)^6*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (7*b^3*sin(c + d*x))/(3*d) + (5*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2)))/(8*d) - (5*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (13*
a^3*cos(c + d*x)^3*sin(c + d*x))/(24*d) - (a^3*cos(c + d*x)^5*sin(c + d*x))
/(6*d) + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) - (b^3*cos(c + d*x)^2*sin(
c + d*x))/(3*d) - (11*a^3*cos(c + d*x)*sin(c + d*x))/(16*d) - (23*a^2*b*sin
(c + d*x))/(5*d) - (45*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(
```

```
4*d) + (6*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (27*a*b^2
*cos(c + d*x)*sin(c + d*x))/(8*d) + (3*a*b^2*sin(c + d*x))/(d*cos(c + d*x))
 + (11*a^2*b*cos(c + d*x)^2*sin(c + d*x))/(5*d) - (3*a*b^2*cos(c + d*x)^3*s
in(c + d*x))/(4*d) - (3*a^2*b*cos(c + d*x)^4*sin(c + d*x))/(5*d)
```

3.191 $\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 236

$$\begin{aligned}\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx &= \frac{3}{8}a\left(a^2-12b^2\right)x+\frac{3b(2a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{2d}\\ &\quad-\frac{b(17a^2-b^2)\sin(c+dx)}{2d}\\ &\quad-\frac{a(21a^2-2b^2)\cos(c+dx)\sin(c+dx)}{8d}\\ &\quad-\frac{(6a^2-b^2)\,(b+a\cos(c+dx))^2\sin(c+dx)}{4bd}\\ &\quad-\frac{(4a^2-b^2)\,(b+a\cos(c+dx))^3\sin(c+dx)}{4b^2d}\\ &\quad+\frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d}\\ &\quad+\frac{(b+a\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2bd}\end{aligned}$$

```
[Out] 3/8*a*(a^2-12*b^2)*x+3/2*b*(2*a^2-b^2)*arctanh(sin(d*x+c))/d-1/2*b*(17*a^2-
b^2)*sin(d*x+c)/d-1/8*a*(21*a^2-2*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/4*(6*a^2-b
^2)*(b+a*cos(d*x+c))^2*sin(d*x+c)/b/d-1/4*(4*a^2-b^2)*(b+a*cos(d*x+c))^3*si
n(d*x+c)/b^2/d+a*(b+a*cos(d*x+c))^4*tan(d*x+c)/b^2/d+1/2*(b+a*cos(d*x+c))^4
*sec(d*x+c)*tan(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2972, 3128, 3112, 3102, 2814, 3855}

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx = \frac{3b(2a^2-b^2)\,\text{arctanh}(\sin(c+dx))}{2d} - \frac{b(17a^2-b^2)\sin(c+dx)}{2d} - \frac{(4a^2-b^2)\sin(c+dx)(a\cos(c+dx)+b)^3}{4b^2d} - \frac{(6a^2-b^2)\sin(c+dx)(a\cos(c+dx)+b)^2}{4bd} - \frac{a(21a^2-2b^2)\sin(c+dx)\cos(c+dx)}{8d} + \frac{3}{8}ax\left(a^2-12b^2\right) + \frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d} + \frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(a^2 - 12*b^2)*x)/8 + (3*b*(2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d)
- (b*(17*a^2 - b^2)*Sin[c + d*x])/(2*d) - (a*(21*a^2 - 2*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(8*d) - ((6*a^2 - b^2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(
4*b*d) - ((4*a^2 - b^2)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*b^2*d) + (a
*(b + a*Cos[c + d*x])^4*Tan[c + d*x])/(b^2*d) + ((b + a*Cos[c + d*x])^4*Sec
[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
```

```
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))^3\sin(c+dx)\tan^3(c+dx)\,dx \\
&= \frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d}+\frac{(b+a\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2bd} \\
&\quad+\frac{\int(-b-a\cos(c+dx))^3\left(-3(2a^2-b^2)+3ab\cos(c+dx)+2(4a^2-b^2)\cos^2(c+dx)\right)\sec(c+dx)\,d}{2b^2} \\
&= -\frac{(4a^2-b^2)(b+a\cos(c+dx))^3\sin(c+dx)}{4b^2d}+\frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d} \\
&\quad+\frac{(b+a\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2bd} \\
&\quad+\frac{\int(-b-a\cos(c+dx))^2\left(12b(2a^2-b^2)-18ab^2\cos(c+dx)-6b(6a^2-b^2)\cos^2(c+dx)\right)\sec(c+d}{8b^2} \\
&= -\frac{(6a^2-b^2)(b+a\cos(c+dx))^2\sin(c+dx)}{4bd}-\frac{(4a^2-b^2)(b+a\cos(c+dx))^3\sin(c+dx)}{4b^2d} \\
&\quad+\frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d}+\frac{(b+a\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2bd} \\
&\quad+\frac{\int(-b-a\cos(c+dx))\left(-36b^2(2a^2-b^2)+78ab^3\cos(c+dx)+6b^2(21a^2-2b^2)\cos^2(c+dx)\right)\sec}{24b^2} \\
&= -\frac{a(21a^2-2b^2)\cos(c+dx)\sin(c+dx)}{8d} \\
&\quad-\frac{(6a^2-b^2)(b+a\cos(c+dx))^2\sin(c+dx)}{4bd} \\
&\quad-\frac{(4a^2-b^2)(b+a\cos(c+dx))^3\sin(c+dx)}{4b^2d}+\frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d} \\
&\quad+\frac{(b+a\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2bd} \\
&\quad+\frac{\int\left(72b^3(2a^2-b^2)+18ab^2(a^2-12b^2)\cos(c+dx)-24b^3(17a^2-b^2)\cos^2(c+dx)\right)\sec(c+dx)\,dx}{48b^2} \\
&= -\frac{b(17a^2-b^2)\sin(c+dx)}{2d}-\frac{a(21a^2-2b^2)\cos(c+dx)\sin(c+dx)}{8d} \\
&\quad-\frac{(6a^2-b^2)(b+a\cos(c+dx))^2\sin(c+dx)}{4bd} \\
&\quad-\frac{(4a^2-b^2)(b+a\cos(c+dx))^3\sin(c+dx)}{4b^2d}+\frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d} \\
&\quad+\frac{(b+a\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2bd} \\
&\quad+\frac{\int\left(72b^3(2a^2-b^2)+18ab^2(a^2-12b^2)\cos(c+dx)\right)\sec(c+dx)\,dx}{48b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3}{8}a\left(a^2-12b^2\right)x - \frac{b(17a^2-b^2)\sin(c+dx)}{2d} - \frac{a(21a^2-2b^2)\cos(c+dx)\sin(c+dx)}{8d} \\
&\quad - \frac{(6a^2-b^2)\,(b+a\cos(c+dx))^2\sin(c+dx)}{4bd} \\
&\quad - \frac{(4a^2-b^2)\,(b+a\cos(c+dx))^3\sin(c+dx)}{4b^2d} + \frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d} \\
&\quad + \frac{(b+a\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2bd} + \frac{1}{2}\left(3b\left(2a^2-b^2\right)\right)\int \sec(c+dx)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3}{8}a\left(a^2-12b^2\right)x + \frac{3b(2a^2-b^2)\,\text{arctanh}(\sin(c+dx))}{2d} \\
&\quad - \frac{b(17a^2-b^2)\sin(c+dx)}{2d} - \frac{a(21a^2-2b^2)\cos(c+dx)\sin(c+dx)}{8d} \\
&\quad - \frac{(6a^2-b^2)\,(b+a\cos(c+dx))^2\sin(c+dx)}{4bd} \\
&\quad - \frac{(4a^2-b^2)\,(b+a\cos(c+dx))^3\sin(c+dx)}{4b^2d} + \frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d} \\
&\quad + \frac{(b+a\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2bd}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 696 vs. $2(236) = 472$.

Time = 6.75 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.95

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$$
$$=\frac{3a(a^2-12b^2)(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^3}{8d(b+a\cos(c+dx))^3}$$
$$+\frac{3(-2a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3}$$
$$-\frac{3(-2a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3}$$
$$+\frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$
$$+\frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$
$$-\frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$
$$+\frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$
$$+\frac{b(-15a^2+4b^2)\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{4d(b+a\cos(c+dx))^3}$$
$$-\frac{a(a^2-3b^2)\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(2(c+dx))}{4d(b+a\cos(c+dx))^3}$$
$$+\frac{a^2b\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(3(c+dx))}{4d(b+a\cos(c+dx))^3}$$
$$+\frac{a^3\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(4(c+dx))}{32d(b+a\cos(c+dx))^3}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(a^2 - 12*b^2)*(c + d*x)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(8*d*(
b + a*Cos[c + d*x])^3) + (3*(-2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])
^3) - (3*(-2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])^3) + (b^3*Cos[c +
d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*S
in[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])) - (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*
x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a
 + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])) + (b*(-15*a^2 + 4*b^2)*Cos[c + d*x]^3*(a + b*S
ec[c + d*x])^3*Sin[c + d*x])/(4*d*(b + a*Cos[c + d*x])^3) - (a*(a^2 - 3*b^2
```

```
)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[2*(c + d*x)])/(4*d*(b + a*Cos[c
 + d*x])^3) + (a^2*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[3*(c + d*x)]
)/(4*d*(b + a*Cos[c + d*x])^3) + (a^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3
*Sin[4*(c + d*x)])/(32*d*(b + a*Cos[c + d*x])^3)
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3}{}$
default	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3}{}$
parts	$\frac{a^3\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^3\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan}{2}\right.}{d}$
parallelrisch	$\frac{-192(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{2}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+192(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{2}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+24adx(a}{}$
risch	$\frac{3a^3x}{8}-\frac{9a\,b^2x}{2}-\frac{i\,e^{i(dx+c)}b^3}{2d}+\frac{i\,e^{-3i(dx+c)}a^2b}{8d}+\frac{i\,e^{2i(dx+c)}a^3}{8d}-\frac{ib^2\left(b\,e^{3i(dx+c)}-6a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-6a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}$
norman	$\frac{(\frac{3}{8}a^3-\frac{9}{2}a\,b^2)x+(-\frac{3}{2}a^3+18a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-\frac{3}{8}a^3+\frac{9}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-\frac{3}{8}a^3+\frac{9}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(\frac{3}{4}a}{}$

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+3*a^
2*b*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*(sin(d
*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+
b^3*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(s
ec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.83

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$$
$$=\frac{3\left(a^3-12\,ab^2\right)dx\cos(dx+c)^2+6\left(2\,a^2b-b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-6\left(2\,a^2b-b^3\right)\cos(dx}{}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a^3 - 12*a*b^2)*d*x*cos(d*x + c)^2 + 6*(2*a^2*b - b^3)*cos(d*x + c)
^2*log(sin(d*x + c) + 1) - 6*(2*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x +
c) + 1) + (2*a^3*cos(d*x + c)^5 + 8*a^2*b*cos(d*x + c)^4 + 24*a*b^2*cos(d*x
 + c) - (5*a^3 - 12*a*b^2)*cos(d*x + c)^3 + 4*b^3 - 8*(4*a^2*b - b^3)*cos(d
*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx=\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.78

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$$
$$=\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))a^3-16\left(2\,\sin(dx+c)^3-3\,\log(\sin(dx+c)+1)+3\,\mathrm{l}\right.}{}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/32*((12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^3 - 16*(2*s
in(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d
*x + c))*a^2*b - 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*ta
n(d*x + c))*a*b^2 - 8*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(
d*x + c) + 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.83

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$$

$$= \frac{3\left(a^3-12\,ab^2\right)(dx+c)+12\left(2\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\left(2\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^3 - 12*a*b^2)*(d*x + c) + 12*(2*a^2*b - b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 12*(2*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 8*
(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(
1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2
 + 2*(3*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 12*a
*b^2*tan(1/2*d*x + 1/2*c)^7 + 8*b^3*tan(1/2*d*x + 1/2*c)^7 + 11*a^3*tan(1/2
*d*x + 1/2*c)^5 - 104*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 12*a*b^2*tan(1/2*d*x +
 1/2*c)^5 + 24*b^3*tan(1/2*d*x + 1/2*c)^5 - 11*a^3*tan(1/2*d*x + 1/2*c)^3 -
 104*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 24*b^
3*tan(1/2*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*x + 1/2*c) - 24*a^2*b*tan(1/2*d*
x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c) + 8*b^3*tan(1/2*d*x + 1/2*c))/(t
an(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.19

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx = \frac{b^3\,\sin(c+d\,x)}{d} + \frac{3\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d} - \frac{3\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{a^3\cos(c+d\,x)^3\,\sin(c+d\,x)}{4\,d} + \frac{b^3\,\sin(c+d\,x)}{2\,d\cos(c+d\,x)^2} - \frac{5\,a^3\,\cos(c+d\,x)\,\sin(c+d\,x)}{8\,d} - \frac{4\,a^2\,b\,\sin(c+d\,x)}{d} - \frac{9\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{3\,a\,b^2\,\cos(c+d\,x)\,\sin(c+d\,x)}{2\,d} + \frac{3\,a\,b^2\,\sin(c+d\,x)}{d\,\cos(c+d\,x)} + \frac{a^2\,b\cos(c+d\,x)^2\,\sin(c+d\,x)}{d}$$

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (b^3*sin(c + d*x))/d + (3*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
(4*d) - (3*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*cos(c
 + d*x)^3*sin(c + d*x))/(4*d) + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) - (
5*a^3*cos(c + d*x)*sin(c + d*x))/(8*d) - (4*a^2*b*sin(c + d*x))/d - (9*a*b^
2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a^2*b*atanh(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*cos(c + d*x)*sin(c + d*x))/(2*d
) + (3*a*b^2*sin(c + d*x))/(d*cos(c + d*x)) + (a^2*b*cos(c + d*x)^2*sin(c +
 d*x))/d
```

3.192 $\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx = \frac{1}{2}a\left(a^2-6b^2\right)x+\frac{b(6a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{15a^2b\sin(c+dx)}{2d}-\frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*a*(a^2-6*b^2)*x+1/2*b*(6*a^2-b^2)*arctanh(sin(d*x+c))/d-15/2*a^2*b*sin(
d*x+c)/d-5/2*a^3*cos(d*x+c)*sin(d*x+c)/d+3/2*a*(b+a*cos(d*x+c))^2*tan(d*x+c
)/d+1/2*(b+a*cos(d*x+c))^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3957, 2968, 3127, 3126, 3112, 3102, 2814, 3855}

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx = -\frac{5a^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{b(6a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{1}{2}ax\left(a^2-6b^2\right)-\frac{15a^2b\sin(c+dx)}{2d} + \frac{3a\tan(c+dx)(a\cos(c+dx)+b)^2}{2d} + \frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}$$

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (a*(a^2 - 6*b^2)*x)/2 + (b*(6*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (15
*a^2*b*Sin[c + d*x])/(2*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a
*(b + a*Cos[c + d*x])^2*Tan[c + d*x])/(2*d) + ((b + a*Cos[c + d*x])^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
```

```
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int (-b-a\cos(c+dx))^3 \sec(c+dx)\tan^2(c+dx)\,dx$$

$$= -\int (-b-a\cos(c+dx))^3 \left(1-\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$

$$= \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{2}\int (-b-a\cos(c+dx))^2\left(-3a+b\cos(c+dx)+4a\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$

$$= \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{2}\int(-b-a\cos(c+dx))\left(6a^2-b^2-5ab\cos(c+dx)-10a^2\cos^2(c+dx)\right)\sec(c+dx)\,dx$$

$$= -\frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{4}\int\left(-2b(6a^2-b^2)-2a(a^2-6b^2)\cos(c+dx)+30a^2b\cos^2(c+dx)\right)\sec(c+dx)\,dx$$

$$= -\frac{15a^2b\sin(c+dx)}{2d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{4}\int\left(-2b(6a^2-b^2)-2a(a^2-6b^2)\cos(c+dx)\right)\sec(c+dx)\,dx$$

$$= \frac{1}{2}a\left(a^2-6b^2\right)x - \frac{15a^2b\sin(c+dx)}{2d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\left(b(6a^2-b^2)\right)\int\sec(c+dx)\,dx$$

$$= \frac{1}{2}a\left(a^2-6b^2\right)x + \frac{b(6a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{15a^2b\sin(c+dx)}{2d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 327 vs. $2(138) = 276$.

Time = 1.20 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.37

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx$$

$$= \frac{\sec^2(c+dx)\left(a^3c-6ab^2c+a^3dx-6ab^2dx-6a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b^3\log\left(\cos\left(\frac{1}{2}(\right.\right.}{}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]^2*(a^3*c - 6*a*b^2*c + a^3*d*x - 6*a*b^2*d*x - 6*a^2*b*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] + b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 6*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - b^3*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + Cos[2*(c + d*x)]*(a*(a^2 - 6*b^2)*(c + d*x)
 + (-6*a^2*b + b^3)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - b*(-6*a^2 +
b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-3*a^2*b + 2*b^3)*Sin[c +
 d*x] - (a^3*Sin[2*(c + d*x)])/2 + 6*a*b^2*Sin[2*(c + d*x)] - 3*a^2*b*Sin[3
*(c + d*x)] - (a^3*Sin[4*(c + d*x)])/4))/(4*d)
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3a\,b^2(\tan(dx+c)-dx-c)+b^3\left(\frac{\sin}{2\cos}\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3a\,b^2(\tan(dx+c)-dx-c)+b^3\left(\frac{\sin}{2\cos}\right.}{d}$
parts	$\frac{a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{3a\,b^2(\tan(dx+c)-dx}{d}$
parallelrisch	$\frac{-24(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{6}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+24(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{6}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4adx(a^2-}{8d(1-}$
risch	$\frac{a^3x}{2}-3a\,b^2x+\frac{i\mathrm{e}^{2i(dx+c)}a^3}{8d}+\frac{3i\mathrm{e}^{i(dx+c)}a^2b}{2d}-\frac{3i\mathrm{e}^{-i(dx+c)}a^2b}{2d}-\frac{i\mathrm{e}^{-2i(dx+c)}a^3}{8d}-\frac{ib^2(b\,\mathrm{e}^{3i(dx+c)}-6a\,\mathrm{e}^{2i(d}}{d(\mathrm{e}^{2i(dx+}}$
norman	$\frac{(\frac{1}{2}a^3-3a\,b^2)x+(\frac{1}{2}a^3-3a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-a^3+6a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{\left(a^3-6a^2b-6a\,b^2+b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{3\left(a^3-\right.}{}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(1+\tan\right.}$

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(-sin(d*x+c)+ln
(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*(tan(d*x+c)-d*x-c)+b^3*(1/2*sin(d*x+c)^3/c
os(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx$$

$$=\frac{2\,(a^3-6\,ab^2)dx\cos\left(dx+c\right)^2+\left(6\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(6\,a^2b-b^3\right)\cos\left(dx+c\right)}{4\,d\cos\left(\right.}$$

[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="fricas")

[Out] 1/4*(2*(a^3 - 6*a*b^2)*d*x*cos(d*x + c)^2 + (6*a^2*b - b^3)*cos(d*x + c)^2*
log(sin(d*x + c) + 1) - (6*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x + c) +
1) - 2*(a^3*cos(d*x + c)^3 + 6*a^2*b*cos(d*x + c)^2 - 6*a*b^2*cos(d*x + c)
- b^3)*sin(d*x + c))/(d*cos(d*x + c)^2)

Sympy [F]

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx=\int (a+b\sec{(c+dx)})^3\sin^2{(c+dx)}\,dx$$

[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**2,x)

[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx$$
$$=\frac{(2\,dx+2\,c-\sin{(2\,dx+2\,c)})a^3-12\,(dx+c-\tan{(dx+c)})ab^2-b^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log{(\sin{(dx+c)}+1)}-\right.}{4\,d}$$

[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^3 - 12*(d*x + c - tan(d*x + c))*a*b
^2 - b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
) - 2*sin(d*x + c)))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(126) = 252.

Time = 0.36 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.51

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx$$
$$=\frac{(a^3-6\,ab^2)(dx+c)+(6\,a^2b-b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(6\,a^2b-b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+}{}$$

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 - 6*a*b^2)*(d*x + c) + (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(
1/2*d*x + 1/2*c)^7 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 6*a*b^2*tan(1/2*d*x +
 1/2*c)^7 + b^3*tan(1/2*d*x + 1/2*c)^7 - 3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a
^2*b*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 3*b^3*tan(1/
2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b*tan(1/2*d*x + 1/2
*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*b^3*tan(1/2*d*x + 1/2*c)^3 - a^3
*tan(1/2*d*x + 1/2*c) - 6*a^2*b*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*d*x
+ 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.46

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx = \frac{a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{b^3\,\sin\left(c+d\,x\right)}{2\,d\cos\left(c+d\,x\right)^2} - \frac{a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d} - \frac{3\,a^2\,b\,\sin\left(c+d\,x\right)}{d} - \frac{6\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (b^3*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) -
 (a^3*cos(c + d*x)*sin(c + d*x))/(2*d) - (3*a^2*b*sin(c + d*x))/d - (6*a*b^
2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a^2*b*atanh(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*sin(c + d*x))/(d*cos(c + d*x))
```

3.193 $\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{3a^2b\,\text{arctanh}(\sin(c+dx))}{d} + \frac{3b^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{a^3\cot(c+dx)}{d} - \frac{3ab^2\cot(c+dx)}{d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{3b^3\csc(c+dx)}{2d} + \frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d}$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d+3/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)/
d-3*a*b^2*cot(d*x+c)/d-3*a^2*b*csc(d*x+c)/d-3/2*b^3*csc(d*x+c)/d+1/2*b^3*cs
c(d*x+c)*sec(d*x+c)^2/d+3*a*b^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3957, 2991, 3852, 8, 2701, 327, 213, 2700, 14, 294}

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx = -\frac{a^3\cot(c+dx)}{d} + \frac{3a^2b\,\text{arctanh}(\sin(c+dx))}{d} - \frac{3a^2b\csc(c+dx)}{d} + \frac{3ab^2\tan(c+dx)}{d} - \frac{3ab^2\cot(c+dx)}{d} + \frac{3b^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{3b^3\csc(c+dx)}{2d} + \frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (3*a*b^2*Cot[c + d*x])/d - (3*a^2*b*Csc[c + d*x])/d -
 (3*b^3*Csc[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d) + (3*
a*b^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))^3\csc^2(c+dx)\sec^3(c+dx)\,dx \\
&= \int \left(a^3\csc^2(c+dx)+3a^2b\csc^2(c+dx)\sec(c+dx)+3ab^2\csc^2(c+dx)\sec^2(c+dx)\right. \\
&\qquad \left.+b^3\csc^2(c+dx)\sec^3(c+dx)\right)\,dx \\
&= a^3\int \csc^2(c+dx)\,dx+\left(3a^2b\right)\int \csc^2(c+dx)\sec(c+dx)\,dx \\
&\quad +\left(3ab^2\right)\int \csc^2(c+dx)\sec^2(c+dx)\,dx+b^3\int \csc^2(c+dx)\sec^3(c+dx)\,dx \\
&= -\frac{a^3\text{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{d}-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d} \\
&\quad +\frac{\left(3ab^2\right)\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\tan(c+dx)\right)}{d}-\frac{b^3\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx,x,\csc(c+dx)\right)}{d}
\end{aligned}$$

$$= -\frac{a^3\cot(c+dx)}{d} - \frac{3a^2b\csc(c+dx)}{d} + \frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d} - \frac{(3a^2b)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} + \frac{(3ab^2)\operatorname{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx, x, \tan(c+dx)\right)}{d} - \frac{(3b^3)\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{2d}$$

$$= \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a^3\cot(c+dx)}{d} - \frac{3ab^2\cot(c+dx)}{d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{3b^3\csc(c+dx)}{2d} + \frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d} - \frac{(3b^3)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{2d}$$

$$= \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{3b^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{a^3\cot(c+dx)}{d} - \frac{3ab^2\cot(c+dx)}{d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{3b^3\csc(c+dx)}{2d} + \frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 406 vs. 2(133) = 266.

Time = 2.44 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.05

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx = -\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(12a^2b+2b^3+6a(a^2+2b^2)\cos(c+dx)+6(2a^2b+b^3)\cos(2(c+dx))\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/16*(Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]*(12*a^2*b + 2*b^3 + 6*a*(a^2 + 2
*b^2)*Cos[c + d*x] + 6*(2*a^2*b + b^3)*Cos[2*(c + d*x)] + 2*a^3*Cos[3*(c +
d*x)] + 12*a*b^2*Cos[3*(c + d*x)] + 6*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]]*Sin[c + d*x] + 3*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin
[c + d*x] - 6*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] -
 3*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 6*a^2*b*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 3*b^3*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 6*a^2*b*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 3*b^3*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]]*Sin[3*(c + d*x)]))/(d*(-1 + Cot[(c + d*x)/2]^2)^2)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-a^3\cot(dx+c)+3a^2b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+3a\,b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+b^3\left(\frac{\cdots}{2\sin(dx+c\cdots}\right.}{d}$
default	$\frac{-a^3\cot(dx+c)+3a^2b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+3a\,b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+b^3\left(\frac{\cdots}{2\sin(dx+c\cdots}\right.}{d}$
parallelrisch	$\frac{-3(1+\cos(2dx+2c))\left(a^2+\frac{b^2}{2}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3(1+\cos(2dx+2c))\left(a^2+\frac{b^2}{2}\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{3\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\csc\cdots}{\cdots}}{d(1+\cos(2dx+2c))}$
norman	$\frac{-\frac{a^3+3a^2b+3a\,b^2+b^3}{2d}+\frac{\left(a^3-3a^2b+3a\,b^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2d}-\frac{3\left(a^3-a^2b+7a\,b^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2d}+\frac{3\left(a^3+a^2b+7a\,b^2+b^3\right)\tan\left(\frac{dx}{2}\cdots\right.}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$
risch	$-\frac{i\left(6a^2b\,e^{5i(dx+c)}+3b^3e^{5i(dx+c)}+2a^3e^{4i(dx+c)}+12a^2b\,e^{3i(dx+c)}+2b^3e^{3i(dx+c)}+4a^3e^{2i(dx+c)}+12a\,b^2e^{2i(dx+c)}+6a^2b\,e^{i(\cdots}\right.}{d\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)^2}$

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*cot(d*x+c)+3*a^2*b*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a*
b^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+b^3*(1/2/sin(d*x+c)/cos(d*x+c)^2
-3/2/sin(d*x+c)+3/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{3\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-3\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\cdots\right.\right.}{4\,d\cos\left(dx+c\right)\cdots}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(2*a^2*b + b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1)*sin(d*x + c) -
3*(2*a^2*b + b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) + 12*a
*b^2*cos(d*x + c) - 4*(a^3 + 6*a*b^2)*cos(d*x + c)^3 + 2*b^3 - 6*(2*a^2*b +
 b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec{(c+dx)})^3\csc^2{(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{b^3\left(\frac{2\left(3\sin(dx+c)^2-2\right)}{\sin(dx+c)^3-\sin(dx+c)} - 3\log\left(\sin\left(dx+c\right)+1\right) + 3\log\left(\sin\left(dx+c\right)-1\right)\right) + 6a^2b\left(\frac{2}{\sin(dx+c)} - \log\left(\sin\right.\right.}{4d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*(3*sin(d*x + c)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 6*a^2*b*(2/sin(d*x + c) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a*b^2*(1/tan(d*x + c) -
tan(d*x + c)) + 4*a^3/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.69

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$= \frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3\left(2a^2b+b^3\right)\mathrm{l}}{}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*tan(1/2*d*x + 1/2*c) - 3*a^2*b*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan(
1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c) + 3*(2*a^2*b + b^3)*log(abs(tan
(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)/tan(1/2*d*x + 1/2*c) - 2*(6*a*b^2*tan
(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/
2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.36

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(6\,a^2\,b+3\,b^3\right)}{d}$$
$$-\frac{3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^3+6\,a^2\,b+18\,a\,b^2+4\,b^3\right)+3\,a^2\,b+a^3+b^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^3+3\,a^2\,b+15\,a\right.}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a-b\right)^3}{2\,d}$$

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^2,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(6*a^2*b + 3*b^3))/d - (3*a*b^2 - tan(c/2 + (d*x
)/2)^2*(18*a*b^2 + 6*a^2*b + 2*a^3 + 4*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2
 + (d*x)/2)^4*(15*a*b^2 + 3*a^2*b + a^3 - b^3))/(d*(2*tan(c/2 + (d*x)/2) -
4*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5)) + (tan(c/2 + (d*x)/2)*(a
- b)^3)/(2*d)
```

3.194 $\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\begin{aligned}\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx = {} & \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} \\ & + \frac{5b^3\text{arctanh}(\sin(c+dx))}{2d} \\ & - \frac{a^3\cot(c+dx)}{d} - \frac{6ab^2\cot(c+dx)}{d} \\ & - \frac{a^3\cot^3(c+dx)}{3d} - \frac{ab^2\cot^3(c+dx)}{d} \\ & - \frac{3a^2b\csc(c+dx)}{d} - \frac{5b^3\csc(c+dx)}{2d} \\ & - \frac{a^2b\csc^3(c+dx)}{d} - \frac{5b^3\csc^3(c+dx)}{6d} \\ & + \frac{b^3\csc^3(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d}\end{aligned}$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d+5/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)/
d-6*a*b^2*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-a*b^2*cot(d*x+c)^3/d-3*a^2*b*
csc(d*x+c)/d-5/2*b^3*csc(d*x+c)/d-a^2*b*csc(d*x+c)^3/d-5/6*b^3*csc(d*x+c)^3
/d+1/2*b^3*csc(d*x+c)^3*sec(d*x+c)^2/d+3*a*b^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3957, 2991, 3852, 2701, 308, 213, 2700, 276, 294}

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx = -\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}-\frac{ab^2\cot^3(c+dx)}{d}-\frac{6ab^2\cot(c+dx)}{d}+\frac{5b^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{5b^3\csc^3(c+dx)}{6d}-\frac{5b^3\csc(c+dx)}{2d}+\frac{b^3\csc^3(c+dx)\sec^2(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (6*a*b^2*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d)
 - (a*b^2*Cot[c + d*x]^3)/d - (3*a^2*b*Csc[c + d*x])/d - (5*b^3*Csc[c + d*x
])/(2*d) - (a^2*b*Csc[c + d*x]^3)/d - (5*b^3*Csc[c + d*x]^3)/(6*d) + (b^3*C
sc[c + d*x]^3*Sec[c + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int (-b - a\cos(c+dx))^3 \csc^4(c+dx)\sec^3(c+dx)\,dx$$

$$= \int \left(a^3\csc^4(c+dx) + 3a^2b\csc^4(c+dx)\sec(c+dx) + 3ab^2\csc^4(c+dx)\sec^2(c+dx) + b^3\csc^4(c+dx)\sec^3(c+dx)\right)\,dx$$

$$= a^3 \int \csc^4(c+dx)\,dx + (3a^2b)\int \csc^4(c+dx)\sec(c+dx)\,dx + (3ab^2)\int \csc^4(c+dx)\sec^2(c+dx)\,dx + b^3\int \csc^4(c+dx)\sec^3(c+dx)\,dx$$

$$= -\frac{a^3\text{Subst}\left(\int (1+x^2)\,dx, x, \cot(c+dx)\right)}{d} - \frac{(3a^2b)\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} + \frac{(3ab^2)\,\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx, x, \tan(c+dx)\right)}{d} - \frac{b^3\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{b^3\csc^3(c+dx)\sec^2(c+dx)}{2d} - \frac{(3a^2b)\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(c+dx)\right)}{d} + \frac{(3ab^2)\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{2d}$$

$$= -\frac{a^3\cot(c+dx)}{d} - \frac{6ab^2\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} - \frac{ab^2\cot^3(c+dx)}{d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{a^2b\csc^3(c+dx)}{d} + \frac{b^3\csc^3(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d} - \frac{(3a^2b)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{(5b^3)\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(c+dx)\right)}{2d}$$

$$= \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} - \frac{a^3\cot(c+dx)}{d} - \frac{6ab^2\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} - \frac{ab^2\cot^3(c+dx)}{d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{5b^3\csc(c+dx)}{2d} - \frac{a^2b\csc^3(c+dx)}{d} - \frac{5b^3\csc^3(c+dx)}{6d} + \frac{b^3\csc^3(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{2d}$$

$$= \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} + \frac{5b^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{a^3\cot(c+dx)}{d} - \frac{6ab^2\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} - \frac{ab^2\cot^3(c+dx)}{d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{5b^3\csc(c+dx)}{2d} - \frac{a^2b\csc^3(c+dx)}{d} - \frac{5b^3\csc^3(c+dx)}{6d} + \frac{b^3\csc^3(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 610 vs. $2(205) = 410$.

Time = 1.44 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.98

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(84a^2b+22b^3+32a(a^2+3b^2)\cos(c+dx)+8(6a^2b+5b^3)\cos(2(c+\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/768*(Csc[(c + d*x)/2]^7*Sec[(c + d*x)/2]^3*(84*a^2*b + 22*b^3 + 32*a*(a^
2 + 3*b^2)*Cos[c + d*x] + 8*(6*a^2*b + 5*b^3)*Cos[2*(c + d*x)] + 4*a^3*Cos[
3*(c + d*x)] + 48*a*b^2*Cos[3*(c + d*x)] - 36*a^2*b*Cos[4*(c + d*x)] - 30*b
^3*Cos[4*(c + d*x)] - 4*a^3*Cos[5*(c + d*x)] - 48*a*b^2*Cos[5*(c + d*x)] +
36*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 30*b^3*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 36*a^2*b*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 30*b^3*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*Sin[c + d*x] + 18*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*Sin[3*(c + d*x)] + 15*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Si
n[3*(c + d*x)] - 18*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c
 + d*x)] - 15*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)]
 - 18*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*
b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 18*a^2*b*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*b^3*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(-1 + Cot[(c + d*x)/2]
^2)^2)
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a^3\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+3a^2b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+3a\,b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(}\right.}{}$
default	$\frac{a^3\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+3a^2b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+3a\,b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(}\right.}{}$
parallelrisch	$\frac{-576(1+\cos(2dx+2c))b\left(a^2+\frac{5b^2}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+576(1+\cos(2dx+2c))b\left(a^2+\frac{5b^2}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\csc\left(\frac{dx}{2}\right.}{}$
norman	$\frac{-\frac{a^3+3a^2b+3a\,b^2+b^3}{24d}+\frac{\left(a^3-3a^2b+3a\,b^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{24d}+\frac{\left(7a^3-39a^2b+57a\,b^2-25b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{24d}-\frac{\left(7a^3+39a^2b+57a\,b^2+25\right.}{24d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(-1+\tan\left(\frac{dx}{2}\right.\right.}$
risch	$-\frac{i\left(18a^2b\,e^{9i(dx+c)}+15b^3e^{9i(dx+c)}-24a^2b\,e^{7i(dx+c)}-20b^3e^{7i(dx+c)}-12a^3e^{6i(dx+c)}-84a^2b\,e^{5i(dx+c)}-22b^3e^{5i(dx+c)}-20\right.}{3d\left(e^{2i}\right.}$

[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+3*a^2*b*(-1/3/sin(d*x+c)^3-1/si
n(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4
/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c))+b^3*(-1/3/sin(d*x+c)^3/cos(d*x+c)^
2+5/6/sin(d*x+c)/cos(d*x+c)^2-5/2/sin(d*x+c)+5/2*ln(sec(d*x+c)+tan(d*x+c)))
)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.27

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx = -\frac{8\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^5+6\left(6\,a^2b+5\,b^3\right)\cos\left(dx+c\right)^4+36\,ab^2\cos\left(dx+c\right)-12\left(a^3+12\,ab^2\right)\cos\left(d\right.}{}$$

[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] -1/12*(8*(a^3 + 12*a*b^2)*cos(d*x + c)^5 + 6*(6*a^2*b + 5*b^3)*cos(d*x + c)
^4 + 36*a*b^2*cos(d*x + c) - 12*(a^3 + 12*a*b^2)*cos(d*x + c)^3 + 6*b^3 - 8
*(6*a^2*b + 5*b^3)*cos(d*x + c)^2 - 3*((6*a^2*b + 5*b^3)*cos(d*x + c)^4 - (
6*a^2*b + 5*b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 3*((6
*a^2*b + 5*b^3)*cos(d*x + c)^4 - (6*a^2*b + 5*b^3)*cos(d*x + c)^2)*log(-sin
(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x
 + c))

Sympy [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec{(c+dx)})^3\csc^4{(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**3,x)
[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.93

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{b^3\left(\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2-2\right)}{\sin(dx+c)^5-\sin(dx+c)^3}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+6a^2b\left(\frac{2\left(3\sin\right.}{\sin}\right.}{}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] -1/12*(b^3*(2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 -
 sin(d*x + c)^3) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6
*a^2*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) +
 3*log(sin(d*x + c) - 1)) + 12*a*b^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3
 - 3*tan(d*x + c)) + 4*(3*tan(d*x + c)^2 + 1)*a^3/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.76

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$=\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^3\tan\left(\frac{1}{2}\right.}{}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^2
*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x +
```

```
1/2*c) - 45*a^2*b*tan(1/2*d*x + 1/2*c) + 63*a*b^2*tan(1/2*d*x + 1/2*c) - 27
*b^3*tan(1/2*d*x + 1/2*c) + 12*(6*a^2*b + 5*b^3)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - 12*(6*a^2*b + 5*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 24*(6*
a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2
*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 -
(9*a^3*tan(1/2*d*x + 1/2*c)^2 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 63*a*b^2*
tan(1/2*d*x + 1/2*c)^2 + 27*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3 + 3*a^2*b + 3*
a*b^2 + b^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 13.85 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.27

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a-b)^3}{24\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3\,b\,(a-b)^2}{4}-\frac{3\,(a-b)^3}{8}\right)}{d}$$
$$- \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{7\,a^3}{3}+13\,a^2\,b+19\,a\,b^2+\frac{25\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{17\,a^3}{3}+29\,a^2\,b+89\,a\,b^2+\frac{77\,b^3}{3}\right)+a\,b^2+}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\tan\left(\frac{c}{2}+\right.\right.}$$
$$- \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2\,b\,6\mathrm{i}+b^3\,5\mathrm{i}\right)\,1\mathrm{i}}{d}$$

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(a - b)^3)/(24*d) - (tan(c/2 + (d*x)/2)*((3*b*(a - b)
^2)/4 - (3*(a - b)^3)/8))/d - (atanh(tan(c/2 + (d*x)/2))*(a^2*b*6i + b^3*5i
)*1i)/d - (tan(c/2 + (d*x)/2)^2*(19*a*b^2 + 13*a^2*b + (7*a^3)/3 + (25*b^3)
/3) - tan(c/2 + (d*x)/2)^4*(89*a*b^2 + 29*a^2*b + (17*a^3)/3 + (77*b^3)/3)
+ a*b^2 + a^2*b + a^3/3 + b^3/3 + tan(c/2 + (d*x)/2)^6*(69*a*b^2 + 15*a^2*b
 + 3*a^3 + b^3))/(d*(8*tan(c/2 + (d*x)/2)^3 - 16*tan(c/2 + (d*x)/2)^5 + 8*t
an(c/2 + (d*x)/2)^7))
```

3.195 $\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 279

$$\begin{aligned}\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx =& \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} \\ &+ \frac{7b^3\text{arctanh}(\sin(c+dx))}{2d} \\ &- \frac{a^3\cot(c+dx)}{d} - \frac{9ab^2\cot(c+dx)}{d} \\ &- \frac{2a^3\cot^3(c+dx)}{3d} - \frac{3ab^2\cot^3(c+dx)}{d} \\ &- \frac{a^3\cot^5(c+dx)}{5d} - \frac{3ab^2\cot^5(c+dx)}{5d} \\ &- \frac{3a^2b\csc(c+dx)}{d} - \frac{7b^3\csc(c+dx)}{2d} \\ &- \frac{a^2b\csc^3(c+dx)}{d} - \frac{7b^3\csc^3(c+dx)}{6d} \\ &- \frac{3a^2b\csc^5(c+dx)}{5d} - \frac{7b^3\csc^5(c+dx)}{10d} \\ &+ \frac{b^3\csc^5(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d}\end{aligned}$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d+7/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)/
d-9*a*b^2*cot(d*x+c)/d-2/3*a^3*cot(d*x+c)^3/d-3*a*b^2*cot(d*x+c)^3/d-1/5*a^
3*cot(d*x+c)^5/d-3/5*a*b^2*cot(d*x+c)^5/d-3*a^2*b*csc(d*x+c)/d-7/2*b^3*csc(
d*x+c)/d-a^2*b*csc(d*x+c)^3/d-7/6*b^3*csc(d*x+c)^3/d-3/5*a^2*b*csc(d*x+c)^5
/d-7/10*b^3*csc(d*x+c)^5/d+1/2*b^3*csc(d*x+c)^5*sec(d*x+c)^2/d+3*a*b^2*tan(
d*x+c)/d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2991, 3852, 2701, 308, 213, 2700, 276, 294}

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx = -\frac{a^3\cot^5(c+dx)}{5d} - \frac{2a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot(c+dx)}{d} + \frac{3a^2b\,\text{arctanh}(\sin(c+dx))}{d} - \frac{3a^2b\csc^5(c+dx)}{5d} - \frac{a^2b\csc^3(c+dx)}{d} - \frac{3a^2b\csc(c+dx)}{d} + \frac{3ab^2\tan(c+dx)}{d} - \frac{3ab^2\cot^5(c+dx)}{5d} - \frac{3ab^2\cot^3(c+dx)}{d} - \frac{9ab^2\cot(c+dx)}{d} + \frac{7b^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{7b^3\csc^5(c+dx)}{10d} - \frac{7b^3\csc^3(c+dx)}{6d} - \frac{7b^3\csc(c+dx)}{2d} + \frac{b^3\csc^5(c+dx)\sec^2(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (7*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/d - (2*a^3*Cot[c + d*x]^3)/(3*
d) - (3*a*b^2*Cot[c + d*x]^3)/d - (a^3*Cot[c + d*x]^5)/(5*d) - (3*a*b^2*Cot
[c + d*x]^5)/(5*d) - (3*a^2*b*Csc[c + d*x])/d - (7*b^3*Csc[c + d*x])/(2*d)
- (a^2*b*Csc[c + d*x]^3)/d - (7*b^3*Csc[c + d*x]^3)/(6*d) - (3*a^2*b*Csc[c
+ d*x]^5)/(5*d) - (7*b^3*Csc[c + d*x]^5)/(10*d) + (b^3*Csc[c + d*x]^5*Sec[c
 + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int (-b - a\cos(c+dx))^3 \csc^6(c+dx)\sec^3(c+dx)\,dx$$

$$= \int \left(a^3\csc^6(c+dx) + 3a^2b\csc^6(c+dx)\sec(c+dx) + 3ab^2\csc^6(c+dx)\sec^2(c+dx) + b^3\csc^6(c+dx)\sec^3(c+dx)\right)\,dx$$

$$= a^3\int \csc^6(c+dx)\,dx + \left(3a^2b\right)\int \csc^6(c+dx)\sec(c+dx)\,dx + \left(3ab^2\right)\int \csc^6(c+dx)\sec^2(c+dx)\,dx + b^3\int \csc^6(c+dx)\sec^3(c+dx)\,dx$$

$$= -\frac{a^3\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, \cot(c+dx)\right)}{d} - \frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} + \frac{\left(3ab^2\right)\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^6}\,dx, x, \tan(c+dx)\right)}{d} - \frac{b^3\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^2}\,dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{a^3\cot(c+dx)}{d} - \frac{2a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{b^3\csc^5(c+dx)\sec^2(c+dx)}{2d} - \frac{\left(3a^2b\right)\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{d} + \frac{\left(3ab^2\right)\text{Subst}\left(\int \left(1+\frac{1}{x^6}+\frac{3}{x^4}+\frac{3}{x^2}\right)\,dx, x, \tan(c+dx)\right)}{d} - \frac{\left(7b^3\right)\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{2d}$$

$$= -\frac{a^3\cot(c+dx)}{d} - \frac{9ab^2\cot(c+dx)}{d} - \frac{2a^3\cot^3(c+dx)}{3d} - \frac{3ab^2\cot^3(c+dx)}{d} - \frac{a^3\cot^5(c+dx)}{5d} - \frac{3ab^2\cot^5(c+dx)}{5d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{a^2b\csc^3(c+dx)}{d} - \frac{3a^2b\csc^5(c+dx)}{5d} + \frac{b^3\csc^5(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d} - \frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{\left(7b^3\right)\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{2d}$$

$$
\begin{aligned}
&= \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a^3\cot(c+dx)}{d} - \frac{9ab^2\cot(c+dx)}{d} \\
&\quad - \frac{2a^3\cot^3(c+dx)}{3d} - \frac{3ab^2\cot^3(c+dx)}{d} - \frac{a^3\cot^5(c+dx)}{5d} - \frac{3ab^2\cot^5(c+dx)}{5d} \\
&\quad - \frac{3a^2b\csc(c+dx)}{d} - \frac{7b^3\csc(c+dx)}{2d} - \frac{a^2b\csc^3(c+dx)}{d} - \frac{7b^3\csc^3(c+dx)}{6d} \\
&\quad - \frac{3a^2b\csc^5(c+dx)}{5d} - \frac{7b^3\csc^5(c+dx)}{10d} + \frac{b^3\csc^5(c+dx)\sec^2(c+dx)}{2d} \\
&\quad + \frac{3ab^2\tan(c+dx)}{d} - \frac{(7b^3)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{2d} \\
&= \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{7b^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{a^3\cot(c+dx)}{d} \\
&\quad - \frac{9ab^2\cot(c+dx)}{d} - \frac{2a^3\cot^3(c+dx)}{3d} - \frac{3ab^2\cot^3(c+dx)}{d} \\
&\quad - \frac{a^3\cot^5(c+dx)}{5d} - \frac{3ab^2\cot^5(c+dx)}{5d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{7b^3\csc(c+dx)}{2d} \\
&\quad - \frac{a^2b\csc^3(c+dx)}{d} - \frac{7b^3\csc^3(c+dx)}{6d} - \frac{3a^2b\csc^5(c+dx)}{5d} \\
&\quad - \frac{7b^3\csc^5(c+dx)}{10d} + \frac{b^3\csc^5(c+dx)\sec^2(c+dx)}{2d} + \frac{3ab^2\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 812 vs. $2(279) = 558$.

Time = 2.16 (sec) , antiderivative size = 812, normalized size of antiderivative = 2.91

$$
\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx =
$$

$$
-\frac{\csc^9\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(1176a^2b+412b^3+80a(5a^2+18b^2)\cos(c+dx)+66(6a^2b+7b^3)\cos\right.}{}
$$

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/61440*(Csc[(c + d*x)/2]^9*Sec[(c + d*x)/2]^5*(1176*a^2*b + 412*b^3 + 80*
a*(5*a^2 + 18*b^2)*Cos[c + d*x] + 66*(6*a^2*b + 7*b^3)*Cos[2*(c + d*x)] + 1
6*a^3*Cos[3*(c + d*x)] + 288*a*b^2*Cos[3*(c + d*x)] - 600*a^2*b*Cos[4*(c +
d*x)] - 700*b^3*Cos[4*(c + d*x)] - 48*a^3*Cos[5*(c + d*x)] - 864*a*b^2*Cos[
5*(c + d*x)] + 180*a^2*b*Cos[6*(c + d*x)] + 210*b^3*Cos[6*(c + d*x)] + 16*a
^3*Cos[7*(c + d*x)] + 288*a*b^2*Cos[7*(c + d*x)] + 450*a^2*b*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 525*b^3*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]]*Sin[c + d*x] - 450*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*Sin[c + d*x] - 525*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[
c + d*x] + 90*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x
)] + 105*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 90
```

```
*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 105*b^3*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 270*a^2*b*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 315*b^3*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 270*a^2*b*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 315*b^3*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]*Sin[5*(c + d*x)] + 90*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]]*Sin[7*(c + d*x)] + 105*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]]*Sin[7*(c + d*x)] - 90*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Si
n[7*(c + d*x)] - 105*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[7*(c
+ d*x)]))/(d*(-1 + Cot[(c + d*x)/2]^2)^2)
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.87

method	result
derivativedivides	$a^3\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c)+3a^2b\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c$
default	$a^3\left(-\frac{8}{15}-\frac{\csc(dx+c)^4}{5}-\frac{4\csc(dx+c)^2}{15}\right)\cot(dx+c)+3a^2b\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c$
parallelrisch	$-46080(1+\cos(2dx+2c))b\left(a^2+\frac{7b^2}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+46080(1+\cos(2dx+2c))b\left(a^2+\frac{7b^2}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+24$
norman	$-\frac{a^3+3a^2b+3a\,b^2+b^3}{160d}+\frac{\left(a^3-3a^2b+3a\,b^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{160d}+\frac{\left(19a^3-87a^2b+117a\,b^2-49b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{480d}-\frac{\left(19a^3+87a^2b+117a\right.}{}$
risch	$-\frac{i\left(90a^2b\,e^{13i(dx+c)}+105b^3e^{13i(dx+c)}-300a^2b\,e^{11i(dx+c)}-350b^3e^{11i(dx+c)}+198a^2b\,e^{9i(dx+c)}+231b^3e^{9i(dx+c)}+160a^3e^{8}\right.}{}$

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+3*a^2*b*(-1/
5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a
*b^2*(-1/5/sin(d*x+c)^5/cos(d*x+c)-2/5/sin(d*x+c)^3/cos(d*x+c)+8/5/sin(d*x+
c)/cos(d*x+c)-16/5*cot(d*x+c))+b^3*(-1/5/sin(d*x+c)^5/cos(d*x+c)^2-7/15/sin
(d*x+c)^3/cos(d*x+c)^2+7/6/sin(d*x+c)/cos(d*x+c)^2-7/2/sin(d*x+c)+7/2*ln(se
c(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.27

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx =$$
$$-\frac{32\,(a^3+18\,ab^2)\cos{(dx+c)}^7+30\,(6\,a^2b+7\,b^3)\cos{(dx+c)}^6-80\,(a^3+18\,ab^2)\cos{(dx+c)}^5-70\,(6\,a^2}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(32*(a^3 + 18*a*b^2)*cos(d*x + c)^7 + 30*(6*a^2*b + 7*b^3)*cos(d*x +
c)^6 - 80*(a^3 + 18*a*b^2)*cos(d*x + c)^5 - 70*(6*a^2*b + 7*b^3)*cos(d*x +
c)^4 - 180*a*b^2*cos(d*x + c) + 60*(a^3 + 18*a*b^2)*cos(d*x + c)^3 - 30*b^3
 + 46*(6*a^2*b + 7*b^3)*cos(d*x + c)^2 - 15*((6*a^2*b + 7*b^3)*cos(d*x + c)
^6 - 2*(6*a^2*b + 7*b^3)*cos(d*x + c)^4 + (6*a^2*b + 7*b^3)*cos(d*x + c)^2)
*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*((6*a^2*b + 7*b^3)*cos(d*x + c)^6
- 2*(6*a^2*b + 7*b^3)*cos(d*x + c)^4 + (6*a^2*b + 7*b^3)*cos(d*x + c)^2)*lo
g(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4
+ d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.82

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx =$$
$$-\frac{b^3\left(\frac{2\left(105\,\sin(dx+c)^6-70\,\sin(dx+c)^4-14\,\sin(dx+c)^2-6\right)}{\sin(dx+c)^7-\sin(dx+c)^5}-105\,\log{(\sin{(dx+c)}+1)}+105\,\log{(\sin{(dx+c)}-1)}\right)}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(b^3*(2*(105*sin(d*x + c)^6 - 70*sin(d*x + c)^4 - 14*sin(d*x + c)^2 -
 6)/(sin(d*x + c)^7 - sin(d*x + c)^5) - 105*log(sin(d*x + c) + 1) + 105*log
(sin(d*x + c) - 1)) + 6*a^2*b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)
/sin(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 36
*a*b^2*((15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d
*x + c)) + 4*(15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^3/tan(d*x + c)^5
)/d
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.78

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$= \frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 25\,a^3\tan}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*
b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*a^3*tan(1/2*
d*x + 1/2*c)^3 - 105*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 135*a*b^2*tan(1/2*d*x +
 1/2*c)^3 - 55*b^3*tan(1/2*d*x + 1/2*c)^3 + 150*a^3*tan(1/2*d*x + 1/2*c) -
990*a^2*b*tan(1/2*d*x + 1/2*c) + 1710*a*b^2*tan(1/2*d*x + 1/2*c) - 870*b^3*
tan(1/2*d*x + 1/2*c) + 240*(6*a^2*b + 7*b^3)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - 240*(6*a^2*b + 7*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 480*(6*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d
*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (1
50*a^3*tan(1/2*d*x + 1/2*c)^4 + 990*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 1710*a*b
^2*tan(1/2*d*x + 1/2*c)^4 + 870*b^3*tan(1/2*d*x + 1/2*c)^4 + 25*a^3*tan(1/2
*d*x + 1/2*c)^2 + 105*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 135*a*b^2*tan(1/2*d*x
+ 1/2*c)^2 + 55*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3 + 9*a^2*b + 9*a*b^2 + 3*
b^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.30

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(a-b)^3}{160\,d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{19\,a^3}{15}+\frac{29\,a^2\,b}{5}+\frac{39\,a\,b^2}{5}+\frac{49\,b^3}{15}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(10\,a^3+66\,a^2\,b+306\,a\,b^2+26\,b^3\right)-\tan\left(\right.}{d\,\left(32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\right.}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{21\,a\,b^2}{16}-\frac{3\,a^2\,b}{8}-\frac{a^3}{16}-\frac{7\,b^3}{8}+\frac{3\,(a-b)^2\,(a-4\,b)}{16}+\frac{3\,(a-b)^3}{16}\right)}{d}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{(a-b)^2\,(a-4\,b)}{48}+\frac{(a-b)^3}{32}\right)}{d}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2\,b\,6\mathrm{i}+b^3\,7\mathrm{i}\right)\,1\mathrm{i}}{d}$$

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a - b)^3)/(160*d) - (tan(c/2 + (d*x)/2)^2*((39*a*b^2
)/5 + (29*a^2*b)/5 + (19*a^3)/15 + (49*b^3)/15) + tan(c/2 + (d*x)/2)^8*(306
*a*b^2 + 66*a^2*b + 10*a^3 + 26*b^3) - tan(c/2 + (d*x)/2)^6*(411*a*b^2 + 12
5*a^2*b + (55*a^3)/3 + (433*b^3)/3) + tan(c/2 + (d*x)/2)^4*((483*a*b^2)/5 +
 (263*a^2*b)/5 + (103*a^3)/15 + (763*b^3)/15) + (3*a*b^2)/5 + (3*a^2*b)/5 +
 a^3/5 + b^3/5)/(d*(32*tan(c/2 + (d*x)/2)^5 - 64*tan(c/2 + (d*x)/2)^7 + 32*
tan(c/2 + (d*x)/2)^9)) - (atanh(tan(c/2 + (d*x)/2))*(a^2*b*6i + b^3*7i)*1i)
/d + (tan(c/2 + (d*x)/2)*((21*a*b^2)/16 - (3*a^2*b)/8 - a^3/16 - (7*b^3)/8
+ (3*(a - b)^2*(a - 4*b))/16 + (3*(a - b)^3)/16))/d + (tan(c/2 + (d*x)/2)^3
*(((a - b)^2*(a - 4*b))/48 + (a - b)^3/32))/d
```

3.196 $\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 223

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{(a^2-b^2)^3\cos(c+dx)}{a^7d} - \frac{b(3a^4-3a^2b^2+b^4)\cos^2(c+dx)}{2a^6d} + \frac{(3a^4-3a^2b^2+b^4)\cos^3(c+dx)}{3a^5d} + \frac{b(3a^2-b^2)\cos^4(c+dx)}{4a^4d} - \frac{(3a^2-b^2)\cos^5(c+dx)}{5a^3d} - \frac{b\cos^6(c+dx)}{6a^2d} + \frac{\cos^7(c+dx)}{7ad} + \frac{b(a^2-b^2)^3\log(b+a\cos(c+dx))}{a^8d}$$

```
[Out] -(a^2-b^2)^3*cos(d*x+c)/a^7/d-1/2*b*(3*a^4-3*a^2*b^2+b^4)*cos(d*x+c)^2/a^6/
d+1/3*(3*a^4-3*a^2*b^2+b^4)*cos(d*x+c)^3/a^5/d+1/4*b*(3*a^2-b^2)*cos(d*x+c)
^4/a^4/d-1/5*(3*a^2-b^2)*cos(d*x+c)^5/a^3/d-1/6*b*cos(d*x+c)^6/a^2/d+1/7*co
s(d*x+c)^7/a/d+b*(a^2-b^2)^3*ln(b+a*cos(d*x+c))/a^8/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

$= \{3957, 2916, 12, 786\}$

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b\cos^6(c+dx)}{6a^2d} + \frac{b(a^2-b^2)^3\log(a\cos(c+dx)+b)}{a^8d} - \frac{(a^2-b^2)^3\cos(c+dx)}{a^7d} + \frac{b(3a^2-b^2)\cos^4(c+dx)}{4a^4d} - \frac{(3a^2-b^2)\cos^5(c+dx)}{5a^3d} - \frac{b(3a^4-3a^2b^2+b^4)\cos^2(c+dx)}{2a^6d} + \frac{(3a^4-3a^2b^2+b^4)\cos^3(c+dx)}{3a^5d} + \frac{\cos^7(c+dx)}{7ad}$$

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^3*Cos[c + d*x])/(a^7*d)) - (b*(3*a^4 - 3*a^2*b^2 + b^4)*Cos[
c + d*x]^2)/(2*a^6*d) + ((3*a^4 - 3*a^2*b^2 + b^4)*Cos[c + d*x]^3)/(3*a^5*d
) + (b*(3*a^2 - b^2)*Cos[c + d*x]^4)/(4*a^4*d) - ((3*a^2 - b^2)*Cos[c + d*x
]^5)/(5*a^3*d) - (b*Cos[c + d*x]^6)/(6*a^2*d) + Cos[c + d*x]^7/(7*a*d) + (b
*(a^2 - b^2)^3*Log[b + a*Cos[c + d*x]])/(a^8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos(c+dx)\sin^7(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)^3}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)^3}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^8 d}$$

$$= \frac{\text{Subst}\left(\int \left((a^2-b^2)^3 + \frac{b(-a^2+b^2)^3}{b-x} - b(3a^4-3a^2b^2+b^4)\,x - (3a^4-3a^2b^2+b^4)\,x^2 - b(-3a^2+b^2)\,x\right.\right.}{a^8 d}$$

$$= -\frac{(a^2-b^2)^3\cos(c+dx)}{a^7 d} - \frac{b(3a^4-3a^2b^2+b^4)\cos^2(c+dx)}{2a^6 d} + \frac{(3a^4-3a^2b^2+b^4)\cos^3(c+dx)}{3a^5 d} + \frac{b(3a^2-b^2)\cos^4(c+dx)}{4a^4 d} - \frac{(3a^2-b^2)\cos^5(c+dx)}{5a^3 d} - \frac{b\cos^6(c+dx)}{6a^2 d} + \frac{\cos^7(c+dx)}{7ad} + \frac{b(a^2-b^2)^3\log(b+a\cos(c+dx))}{a^8 d}$$

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.26

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-105a(35a^6-152a^4b^2+176a^2b^4-64b^6)\cos(c+dx) - 105(29a^6b-40a^4b^3+16a^2b^5)\cos(2(c+dx)) + 73}{}$$

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-105*a*(35*a^6 - 152*a^4*b^2 + 176*a^2*b^4 - 64*b^6)*Cos[c + d*x] - 105*(2
9*a^6*b - 40*a^4*b^3 + 16*a^2*b^5)*Cos[2*(c + d*x)] + 735*a^7*Cos[3*(c + d*
x)] - 1260*a^5*b^2*Cos[3*(c + d*x)] + 560*a^3*b^4*Cos[3*(c + d*x)] + 420*a^
6*b*Cos[4*(c + d*x)] - 210*a^4*b^3*Cos[4*(c + d*x)] - 147*a^7*Cos[5*(c + d*
x)] + 84*a^5*b^2*Cos[5*(c + d*x)] - 35*a^6*b*Cos[6*(c + d*x)] + 15*a^7*Cos[
7*(c + d*x)] + 6720*a^6*b*Log[b + a*Cos[c + d*x]] - 20160*a^4*b^3*Log[b + a
*Cos[c + d*x]] + 20160*a^2*b^5*Log[b + a*Cos[c + d*x]] - 6720*b^7*Log[b + a
*Cos[c + d*x]])/(6720*a^8*d)
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^7 a^6}{7}-\frac{b\cos(dx+c)^6 a^5}{6}-\frac{3a^6\cos(dx+c)^5}{5}+\frac{a^4b^2\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4 a^5 b}{4}-\frac{\cos(dx+c)^4 a^3b^3}{4}+\cos(dx+c)^3a^6-\cos(dx+\cdots}{\cdots}$
default	$\frac{\frac{\cos(dx+c)^7 a^6}{7}-\frac{b\cos(dx+c)^6 a^5}{6}-\frac{3a^6\cos(dx+c)^5}{5}+\frac{a^4b^2\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4 a^5 b}{4}-\frac{\cos(dx+c)^4 a^3b^3}{4}+\cos(dx+c)^3a^6-\cos(dx+\cdots}{\cdots}$
parallelrisch	$\frac{320b(a-b)^3(a+b)^3\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-320b(a-b)^3(a+b)^3\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-7\left(\frac{5\left(29a^5b-40a^3b^3+16ab^5\right)}{7}\cdots\right.}{\cdots}$
norman	$\frac{\frac{\left(2a^5b+2a^4b^2-4a^3b^3-4a^2b^4+2ab^5+2b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{a^7d}+\frac{\left(14a^5b+16a^4b^2-24a^3b^3-28a^2b^4+10ab^5+12b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{a^7d}+\frac{-96\cdots}{\cdots}}{\cdots}$
risch	$-\frac{7\cos(5dx+5c)}{320da}-\frac{b\cos(6dx+6c)}{192d\,a^2}+\frac{7\cos(3dx+3c)}{64ad}+\frac{\cos(7dx+7c)}{448ad}+\frac{\cos(5dx+5c)b^2}{80d\,a^3}+\frac{b\cos(4dx+4c)}{16d\,a^2}-\frac{b^3 \cdots}{\cdots}$

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^7*(1/7*cos(d*x+c)^7*a^6-1/6*b*cos(d*x+c)^6*a^5-3/5*a^6*cos(d*x+c)^
5+1/5*a^4*b^2*cos(d*x+c)^5+3/4*cos(d*x+c)^4*a^5*b-1/4*cos(d*x+c)^4*a^3*b^3+
cos(d*x+c)^3*a^6-cos(d*x+c)^3*a^4*b^2+1/3*cos(d*x+c)^3*a^2*b^4-3/2*cos(d*x+
c)^2*a^5*b+3/2*cos(d*x+c)^2*a^3*b^3-1/2*cos(d*x+c)^2*a*b^5-cos(d*x+c)*a^6+3
*cos(d*x+c)*a^4*b^2-3*cos(d*x+c)*a^2*b^4+b^6*cos(d*x+c))+b*(a^6-3*a^4*b^2+3
*a^2*b^4-b^6)/a^8*ln(b+a*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$=\frac{60\,a^7\cos(dx+c)^7-70\,a^6b\cos(dx+c)^6-84\left(3\,a^7-a^5b^2\right)\cos(dx+c)^5+105\left(3\,a^6b-a^4b^3\right)\cos(dx+c)^{\cdots}}{\cdots}$$

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(60*a^7*cos(d*x + c)^7 - 70*a^6*b*cos(d*x + c)^6 - 84*(3*a^7 - a^5*b^
2)*cos(d*x + c)^5 + 105*(3*a^6*b - a^4*b^3)*cos(d*x + c)^4 + 140*(3*a^7 - 3
*a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 - 210*(3*a^6*b - 3*a^4*b^3 + a^2*b^5)*co
s(d*x + c)^2 - 420*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c) + 420
*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*log(a*cos(d*x + c) + b))/(a^8*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{60\,a^6\cos(dx+c)^7-70\,a^5b\cos(dx+c)^6-84\,(3\,a^6-a^4b^2)\cos(dx+c)^5+105\,(3\,a^5b-a^3b^3)\cos(dx+c)^4+140\,(3\,a^6-3\,a^4b^2+a^2b^4)\cos(dx+c)^3-210\,(3\,a^5}{a^7}}{420\,d}$$

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/420*((60*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6 - 84*(3*a^6 - a^4*b
^2)*cos(d*x + c)^5 + 105*(3*a^5*b - a^3*b^3)*cos(d*x + c)^4 + 140*(3*a^6 -
3*a^4*b^2 + a^2*b^4)*cos(d*x + c)^3 - 210*(3*a^5*b - 3*a^3*b^3 + a*b^5)*cos
(d*x + c)^2 - 420*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(d*x + c))/a^7 + 4
20*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*log(a*cos(d*x + c) + b)/a^8)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. 2(211) = 422.

Time = 0.36 (sec) , antiderivative size = 1559, normalized size of antiderivative = 6.99

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] 1/420*(420*(a^7*b - a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 - 3*a^2*b^6
 - a*b^7 + b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^9 - a^8*b) - 420*(a^6*b - 3*a^4
*b^3 + 3*a^2*b^5 - b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)
)/a^8 + (384*a^7 - 1089*a^6*b - 1848*a^5*b^2 + 3267*a^4*b^3 + 2240*a^3*b^4
- 3267*a^2*b^5 - 840*a*b^6 + 1089*b^7 - 2688*a^7*(cos(d*x + c) - 1)/(cos(d*
```

x + c) + 1) + 8463*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12096*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 24549*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 14000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 23709*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 5040*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 7623*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8064*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 32088*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 78687*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 35280*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 72807*a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12600*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 22869*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 13440*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 40320*a^5*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 136185*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 45920*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 122745*a^2*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 16800*a*b^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 38115*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 56035*a^6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 24360*a^5*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 136185*a^4*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 32480*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 122745*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 12600*a*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 38115*b^7*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 6720*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 78687*a^4*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 11760*a^3*b^4*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 72807*a^2*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 5040*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 22869*b^7*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 840*a^5*b^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 24549*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1680*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 23709*a^2*b^5*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 840*a*b^6*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 7623*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*a^6*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 3267*a^4*b^3*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 3267*a^2*b^5*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 1089*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/(a^8*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7))/d

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.12

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{\cos(c+d\,x)^3\left(\frac{1}{a}-\frac{b^2\left(\frac{1}{a}-\frac{b^2}{3\,a^3}\right)}{a^2}\right)-\cos(c+d\,x)^5\left(\frac{3}{5\,a}-\frac{b^2}{5\,a^3}\right)-\cos(c+d\,x)\left(\frac{1}{a}-\frac{b^2\left(\frac{3}{a}-\frac{b^2\left(\frac{3}{a}-\frac{b^2}{a^3}\right)}{a^2}\right)}{a^2}\right)+\mathrm{cos}}{d}$$

[In] int(sin(c + d*x)^7/(a + b/cos(c + d*x)),x)

[Out] (cos(c + d*x)^3*(1/a - (b^2*(1/a - b^2/(3*a^3)))/a^2) - cos(c + d*x)^5*(3/(5*a) - b^2/(5*a^3)) - cos(c + d*x)*(1/a - (b^2*(3/a - (b^2*(3/a - b^2/a^3))/a^2))/a^2) + cos(c + d*x)^7/(7*a) + (log(b + a*cos(c + d*x))*(a^6*b - b^7 + 3*a^2*b^5 - 3*a^4*b^3))/a^8 - (b*cos(c + d*x)^6)/(6*a^2) - (b*cos(c + d*x)^2*(3/a - (b^2*(3/a - b^2/a^3))/a^2))/(2*a) + (b*cos(c + d*x)^4*(3/a - b^2/a^3))/(4*a))/d

3.197 $\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{(a^2-b^2)^2\cos(c+dx)}{a^5d} - \frac{b(2a^2-b^2)\cos^2(c+dx)}{2a^4d} + \frac{(2a^2-b^2)\cos^3(c+dx)}{3a^3d} + \frac{b\cos^4(c+dx)}{4a^2d} - \frac{\cos^5(c+dx)}{5ad} + \frac{b(a^2-b^2)^2\log(b+a\cos(c+dx))}{a^6d}$$

```
[Out] -(a^2-b^2)^2*cos(d*x+c)/a^5/d-1/2*b*(2*a^2-b^2)*cos(d*x+c)^2/a^4/d+1/3*(2*a
^2-b^2)*cos(d*x+c)^3/a^3/d+1/4*b*cos(d*x+c)^4/a^2/d-1/5*cos(d*x+c)^5/a/d+b*
(a^2-b^2)^2*ln(b+a*cos(d*x+c))/a^6/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2916, 12, 786}

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{b\cos^4(c+dx)}{4a^2d} + \frac{b(a^2-b^2)^2\log(a\cos(c+dx)+b)}{a^6d} - \frac{(a^2-b^2)^2\cos(c+dx)}{a^5d} - \frac{b(2a^2-b^2)\cos^2(c+dx)}{2a^4d} + \frac{(2a^2-b^2)\cos^3(c+dx)}{3a^3d} - \frac{\cos^5(c+dx)}{5ad}$$

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Cos[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Cos[c + d*x]^2)/
(2*a^4*d) + ((2*a^2 - b^2)*Cos[c + d*x]^3)/(3*a^3*d) + (b*Cos[c + d*x]^4)/(
4*a^2*d) - Cos[c + d*x]^5/(5*a*d) + (b*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]
])/(a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos(c+dx)\sin^5(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)^2}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)^2}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^6 d}$$

$$= \frac{\text{Subst}\left(\int \left((a^2-b^2)^2 - \frac{b(-a^2+b^2)^2}{b-x} + b(-2a^2+b^2)\,x - (2a^2-b^2)\,x^2 + bx^3 + x^4\right)\,dx, x, -a\cos(c+d\right.}{a^6 d}$$

$$= -\frac{(a^2-b^2)^2\cos(c+dx)}{a^5 d} - \frac{b(2a^2-b^2)\cos^2(c+dx)}{2a^4 d} + \frac{(2a^2-b^2)\cos^3(c+dx)}{3a^3 d} + \frac{b\cos^4(c+dx)}{4a^2 d} - \frac{\cos^5(c+dx)}{5ad} + \frac{b(a^2-b^2)^2\log(b+a\cos(c+dx))}{a^6 d}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.13

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{-60a(5a^4-14a^2b^2+8b^4)\cos(c+dx)-60(3a^4b-2a^2b^3)\cos(2(c+dx))+50a^5\cos(3(c+dx))-40a^3b^2}{}$$

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-60*a*(5*a^4 - 14*a^2*b^2 + 8*b^4)*Cos[c + d*x] - 60*(3*a^4*b - 2*a^2*b^3)
*Cos[2*(c + d*x)] + 50*a^5*Cos[3*(c + d*x)] - 40*a^3*b^2*Cos[3*(c + d*x)] +
 15*a^4*b*Cos[4*(c + d*x)] - 6*a^5*Cos[5*(c + d*x)] + 480*a^4*b*Log[b + a*C
os[c + d*x]] - 960*a^2*b^3*Log[b + a*Cos[c + d*x]] + 480*b^5*Log[b + a*Cos[
c + d*x]])/(480*a^6*d)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{-\frac{\frac{\cos(dx+c)^5a^4}{5}-\frac{b\cos(dx+c)^4a^3}{4}-\frac{2\cos(dx+c)^3a^4}{3}+\frac{\cos(dx+c)^3a^2b^2}{3}+\cos(dx+c)^2a^3b-\frac{\cos(dx+c)^2a\,b^3}{2}+\cos(dx+c)a^4-2\cos(dx+c}{a^5}}{d}$
default	$\frac{-\frac{\frac{\cos(dx+c)^5a^4}{5}-\frac{b\cos(dx+c)^4a^3}{4}-\frac{2\cos(dx+c)^3a^4}{3}+\frac{\cos(dx+c)^3a^2b^2}{3}+\cos(dx+c)^2a^3b-\frac{\cos(dx+c)^2a\,b^3}{2}+\cos(dx+c)a^4-2\cos(dx+c}{a^5}}{d}$
parallelrisch	$\frac{480b(a-b)^2(a+b)^2\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-480b(a-b)^2(a+b)^2\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-6\left(\left(30a^3b-20a\,b^3\right)\cos(2dx\right.\right.}{}$
norman	$\frac{\frac{\left(2a^3b+2a^2b^2-2a\,b^3-2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d\,a^5}+\frac{-16a^4+50a^2b^2-30b^4}{15d\,a^5}+\frac{2\left(5a^3b+6a^2b^2-3a\,b^3-4b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d\,a^5}+\frac{\left(-16a^4+6a^3b+44\right.}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$
risch	$-\frac{\cos(5dx+5c)}{80da}-\frac{5\,\mathrm{e}^{-i(dx+c)}}{16ad}-\frac{2ib^5c}{a^6d}+\frac{4ib^3c}{a^4d}-\frac{ix\,b^5}{a^6}+\frac{5\cos(3dx+3c)}{48ad}-\frac{2ibc}{a^2d}-\frac{ixb}{a^2}-\frac{5\,\mathrm{e}^{i(dx+c)}}{16da}+\frac{b\ln}{}$

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^5*(1/5*cos(d*x+c)^5*a^4-1/4*b*cos(d*x+c)^4*a^3-2/3*cos(d*x+c)^3*a
^4+1/3*cos(d*x+c)^3*a^2*b^2+cos(d*x+c)^2*a^3*b-1/2*cos(d*x+c)^2*a*b^3+cos(d
*x+c)*a^4-2*cos(d*x+c)*a^2*b^2+cos(d*x+c)*b^4)+b*(a^4-2*a^2*b^2+b^4)/a^6*ln
(b+a*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{12\,a^5\cos\left(dx+c\right)^5 - 15\,a^4b\cos\left(dx+c\right)^4 - 20\,\left(2\,a^5 - a^3b^2\right)\cos\left(dx+c\right)^3 + 30\,\left(2\,a^4b - a^2b^3\right)\cos\left(dx+c\right)^2}{60\,a^6d}$$

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(12*a^5*cos(d*x + c)^5 - 15*a^4*b*cos(d*x + c)^4 - 20*(2*a^5 - a^3*b^
2)*cos(d*x + c)^3 + 30*(2*a^4*b - a^2*b^3)*cos(d*x + c)^2 + 60*(a^5 - 2*a^3
*b^2 + a*b^4)*cos(d*x + c) - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(a*cos(d*x + c
) + b))/(a^6*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{12\,a^4\cos(dx+c)^5 - 15\,a^3b\cos(dx+c)^4 - 20\,\left(2\,a^4 - a^2b^2\right)\cos(dx+c)^3 + 30\,\left(2\,a^3b - ab^3\right)\cos(dx+c)^2 + 60\,\left(a^4 - 2\,a^2b^2 + b^4\right)\cos(dx+c)}{a^5} - \frac{60\,\left(a^4b - 2\,a\right.}{}}{60\,d}$$

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((12*a^4*cos(d*x + c)^5 - 15*a^3*b*cos(d*x + c)^4 - 20*(2*a^4 - a^2*b
^2)*cos(d*x + c)^3 + 30*(2*a^3*b - a*b^3)*cos(d*x + c)^2 + 60*(a^4 - 2*a^2*
b^2 + b^4)*cos(d*x + c))/a^5 - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(a*cos(d*x +
 c) + b)/a^6)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(144) = 288$.

Time = 0.33 (sec) , antiderivative size = 867, normalized size of antiderivative = 5.70

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*log(abs(a
+ b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1)))/(a^7 - a^6*b) - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^6 + (64*a^5 - 137*a^4*b - 200*a^3*b
^2 + 274*a^2*b^3 + 120*a*b^4 - 137*b^5 - 320*a^5*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 805*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 880*a^3*b^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1490*a^2*b^3*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 480*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 685*b^
5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 640*a^5*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 - 1970*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1
280*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3100*a^2*b^3*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 720*a*b^4*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 1370*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*
a^4*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 720*a^3*b^2*(cos(d*x + c)
 - 1)^3/(cos(d*x + c) + 1)^3 - 3100*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 - 480*a*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1370*b^5
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*a^4*b*(cos(d*x + c) - 1)^4
/(cos(d*x + c) + 1)^4 - 120*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)
^4 + 1490*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 120*a*b^4*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 685*b^5*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 + 137*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 274
*a^2*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 137*b^5*(cos(d*x + c)
- 1)^5/(cos(d*x + c) + 1)^5)/(a^6*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\cos(c+d\,x)\left(\frac{1}{a}-\frac{b^2\left(\frac{2}{a}-\frac{b^2}{a^3}\right)}{a^2}\right)-\cos(c+d\,x)^3\left(\frac{2}{3\,a}-\frac{b^2}{3\,a^3}\right)+\frac{\cos(c+d\,x)^5}{5\,a}-\frac{b\cos(c+d\,x)^4}{4\,a^2}-\frac{\ln(b+a\,\cos(c+d\,x))\,(a}{a^6}}{d}$$

```
[In] int(sin(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)*(1/a - (b^2*(2/a - b^2/a^3))/a^2) - cos(c + d*x)^3*(2/(3*a)
- b^2/(3*a^3)) + cos(c + d*x)^5/(5*a) - (b*cos(c + d*x)^4)/(4*a^2) - (log(b
 + a*cos(c + d*x))*(a^4*b + b^5 - 2*a^2*b^3))/a^6 + (b*cos(c + d*x)^2*(2/a
- b^2/a^3))/(2*a))/d
```

3.198 $\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{(a^2-b^2)\cos(c+dx)}{a^3d}-\frac{b\cos^2(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{3ad}+\frac{b(a^2-b^2)\log(b+a\cos(c+dx))}{a^4d}$$

```
[Out] -(a^2-b^2)*cos(d*x+c)/a^3/d-1/2*b*cos(d*x+c)^2/a^2/d+1/3*cos(d*x+c)^3/a/d+b
*(a^2-b^2)*ln(b+a*cos(d*x+c))/a^4/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2916, 12, 786}

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b\cos^2(c+dx)}{2a^2d}+\frac{b(a^2-b^2)\log(a\cos(c+dx)+b)}{a^4d}-\frac{(a^2-b^2)\cos(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3ad}$$

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*Cos[c + d*x])/(a^3*d)) - (b*Cos[c + d*x]^2)/(2*a^2*d) + Cos[
c + d*x]^3/(3*a*d) + (b*(a^2 - b^2)*Log[b + a*Cos[c + d*x]])/(a^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin^3(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{a^3 d} \\
&= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^4 d} \\
&= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{-a^2b+b^3}{b-x}-bx-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{a^4 d} \\
&= -\frac{(a^2-b^2)\cos(c+dx)}{a^3 d}-\frac{b\cos^2(c+dx)}{2a^2 d}+\frac{\cos^3(c+dx)}{3ad}+\frac{b(a^2-b^2)\log(b+a\cos(c+dx))}{a^4 d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{(-9a^3+12ab^2)\cos(c+dx)-3a^2b\cos(2(c+dx))+a^3\cos(3(c+dx))+12a^2b\log(b+a\cos(c+dx))-12}{12a^4d}$$

[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x]),x]

[Out] ((-9*a^3 + 12*a*b^2)*Cos[c + d*x] - 3*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
 + d*x)] + 12*a^2*b*Log[b + a*Cos[c + d*x]] - 12*b^3*Log[b + a*Cos[c + d*x]
])/(12*a^4*d)

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{\frac{\cos(dx+c)^3a^2}{3}-\frac{ab\cos(dx+c)^2}{2}-\cos(dx+c)a^2+\cos(dx+c)b^2}{a^3}+\frac{b\left(a^2-b^2\right)\ln(b+a\cos(dx+c))}{a^4}}{d}$
default	$\frac{\frac{\frac{\cos(dx+c)^3a^2}{3}-\frac{ab\cos(dx+c)^2}{2}-\cos(dx+c)a^2+\cos(dx+c)b^2}{a^3}+\frac{b\left(a^2-b^2\right)\ln(b+a\cos(dx+c))}{a^4}}{d}$
parallelrisch	$\frac{-9a^3\cos(dx+c)+12\cos(dx+c)a\,b^2+a^3\cos(3dx+3c)-3a^2b\cos(2dx+2c)-12\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a^2b+12\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{12a^4d}$
norman	$\frac{\frac{\left(4a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a^3d}+\frac{-2ab+2b^2}{3d\,a^3}+\frac{\left(4a^2-2ab-4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3d\,a^3}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{(a+b)b(a-b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b\right.}{a^4d}$
risch	$-\frac{ixb}{a^2}+\frac{ix\,b^3}{a^4}-\frac{b\,e^{2i(dx+c)}}{8a^2d}-\frac{3\,e^{i(dx+c)}}{8da}+\frac{e^{i(dx+c)}b^2}{2a^3d}-\frac{3\,e^{-i(dx+c)}}{8ad}+\frac{e^{-i(dx+c)}b^2}{2a^3d}-\frac{b\,e^{-2i(dx+c)}}{8a^2d}-\frac{2ibc}{a^2d}$

[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/a^3*(1/3*cos(d*x+c)^3*a^2-1/2*a*b*cos(d*x+c)^2-cos(d*x+c)*a^2+cos(d*
x+c)*b^2)+b*(a^2-b^2)/a^4*ln(b+a*cos(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\,a^3\cos\left(dx+c\right)^3-3\,a^2b\cos\left(dx+c\right)^2-6\left(a^3-ab^2\right)\cos\left(dx+c\right)+6\left(a^2b-b^3\right)\log\left(a\cos\left(dx+c\right)+b\right)}{6\,a^4d}$$

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 - 3*a^2*b*cos(d*x + c)^2 - 6*(a^3 - a*b^2)*cos(d*
x + c) + 6*(a^2*b - b^3)*log(a*cos(d*x + c) + b))/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2\,a^2\cos(dx+c)^3-3\,ab\cos(dx+c)^2-6\,(a^2-b^2)\cos(dx+c)}{a^3}+\frac{6\,(a^2b-b^3)\log(a\cos(dx+c)+b)}{a^4}}{6\,d}$$

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*a^2*cos(d*x + c)^3 - 3*a*b*cos(d*x + c)^2 - 6*(a^2 - b^2)*cos(d*x +
 c))/a^3 + 6*(a^2*b - b^3)*log(a*cos(d*x + c) + b)/a^4)/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(a^2b-b^3)\log(|-a\cos(dx+c)-b|)}{a^4d} + \frac{2\,a^2d^2\cos(dx+c)^3-3\,abd^2\cos(dx+c)^2-6\,a^2d^2\cos(dx+c)+6\,b^2d^2\cos(dx+c)}{6\,a^3d^3}$$

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a^2*b - b^3)*log(abs(-a*cos(d*x + c) - b))/(a^4*d) + 1/6*(2*a^2*d^2*cos(d*
x + c)^3 - 3*a*b*d^2*cos(d*x + c)^2 - 6*a^2*d^2*cos(d*x + c) + 6*b^2*d^2*co
s(d*x + c))/(a^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\cos(c+d\,x)\,\left(\frac{1}{a}-\frac{b^2}{a^3}\right)-\frac{\cos(c+d\,x)^3}{3\,a}+\frac{b\cos(c+d\,x)^2}{2\,a^2}-\frac{\ln(b+a\,\cos(c+d\,x))\,(a^2\,b-b^3)}{a^4}}{d}$$

```
[In] int(sin(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)*(1/a - b^2/a^3) - cos(c + d*x)^3/(3*a) + (b*cos(c + d*x)^2)/
(2*a^2) - (log(b + a*cos(c + d*x))*(a^2*b - b^3))/a^4)/d
```

3.199 $\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx=-\frac{\cos(c+dx)}{ad}+\frac{b\log(b+a\cos(c+dx))}{a^2d}$$

```
[Out] -cos(d*x+c)/a/d+b*ln(b+a*cos(d*x+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2912, 12, 45}

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx=\frac{b\log(a\cos(c+dx)+b)}{a^2d}-\frac{\cos(c+dx)}{ad}$$

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Cos[c + d*x]/(a*d)) + (b*Log[b + a*Cos[c + d*x]])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \frac{x}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^2d} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{b}{b-x}\right)\,dx, x, -a\cos(c+dx)\right)}{a^2d} \\
&= -\frac{\cos(c+dx)}{ad} + \frac{b\log(b+a\cos(c+dx))}{a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$
\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-a\cos(c+dx)+b\log(b+a\cos(c+dx))}{a^2d}
$$

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x]),x]
[Out] (-(a*Cos[c + d*x]) + b*Log[b + a*Cos[c + d*x]])/(a^2*d)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{-\frac{1}{a\sec(dx+c)}-\frac{b\ln(\sec(dx+c))}{a^2}+\frac{b\ln(a+b\sec(dx+c))}{a^2}}{d}$	48
default	$\frac{-\frac{1}{a\sec(dx+c)}-\frac{b\ln(\sec(dx+c))}{a^2}+\frac{b\ln(a+b\sec(dx+c))}{a^2}}{d}$	48
parallelrisch	$\frac{b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-a(\cos(dx+c)+1)}{a^2d}$	59
risch	$-\frac{ixb}{a^2}-\frac{e^{i(dx+c)}}{2da}-\frac{e^{-i(dx+c)}}{2ad}-\frac{2ibc}{a^2d}+\frac{b\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right)}{a^2d}$	90
norman	$-\frac{2}{da\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}{a^2d}-\frac{b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{a^2d}$	91

```
[In] int(sin(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/a/sec(d*x+c)-1/a^2*b*ln(sec(d*x+c))+1/a^2*b*ln(a+b*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx=-\frac{a\cos(dx+c)-b\log(a\cos(dx+c)+b)}{a^2d}$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] -(a*cos(d*x + c) - b*log(a*cos(d*x + c) + b))/(a^2*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x)
[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{\cos(dx+c)}{a} - \frac{b\log(a\cos(dx+c)+b)}{a^2}}{d}$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] -(cos(d*x + c)/a - b*log(a*cos(d*x + c) + b)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\cos(dx+c)}{ad} + \frac{b\log(|-a\cos(dx+c)-b|)}{a^2d}$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] -cos(d*x + c)/(a*d) + b*log(abs(-a*cos(d*x + c) - b))/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{b\ln(b+a\cos(c+dx)) - a\cos(c+dx)}{a^2\,d}$$

```
[In] int(sin(c + d*x)/(a + b/cos(c + d*x)),x)
[Out] (b*log(b + a*cos(c + d*x)) - a*cos(c + d*x))/(a^2*d)
```

3.200 $\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\log(1-\cos(c+dx))}{2(a+b)d} - \frac{\log(1+\cos(c+dx))}{2(a-b)d} + \frac{b\log(b+a\cos(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d-1/2*ln(1+cos(d*x+c))/(a-b)/d+b*ln(b+a*cos(d*x+
c))/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3957, 2800, 815}

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{b\log(a\cos(c+dx)+b)}{d\,(a^2-b^2)} + \frac{\log(1-\cos(c+dx))}{2d(a+b)} - \frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)*d) +
 (b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)}{-b-a\cos(c+dx)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x}{(-b+x)(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a-b)(a-x)}-\frac{b}{(a-b)(a+b)(b-x)}+\frac{1}{2(a+b)(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{b\log(b+a\cos(c+dx))}{(a^2-b^2)\,d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$
\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-\left((a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\log(b+a\cos(c+dx))+(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)(a+b)d}
$$

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x]),x]
[Out] (-((a + b)*Log[Cos[(c + d*x)/2]]) + b*Log[b + a*Cos[c + d*x]] + (a - b)*Log
[Sin[(c + d*x)/2]])/((a - b)*(a + b)*d)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

method	result
parallelrisch	$\frac{b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}(a-b)\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)(a-b)}{d(a^{2}-b^{2})}$
derivativedivides	$\frac{-\frac{\ln(\cos(dx+c)+1)}{2a-2b}+\frac{b\ln(b+a\cos(dx+c))}{(a-b)(a+b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}}{d}$
default	$\frac{-\frac{\ln(\cos(dx+c)+1)}{2a-2b}+\frac{b\ln(b+a\cos(dx+c))}{(a-b)(a+b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}}{d}$
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a+b)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)}{d(a^{2}-b^{2})}$
risch	$\frac{ix}{a-b}+\frac{ic}{d(a-b)}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{2ibx}{a^{2}-b^{2}}-\frac{2ibc}{d(a^{2}-b^{2})}-\frac{\ln(e^{i(dx+c)}+1)}{d(a-b)}+\frac{\ln(e^{i(dx+c)}-1)}{d(a+b)}+\frac{b\ln\left(e^{2i(dx+c)}\right.}{d(a}$

[In] int(csc(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] (b*ln(-2*a+sec(1/2*d*x+1/2*c)^2*(a-b))+ln(tan(1/2*d*x+1/2*c))*(a-b))/d/(a^2
-b^2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$=\frac{2\,b\log\left(a\cos\left(dx+c\right)+b\right)-(a+b)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+(a-b)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^{2}-b^{2}\right)d}$$

[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(2*b*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) + (a
 - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)

Sympy [F]

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2\,b\log(a\cos(dx+c)+b)}{a^2-b^2} - \frac{\log(\cos(dx+c)+1)}{a-b} + \frac{\log(\cos(dx+c)-1)}{a+b}}{2\,d}$$

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*log(a*cos(d*x + c) + b)/(a^2 - b^2) - log(cos(d*x + c) + 1)/(a - b
) + log(cos(d*x + c) - 1)/(a + b))/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2\,b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2} + \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) + log(abs(-cos(d*x + c) + 1)
/abs(cos(d*x + c) + 1))/(a + b))/d
```

Mupad [B] (verification not implemented)

Time = 13.67 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)}{2\,d\,\left(a+b\right)} - \frac{\ln\left(\cos\left(c+d\,x\right)+1\right)}{2\,d\,\left(a-b\right)} + \frac{b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{d\,\left(a^2-b^2\right)}$$

```
[In] int(1/(sin(c + d*x)*(a + b/cos(c + d*x))),x)
```

```
[Out] log(cos(c + d*x) - 1)/(2*d*(a + b)) - log(cos(c + d*x) + 1)/(2*d*(a - b)) +
 (b*log(b + a*cos(c + d*x)))/(d*(a^2 - b^2))
```

3.201 $\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)d} + \frac{a\log(1-\cos(c+dx))}{4(a+b)^2d} - \frac{a\log(1+\cos(c+dx))}{4(a-b)^2d} + \frac{a^2b\log(b+a\cos(c+dx))}{(a^2-b^2)^2d}$$

```
[Out] 1/2*(b-a*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)/d+1/4*a*ln(1-cos(d*x+c))/(a+b)^
2/d-1/4*a*ln(1+cos(d*x+c))/(a-b)^2/d+a^2*b*ln(b+a*cos(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2916, 12, 837, 815}

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a^2b\log(a\cos(c+dx)+b)}{d(a^2-b^2)^2} + \frac{\csc^2(c+dx)(b-a\cos(c+dx))}{2d(a^2-b^2)} + \frac{a\log(1-\cos(c+dx))}{4d(a+b)^2} - \frac{a\log(\cos(c+dx)+1)}{4d(a-b)^2}$$

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b - a*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)*d) + (a*Log[1 - Cos[c
+ d*x]])/(4*(a + b)^2*d) - (a*Log[1 + Cos[c + d*x]])/(4*(a - b)^2*d) + (a^2
*b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cot(c+dx)\csc^2(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{a^3\text{Subst}\left(\int \frac{x}{a(-b+x)(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^2\text{Subst}\left(\int \frac{x}{(-b+x)(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$=\frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)d}+\frac{\text{Subst}\left(\int\frac{a^2b+a^2x}{(-b+x)(a^2-x^2)}dx,x,-a\cos(c+dx)\right)}{2(a^2-b^2)d}$$

$$=\frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)d}+\frac{\text{Subst}\left(\int\left(\frac{a(a+b)}{2(a-b)(a-x)}-\frac{2a^2b}{(a-b)(a+b)(b-x)}+\frac{a(a-b)}{2(a+b)(a+x)}\right)dx,x,-a\cos(c+dx)\right)}{2(a^2-b^2)d}$$

$$=\frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)d}+\frac{a\log(1-\cos(c+dx))}{4(a+b)^2d}-\frac{a\log(1+\cos(c+dx))}{4(a-b)^2d}+\frac{a^2b\log(b+a\cos(c+dx))}{(a^2-b^2)^2d}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int\frac{\csc^3(c+dx)}{a+b\sec(c+dx)}dx=\frac{-(a-b)^2(a+b)\csc^2\left(\frac{1}{2}(c+dx)\right)-4a\left((a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2ab\log(b+a\cos(c+dx))-(a-\right.}{8(a-b)^2(a+b)^2d}$$

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-((a - b)^2*(a + b)*Csc[(c + d*x)/2]^2) - 4*a*((a + b)^2*Log[Cos[(c + d*x)
/2]] - 2*a*b*Log[b + a*Cos[c + d*x]] - (a - b)^2*Log[Sin[(c + d*x)/2]]) + (
a - b)*(a + b)^2*Sec[(c + d*x)/2]^2)/(8*(a - b)^2*(a + b)^2*d)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{1}{(4a+4b)(\cos(dx+c)-1)}+\frac{a\ln(\cos(dx+c)-1)}{4(a+b)^2}+\frac{1}{(4a-4b)(\cos(dx+c)+1)}-\frac{a\ln(\cos(dx+c)+1)}{4(a-b)^2}+\frac{b\,a^2\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$
default	$\frac{\frac{1}{(4a+4b)(\cos(dx+c)-1)}+\frac{a\ln(\cos(dx+c)-1)}{4(a+b)^2}+\frac{1}{(4a-4b)(\cos(dx+c)+1)}-\frac{a\ln(\cos(dx+c)+1)}{4(a-b)^2}+\frac{b\,a^2\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$
parallelrisch	$\frac{8\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)a^2b-(a-b)\left(-4a(a-b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(a+b)\left((a-b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-\right.\right.}{8d(a-b)^2(a+b)^2}$
norman	$\frac{-\frac{1}{8d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8d(a-b)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{a^2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}{d(a^4-2a^2b^2+b^4)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d(a^2+2ab+b^2)}$
risch	$-\frac{iax}{2(a^2+2ab+b^2)}-\frac{iac}{2d(a^2+2ab+b^2)}+\frac{iax}{2a^2-4ab+2b^2}+\frac{iac}{2d(a^2-2ab+b^2)}-\frac{2ia^2bx}{a^4-2a^2b^2+b^4}-\frac{2ia^2bc}{d(a^4-2a^2b^2+b^4)}-$

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(4*a+4*b)/(cos(d*x+c)-1)+1/4*a/(a+b)^2*ln(cos(d*x+c)-1)+1/(4*a-4*b)/
(cos(d*x+c)+1)-1/4*a/(a-b)^2*ln(cos(d*x+c)+1)+b*a^2/(a+b)^2/(a-b)^2*ln(b+a*
cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.86

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2\,a^2b-2\,b^3-2\,(a^3-ab^2)\cos{(dx+c)}-4\left(a^2b\cos{(dx+c)}^2-a^2b\right)\log{(a\cos{(dx+c)}+b)}-\left(a^3+2\,a^2\right.}{4\left((a^4\right.}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(d*x + c) - 4*(a^2*b*cos(d*x + c
)^2 - a^2*b)*log(a*cos(d*x + c) + b) - (a^3 + 2*a^2*b + a*b^2 - (a^3 + 2*a^
2*b + a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - 2*a^2*b +
 a*b^2 - (a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/
2))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2 - (a^4 - 2*a^2*b^2 + b^4)*d)
```

Sympy [F]

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc^3{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{4\,a^2b\log(a\cos(dx+c)+b)}{a^4-2\,a^2b^2+b^4} - \frac{a\log(\cos(dx+c)+1)}{a^2-2\,ab+b^2} + \frac{a\log(\cos(dx+c)-1)}{a^2+2\,ab+b^2} + \frac{2\,(a\cos(dx+c)-b)}{(a^2-b^2)\cos(dx+c)^2-a^2+b^2}}{4\,d}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*b*log(a*cos(d*x + c) + b)/(a^4 - 2*a^2*b^2 + b^4) - a*log(cos(d*
x + c) + 1)/(a^2 - 2*a*b + b^2) + a*log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^
2) + 2*(a*cos(d*x + c) - b)/((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.74

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{8\,a^2b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-2\,a^2b^2+b^4} + \frac{2\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2} + \frac{\left(a+b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^2+2\,ab+b^2)(\cos(dx+c)-1)} - \frac{\cos(dx+c}{(a-b)(\cos(dx}}{8\,d}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - 2*a^2*b^2 + b^4) + 2*a*log(ab
s(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) + (a + b -
2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^2 + 2*a*b
 + b^2)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a - b)*(cos(d*x + c) + 1
)))/d
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{4\,d\,{\left(a+b\right)}^2} - \frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(\frac{a}{4\,{\left(a+b\right)}^2}-\frac{a}{4\,{\left(a-b\right)}^2}\right)}{d} - \frac{\frac{b}{2\,\left(a^2-b^2\right)}-\frac{a\,\cos\left(c+d\,x\right)}{2\,\left(a^2-b^2\right)}}{d\,\left({\cos\left(c+d\,x\right)}^2-1\right)} - \frac{a\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{4\,d\,{\left(a-b\right)}^2}$$

```
[In] int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))),x)
```

```
[Out] (a*log(cos(c + d*x) - 1))/(4*d*(a + b)^2) - (log(b + a*cos(c + d*x))*(a/(4*
(a + b)^2) - a/(4*(a - b)^2)))/d - (b/(2*(a^2 - b^2)) - (a*cos(c + d*x))/(2
*(a^2 - b^2)))/(d*(cos(c + d*x)^2 - 1)) - (a*log(cos(c + d*x) + 1))/(4*d*(a
 - b)^2)
```

3.202 $\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(4a^2b-a(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{8\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)d} + \frac{a(3a+b)\log(1-\cos(c+dx))}{16(a+b)^3d} - \frac{a(3a-b)\log(1+\cos(c+dx))}{16(a-b)^3d} + \frac{a^4b\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^3 d}$$

```
[Out] 1/8*(4*a^2*b-a*(3*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^2/d+1/4*(b-a*
cos(d*x+c))*csc(d*x+c)^4/(a^2-b^2)/d+1/16*a*(3*a+b)*ln(1-cos(d*x+c))/(a+b)^
3/d-1/16*a*(3*a-b)*ln(1+cos(d*x+c))/(a-b)^3/d+a^4*b*ln(b+a*cos(d*x+c))/(a^2
-b^2)^3/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used

$= \{3957, 2916, 12, 837, 815\}$

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\csc^4(c+dx)(b-a\cos(c+dx))}{4d\,(a^2-b^2)} + \frac{\csc^2(c+dx)\,(4a^2b-a(3a^2+b^2)\cos(c+dx))}{8d\,(a^2-b^2)^2} + \frac{a^4b\log(a\cos(c+dx)+b)}{d\,(a^2-b^2)^3} + \frac{a(3a+b)\log(1-\cos(c+dx))}{16d(a+b)^3} - \frac{a(3a-b)\log(\cos(c+dx)+1)}{16d(a-b)^3}$$

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((4*a^2*b - a*(3*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 - b^2)^2*
d) + ((b - a*Cos[c + d*x])*Csc[c + d*x]^4)/(4*(a^2 - b^2)*d) + (a*(3*a + b)
*Log[1 - Cos[c + d*x]])/(16*(a + b)^3*d) - (a*(3*a - b)*Log[1 + Cos[c + d*x
]])/(16*(a - b)^3*d) + (a^4*b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)\csc^4(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{a^5\text{Subst}\left(\int \frac{x}{a(-b+x)(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^4\text{Subst}\left(\int \frac{x}{(-b+x)(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)d} + \frac{a^2\text{Subst}\left(\int \frac{a^2b+3a^2x}{(-b+x)(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{4(a^2-b^2)d} \\
&= \frac{(4a^2b-a(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^2d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{a^2b(5a^2-b^2)+a^2(3a^2+b^2)x}{(-b+x)(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{8(a^2-b^2)^2d} \\
&= \frac{(4a^2b-a(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^2d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)d} \\
&\quad + \frac{\text{Subst}\left(\int \left(\frac{a(3a-b)(a+b)^2}{2(a-b)(a-x)} - \frac{8a^4b}{(a-b)(a+b)(b-x)} + \frac{a(a-b)^2(3a+b)}{2(a+b)(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{8(a^2-b^2)^2d} \\
&= \frac{(4a^2b-a(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^2d} \\
&\quad + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)d} + \frac{a(3a+b)\log(1-\cos(c+dx))}{16(a+b)^3d} \\
&\quad - \frac{a(3a-b)\log(1+\cos(c+dx))}{16(a-b)^3d} + \frac{a^4b\log(b+a\cos(c+dx))}{(a^2-b^2)^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.16

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-2(a-b)^3\left(3a^2+4ab+b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)-(a-b)^3(a+b)^2\csc^4\left(\frac{1}{2}(c+dx)\right)+8a\left(-\left((3a-b)(a+b)^3\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)^3*(3*a^2 + 4*a*b + b^2)*Csc[(c + d*x)/2]^2 - (a - b)^3*(a + b)^
2*Csc[(c + d*x)/2]^4 + 8*a*(-((3*a - b)*(a + b)^3*Log[Cos[(c + d*x)/2]]) +
8*a^3*b*Log[b + a*Cos[c + d*x]] + (a - b)^3*(3*a + b)*Log[Sin[(c + d*x)/2]]
) + 2*(a + b)^3*(3*a^2 - 4*a*b + b^2)*Sec[(c + d*x)/2]^2 + (a - b)^2*(a + b
)^3*Sec[(c + d*x)/2]^4)/(64*(a - b)^3*(a + b)^3*d)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{b a^4 \ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{2(8a-8b)(\cos(dx+c)+1)^2}-\frac{-3a+b}{16(a-b)^2(\cos(dx+c)+1)}-\frac{(3a-b)a\ln(\cos(dx+c)+1)}{16(a-b)^3}-\frac{1}{2(8a+8b)(\cos(dx+c)-1)^2}}{d}$
default	$\frac{\frac{b a^4 \ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{2(8a-8b)(\cos(dx+c)+1)^2}-\frac{-3a+b}{16(a-b)^2(\cos(dx+c)+1)}-\frac{(3a-b)a\ln(\cos(dx+c)+1)}{16(a-b)^3}-\frac{1}{2(8a+8b)(\cos(dx+c)-1)^2}}{d}$
parallelrisch	$\frac{64\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)a^4b-(a-b)\left(-24(a-b)^2a\left(a+\frac{b}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(a+b)\left((a+b)(a-b)^2\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4\right.\right.}{64d(a-b)^3(a+b)^3}$
norman	$\frac{-\frac{1}{64d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{64d(a-b)}+\frac{(2a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16d\left(a^2-2ab+b^2\right)}-\frac{(2a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{16d\left(a^2+2ab+b^2\right)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{a^4b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}{d(a^6-3a^4b^2+3a^2b^4-b^6)}$
risch	$-\frac{iabx}{8(a^3-3a^2b+3ab^2-b^3)}-\frac{iabc}{8d(a^3-3a^2b+3ab^2-b^3)}+\frac{3ia^2c}{8d(a^3-3a^2b+3ab^2-b^3)}-\frac{3ia^2x}{8(a^3+3a^2b+3ab^2+b^3)}-\frac{}{d(a^6-}$

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*a^4/(a+b)^3/(a-b)^3*ln(b+a*cos(d*x+c))+1/2/(8*a-8*b)/(cos(d*x+c)+1)^
2-1/16*(-3*a+b)/(a-b)^2/(cos(d*x+c)+1)-1/16*(3*a-b)*a/(a-b)^3*ln(cos(d*x+c)
+1)-1/2/(8*a+8*b)/(cos(d*x+c)-1)^2-1/16*(-3*a-b)/(a+b)^2/(cos(d*x+c)-1)+1/1
6*(3*a+b)/(a+b)^3*a*ln(cos(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(172) = 344$.

Time = 0.37 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.62

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{12\,a^4b - 16\,a^2b^3 + 4\,b^5 + 2\,(3\,a^5 - 2\,a^3b^2 - ab^4)\cos(dx+c)^3 - 8\,(a^4b - a^2b^3)\cos(dx+c)^2 - 2\,(5\,a^5 - 6\,a}{}$$

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(12*a^4*b - 16*a^2*b^3 + 4*b^5 + 2*(3*a^5 - 2*a^3*b^2 - a*b^4)*cos(d*x
 + c)^3 - 8*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 2*(5*a^5 - 6*a^3*b^2 + a*b^4
)*cos(d*x + c) + 16*(a^4*b*cos(d*x + c)^4 - 2*a^4*b*cos(d*x + c)^2 + a^4*b)
*log(a*cos(d*x + c) + b) - (3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4 + (3*a^5 +
8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 + 8*a^4*b + 6*a^3*b^
2 - a*b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (3*a^5 - 8*a^4*b +
 6*a^3*b^2 - a*b^4 + (3*a^5 - 8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4 -
 2*(3*a^5 - 8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x +
 c) + 1/2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4 - 2*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*d)
```

Sympy [F]

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.50

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{16\,a^4b\log(a\cos(dx+c)+b)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{(3\,a^2-ab)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3} + \frac{(3\,a^2+ab)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\left(4\,a^2b\cos(dx+c)^2-(3\,a^3+ab^2)\cos(dx+c)^3-\right.}{(a^4-2\,a^2b^2+b^4)\cos(dx+c)^4+a^4-2\,a^2b^2+b}}{16\,d}$$

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^4*b*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
- (3*a^2 - a*b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (3*
a^2 + a*b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(4*a^2
*b*cos(d*x + c)^2 - (3*a^3 + a*b^2)*cos(d*x + c)^3 - 6*a^2*b + 2*b^3 + (5*a
^3 - a*b^2)*cos(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2
*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(172) = 344$.

Time $= 0.35$ (sec) , antiderivative size $= 419$, normalized size of antiderivative $= 2.34$

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{64\,a^4 b \log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{4\,(3\,a^2+ab)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1}{(\cos(dx+c)+1)}}{a^2-2\,ab+b^2}}{}$$

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(64*a^4*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
) + 4*(3*a^2 + a*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3
+ 3*a^2*b + 3*a*b^2 + b^3) - (8*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4
*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^2 - 2*a*b + b
^2) - (a^2 + 2*a*b + b^2 - 8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12
*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b^2*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 18*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 6*a*b*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*(cos(d*x + c) - 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 14.03 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.66

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{3\,a^2\,b-b^3}{4\,(a^2-b^2)^2}+\frac{\cos(c+d\,x)^3\,(3\,a^3+a\,b^2)}{8\,(a^4-2\,a^2\,b^2+b^4)}-\frac{a^2\,b\cos(c+d\,x)^2}{2\,(a^2-b^2)^2}-\frac{a\,\cos(c+d\,x)\,(5\,a^2-b^2)}{8\,(a^4-2\,a^2\,b^2+b^4)}}{d\,\left(\cos\left(c+d\,x\right)^4-2\cos\left(c+d\,x\right)^2+1\right)} + \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3}{16\,(a+b)}-\frac{5\,b}{16\,(a+b)^2}+\frac{b^2}{8\,(a+b)^3}\right)}{d} - \frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{b^2}{8\,(a-b)^3}+\frac{5\,b}{16\,(a-b)^2}+\frac{3}{16\,(a-b)}\right)}{d} + \frac{a^4\,b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}$$

```
[In] int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))),x)
```

```
[Out] ((3*a^2*b - b^3)/(4*(a^2 - b^2)^2) + (cos(c + d*x)^3*(a*b^2 + 3*a^3))/(8*(a
^4 + b^4 - 2*a^2*b^2)) - (a^2*b*cos(c + d*x)^2)/(2*(a^2 - b^2)^2) - (a*cos(
c + d*x)*(5*a^2 - b^2))/(8*(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c + d*x)^4 - 2
*cos(c + d*x)^2 + 1)) + (log(cos(c + d*x) - 1)*(3/(16*(a + b)) - (5*b)/(16*
(a + b)^2) + b^2/(8*(a + b)^3)))/d - (log(cos(c + d*x) + 1)*(b^2/(8*(a - b)
^3) + (5*b)/(16*(a - b)^2) + 3/(16*(a - b))))/d + (a^4*b*log(b + a*cos(c +
d*x)))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
```

3.203 $\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 230

$$\begin{aligned}\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\,x}{16a^7}\\ &\quad-\frac{2(a-b)^{5/2}b(a+b)^{5/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7d}\\ &\quad+\frac{\left(16b(a^2-b^2)^2-a(5a^4-14a^2b^2+8b^4)\cos(c+dx)\right)\sin(c+dx)}{16a^6d}\\ &\quad+\frac{(8b(a^2-b^2)-a(5a^2-6b^2)\cos(c+dx))\sin^3(c+dx)}{24a^4d}\\ &\quad+\frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d}\end{aligned}$$

```
[Out] 1/16*(5*a^6-30*a^4*b^2+40*a^2*b^4-16*b^6)*x/a^7-2*(a-b)^(5/2)*b*(a+b)^(5/2)
*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^7/d+1/16*(16*b*(a^2-
b^2)^2-a*(5*a^4-14*a^2*b^2+8*b^4)*cos(d*x+c))*sin(d*x+c)/a^6/d+1/24*(8*b*(a
^2-b^2)-a*(5*a^2-6*b^2)*cos(d*x+c))*sin(d*x+c)^3/a^4/d+1/30*(6*b-5*a*cos(d*
x+c))*sin(d*x+c)^5/a^2/d
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2944, 2814, 2738, 214}

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2b(a-b)^{5/2}(a+b)^{5/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7d} + \frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d} + \frac{\sin^3(c+dx)\left(8b(a^2-b^2)-a(5a^2-6b^2)\cos(c+dx)\right)}{24a^4d} + \frac{\sin(c+dx)\left(16b(a^2-b^2)^2-a(5a^4-14a^2b^2+8b^4)\cos(c+dx)\right)}{16a^6d} + \frac{x(5a^6-30a^4b^2+40a^2b^4-16b^6)}{16a^7}$$

```
[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*x)/(16*a^7) - (2*(a - b)^(5/2)*
b*(a + b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^7*d
) + ((16*b*(a^2 - b^2)^2 - a*(5*a^4 - 14*a^2*b^2 + 8*b^4)*Cos[c + d*x])*Sin
[c + d*x])/(16*a^6*d) + ((8*b*(a^2 - b^2) - a*(5*a^2 - 6*b^2)*Cos[c + d*x])
*Sin[c + d*x]^3)/(24*a^4*d) + ((6*b - 5*a*Cos[c + d*x])*Sin[c + d*x]^5)/(30
*a^2*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin^6(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d} - \frac{\int \frac{(-ab+(5a^2-6b^2)\cos(c+dx))\sin^4(c+dx)}{-b-a\cos(c+dx)}\,dx}{6a^2} \\
&= \frac{(8b(a^2-b^2)-a(5a^2-6b^2)\cos(c+dx))\sin^3(c+dx)}{24a^4d} \\
&\quad + \frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d} \\
&\quad - \frac{\int \frac{(-3ab(3a^2-2b^2)+3(5a^4-14a^2b^2+8b^4)\cos(c+dx))\sin^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{24a^4} \\
&= \frac{\left(16b(a^2-b^2)^2-a(5a^4-14a^2b^2+8b^4)\cos(c+dx)\right)\sin(c+dx)}{16a^6d} \\
&\quad + \frac{(8b(a^2-b^2)-a(5a^2-6b^2)\cos(c+dx))\sin^3(c+dx)}{24a^4d} \\
&\quad + \frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d} \\
&\quad - \frac{\int \frac{-3ab(11a^4-18a^2b^2+8b^4)+3(5a^6-30a^4b^2+40a^2b^4-16b^6)\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{48a^6}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\,x}{16a^7} \\
&\quad + \frac{\left(16b(a^2-b^2)^2-a(5a^4-14a^2b^2+8b^4)\cos(c+dx)\right)\sin(c+dx)}{16a^6d} \\
&\quad + \frac{(8b(a^2-b^2)-a(5a^2-6b^2)\cos(c+dx))\sin^3(c+dx)}{24a^4d} \\
&\quad + \frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d} + \frac{\left(b(a^2-b^2)^3\right)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^7} \\
&= \frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\,x}{16a^7} \\
&\quad + \frac{\left(16b(a^2-b^2)^2-a(5a^4-14a^2b^2+8b^4)\cos(c+dx)\right)\sin(c+dx)}{16a^6d} \\
&\quad + \frac{(8b(a^2-b^2)-a(5a^2-6b^2)\cos(c+dx))\sin^3(c+dx)}{24a^4d} \\
&\quad + \frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d} \\
&\quad + \frac{\left(2b(a^2-b^2)^3\right)\mathrm{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^7d} \\
&= \frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\,x}{16a^7} \\
&\quad - \frac{2(a-b)^{5/2}b(a+b)^{5/2}\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7d} \\
&\quad + \frac{\left(16b(a^2-b^2)^2-a(5a^4-14a^2b^2+8b^4)\cos(c+dx)\right)\sin(c+dx)}{16a^6d} \\
&\quad + \frac{(8b(a^2-b^2)-a(5a^2-6b^2)\cos(c+dx))\sin^3(c+dx)}{24a^4d} \\
&\quad + \frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.17

$$
\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx
= \frac{300a^6c-1800a^4b^2c+2400a^2b^4c-960b^6c+300a^6dx-1800a^4b^2dx+2400a^2b^4dx-960b^6dx+1920b(a^2}{}
$$

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

[Out] (300*a^6*c - 1800*a^4*b^2*c + 2400*a^2*b^4*c - 960*b^6*c + 300*a^6*d*x - 18
00*a^4*b^2*d*x + 2400*a^2*b^4*d*x - 960*b^6*d*x + 1920*b*(a^2 - b^2)^(5/2)*
ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 120*a*b*(11*a^4 - 18
*a^2*b^2 + 8*b^4)*Sin[c + d*x] - 15*(15*a^6 - 32*a^4*b^2 + 16*a^2*b^4)*Sin[
2*(c + d*x)] - 140*a^5*b*Sin[3*(c + d*x)] + 80*a^3*b^3*Sin[3*(c + d*x)] + 4
5*a^6*Sin[4*(c + d*x)] - 30*a^4*b^2*Sin[4*(c + d*x)] + 12*a^5*b*Sin[5*(c +
d*x)] - 5*a^6*Sin[6*(c + d*x)])/(960*a^7*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(213) = 426.

Time = 1.56 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.92

method	result
derivativedivides	$-\frac{2b(a+b)^3(a-b)^3 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^7\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(\frac{5}{16}a^6+a^5b-\frac{7}{8}a^4b^2-2a^3b^3+\frac{1}{2}a^2b^4+ab^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(\frac{19}{3}a^5b-\frac{29}{8}a^4b^2\right.}{}$
default	$-\frac{2b(a+b)^3(a-b)^3 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^7\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(\frac{5}{16}a^6+a^5b-\frac{7}{8}a^4b^2-2a^3b^3+\frac{1}{2}a^2b^4+ab^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(\frac{19}{3}a^5b-\frac{29}{8}a^4b^2\right.}{}$
risch	$\frac{3\sin(4dx+4c)}{64ad}-\frac{15x\,b^2}{8a^3}+\frac{5x\,b^4}{2a^5}-\frac{x\,b^6}{a^7}-\frac{\sin(6dx+6c)}{192da}-\frac{15\sin(2dx+2c)}{64da}-\frac{\sin(4dx+4c)b^2}{32a^3d}-\frac{7b\sin(3dx+3c)}{48a^2d}$

[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(-2*b*(a+b)^3*(a-b)^3/a^7/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x
+1/2*c)/((a-b)*(a+b))^(1/2))+2/a^7*(((5/16*a^6+a^5*b-7/8*a^4*b^2-2*a^3*b^3+
1/2*a^2*b^4+a*b^5)*tan(1/2*d*x+1/2*c)^11+(19/3*a^5*b-29/8*a^4*b^2+3/2*a^2*b
^4+5*a*b^5+85/48*a^6-34/3*a^3*b^3)*tan(1/2*d*x+1/2*c)^9+(86/5*a^5*b-11/4*a^
4*b^2-24*a^3*b^3+a^2*b^4+10*a*b^5+33/8*a^6)*tan(1/2*d*x+1/2*c)^7+(-33/8*a^6
+11/4*a^4*b^2-a^2*b^4+86/5*a^5*b-24*a^3*b^3+10*a*b^5)*tan(1/2*d*x+1/2*c)^5+
(19/3*a^5*b+29/8*a^4*b^2-34/3*a^3*b^3-3/2*a^2*b^4+5*a*b^5-85/48*a^6)*tan(1/
2*d*x+1/2*c)^3+(a^5*b-2*a^3*b^3+a*b^5-5/16*a^6+7/8*a^4*b^2-1/2*a^2*b^4)*tan
(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^6+1/16*(5*a^6-30*a^4*b^2+40*a^2*b
^4-16*b^6)*arctan(tan(1/2*d*x+1/2*c))))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.40

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{15\,(5\,a^6-30\,a^4b^2+40\,a^2b^4-16\,b^6)dx+120\,(a^4b-2\,a^2b^3+b^5)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+}{a^2\cos(d}\right.}{}\right.$$

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*d*x + 120*(a^4*b - 2*
a^2*b^3 + b^5)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(
d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 -
b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (40*a^6*cos(d*x + c
)^5 - 48*a^5*b*cos(d*x + c)^4 - 368*a^5*b + 560*a^3*b^3 - 240*a*b^5 - 10*(1
3*a^6 - 6*a^4*b^2)*cos(d*x + c)^3 + 16*(11*a^5*b - 5*a^3*b^3)*cos(d*x + c)^
2 + 15*(11*a^6 - 18*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^7*d
), 1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*d*x - 240*(a^4*b -
2*a^2*b^3 + b^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c))) - (40*a^6*cos(d*x + c)^5 - 48*a^5*b*cos(d*
x + c)^4 - 368*a^5*b + 560*a^3*b^3 - 240*a*b^5 - 10*(13*a^6 - 6*a^4*b^2)*co
s(d*x + c)^3 + 16*(11*a^5*b - 5*a^3*b^3)*cos(d*x + c)^2 + 15*(11*a^6 - 18*a
^4*b^2 + 8*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^7*d)]
```

Sympy [F]

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin^6{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(212) = 424$.

Time = 0.35 (sec) , antiderivative size = 781, normalized size of antiderivative = 3.40

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*(d*x + c)/a^7 - 480*(a
^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^7) + 2*(75*a^5*tan(1/2*d*x + 1/2*c)^11 +
 240*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 210*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 -
480*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 2
40*b^5*tan(1/2*d*x + 1/2*c)^11 + 425*a^5*tan(1/2*d*x + 1/2*c)^9 + 1520*a^4*
b*tan(1/2*d*x + 1/2*c)^9 - 870*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 2720*a^2*b^
3*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^4*tan(1/2*d*x + 1/2*c)^9 + 1200*b^5*tan(
1/2*d*x + 1/2*c)^9 + 990*a^5*tan(1/2*d*x + 1/2*c)^7 + 4128*a^4*b*tan(1/2*d*
x + 1/2*c)^7 - 660*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*a^2*b^3*tan(1/2*d*
x + 1/2*c)^7 + 240*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 2400*b^5*tan(1/2*d*x + 1/
2*c)^7 - 990*a^5*tan(1/2*d*x + 1/2*c)^5 + 4128*a^4*b*tan(1/2*d*x + 1/2*c)^5
 + 660*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 5760*a^2*b^3*tan(1/2*d*x + 1/2*c)^5
 - 240*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 2400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425
*a^5*tan(1/2*d*x + 1/2*c)^3 + 1520*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 870*a^3*b
^2*tan(1/2*d*x + 1/2*c)^3 - 2720*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*a*b^4
*tan(1/2*d*x + 1/2*c)^3 + 1200*b^5*tan(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*
d*x + 1/2*c) + 240*a^4*b*tan(1/2*d*x + 1/2*c) + 210*a^3*b^2*tan(1/2*d*x + 1
/2*c) - 480*a^2*b^3*tan(1/2*d*x + 1/2*c) - 120*a*b^4*tan(1/2*d*x + 1/2*c) +
 240*b^5*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^6))/d
```

Mupad [B] (verification not implemented)

Time = 16.12 (sec) , antiderivative size = 3341, normalized size of antiderivative = 14.53

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] int(sin(c + d*x)^6/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan((((((((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 +
 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 - (tan(c/2 +
(d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2)*(a^6*5i - b^6*16i + a^
2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4
*b^2*30i))/(16*a^7) + (tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^1
5 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^
10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^1
1*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(a^6*5i - b^6*16i + a^2*b^4*
40i - a^4*b^2*30i)*1i)/(16*a^7) - (((((42*a^21*b - 10*a^22 + 32*a^14*b^8 -
48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a
^20*b^2)/a^18 + (tan(c/2 + (d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*
b^2)*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i -
b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) - (tan(c/2 + (d*x)/2)*(1024*
a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2
560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6
 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(
a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i)*1i)/(16*a^7))/(((25*a^19*b)/4
 - 96*a*b^19 + 64*b^20 - 480*a^2*b^18 + 760*a^3*b^17 + 1544*a^4*b^16 - 2628
*a^5*b^15 - 2748*a^6*b^14 + 5179*a^7*b^13 + 2890*a^8*b^12 - 6359*a^9*b^11 -
 1736*a^10*b^10 + (19951*a^11*b^9)/4 + (937*a^12*b^8)/2 - (4915*a^13*b^7)/2
 + (85*a^14*b^6)/2 + 715*a^15*b^5 - (105*a^16*b^4)/2 - (215*a^17*b^3)/2 + (
15*a^18*b^2)/2)/a^18 + (((((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7
 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^
18 - (tan(c/2 + (d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2)*(a^6*5
i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i - b^6*16i + a
^2*b^4*40i - a^4*b^2*30i))/(16*a^7) + (tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75
*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^1
1 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^1
0*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(a^6*5i - b^
6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) + (((((42*a^21*b - 10*a^22 + 3
2*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a
^19*b^3 + 6*a^20*b^2)/a^18 + (tan(c/2 + (d*x)/2)*(512*a^16*b + 512*a^14*b^3
 - 1024*a^15*b^2)*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(128*a^19
))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) - (tan(c/2 + (d
*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120
*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 +
```

5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2)
)/(8*a^12))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7)))*(a^6
*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i)*1i)/(8*a^7*d) - ((tan(c/2 + (d*x
)/2)*(8*a*b^4 - 16*a^4*b + 5*a^5 - 16*b^5 + 32*a^2*b^3 - 14*a^3*b^2))/(8*a^
6) - (tan(c/2 + (d*x)/2)^11*(8*a*b^4 + 16*a^4*b + 5*a^5 + 16*b^5 - 32*a^2*b
^3 - 14*a^3*b^2))/(8*a^6) + (tan(c/2 + (d*x)/2)^3*(72*a*b^4 - 304*a^4*b + 8
5*a^5 - 240*b^5 + 544*a^2*b^3 - 174*a^3*b^2))/(24*a^6) - (tan(c/2 + (d*x)/2
)^9*(72*a*b^4 + 304*a^4*b + 85*a^5 + 240*b^5 - 544*a^2*b^3 - 174*a^3*b^2))/
(24*a^6) + (tan(c/2 + (d*x)/2)^5*(40*a*b^4 - 688*a^4*b + 165*a^5 - 400*b^5
+ 960*a^2*b^3 - 110*a^3*b^2))/(20*a^6) - (tan(c/2 + (d*x)/2)^7*(40*a*b^4 +
688*a^4*b + 165*a^5 + 400*b^5 - 960*a^2*b^3 - 110*a^3*b^2))/(20*a^6))/(d*(6
*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 +
15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12
+ 1)) + (b*atan(((b*((tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15
- 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^1
0 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11
*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12) + (b*((a + b)^5*(a - b)^5)^(1/
2)*((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^
17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 - (b*tan(c/2 + (d*x)
/2)*((a + b)^5*(a - b)^5)^(1/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2)
)/(8*a^19)))/a^7)*((a + b)^5*(a - b)^5)^(1/2)*1i)/a^7 + (b*((tan(c/2 + (d*x
)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a
^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5
180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/
(8*a^12) - (b*((a + b)^5*(a - b)^5)^(1/2)*((42*a^21*b - 10*a^22 + 32*a^14*b
^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3
+ 6*a^20*b^2)/a^18 + (b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(512
*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2))/(8*a^19)))/a^7)*((a + b)^5*(a - b)
^5)^(1/2)*1i)/a^7)/(((25*a^19*b)/4 - 96*a*b^19 + 64*b^20 - 480*a^2*b^18 + 7
60*a^3*b^17 + 1544*a^4*b^16 - 2628*a^5*b^15 - 2748*a^6*b^14 + 5179*a^7*b^13
+ 2890*a^8*b^12 - 6359*a^9*b^11 - 1736*a^10*b^10 + (19951*a^11*b^9)/4 + (9
37*a^12*b^8)/2 - (4915*a^13*b^7)/2 + (85*a^14*b^6)/2 + 715*a^15*b^5 - (105*
a^16*b^4)/2 - (215*a^17*b^3)/2 + (15*a^18*b^2)/2)/a^18 + (b*((tan(c/2 + (d*
x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*
a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 +
5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))
/(8*a^12) + (b*((a + b)^5*(a - b)^5)^(1/2)*((42*a^21*b - 10*a^22 + 32*a^14*
b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3
+ 6*a^20*b^2)/a^18 - (b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(51
2*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2))/(8*a^19)))/a^7)*((a + b)^5*(a - b
)^5)^(1/2))/a^7 - (b*((tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^1
5 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^
10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^1
1*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12) - (b*((a + b)^5*(a - b)^5)^(1
/2)*((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a

```
^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 + (b*tan(c/2 + (d*x
)/2)*((a + b)^5*(a - b)^5)^(1/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2
))/(8*a^19)))/a^7)*((a + b)^5*(a - b)^5)^(1/2))/a^7))*((a + b)^5*(a - b)^5)
^(1/2)*2i)/(a^7*d)
```

3.204 $\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d} + \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))\sin(c+dx)}{8a^4d} + \frac{(4b-3a\cos(c+dx))\sin^3(c+dx)}{12a^2d}$$

```
[Out] 1/8*(3*a^4-12*a^2*b^2+8*b^4)*x/a^5-2*(a-b)^(3/2)*b*(a+b)^(3/2)*arctanh((a-b
)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/d+1/8*(8*b*(a^2-b^2)-a*(3*a^2-4
*b^2)*cos(d*x+c))*sin(d*x+c)/a^4/d+1/12*(4*b-3*a*cos(d*x+c))*sin(d*x+c)^3/a
^2/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used

$= \{3957, 2944, 2814, 2738, 214\}$

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2b(a-b)^{3/2}(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d} + \frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d} + \frac{\sin(c+dx)\left(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx)\right)}{8a^4d} + \frac{x(3a^4-12a^2b^2+8b^4)}{8a^5}$$

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((3*a^4 - 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*(a - b)^(3/2)*b*(a + b)^(3/2)
*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*d) + ((8*b*(a^2
- b^2) - a*(3*a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/(8*a^4*d) + ((4*b -
3*a*Cos[c + d*x])*Sin[c + d*x]^3)/(12*a^2*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
```

```
0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin^4(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{(4b-3a\cos(c+dx))\sin^3(c+dx)}{12a^2d} - \frac{\int \frac{(-ab+(3a^2-4b^2)\cos(c+dx))\sin^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{4a^2} \\
&= \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))\sin(c+dx)}{8a^4d} \\
&\quad + \frac{(4b-3a\cos(c+dx))\sin^3(c+dx)}{12a^2d} - \frac{\int \frac{-ab(5a^2-4b^2)+(3a^4-12a^2b^2+8b^4)\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{8a^4} \\
&= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} + \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))\sin(c+dx)}{8a^4d} \\
&\quad + \frac{(4b-3a\cos(c+dx))\sin^3(c+dx)}{12a^2d} + \frac{\left(b(a^2-b^2)^2\right)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^5} \\
&= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} + \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))\sin(c+dx)}{8a^4d} \\
&\quad + \frac{(4b-3a\cos(c+dx))\sin^3(c+dx)}{12a^2d} \\
&\quad + \frac{\left(2b(a^2-b^2)^2\right)\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} \\
&= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d} \\
&\quad + \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))\sin(c+dx)}{8a^4d} \\
&\quad + \frac{(4b-3a\cos(c+dx))\sin^3(c+dx)}{12a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.07

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{36a^4c-144a^2b^2c+96b^4c+36a^4dx-144a^2b^2dx+96b^4dx+192b(a^2-b^2)^{3/2}\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\cdots}{96a\cdots}$$

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (36*a^4*c - 144*a^2*b^2*c + 96*b^4*c + 36*a^4*d*x - 144*a^2*b^2*d*x + 96*b^
4*d*x + 192*b*(a^2 - b^2)^(3/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]] + 24*a*b*(5*a^2 - 4*b^2)*Sin[c + d*x] - 24*(a^4 - a^2*b^2)*Sin[2*
(c + d*x)] - 8*a^3*b*Sin[3*(c + d*x)] + 3*a^4*Sin[4*(c + d*x)])/(96*a^5*d)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.64

method	result
derivativedivides	$-\frac{2(a-b)^2(a+b)^2 b\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^5\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(\frac{3}{8}a^4+a^3b-\frac{1}{2}a^2b^2-ab^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(\frac{13}{3}a^3b-\frac{1}{2}a^2b^2-3ab^3+\frac{11}{8}a^4\right)\mathrm{ta}\cdots}{\cdots}$
default	$-\frac{2(a-b)^2(a+b)^2 b\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^5\sqrt{(a-b)(a+b)}}+\frac{2\left(\left(\frac{3}{8}a^4+a^3b-\frac{1}{2}a^2b^2-ab^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(\frac{13}{3}a^3b-\frac{1}{2}a^2b^2-3ab^3+\frac{11}{8}a^4\right)\mathrm{ta}\cdots}{\cdots}$
risch	$\frac{3x}{8a}-\frac{3xb^2}{2a^3}+\frac{xb^4}{a^5}-\frac{5ib\,e^{i(dx+c)}}{8d\,a^2}+\frac{ib^3e^{i(dx+c)}}{2d\,a^4}+\frac{5ib\,e^{-i(dx+c)}}{8d\,a^2}-\frac{ib^3e^{-i(dx+c)}}{2d\,a^4}+\frac{\sqrt{a^2-b^2}\,b\ln\left(e^{i(dx+c)}+\cdots\right.}{d\,a^3}$

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(a-b)^2*(a+b)^2*b/a^5/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x
+1/2*c)/((a-b)*(a+b))^(1/2))+2/a^5*(((3/8*a^4+a^3*b-1/2*a^2*b^2-a*b^3)*tan(
1/2*d*x+1/2*c)^7+(13/3*a^3*b-1/2*a^2*b^2-3*a*b^3+11/8*a^4)*tan(1/2*d*x+1/2*
c)^5+(-11/8*a^4+1/2*a^2*b^2+13/3*a^3*b-3*a*b^3)*tan(1/2*d*x+1/2*c)^3+(a^3*b
-a*b^3-3/8*a^4+1/2*a^2*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4+
1/8*(3*a^4-12*a^2*b^2+8*b^4)*arctan(tan(1/2*d*x+1/2*c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.44

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = \left[\frac{3\,(3\,a^4-12\,a^2b^2+8\,b^4)dx-12\,(a^2b-b^3)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{}\right.$$

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*d*x - 12*(a^2*b - b^3)*sqrt(a^2 - b^2
)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2
)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*
a*b*cos(d*x + c) + b^2)) + (6*a^4*cos(d*x + c)^3 - 8*a^3*b*cos(d*x + c)^2 +
 32*a^3*b - 24*a*b^3 - 3*(5*a^4 - 4*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/(a
^5*d), 1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*d*x - 24*(a^2*b - b^3)*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x
 + c))) + (6*a^4*cos(d*x + c)^3 - 8*a^3*b*cos(d*x + c)^2 + 32*a^3*b - 24*a*
b^3 - 3*(5*a^4 - 4*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/(a^5*d)]
```

Sympy [F]

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(145) = 290$.

Time = 0.31 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.53

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{3\,(3\,a^4-12\,a^2b^2+8\,b^4)(dx+c)}{a^5} - \frac{48\,(a^4b-2\,a^2b^3+b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}a^5} + \frac{2\,(9\,a^3}{}}{}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*(d*x + c)/a^5 - 48*(a^4*b - 2*a^2*b^3
+ b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b
^2)*a^5) + 2*(9*a^3*tan(1/2*d*x + 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^
7 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^
3*tan(1/2*d*x + 1/2*c)^5 + 104*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 12*a*b^2*tan(
1/2*d*x + 1/2*c)^5 - 72*b^3*tan(1/2*d*x + 1/2*c)^5 - 33*a^3*tan(1/2*d*x + 1
/2*c)^3 + 104*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^
3 - 72*b^3*tan(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*t
an(1/2*d*x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c) - 24*b^3*tan(1/2*d*x +
1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^4))/d
```

Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.97

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{5\,b\,\sin(c+d\,x)}{4}-\frac{b\,\sin(3\,c+3\,d\,x)}{12}}{a^2\,d} - \frac{3\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{b^2\,\sin(2\,c+2\,d\,x)}{4}}{a^3\,d}$$
$$+ \frac{\frac{3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}-\frac{\sin(2\,c+2\,d\,x)}{4}+\frac{\sin(4\,c+4\,d\,x)}{32}}{a\,d} - \frac{b^3\,\sin\left(c+d\,x\right)}{a^4\,d} + \frac{2\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^5\,d}$$
$$- \frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^3+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b^2-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^3}\right)\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{a^5\,d}$$

```
[In] int(sin(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

```
[Out] ((5*b*sin(c + d*x))/4 - (b*sin(3*c + 3*d*x))/12)/(a^2*d) - (3*b^2*atan(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (b^2*sin(2*c + 2*d*x))/4)/(a^3*d) + ((
```

```
3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 - sin(2*c + 2*d*x)/4 + sin
(4*c + 4*d*x)/32)/(a*d) - (b^3*sin(c + d*x))/(a^4*d) + (2*b^4*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^5*d) - (2*b*atanh((sin(c/2 + (d*x)/2)*(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^3*cos(c/2 + (d*x)/2) - b^3*cos(
c/2 + (d*x)/2) - a*b^2*cos(c/2 + (d*x)/2) + a^2*b*cos(c/2 + (d*x)/2)))*(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^5*d)
```

3.205 $\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(a^2-2b^2)\,x}{2a^3} - \frac{2\sqrt{a-b}b\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d} + \frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d}$$

```
[Out] 1/2*(a^2-2*b^2)*x/a^3+1/2*(2*b-a*cos(d*x+c))*sin(d*x+c)/a^2/d-2*b*arctanh((
a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2944, 2814, 2738, 214}

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2b\sqrt{a-b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d} + \frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d} + \frac{x(a^2-2b^2)}{2a^3}$$

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^2 - 2*b^2)*x)/(2*a^3) - (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*d) + ((2*b - a*Cos[c + d*x])*Sin[c
+ d*x])/(2*a^2*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sin^2(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d} - \frac{\int \frac{-ab+(a^2-2b^2)\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{2a^2} \\
&= \frac{(a^2-2b^2)\,x}{2a^3} + \frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d} + \frac{(b(a^2-b^2))\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(a^2-2b^2)\,x}{2a^3}+\frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d} \\
&\quad +\frac{(2b(a^2-b^2))\,\text{Subst}\left(\int\frac{1}{-a-b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3d} \\
&= \frac{(a^2-2b^2)\,x}{2a^3}-\frac{2\sqrt{a-b}b\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d}+\frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$
\int\frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx
=\frac{2(a^2-2b^2)\,(c+dx)+8b\sqrt{a^2-b^2}\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+4ab\sin(c+dx)-a^2\sin(2(c+dx))}{4a^3d}
$$

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - 2*b^2)*(c + d*x) + 8*b*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]] + 4*a*b*Sin[c + d*x] - a^2*Sin[2*(c + d*x)])/(4*
a^3*d)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{-\frac{2(a+b)b(a-b)\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}+\frac{\frac{2\left(\left(\frac{1}{2}a^2+ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(ab-\frac{1}{2}a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2-2b^2\right)\arctan\left(\tan\left(\cdots\right)\right)}{a^3}}{d}$
default	$\frac{-\frac{2(a+b)b(a-b)\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}+\frac{\frac{2\left(\left(\frac{1}{2}a^2+ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(ab-\frac{1}{2}a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2-2b^2\right)\arctan\left(\tan\left(\cdots\right)\right)}{a^3}}{d}$
risch	$\frac{x}{2a}-\frac{x\,b^2}{a^3}-\frac{ib\,e^{i(dx+c)}}{2d\,a^2}+\frac{ib\,e^{-i(dx+c)}}{2d\,a^2}-\frac{\sqrt{a^2-b^2}\,b\ln\left(e^{i(dx+c)}+\frac{b+i\sqrt{a^2-b^2}}{a}\right)}{d\,a^3}+\frac{\sqrt{a^2-b^2}\,b\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2\cdots}}{\cdots}\right)}{d\,a^3}$

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(a+b)*b*(a-b)/a^3/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a-b)*(a+b))^(1/2))+2/a^3*((((1/2*a^2+a*b)*tan(1/2*d*x+1/2*c)^3+(a*b-1/
2*a^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2-2*b^2)*arcta
n(tan(1/2*d*x+1/2*c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.58

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{(a^2-2\,b^2)dx + \sqrt{a^2-b^2}b\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right) - (a^2\,\mathrm{co}}{2\,a^3d} \right.$$

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 - 2*b^2)*d*x + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (a^2*c
os(d*x + c) - 2*a*b)*sin(d*x + c))/(a^3*d), 1/2*((a^2 - 2*b^2)*d*x - 2*sqrt
(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*s
in(d*x + c))) - (a^2*cos(d*x + c) - 2*a*b)*sin(d*x + c))/(a^3*d)]
```

Sympy [F]

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(86) = 172$.

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.85

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{(a^2-2b^2)(dx+c)}{a^3} - \frac{4(a^2b-b^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^3} + \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2b\tan\cdots\right.}{\cdots}}{2d}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((a^2 - 2*b^2)*(d*x + c)/a^3 - 4*(a^2*b - b^3)*(pi*floor(1/2*(d*x + c)/
pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
 + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^3) + 2*(a*tan(1/2*d*x + 1
/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 2*b*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{\sin(2\,c+2\,d\,x)}{4}}{a\,d} - \frac{2\,b^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^3\,d} + \frac{b\,\sin\left(c+d\,x\right)}{a^2\,d} - \frac{2\,b\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a+b)}\right)\sqrt{a^2-b^2}}{a^3\,d}$$

```
[In] int(sin(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - sin(2*c + 2*d*x)/4)/(a*d) -
(2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^3*d) + (b*sin(c + d*
x))/(a^2*d) - (2*b*atanh((sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2 +
(d*x)/2)*(a + b)))*(a^2 - b^2)^(1/2))/(a^3*d)
```

3.206 $\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2ab\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}+\frac{(b-a\cos(c+dx))\csc(c+dx)}{(a^2-b^2)\,d}$$

```
[Out] -2*a*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b
)^(3/2)/d+(b-a*cos(d*x+c))*csc(d*x+c)/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2945, 12, 2738, 214}

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\csc(c+dx)(b-a\cos(c+dx))}{d\,(a^2-b^2)}-\frac{2ab\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)
*(a + b)^(3/2)*d) + ((b - a*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)\csc(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{(b-a\cos(c+dx))\csc(c+dx)}{(a^2-b^2)\,d}+\frac{\int \frac{ab}{-b-a\cos(c+dx)}\,dx}{a^2-b^2} \\
&= \frac{(b-a\cos(c+dx))\csc(c+dx)}{(a^2-b^2)\,d}+\frac{(ab)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^2-b^2} \\
&= \frac{(b-a\cos(c+dx))\csc(c+dx)}{(a^2-b^2)\,d}+\frac{(2ab)\mathrm{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)\,d} \\
&= -\frac{2ab\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}+\frac{(b-a\cos(c+dx))\csc(c+dx)}{(a^2-b^2)\,d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{a^2-b^2}(b-a\cos(c+dx))+2ab\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\sin(c+dx)\right)}{2(a-b)(a+b)\sqrt{a^2-b^2}d}$$

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Sqrt[a^2 - b^2]*(b - a*Cos[c + d*x]) +
2*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Sin[c + d*x]))/(
2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2ab\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	96
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2ab\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	96
risch	$-\frac{2i\left(-b\,\mathrm{e}^{i(dx+c)}+a\right)}{d(a^2-b^2)\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{ba\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}+\frac{ba\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}$	209

```
[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*tan(1/2*d*x+1/2*c)/(a-b)-2*a/(a-b)/(a+b)*b/((a-b)*(a+b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/2/(a+b)/tan(1/2*d*x+1/
2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.57

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{a^2-b^2}ab\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin(dx+c)-2\,a^2b}{2\,(a^4-2\,a^2b^2+b^4)d\sin(dx+c)}\right.$$
$$\left.-\frac{\sqrt{-a^2+b^2}ab\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin(dx+c)-a^2b+b^3+(a^3-ab^2)\cos(dx+c)}{(a^4-2\,a^2b^2+b^4)d\sin(dx+c)}\right]$$

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 - b^2)*a*b*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)
/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*a^2*b +
2*b^3 + 2*(a^3 - a*b^2)*cos(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*sin(d*x +
c)), -(sqrt(-a^2 + b^2)*a*b*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/(
(a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*cos(d
*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc^2{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab}{(a^2-b^2)\sqrt{-a^2+b^2}} - \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a-b} + \frac{1}{(a+b)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b/((a^2 - b
^2)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/(a - b) + 1/((a + b)*tan(1/2*d
*x + 1/2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,(2a-2b)} - \frac{a-b}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a+b)\,(2a-2b)} - \frac{2ab\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a^2-b^2)}{(a+b)^{3/2}\,\sqrt{a-b}}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

```
[In] int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(d*(2*a - 2*b)) - (a - b)/(d*tan(c/2 + (d*x)/2)*(a + b)*
(2*a - 2*b)) - (2*a*b*atanh((tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)
*(a - b)^(1/2))))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.207 $\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2a^3b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{(3a^2b-a(2a^2+b^2)\cos(c+dx))\csc(c+dx)}{3\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3\left(a^2-b^2\right)d}$$

```
[Out] -2*a^3*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a
+b)^(5/2)/d+1/3*(3*a^2*b-a*(2*a^2+b^2)*cos(d*x+c))*csc(d*x+c)/(a^2-b^2)^2/d
+1/3*(b-a*cos(d*x+c))*csc(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2945, 12, 2738, 214}

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2a^3b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}} + \frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d\left(a^2-b^2\right)} + \frac{\csc(c+dx)\left(3a^2b-a(2a^2+b^2)\cos(c+dx)\right)}{3d\left(a^2-b^2\right)^2}$$

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a^3*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) + ((3*a^2*b - a*(2*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x
])/(3*(a^2 - b^2)^2*d) + ((b - a*Cos[c + d*x])*Csc[c + d*x]^3)/(3*(a^2 - b^
2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)\csc^3(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3\left(a^2-b^2\right)d}+\frac{\int \frac{\left(ab-2a^2\cos(c+dx)\right)\csc^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{3\left(a^2-b^2\right)}
\end{aligned}
$$

$$= \frac{(3a^2b - a(2a^2+b^2)\cos(c+dx))\csc(c+dx)}{3(a^2-b^2)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3(a^2-b^2)d} + \frac{\int \frac{3a^3b}{-b-a\cos(c+dx)}\,dx}{3(a^2-b^2)^2}$$

$$= \frac{(3a^2b - a(2a^2+b^2)\cos(c+dx))\csc(c+dx)}{3(a^2-b^2)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3(a^2-b^2)d} + \frac{(a^3b)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{(a^2-b^2)^2}$$

$$= \frac{(3a^2b - a(2a^2+b^2)\cos(c+dx))\csc(c+dx)}{3(a^2-b^2)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3(a^2-b^2)d} + \frac{(2a^3b)\,\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)^2 d}$$

$$= -\frac{2a^3b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{(3a^2b - a(2a^2+b^2)\cos(c+dx))\csc(c+dx)}{3(a^2-b^2)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3(a^2-b^2)d}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.16

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{24a^3b\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + \sqrt{a^2-b^2}(10a^2b - 4b^3 + (-6a^3+3ab^2)\cos(c+dx) - 6a^2b\cos(2(c+}{12(a-b)^2(a+b)^2\sqrt{a^2-b^2}d}$$

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (24*a^3*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + Sqrt[a^2 -
 b^2]*(10*a^2*b - 4*b^3 + (-6*a^3 + 3*a*b^2)*Cos[c + d*x] - 6*a^2*b*Cos[2*(
c + d*x)] + 2*a^3*Cos[3*(c + d*x)] + a*b^2*Cos[3*(c + d*x)])*Csc[c + d*x]^3
)/(12*(a - b)^2*(a + b)^2*Sqrt[a^2 - b^2]*d)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\frac{\tan(\frac{dx}{2}+\frac{c}{2})^3 a}{3}-\frac{\tan(\frac{dx}{2}+\frac{c}{2})^3 b}{3}+3\tan(\frac{dx}{2}+\frac{c}{2})a-\tan(\frac{dx}{2}+\frac{c}{2})b}{8(a-b)^2}-\frac{2a^3b\,\text{arctanh}\left(\frac{(a-b)\tan(\frac{dx}{2}+\frac{c}{2})}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2\sqrt{(a-b)(a+b)}}-\frac{1}{24(a+b)\tan(\frac{dx}{2}+\frac{c}{2})^3}-}{d}$
default	$\frac{\frac{\frac{\tan(\frac{dx}{2}+\frac{c}{2})^3 a}{3}-\frac{\tan(\frac{dx}{2}+\frac{c}{2})^3 b}{3}+3\tan(\frac{dx}{2}+\frac{c}{2})a-\tan(\frac{dx}{2}+\frac{c}{2})b}{8(a-b)^2}-\frac{2a^3b\,\text{arctanh}\left(\frac{(a-b)\tan(\frac{dx}{2}+\frac{c}{2})}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2\sqrt{(a-b)(a+b)}}-\frac{1}{24(a+b)\tan(\frac{dx}{2}+\frac{c}{2})^3}-}{d}$
risch	$\frac{2i(3a^2b\,e^{5i(dx+c)}-3a\,b^2e^{4i(dx+c)}-10a^2b\,e^{3i(dx+c)}+4b^3e^{3i(dx+c)}+6a^3e^{2i(dx+c)}+3a^2b\,e^{i(dx+c)}-2a^3-a\,b^2)}{3d(-a^2+b^2)^2(e^{2i(dx+c)}-1)^3}+\frac{b\,a^3\ln(\mathrm{e}}{\sqrt{}}$

```
[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/(a-b)^2*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*tan(1/2*d*x+1/2*c)^3*b+3*t
an(1/2*d*x+1/2*c)*a-tan(1/2*d*x+1/2*c)*b)-2/(a-b)^2/(a+b)^2*a^3*b/((a-b)*(a
+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/24/(a+b)
/tan(1/2*d*x+1/2*c)^3-1/8*(3*a+b)/(a+b)^2/tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 558, normalized size of antiderivative = 3.99

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$=\left[-\frac{8\,a^4b-10\,a^2b^3+2\,b^5+2\,(2\,a^5-a^3b^2-ab^4)\cos{(dx+c)}^3-3\,(a^3b\cos{(dx+c)}^2-a^3b)\sqrt{a^2-b^2}\log\left(\frac{2}{}\right.}{6\,((a^6-3\,a^4b^2+3\,a^2b^4-b^6)}\right.$$
$$-\frac{4\,a^4b-5\,a^2b^3+b^5+(2\,a^5-a^3b^2-ab^4)\cos{(dx+c)}^3+3\,(a^3b\cos{(dx+c)}^2-a^3b)\sqrt{-a^2+b^2}\arctan\left(-\right.}{3\,((a^6-3\,a^4b^2+3\,a^2b^4-b^6)d\cos{(dx+c)}^2-(a^6}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(8*a^4*b - 10*a^2*b^3 + 2*b^5 + 2*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x +
 c)^3 - 3*(a^3*b*cos(d*x + c)^2 - a^3*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x
 + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b
^2))*sin(d*x + c) - 6*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 6*(a^5 - a^3*b^2)*
cos(d*x + c))/(((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^2 - (a^6
```

```
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)*sin(d*x + c)), -1/3*(4*a^4*b - 5*a^2*b^3
 + b^5 + (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3 + 3*(a^3*b*cos(d*x + c)^2
 - a^3*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((
a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^4*b - a^2*b^3)*cos(d*x + c)^2
 - 3*(a^5 - a^3*b^2)*cos(d*x + c))/(((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*
cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc^4{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c)),x)
[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(128) = 256$.

Time = 0.33 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.92

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^3b}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} + \frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{a^3-3}}{24d}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^3*b/((a^4 -
2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*ta
n(1/2*d*x + 1/2*c)^3 + b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2
*c) - 12*a*b*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(a^3 - 3*a^
2*b + 3*a*b^2 - b^3) - (9*a*tan(1/2*d*x + 1/2*c)^2 + 3*b*tan(1/2*d*x + 1/2*
c)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.56

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{2}{8\,a-8\,b}+\frac{8\,a+8\,b}{{\left(8\,a-8\,b\right)}^2}\right)}{d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{3\,d\,\left(8\,a-8\,b\right)}-\frac{\frac{a^2-2\,a\,b+b^2}{3\,\left(a+b\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(3\,a^3-5\,a^2\,b+a\,b^2+b^3\right)}{{\left(a+b\right)}^2}}{d\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(8\,a^2-16\,a\,b+8\,b^2\right)}-\frac{2\,a^3\,b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{3/2}}\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

```
[In] int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(2/(8*a - 8*b) + (8*a + 8*b)/(8*a - 8*b)^2))/d + tan(c/
2 + (d*x)/2)^3/(3*d*(8*a - 8*b)) - ((a^2 - 2*a*b + b^2)/(3*(a + b)) + (tan(
c/2 + (d*x)/2)^2*(a*b^2 - 5*a^2*b + 3*a^3 + b^3))/(a + b)^2)/(d*tan(c/2 + (
d*x)/2)^3*(8*a^2 - 16*a*b + 8*b^2)) - (2*a^3*b*atanh((tan(c/2 + (d*x)/2)*(a
^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/(d*(a + b)^(5/2)*(a
- b)^(5/2))
```

3.208 $\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 201

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2a^5 b\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx))\csc(c+dx)}{15\left(a^2-b^2\right)^3 d} + \frac{(5a^2b-a(4a^2+b^2)\cos(c+dx))\csc^3(c+dx)}{15\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right) d}$$

```
[Out] -2*a^5*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a
+b)^(7/2)/d+1/15*(15*a^4*b-a*(8*a^4+9*a^2*b^2-2*b^4)*cos(d*x+c))*csc(d*x+c)
/(a^2-b^2)^3/d+1/15*(5*a^2*b-a*(4*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^3/(a^2-b^
2)^2/d+1/5*(b-a*cos(d*x+c))*csc(d*x+c)^5/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used

$= \{3957, 2945, 12, 2738, 214\}$

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2a^5b\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} + \frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)} + \frac{\csc^3(c+dx)\left(5a^2b-a(4a^2+b^2)\cos(c+dx)\right)}{15d\left(a^2-b^2\right)^2} + \frac{\csc(c+dx)\left(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx)\right)}{15d\left(a^2-b^2\right)^3}$$

```
[In] Int[Csc[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a^5*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/
2)*(a + b)^(7/2)*d) + ((15*a^4*b - a*(8*a^4 + 9*a^2*b^2 - 2*b^4)*Cos[c + d*
x])*Csc[c + d*x])/(15*(a^2 - b^2)^3*d) + ((5*a^2*b - a*(4*a^2 + b^2)*Cos[c
+ d*x])*Csc[c + d*x]^3)/(15*(a^2 - b^2)^2*d) + ((b - a*Cos[c + d*x])*Csc[c
+ d*x]^5)/(5*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot(c+dx)\csc^5(c+dx)}{-b-a\cos(c+dx)}\,dx\\
&= \frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d}+\frac{\int \frac{\left(ab-4a^2\cos(c+dx)\right)\csc^4(c+dx)}{-b-a\cos(c+dx)}\,dx}{5\left(a^2-b^2\right)}\\
&= \frac{\left(5a^2b-a(4a^2+b^2)\cos(c+dx)\right)\csc^3(c+dx)}{15\left(a^2-b^2\right)^2d}+\frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d}\\
&\quad+\frac{\int \frac{\left(ab(7a^2-2b^2)-2a^2(4a^2+b^2)\cos(c+dx)\right)\csc^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{15\left(a^2-b^2\right)^2}\\
&= \frac{\left(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3d}\\
&\quad+\frac{\left(5a^2b-a(4a^2+b^2)\cos(c+dx)\right)\csc^3(c+dx)}{15\left(a^2-b^2\right)^2d}\\
&\quad+\frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d}+\frac{\int \frac{15a^5b}{-b-a\cos(c+dx)}\,dx}{15\left(a^2-b^2\right)^3}\\
&= \frac{\left(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3d}\\
&\quad+\frac{\left(5a^2b-a(4a^2+b^2)\cos(c+dx)\right)\csc^3(c+dx)}{15\left(a^2-b^2\right)^2d}\\
&\quad+\frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d}+\frac{\left(a^5b\right)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{\left(a^2-b^2\right)^3}\\
&= \frac{\left(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3d}\\
&\quad+\frac{\left(5a^2b-a(4a^2+b^2)\cos(c+dx)\right)\csc^3(c+dx)}{15\left(a^2-b^2\right)^2d}+\frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d}\\
&\quad+\frac{\left(2a^5b\right)\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)^3d}\\
&= -\frac{2a^5b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}+\frac{\left(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3d}\\
&\quad+\frac{\left(5a^2b-a(4a^2+b^2)\cos(c+dx)\right)\csc^3(c+dx)}{15\left(a^2-b^2\right)^2d}+\frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.38

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{(b+a\cos(c+dx))\sec(c+dx)\left(\frac{960a^5b\,\mathrm{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}-\frac{2(64a^2+43ab+9b^2)\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3}+\frac{8(19a-9b)\csc^3}{}\right.}{48}$$

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((960*a^5*b*ArcTanh[((-a + b)*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - (2*(64*a^2 + 43*a*b + 9*b^2)*
Cot[(c + d*x)/2])/(a + b)^3 + (8*(19*a - 9*b)*Csc[c + d*x]^3*Sin[(c + d*x)/
2]^4)/(a - b)^2 + (96*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6)/(a - b) - ((19*a +
 9*b)*Csc[(c + d*x)/2]^4*Sin[c + d*x])/(2*(a + b)^2) - (3*Csc[(c + d*x)/2]^
6*Sin[c + d*x])/(2*(a + b)) + (2*(64*a^2 - 43*a*b + 9*b^2)*Tan[(c + d*x)/2]
)/(a - b)^3))/(480*d*(a + b*Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{\frac{\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{2ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{8ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+10a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{32(a-b)^3}}{}$
default	$\frac{\frac{\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{2ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{8ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+10a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{32(a-b)^3}}{}$
risch	$-\frac{2i\left(15a^4b\,e^{9i(dx+c)}-15a^3b^2e^{8i(dx+c)}-80a^4b\,e^{7i(dx+c)}+20a^2b^3e^{7i(dx+c)}+90a^3b^2e^{6i(dx+c)}-30a\,b^4e^{6i(dx+c)}+178a^4b\,e^{5}\right.}{}$

```
[In] int(csc(d*x+c)^6/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32/(a-b)^3*(1/5*a^2*tan(1/2*d*x+1/2*c)^5-2/5*a*b*tan(1/2*d*x+1/2*c)^
5+1/5*b^2*tan(1/2*d*x+1/2*c)^5+5/3*a^2*tan(1/2*d*x+1/2*c)^3-8/3*a*b*tan(1/2
*d*x+1/2*c)^3+b^2*tan(1/2*d*x+1/2*c)^3+10*a^2*tan(1/2*d*x+1/2*c)-8*a*b*tan(
1/2*d*x+1/2*c)+2*b^2*tan(1/2*d*x+1/2*c))-2/(a-b)^3/(a+b)^3*a^5*b/((a-b)*(a+
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/160/(a+b)
```

```
/tan(1/2*d*x+1/2*c)^5-1/96*(5*a+3*b)/(a+b)^2/tan(1/2*d*x+1/2*c)^3-1/32/(a+b
)^3*(10*a^2+8*a*b+2*b^2)/tan(1/2*d*x+1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(187) = 374$.

Time = 0.32 (sec) , antiderivative size = 861, normalized size of antiderivative = 4.28

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{46\,a^6b - 68\,a^4b^3 + 28\,a^2b^5 - 6\,b^7 - 2\left(8\,a^7 + a^5b^2 - 11\,a^3b^4 + 2\,ab^6\right)\cos\left(dx+c\right)^5 + 30\left(a^6b - a^4b^3\right)\cos\left(c\right.}{}\right.$$

```
[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/30*(46*a^6*b - 68*a^4*b^3 + 28*a^2*b^5 - 6*b^7 - 2*(8*a^7 + a^5*b^2 - 11
*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^5 + 30*(a^6*b - a^4*b^3)*cos(d*x + c)^4 +
10*(4*a^7 - a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 15*(a^5*b*cos(d*x
 + c)^4 - 2*a^5*b*cos(d*x + c)^2 + a^5*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*
x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
 a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
b^2))*sin(d*x + c) - 10*(7*a^6*b - 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 30
*(a^7 - a^5*b^2)*cos(d*x + c))/(((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 +
 b^8)*d*cos(d*x + c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*
d*cos(d*x + c)^2 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sin(d
*x + c)), 1/15*(23*a^6*b - 34*a^4*b^3 + 14*a^2*b^5 - 3*b^7 - (8*a^7 + a^5*b
^2 - 11*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^5 + 15*(a^6*b - a^4*b^3)*cos(d*x +
c)^4 + 5*(4*a^7 - a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 15*(a^5*b*c
os(d*x + c)^4 - 2*a^5*b*cos(d*x + c)^2 + a^5*b)*sqrt(-a^2 + b^2)*arctan(-sq
rt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c
) - 5*(7*a^6*b - 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 15*(a^7 - a^5*b^2)*c
os(d*x + c))/(((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d*cos(d*x +
c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d*cos(d*x + c)^2 +
 (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\csc^6\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

```
[In] integrate(csc(d*x+c)**6/(a+b*sec(d*x+c)),x)
[Out] Integral(csc(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(187) = 374$.

Time = 0.34 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.69

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{960\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^5b}{(a^6-3a^4b^2+3a^2b^4-b^6)\sqrt{-a^2+b^2}}-\frac{3a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-12a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+18a^2b}{}}{}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] -1/480*(960*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^5*b/((a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (3*a^4*tan(1/2*d*x +
1/2*c)^5 - 12*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c
)^5 - 12*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2*d*x + 1/2*c)^5 + 25*a
^4*tan(1/2*d*x + 1/2*c)^3 - 90*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^2*t
an(1/2*d*x + 1/2*c)^3 - 70*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*b^4*tan(1/2*d*
```

```
x + 1/2*c)^3 + 150*a^4*tan(1/2*d*x + 1/2*c) - 420*a^3*b*tan(1/2*d*x + 1/2*c
) + 420*a^2*b^2*tan(1/2*d*x + 1/2*c) - 180*a*b^3*tan(1/2*d*x + 1/2*c) + 30*
b^4*tan(1/2*d*x + 1/2*c))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^
4 - b^5) + (150*a^2*tan(1/2*d*x + 1/2*c)^4 + 120*a*b*tan(1/2*d*x + 1/2*c)^4
 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 + 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 40*a*b*t
an(1/2*d*x + 1/2*c)^2 + 15*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b
^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)^5))/d
```

Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.93

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5\,d\,\left(32\,a-32\,b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{4}{3\,\left(32\,a-32\,b\right)}+\frac{32\,a+32\,b}{3\,{\left(32\,a-32\,b\right)}^2}\right)}{d}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5}{32\,a-32\,b}+\frac{\left(\frac{4}{32\,a-32\,b}+\frac{32\,a+32\,b}{{\left(32\,a-32\,b\right)}^2}\right)\,\left(32\,a+32\,b\right)}{32\,a-32\,b}\right)}{d}$$
$$-\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{5\,\left(a+b\right)}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-5\,a^5+11\,a^4\,b-4\,a^3\,b^2-4\,a^2\,b^3+a\,b^4+b^5\right)}{{\left(a+b\right)}^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(5\,a^4-12\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3-3\,b^4\right)}{3\,{\left(a+b\right)}^2}}{d\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(32\,a^3-96\,a^2\,b+96\,a\,b^2-32\,b^3\right)}$$
$$-\frac{2\,a^5\,b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{5/2}}\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

```
[In] int(1/(sin(c + d*x)^6*(a + b/cos(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(5*d*(32*a - 32*b)) + (tan(c/2 + (d*x)/2)^3*(4/(3*(32*
a - 32*b)) + (32*a + 32*b)/(3*(32*a - 32*b)^2)))/d + (tan(c/2 + (d*x)/2)*(5
/(32*a - 32*b) + ((4/(32*a - 32*b) + (32*a + 32*b)/(32*a - 32*b)^2)*(32*a +
 32*b))/(32*a - 32*b)))/d - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(5*(a + b)) -
(2*tan(c/2 + (d*x)/2)^4*(a*b^4 + 11*a^4*b - 5*a^5 + b^5 - 4*a^2*b^3 - 4*a^3
*b^2))/(a + b)^3 + (tan(c/2 + (d*x)/2)^2*(4*a*b^3 - 12*a^3*b + 5*a^4 - 3*b^
4 + 6*a^2*b^2))/(3*(a + b)^2))/(d*tan(c/2 + (d*x)/2)^5*(96*a*b^2 - 96*a^2*b
 + 32*a^3 - 32*b^3)) - (2*a^5*b*atanh((tan(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^
2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(5/2))))/(d*(a + b)^(7/2)*(a - b
)^(7/2))
```

3.209 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 267

$$\begin{aligned}\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx =& -\frac{(a^2-7b^2)\,(a^2-b^2)^2\cos(c+dx)}{a^8d}-\frac{3b(a^2-b^2)^2\cos^2(c+dx)}{a^7d}\\&+\frac{(3a^4-9a^2b^2+5b^4)\cos^3(c+dx)}{3a^6d}+\frac{b(3a^2-2b^2)\cos^4(c+dx)}{2a^5d}\\&-\frac{3(a^2-b^2)\cos^5(c+dx)}{5a^4d}-\frac{b\cos^6(c+dx)}{3a^3d}\\&+\frac{\cos^7(c+dx)}{7a^2d}+\frac{b^2(a^2-b^2)^3}{a^9d(b+a\cos(c+dx))}\\&+\frac{2b(a^2-4b^2)\,(a^2-b^2)^2\log(b+a\cos(c+dx))}{a^9d}\end{aligned}$$

```
[Out] -(a^2-7*b^2)*(a^2-b^2)^2*cos(d*x+c)/a^8/d-3*b*(a^2-b^2)^2*cos(d*x+c)^2/a^7/
d+1/3*(3*a^4-9*a^2*b^2+5*b^4)*cos(d*x+c)^3/a^6/d+1/2*b*(3*a^2-2*b^2)*cos(d*
x+c)^4/a^5/d-3/5*(a^2-b^2)*cos(d*x+c)^5/a^4/d-1/3*b*cos(d*x+c)^6/a^3/d+1/7*
cos(d*x+c)^7/a^2/d+b^2*(a^2-b^2)^3/a^9/d/(b+a*cos(d*x+c))+2*b*(a^2-4*b^2)*(
a^2-b^2)^2*ln(b+a*cos(d*x+c))/a^9/d
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2916, 12, 962}

$$\begin{aligned}\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &-\frac{b\cos^6(c+dx)}{3a^3d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{b^2(a^2-b^2)^3}{a^9d(a\cos(c+dx)+b)}\\ &+\frac{2b(a^2-4b^2)\,(a^2-b^2)^2\log(a\cos(c+dx)+b)}{a^9d}\\ &-\frac{(a^2-7b^2)\,(a^2-b^2)^2\cos(c+dx)}{a^8d}-\frac{3b(a^2-b^2)^2\cos^2(c+dx)}{a^7d}\\ &+\frac{b(3a^2-2b^2)\cos^4(c+dx)}{2a^5d}-\frac{3(a^2-b^2)\cos^5(c+dx)}{5a^4d}\\ &+\frac{(3a^4-9a^2b^2+5b^4)\cos^3(c+dx)}{3a^6d}\end{aligned}$$

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 7*b^2)*(a^2 - b^2)^2*Cos[c + d*x])/(a^8*d)) - (3*b*(a^2 - b^2)^2*
Cos[c + d*x]^2)/(a^7*d) + ((3*a^4 - 9*a^2*b^2 + 5*b^4)*Cos[c + d*x]^3)/(3*a
^6*d) + (b*(3*a^2 - 2*b^2)*Cos[c + d*x]^4)/(2*a^5*d) - (3*(a^2 - b^2)*Cos[c
 + d*x]^5)/(5*a^4*d) - (b*Cos[c + d*x]^6)/(3*a^3*d) + Cos[c + d*x]^7/(7*a^2
*d) + (b^2*(a^2 - b^2)^3)/(a^9*d*(b + a*Cos[c + d*x])) + (2*b*(a^2 - 4*b^2)
*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]])/(a^9*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^7(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)^3}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^7 d} \\
&= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)^3}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^9 d} \\
&= \frac{\text{Subst}\left(\int \left((a^2-7b^2)(a^2-b^2)^2 - \frac{b^2(-a^2+b^2)^3}{(b-x)^2} + \frac{2b(-a^2+b^2)^2(-a^2+4b^2)}{b-x} - 6b(-a^2+b^2)^2 x - (3a^4-9a^2
\end{aligned}$$

$$\cdots \over a^9 d$$

$$\begin{aligned}
&= -\frac{(a^2-7b^2)(a^2-b^2)^2\cos(c+dx)}{a^8 d} - \frac{3b(a^2-b^2)^2\cos^2(c+dx)}{a^7 d} \\
&\quad + \frac{(3a^4-9a^2b^2+5b^4)\cos^3(c+dx)}{3a^6 d} + \frac{b(3a^2-2b^2)\cos^4(c+dx)}{2a^5 d} \\
&\quad - \frac{3(a^2-b^2)\cos^5(c+dx)}{5a^4 d} - \frac{b\cos^6(c+dx)}{3a^3 d} + \frac{\cos^7(c+dx)}{7a^2 d} \\
&\quad + \frac{b^2(a^2-b^2)^3}{a^9 d(b+a\cos(c+dx))} + \frac{2b(a^2-4b^2)(a^2-b^2)^2\log(b+a\cos(c+dx))}{a^9 d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.56

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{-3675a^8 + 61320a^6b^2 - 132720a^4b^4 + 87360a^2b^6 - 13440b^8 - 140(21a^8 - 228a^6b^2 + 400a^4b^4 - 192a^2b^6)\,\text{co}}{\cdots}$$

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-3675*a^8 + 61320*a^6*b^2 - 132720*a^4*b^4 + 87360*a^2*b^6 - 13440*b^8 - 1
40*(21*a^8 - 228*a^6*b^2 + 400*a^4*b^4 - 192*a^2*b^6)*Cos[2*(c + d*x)] - 37
80*a^7*b*Cos[3*(c + d*x)] + 8400*a^5*b^3*Cos[3*(c + d*x)] - 4480*a^3*b^5*Co
s[3*(c + d*x)] + 588*a^8*Cos[4*(c + d*x)] - 1848*a^6*b^2*Cos[4*(c + d*x)] +
```

```
 1120*a^4*b^4*Cos[4*(c + d*x)] + 476*a^7*b*Cos[5*(c + d*x)] - 336*a^5*b^3*C
os[5*(c + d*x)] - 132*a^8*Cos[6*(c + d*x)] + 112*a^6*b^2*Cos[6*(c + d*x)] -
 40*a^7*b*Cos[7*(c + d*x)] + 15*a^8*Cos[8*(c + d*x)] + 26880*a^6*b^2*Log[b
+ a*Cos[c + d*x]] - 161280*a^4*b^4*Log[b + a*Cos[c + d*x]] + 241920*a^2*b^6
*Log[b + a*Cos[c + d*x]] - 107520*b^8*Log[b + a*Cos[c + d*x]] + 1680*a*b*Co
s[c + d*x]*(-8*a^6 + 67*a^4*b^2 - 116*a^2*b^4 + 56*b^6 + 16*(a^2 - 4*b^2)*(
a^2 - b^2)^2*Log[b + a*Cos[c + d*x]]))/(13440*a^9*d*(b + a*Cos[c + d*x]))
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^7a^6}{7}-\frac{b\cos(dx+c)^6a^5}{3}-\frac{3a^6\cos(dx+c)^5}{5}+\frac{3a^4b^2\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4a^5b}{2}-\cos(dx+c)^4a^3b^3+\cos(dx+c)^3a^6-3\cos(d\cdots}{\cdots}$
default	$\frac{\frac{\cos(dx+c)^7a^6}{7}-\frac{b\cos(dx+c)^6a^5}{3}-\frac{3a^6\cos(dx+c)^5}{5}+\frac{3a^4b^2\cos(dx+c)^5}{5}+\frac{3\cos(dx+c)^4a^5b}{2}-\cos(dx+c)^4a^3b^3+\cos(dx+c)^3a^6-3\cos(d\cdots}{\cdots}$
parallelrisch	$\frac{2240b(a-2b)(a+2b)(a-b)^2(a+b)^2(b+a\cos(dx+c))\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-2240b(a-2b)(a+2b)(a-b)^2(a+b)^2(\cdots}{\cdots}$
risch	$\frac{7\cos(3dx+3c)}{64da^2}-\frac{35\,e^{i(dx+c)}}{128a^2d}-\frac{35\,e^{-i(dx+c)}}{128a^2d}+\frac{\cos(7dx+7c)}{448a^2d}-\frac{2b^2(-a^6+3a^4b^2-3a^2b^4+b^6)e^{i(dx+c)}}{a^9d(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a)}+\frac{24ib^3c}{a^5d}\cdots$
norman	$\frac{(96a^7+96ba^6+380a^5b^2-1596a^4b^3-1188b^4a^3+3220b^5a^2+720ab^6-1680b^7)(a+b)}{210a^8db}-\frac{(32a^8-160a^7b-32a^6b^2+172a^5b^3-644a^4b^4-19\cdots}{5a\cdots}$

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^8*(1/7*cos(d*x+c)^7*a^6-1/3*b*cos(d*x+c)^6*a^5-3/5*a^6*cos(d*x+c)^
5+3/5*a^4*b^2*cos(d*x+c)^5+3/2*cos(d*x+c)^4*a^5*b-cos(d*x+c)^4*a^3*b^3+cos(
d*x+c)^3*a^6-3*cos(d*x+c)^3*a^4*b^2+5/3*cos(d*x+c)^3*a^2*b^4-3*cos(d*x+c)^2
*a^5*b+6*cos(d*x+c)^2*a^3*b^3-3*cos(d*x+c)^2*a*b^5-cos(d*x+c)*a^6+9*cos(d*x
+c)*a^4*b^2-15*cos(d*x+c)*a^2*b^4+7*b^6*cos(d*x+c))+b^2*(a^6-3*a^4*b^2+3*a^
2*b^4-b^6)/a^9/(b+a*cos(d*x+c))+2/a^9*b*(a^6-6*a^4*b^2+9*a^2*b^4-4*b^6)*ln(
b+a*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.29

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{120\,a^8\cos(dx+c)^8 - 160\,a^7b\cos(dx+c)^7 + 1715\,a^6b^2 - 4725\,a^4b^4 + 3780\,a^2b^6 - 840\,b^8 - 56\,(9\,a^8 - 4\,a^6b^2}{}$$

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*a^8*cos(d*x + c)^8 - 160*a^7*b*cos(d*x + c)^7 + 1715*a^6*b^2 - 4
725*a^4*b^4 + 3780*a^2*b^6 - 840*b^8 - 56*(9*a^8 - 4*a^6*b^2)*cos(d*x + c)^
6 + 84*(9*a^7*b - 4*a^5*b^3)*cos(d*x + c)^5 + 140*(6*a^8 - 9*a^6*b^2 + 4*a^
4*b^4)*cos(d*x + c)^4 - 280*(6*a^7*b - 9*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^
3 - 840*(a^8 - 6*a^6*b^2 + 9*a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^2 + 35*(a^7*
b + 153*a^5*b^3 - 324*a^3*b^5 + 168*a*b^7)*cos(d*x + c) + 1680*(a^6*b^2 - 6
*a^4*b^4 + 9*a^2*b^6 - 4*b^8 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*co
s(d*x + c))*log(a*cos(d*x + c) + b))/(a^10*d*cos(d*x + c) + a^9*b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.01

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{210\,(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8)}{a^{10}\cos(dx+c)+a^9b} + \frac{30\,a^6\cos(dx+c)^7-70\,a^5b\cos(dx+c)^6-126\,(a^6-a^4b^2)\cos(dx+c)^5+105\,(3\,a^5b-2\,a^3b^3)\cos(dx+c)^4+70\,(3\,a^6-9}{a^8}}{}$$

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/210*(210*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)/(a^10*cos(d*x + c) + a^9
*b) + (30*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6 - 126*(a^6 - a^4*b^2
)*cos(d*x + c)^5 + 105*(3*a^5*b - 2*a^3*b^3)*cos(d*x + c)^4 + 70*(3*a^6 - 9
*a^4*b^2 + 5*a^2*b^4)*cos(d*x + c)^3 - 630*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(
d*x + c)^2 - 210*(a^6 - 9*a^4*b^2 + 15*a^2*b^4 - 7*b^6)*cos(d*x + c))/a^8 +
 420*(a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*log(a*cos(d*x + c) + b)/a^9)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1861 vs. $2(257) = 514$.

Time = 0.37 (sec) , antiderivative size = 1861, normalized size of antiderivative = 6.97

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/210*(420*(a^7*b - a^6*b^2 - 6*a^5*b^3 + 6*a^4*b^4 + 9*a^3*b^5 - 9*a^2*b^6
 - 4*a*b^7 + 4*b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^10 - a^9*b) - 420*(a^6*b -
6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1))/a^9 - 420*(a^7*b - 7*a^5*b^3 - 4*a^4*b^4 + 11*a^3*b^5 + 8*a^2*b^6
- 5*a*b^7 - 4*b^8 + a^7*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^6*b^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*a^5*b^3*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 6*a^4*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^3*b^5*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 9*a^2*b^6*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 4*a*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b^8*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^9) + (192*a^7 - 1089*a^
6*b - 2772*a^5*b^2 + 6534*a^4*b^3 + 5600*a^3*b^4 - 9801*a^2*b^5 - 2940*a*b^
6 + 4356*b^7 - 1344*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8463*a^6*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18144*a^5*b^2*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 49098*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 35
000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 71127*a^2*b^5*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 17640*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 30492*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4032*a^7*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 48132*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
157374*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 88200*a^3*b^4*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 218421*a^2*b^5*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 - 44100*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 91476*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6720*a^7*(cos(
d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 60480*a^5*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 - 272370*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 114800*a^3*b^
```

```
4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 368235*a^2*b^5*(cos(d*x + c)
 - 1)^3/(cos(d*x + c) + 1)^3 + 58800*a*b^6*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 - 152460*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 56035*a^6
*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 36540*a^5*b^2*(cos(d*x + c)
 - 1)^4/(cos(d*x + c) + 1)^4 + 272370*a^4*b^3*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4 + 81200*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 368
235*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 44100*a*b^6*(cos(d*
x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 152460*b^7*(cos(d*x + c) - 1)^4/(cos(d
*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 10
080*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 157374*a^4*b^3*(cos
(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 29400*a^3*b^4*(cos(d*x + c) - 1)^5/
(cos(d*x + c) + 1)^5 + 218421*a^2*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) +
1)^5 + 17640*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 91476*b^7*(c
os(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(cos(d*x + c) - 1)^6/(
cos(d*x + c) + 1)^6 - 1260*a^5*b^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6 + 49098*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 4200*a^3*b^4*
(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 71127*a^2*b^5*(cos(d*x + c) - 1
)^6/(cos(d*x + c) + 1)^6 - 2940*a*b^6*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 + 30492*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*a^6*b*(co
s(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 6534*a^4*b^3*(cos(d*x + c) - 1)^7/
(cos(d*x + c) + 1)^7 + 9801*a^2*b^5*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)
^7 - 4356*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/(a^9*((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 1)^7))/d
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.20

$$
\begin{aligned}
\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = {} & \frac{\cos(c+d\,x)^4\left(\frac{b^3}{2\,a^5}+\frac{b\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{2\,a}\right)}{d} \\
& - \frac{\cos(c+d\,x)^2\left(\frac{b^2\left(\frac{2\,b^3}{a^5}+\frac{2\,b\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{2\,a^2}+\frac{b\left(\frac{3}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{a^2}-\frac{2\,b\left(\frac{2\,b^3}{a^5}+\frac{2\,b\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{a}\right)}{a}\right)}{d} \\
& - \frac{\cos(c+d\,x)^5\left(\frac{3}{5\,a^2}-\frac{3\,b^2}{5\,a^4}\right)}{d} + \frac{\cos(c+d\,x)^7}{7\,a^2\,d} \\
& - \frac{\cos(c+d\,x)\left(\frac{1}{a^2}+\frac{b^2\left(\frac{3}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{a^2}-\frac{2\,b\left(\frac{2\,b^3}{a^5}+\frac{2\,b\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{a}\right)}{a^2}-\frac{2\,b\left(\frac{b^2\left(\frac{2\,b^3}{a^5}+\frac{2\,b\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{a^2}+\frac{2\,b\left(\frac{3}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\cdots\right)}{a^2}\cdots\right)}{\cdots}\right)}{a}\cdots\right)}{d} \\
& + \frac{\cos(c+d\,x)^3\left(\frac{1}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{3\,a^2}-\frac{2\,b\left(\frac{2\,b^3}{a^5}+\frac{2\,b\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{3\,a}\right)}{d} \\
& - \frac{b\cos(c+d\,x)^6}{3\,a^3\,d} - \frac{-a^6\,b^2+3\,a^4\,b^4-3\,a^2\,b^6+b^8}{a\,d\,\left(\cos(c+d\,x)\,a^9+b\,a^8\right)} \\
& + \frac{\ln(b+a\,\cos(c+d\,x))\,\left(2\,a^6\,b-12\,a^4\,b^3+18\,a^2\,b^5-8\,b^7\right)}{a^9\,d}
\end{aligned}
$$

```
[In] int(sin(c + d*x)^7/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^4*(b^3/(2*a^5) + (b*(3/a^2 - (3*b^2)/a^4))/(2*a)))/d - (cos(c
 + d*x)^2*((b^2*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a^4))/a))/(2*a^2) + (b
*(3/a^2 + (b^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(3/a^2
 - (3*b^2)/a^4))/a))/a))/a))/d - (cos(c + d*x)^5*(3/(5*a^2) - (3*b^2)/(5*a^
4)))/d + cos(c + d*x)^7/(7*a^2*d) - (cos(c + d*x)*(1/a^2 + (b^2*(3/a^2 + (b
^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a
^4))/a))/a))/a^2 - (2*b*((b^2*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a^4))/a)
)/a^2 + (2*b*(3/a^2 + (b^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^3)/a^5 +
 (2*b*(3/a^2 - (3*b^2)/a^4))/a))/a))/a))/a))/d + (cos(c + d*x)^3*(1/a^2 + (
b^2*(3/a^2 - (3*b^2)/a^4))/(3*a^2) - (2*b*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b
^2)/a^4))/a))/(3*a)))/d - (b*cos(c + d*x)^6)/(3*a^3*d) - (b^8 - 3*a^2*b^6 +
 3*a^4*b^4 - a^6*b^2)/(a*d*(a^9*cos(c + d*x) + a^8*b)) + (log(b + a*cos(c +
 d*x))*(2*a^6*b - 8*b^7 + 18*a^2*b^5 - 12*a^4*b^3))/(a^9*d)
```

3.210 $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 194

$$\begin{aligned}\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{(a^4-6a^2b^2+5b^4)\cos(c+dx)}{a^6d}-\frac{2b(a^2-b^2)\cos^2(c+dx)}{a^5d}\\ &\quad+\frac{(2a^2-3b^2)\cos^3(c+dx)}{3a^4d}+\frac{b\cos^4(c+dx)}{2a^3d}\\ &\quad-\frac{\cos^5(c+dx)}{5a^2d}+\frac{b^2(a^2-b^2)^2}{a^7d(b+a\cos(c+dx))}\\ &\quad+\frac{2b(a^4-4a^2b^2+3b^4)\log(b+a\cos(c+dx))}{a^7d}\end{aligned}$$

```
[Out] -(a^4-6*a^2*b^2+5*b^4)*cos(d*x+c)/a^6/d-2*b*(a^2-b^2)*cos(d*x+c)^2/a^5/d+1/
3*(2*a^2-3*b^2)*cos(d*x+c)^3/a^4/d+1/2*b*cos(d*x+c)^4/a^3/d-1/5*cos(d*x+c)^
5/a^2/d+b^2*(a^2-b^2)^2/a^7/d/(b+a*cos(d*x+c))+2*b*(a^4-4*a^2*b^2+3*b^4)*ln
(b+a*cos(d*x+c))/a^7/d
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

$= \{3957, 2916, 12, 962\}$

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b\cos^4(c+dx)}{2a^3d} - \frac{\cos^5(c+dx)}{5a^2d} + \frac{b^2(a^2-b^2)^2}{a^7d(a\cos(c+dx)+b)} - \frac{2b(a^2-b^2)\cos^2(c+dx)}{a^5d} + \frac{(2a^2-3b^2)\cos^3(c+dx)}{3a^4d} + \frac{2b(a^4-4a^2b^2+3b^4)\log(a\cos(c+dx)+b)}{a^7d} - \frac{(a^4-6a^2b^2+5b^4)\cos(c+dx)}{a^6d}$$

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^4 - 6*a^2*b^2 + 5*b^4)*Cos[c + d*x])/(a^6*d)) - (2*b*(a^2 - b^2)*Cos[
c + d*x]^2)/(a^5*d) + ((2*a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*a^4*d) + (b*Cos[c
 + d*x]^4)/(2*a^3*d) - Cos[c + d*x]^5/(5*a^2*d) + (b^2*(a^2 - b^2)^2)/(a^7*
d*(b + a*Cos[c + d*x])) + (2*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c +
d*x]])/(a^7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \frac{\cos^2(c+dx)\sin^5(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)^2}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)^2}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{-6a^2b^2+5b^4}{a^4}\right)+\frac{b^2(a^2-b^2)^2}{(b-x)^2}-\frac{2b(a^4-4a^2b^2+3b^4)}{b-x}+4b(-a^2+b^2)x-(2a^2-3b^2)x^2+2 \right.}{a^7 d}$$
$$= -\frac{(a^4-6a^2b^2+5b^4)\cos(c+dx)}{a^6 d} - \frac{2b(a^2-b^2)\cos^2(c+dx)}{a^5 d} + \frac{(2a^2-3b^2)\cos^3(c+dx)}{3a^4 d} + \frac{b\cos^4(c+dx)}{2a^3 d} - \frac{\cos^5(c+dx)}{5a^2 d} + \frac{b^2(a^2-b^2)^2}{a^7 d(b+a\cos(c+dx))} + \frac{2b(a^4-4a^2b^2+3b^4)\log(b+a\cos(c+dx))}{a^7 d}$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.44

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-150a^6+1740a^4b^2-2160a^2b^4+480b^6-5(25a^6-168a^4b^2+144a^2b^4)\cos(2(c+dx))-115a^5b\cos(3(c+ }{}$$

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-150*a^6 + 1740*a^4*b^2 - 2160*a^2*b^4 + 480*b^6 - 5*(25*a^6 - 168*a^4*b^2
 + 144*a^2*b^4)*Cos[2*(c + d*x)] - 115*a^5*b*Cos[3*(c + d*x)] + 120*a^3*b^3
*Cos[3*(c + d*x)] + 22*a^6*Cos[4*(c + d*x)] - 30*a^4*b^2*Cos[4*(c + d*x)] +
 9*a^5*b*Cos[5*(c + d*x)] - 3*a^6*Cos[6*(c + d*x)] + 960*a^4*b^2*Log[b + a*
Cos[c + d*x]] - 3840*a^2*b^4*Log[b + a*Cos[c + d*x]] + 2880*b^6*Log[b + a*C
os[c + d*x]] + 120*a*b*Cos[c + d*x]*(-4*a^4 + 23*a^2*b^2 - 20*b^4 + 8*(a^4
- 4*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c + d*x]]))/(480*a^7*d*(b + a*Cos[c + d*
x]))
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{-\frac{\cos(dx+c)^5 a^4}{5}-\frac{b\cos(dx+c)^4 a^3}{2}-\frac{2\cos(dx+c)^3 a^4}{3}+\cos(dx+c)^3 a^2 b^2+2\cos(dx+c)^2 a^3 b-2\cos(dx+c)^2 a\,b^3+\cos(dx+c) a^4-6\cos(dx+c)}{a^6}}{d}$
default	$\frac{-\frac{-\frac{\cos(dx+c)^5 a^4}{5}-\frac{b\cos(dx+c)^4 a^3}{2}-\frac{2\cos(dx+c)^3 a^4}{3}+\cos(dx+c)^3 a^2 b^2+2\cos(dx+c)^2 a^3 b-2\cos(dx+c)^2 a\,b^3+\cos(dx+c) a^4-6\cos(dx+c)}{a^6}}{d}$
parallelrisch	$\frac{960b^2(a-b)(a+b)(a^2-3b^2)(b+a\cos(dx+c))\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-960b^2(a-b)(a+b)(a^2-3b^2)(b+a\cos(dx+c))}{}$
risch	$-\frac{4ibc}{a^3d}+\frac{16ib^3c}{a^5d}-\frac{12ib^5c}{a^7d}+\frac{5\cos(3dx+3c)}{48d\,a^2}+\frac{2b^2(a^4-2a^2b^2+b^4)\mathrm{e}^{i(dx+c)}}{a^7d(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a)}-\frac{5\,\mathrm{e}^{i(dx+c)}}{16a^2d}+\frac{8ib^3x}{a^5}-\frac{6ib^5x}{a^7}$
norman	$\frac{\frac{(32a^5+32a^4b+96a^3b^2-360a^2b^3-144a\,b^4+360b^5)(a+b)}{60a^6bd}-\frac{(32a^6-128a^5b+120a^3b^3-408a^2b^4-72a\,b^5+360b^6)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{12a^6bd}+\frac{(32a^6-}{}}{}$

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^6*(1/5*cos(d*x+c)^5*a^4-1/2*b*cos(d*x+c)^4*a^3-2/3*cos(d*x+c)^3*a
^4+cos(d*x+c)^3*a^2*b^2+2*cos(d*x+c)^2*a^3*b-2*cos(d*x+c)^2*a*b^3+cos(d*x+c
)*a^4-6*cos(d*x+c)*a^2*b^2+5*cos(d*x+c)*b^4)+b^2*(a^4-2*a^2*b^2+b^4)/a^7/(b
+a*cos(d*x+c))+2/a^7*b*(a^4-4*a^2*b^2+3*b^4)*ln(b+a*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.24

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{48\,a^6\cos{(dx+c)}^6-72\,a^5b\cos{(dx+c)}^5-435\,a^4b^2+720\,a^2b^4-240\,b^6-40\,(4\,a^6-3\,a^4b^2)\cos{(dx+c)}^4}{}$$

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^6*cos(d*x + c)^6 - 72*a^5*b*cos(d*x + c)^5 - 435*a^4*b^2 + 720
*a^2*b^4 - 240*b^6 - 40*(4*a^6 - 3*a^4*b^2)*cos(d*x + c)^4 + 80*(4*a^5*b -
3*a^3*b^3)*cos(d*x + c)^3 + 240*(a^6 - 4*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^
2 + 15*(3*a^5*b - 80*a^3*b^3 + 80*a*b^5)*cos(d*x + c) - 480*(a^4*b^2 - 4*a^
2*b^4 + 3*b^6 + (a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*log(a*cos(d*x +
 c) + b))/(a^8*d*cos(d*x + c) + a^7*b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.95

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{30\,(a^4b^2-2\,a^2b^4+b^6)}{a^8\cos(dx+c)+a^7b} - \frac{6\,a^4\cos(dx+c)^5-15\,a^3b\cos(dx+c)^4-10\,(2\,a^4-3\,a^2b^2)\cos(dx+c)^3+60\,(a^3b-ab^3)\cos(dx+c)^2+30\,(a^4-6\,a^2b^2+5\,b^4)\,\mathrm{co}}{a^6}}{30\,d}$$

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] 1/30*(30*(a^4*b^2 - 2*a^2*b^4 + b^6)/(a^8*cos(d*x + c) + a^7*b) - (6*a^4*co
s(d*x + c)^5 - 15*a^3*b*cos(d*x + c)^4 - 10*(2*a^4 - 3*a^2*b^2)*cos(d*x + c
)^3 + 60*(a^3*b - a*b^3)*cos(d*x + c)^2 + 30*(a^4 - 6*a^2*b^2 + 5*b^4)*cos(
d*x + c))/a^6 + 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(a*cos(d*x + c) + b)/a^7)
/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. 2(188) = 376.

Time = 0.34 (sec) , antiderivative size = 1102, normalized size of antiderivative = 5.68

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] 1/30*(60*(a^5*b - a^4*b^2 - 4*a^3*b^3 + 4*a^2*b^4 + 3*a*b^5 - 3*b^6)*log(ab
s(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - a^7*b) - 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^7 - 60*(a^5*b - 5*a^3*b^3 - 3
*a^2*b^4 + 4*a*b^5 + 3*b^6 + a^5*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
a^4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^3*(cos(d*x + c) - 1
```

```
)/(cos(d*x + c) + 1) + 4*a^2*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*
a*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*b^6*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1))*a^7) + (32*a^5 - 137*a^4*b - 300*a^3*b^2
+ 548*a^2*b^3 + 300*a*b^4 - 411*b^5 - 160*a^5*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 805*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1320*a^3*b^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2980*a^2*b^3*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 1200*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2055*b^
5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*a^5*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 - 1970*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1
920*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 6200*a^2*b^3*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1800*a*b^4*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 4110*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970
*a^4*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1080*a^3*b^2*(cos(d*x +
c) - 1)^3/(cos(d*x + c) + 1)^3 - 6200*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 - 1200*a*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 4110*
b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*a^4*b*(cos(d*x + c) - 1
)^4/(cos(d*x + c) + 1)^4 - 180*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
 1)^4 + 2980*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 300*a*b^4*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2055*b^5*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 + 137*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 -
 548*a^2*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 411*b^5*(cos(d*x +
 c) - 1)^5/(cos(d*x + c) + 1)^5)/(a^7*((cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.30

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\cos(c+d\,x)^3\,\left(\frac{2}{3\,a^2}-\frac{b^2}{a^4}\right)}{d}-\frac{\cos(c+d\,x)^2\,\left(\frac{b^3}{a^5}+\frac{b\,\left(\frac{2}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{d}-\frac{\cos(c+d\,x)\,\left(\frac{1}{a^2}+\frac{b^2\,\left(\frac{2}{a^2}-\frac{3\,b^2}{a^4}\right)}{a^2}-\frac{2\,b\,\left(\frac{2\,b^3}{a^5}+\frac{2\,b\,\left(\frac{2}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{a}\right)}{d}-\frac{\cos(c+d\,x)^5}{5\,a^2\,d}+\frac{b\cos(c+d\,x)^4}{2\,a^3\,d}+\frac{\ln\left(b+a\,\cos(c+d\,x)\right)\,\left(2\,a^4\,b-8\,a^2\,b^3+6\,b^5\right)}{a^7\,d}+\frac{a^4\,b^2-2\,a^2\,b^4+b^6}{a\,d\,\left(\cos(c+d\,x)\,a^7+b\,a^6\right)}$$

```
[In] int(sin(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^3*(2/(3*a^2) - b^2/a^4))/d - (cos(c + d*x)^2*(b^3/a^5 + (b*(2
/a^2 - (3*b^2)/a^4))/a))/d - (cos(c + d*x)*(1/a^2 + (b^2*(2/a^2 - (3*b^2)/a
^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(2/a^2 - (3*b^2)/a^4))/a))/a))/d - cos(
c + d*x)^5/(5*a^2*d) + (b*cos(c + d*x)^4)/(2*a^3*d) + (log(b + a*cos(c + d*
x))*(2*a^4*b + 6*b^5 - 8*a^2*b^3))/(a^7*d) + (b^6 - 2*a^2*b^4 + a^4*b^2)/(a
*d*(a^7*cos(c + d*x) + a^6*b))
```

3.211 $\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{(a^2-3b^2)\cos(c+dx)}{a^4d} - \frac{b\cos^2(c+dx)}{a^3d} + \frac{\cos^3(c+dx)}{3a^2d} + \frac{b^2(a^2-b^2)}{a^5d(b+a\cos(c+dx))} + \frac{2b(a^2-2b^2)\log(b+a\cos(c+dx))}{a^5d}$$

```
[Out] -(a^2-3*b^2)*cos(d*x+c)/a^4/d-b*cos(d*x+c)^2/a^3/d+1/3*cos(d*x+c)^3/a^2/d+b
^2*(a^2-b^2)/a^5/d/(b+a*cos(d*x+c))+2*b*(a^2-2*b^2)*ln(b+a*cos(d*x+c))/a^5/
d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2916, 12, 908}

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b\cos^2(c+dx)}{a^3d} + \frac{\cos^3(c+dx)}{3a^2d} + \frac{b^2(a^2-b^2)}{a^5d(a\cos(c+dx)+b)} + \frac{2b(a^2-2b^2)\log(a\cos(c+dx)+b)}{a^5d} - \frac{(a^2-3b^2)\cos(c+dx)}{a^4d}$$

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 3*b^2)*Cos[c + d*x])/(a^4*d)) - (b*Cos[c + d*x]^2)/(a^3*d) + Cos[
c + d*x]^3/(3*a^2*d) + (b^2*(a^2 - b^2))/(a^5*d*(b + a*Cos[c + d*x])) + (2*
b*(a^2 - 2*b^2)*Log[b + a*Cos[c + d*x]])/(a^5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(-b-a\cos(c+dx))^2}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3 d}\\
&= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}\\
&= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{3b^2}{a^2}\right)-\frac{b^2(-a^2+b^2)}{(b-x)^2}+\frac{2b(-a^2+2b^2)}{b-x}-2bx-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{a^5 d}\\
&= -\frac{(a^2-3b^2)\cos(c+dx)}{a^4 d}-\frac{b\cos^2(c+dx)}{a^3 d}+\frac{\cos^3(c+dx)}{3a^2 d}\\
&\quad+\frac{b^2(a^2-b^2)}{a^5 d(b+a\cos(c+dx))}+\frac{2b(a^2-2b^2)\log(b+a\cos(c+dx))}{a^5 d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.40

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{-9a^4+60a^2b^2-24b^4-8(a^4-3a^2b^2)\cos(2(c+dx))-4a^3b\cos(3(c+dx))+a^4\cos(4(c+dx))+48a^2b^2\,\mathrm{lo}\ldots}{24a^5d(\ldots}$$

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-9*a^4 + 60*a^2*b^2 - 24*b^4 - 8*(a^4 - 3*a^2*b^2)*Cos[2*(c + d*x)] - 4*a^
3*b*Cos[3*(c + d*x)] + a^4*Cos[4*(c + d*x)] + 48*a^2*b^2*Log[b + a*Cos[c +
d*x]] - 96*b^4*Log[b + a*Cos[c + d*x]] + 24*a*b*Cos[c + d*x]*(-a^2 + 3*b^2
+ 2*(a^2 - 2*b^2)*Log[b + a*Cos[c + d*x]]))/(24*a^5*d*(b + a*Cos[c + d*x]))
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\frac{\cos(dx+c)^3a^2}{3}-ab\cos(dx+c)^2-\cos(dx+c)a^2+3\cos(dx+c)b^2}{a^4}+\frac{b^2\left(a^2-b^2\right)}{a^5(b+a\cos(dx+c))}+\frac{2b\left(a^2-2b^2\right)\ln(b+a\cos(dx+c))}{a^5}}{d}$
default	$\frac{\frac{\frac{\cos(dx+c)^3a^2}{3}-ab\cos(dx+c)^2-\cos(dx+c)a^2+3\cos(dx+c)b^2}{a^4}+\frac{b^2\left(a^2-b^2\right)}{a^5(b+a\cos(dx+c))}+\frac{2b\left(a^2-2b^2\right)\ln(b+a\cos(dx+c))}{a^5}}{d}$
parallelrisch	$\frac{48b^2(a^2-2b^2)(b+a\cos(dx+c))\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-48b^2(a^2-2b^2)(b+a\cos(dx+c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+(-8a\ldots}{24d\,a^5b(b+a\ldots}$
risch	$-\frac{2ibx}{a^3}+\frac{4ib^3x}{a^5}-\frac{b\,\mathrm{e}^{2i(dx+c)}}{4a^3d}-\frac{3\,\mathrm{e}^{i(dx+c)}}{8a^2d}+\frac{3\,\mathrm{e}^{i(dx+c)}b^2}{2a^4d}-\frac{3\,\mathrm{e}^{-i(dx+c)}}{8a^2d}+\frac{3\,\mathrm{e}^{-i(dx+c)}b^2}{2a^4d}-\frac{b\,\mathrm{e}^{-2i(dx+c)}}{4a^3d}+\frac{8\ldots}{\ldots}$
norman	$\frac{-\frac{\left(4a^4-8a^3b+12a^2b^2+16a\,b^3-24b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{6d\,a^4b}+\frac{\left(4a^3+4a^2b+8a\,b^2-24b^3\right)(a+b)}{6a^4bd}-\frac{\left(4a^4-12a^3b+4a^2b^2+8a\,b^3-24b^4\right)\tan\left(\frac{dx}{2}+\ldots\right.}{3a^4bd}\ldots}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}$

```
[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^4*(1/3*cos(d*x+c)^3*a^2-a*b*cos(d*x+c)^2-cos(d*x+c)*a^2+3*cos(d*x+
c)*b^2)+b^2*(a^2-b^2)/a^5/(b+a*cos(d*x+c))+2/a^5*b*(a^2-2*b^2)*ln(b+a*cos(d
*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2\,a^4\cos\left(dx+c\right)^4-4\,a^3b\cos\left(dx+c\right)^3+9\,a^2b^2-6\,b^4-6\,\left(a^4-2\,a^2b^2\right)\cos\left(dx+c\right)^2-3\,\left(a^3b-6\,ab^3\right)\cos}{6\,\left(a^6d\cos\left(dx+c\right)+a^5bd\right)}$$

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^4*cos(d*x + c)^4 - 4*a^3*b*cos(d*x + c)^3 + 9*a^2*b^2 - 6*b^4 - 6*
(a^4 - 2*a^2*b^2)*cos(d*x + c)^2 - 3*(a^3*b - 6*a*b^3)*cos(d*x + c) + 12*(a
^2*b^2 - 2*b^4 + (a^3*b - 2*a*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b))/(
a^6*d*cos(d*x + c) + a^5*b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.94

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{3\,\left(a^2b^2-b^4\right)}{a^6\cos\left(dx+c\right)+a^5b}+\frac{a^2\cos\left(dx+c\right)^3-3\,ab\cos\left(dx+c\right)^2-3\,\left(a^2-3\,b^2\right)\cos\left(dx+c\right)}{a^4}+\frac{6\,\left(a^2b-2\,b^3\right)\log\left(a\cos\left(dx+c\right)+b\right)}{a^5}}{3\,d}$$

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^2*b^2 - b^4)/(a^6*cos(d*x + c) + a^5*b) + (a^2*cos(d*x + c)^3 - 3
*a*b*cos(d*x + c)^2 - 3*(a^2 - 3*b^2)*cos(d*x + c))/a^4 + 6*(a^2*b - 2*b^3)
*log(a*cos(d*x + c) + b)/a^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{2\,(a^2b-2\,b^3)\log\left(|-a\cos\left(dx+c\right)-b|\right)}{a^5d}+\frac{a^2b^2-b^4}{(a\cos\left(dx+c\right)+b)a^5d}$$
$$+\frac{a^4d^5\cos\left(dx+c\right)^3-3\,a^3bd^5\cos\left(dx+c\right)^2-3\,a^4d^5\cos\left(dx+c\right)+9\,a^2b^2d^5\cos\left(dx+c\right)}{3\,a^6d^6}$$

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*b - 2*b^3)*log(abs(-a*cos(d*x + c) - b))/(a^5*d) + (a^2*b^2 - b^4)/(
(a*cos(d*x + c) + b)*a^5*d) + 1/3*(a^4*d^5*cos(d*x + c)^3 - 3*a^3*b*d^5*cos
(d*x + c)^2 - 3*a^4*d^5*cos(d*x + c) + 9*a^2*b^2*d^5*cos(d*x + c))/(a^6*d^6
)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{\cos\left(c+d\,x\right)\,\left(\frac{1}{a^2}-\frac{3\,b^2}{a^4}\right)-\frac{\cos(c+d\,x)^3}{3\,a^2}+\frac{b\cos(c+d\,x)^2}{a^3}-\frac{\ln(b+a\,\cos(c+d\,x))\,\left(2\,a^2\,b-4\,b^3\right)}{a^5}+\frac{b^4-a^2\,b^2}{a\,\left(\cos(c+d\,x)\,a^5+b\,a^4\right)}}{d}$$

```
[In] int(sin(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)*(1/a^2 - (3*b^2)/a^4) - cos(c + d*x)^3/(3*a^2) + (b*cos(c +
d*x)^2)/a^3 - (log(b + a*cos(c + d*x))*(2*a^2*b - 4*b^3))/a^5 + (b^4 - a^2*
b^2)/(a*(a^5*cos(c + d*x) + a^4*b)))/d
```

3.212 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\cos(c+dx)}{a^2d}+\frac{b^2}{a^3d(b+a\cos(c+dx))}+\frac{2b\log(b+a\cos(c+dx))}{a^3d}$$

```
[Out] -cos(d*x+c)/a^2/d+b^2/a^3/d/(b+a*cos(d*x+c))+2*b*ln(b+a*cos(d*x+c))/a^3/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2912, 12, 45}

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2}{a^3d(a\cos(c+dx)+b)}+\frac{2b\log(a\cos(c+dx)+b)}{a^3d}-\frac{\cos(c+dx)}{a^2d}$$

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]/(a^2*d)) + b^2/(a^3*d*(b + a*Cos[c + d*x])) + (2*b*Log[b + a
*Cos[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{b^2}{(b-x)^2}-\frac{2b}{b-x}\right)\,dx, x, -a\cos(c+dx)\right)}{a^3d} \\
&= -\frac{\cos(c+dx)}{a^2d}+\frac{b^2}{a^3d(b+a\cos(c+dx))}+\frac{2b\log(b+a\cos(c+dx))}{a^3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-a^2\cos^2(c+dx)+ab\cos(c+dx)(-1+2\log(b+a\cos(c+dx)))+b^2(1+2\log(b+a\cos(c+dx)))}{a^3d(b+a\cos(c+dx))}$$

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x])^2,x]
[Out] (-(a^2*Cos[c + d*x]^2) + a*b*Cos[c + d*x]*(-1 + 2*Log[b + a*Cos[c + d*x]])
+ b^2*(1 + 2*Log[b + a*Cos[c + d*x]]))/(a^3*d*(b + a*Cos[c + d*x]))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{-\frac{1}{a^{2}\sec(dx+c)}-\frac{2b\ln(\sec(dx+c))}{a^{3}}-\frac{b}{a^{2}(a+b\sec(dx+c))}+\frac{2b\ln(a+b\sec(dx+c))}{a^{3}}}{d}$
default	$\frac{-\frac{1}{a^{2}\sec(dx+c)}-\frac{2b\ln(\sec(dx+c))}{a^{3}}-\frac{b}{a^{2}(a+b\sec(dx+c))}+\frac{2b\ln(a+b\sec(dx+c))}{a^{3}}}{d}$
risch	$-\frac{2ibx}{a^{3}}-\frac{e^{i(dx+c)}}{2a^{2}d}-\frac{e^{-i(dx+c)}}{2a^{2}d}-\frac{4ibc}{a^{3}d}+\frac{2b^{2}e^{i(dx+c)}}{a^{3}d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{2b\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right)}{a^{3}d}$
parallelrisch	$\frac{4\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}(a-b)\right)a\,b^{2}\cos(dx+c)-4\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a\,b^{2}\cos(dx+c)-2a^{3}\cos(dx+c)-4\cos(dx+c)a\,b^{2}+}{2a^{3}db(b+a\cos(dx+c))}$
norman	$\frac{\frac{2a^{2}+4ab+4b^{2}}{2d\,a^{2}b}-\frac{\left(2a^{2}-4ab+4b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2d\,a^{2}b}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)}-\frac{2b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}{a^{3}d}+\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\right.}{a^{3}}$

```
[In] int(sin(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2/sec(d*x+c)-2/a^3*b*ln(sec(d*x+c))-b/a^2/(a+b*sec(d*x+c))+2/a^3*
b*ln(a+b*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$=-\frac{a^{2}\cos\left(dx+c\right)^{2}+ab\cos\left(dx+c\right)-b^{2}-2\left(ab\cos\left(dx+c\right)+b^{2}\right)\log\left(a\cos\left(dx+c\right)+b\right)}{a^{4}d\cos\left(dx+c\right)+a^{3}bd}$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*cos(d*x + c)^2 + a*b*cos(d*x + c) - b^2 - 2*(a*b*cos(d*x + c) + b^2)*
log(a*cos(d*x + c) + b))/(a^4*d*cos(d*x + c) + a^3*b*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))**2,x)
[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{b^2}{a^4\cos(dx+c)+a^3b} - \frac{\cos(dx+c)}{a^2} + \frac{2\,b\log(a\cos(dx+c)+b)}{a^3}}{d}$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] (b^2/(a^4*cos(d*x + c) + a^3*b) - cos(d*x + c)/a^2 + 2*b*log(a*cos(d*x + c)
 + b)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\cos(dx+c)}{a^2d} + \frac{2\,b\log(|-a\cos(dx+c)-b|)}{a^3d} + \frac{b^2}{(a\cos(dx+c)+b)a^3d}$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] -cos(d*x + c)/(a^2*d) + 2*b*log(abs(-a*cos(d*x + c) - b))/(a^3*d) + b^2/((a
*cos(d*x + c) + b)*a^3*d)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2}{d\,\left(\cos\left(c+d\,x\right)\,a^4+b\,a^3\right)} - \frac{\cos\left(c+d\,x\right)}{a^2\,d} + \frac{2\,b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{a^3\,d}$$

```
[In] int(sin(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] b^2/(d*(a^4*cos(c + d*x) + a^3*b)) - cos(c + d*x)/(a^2*d) + (2*b*log(b + a*
cos(c + d*x)))/(a^3*d)
```

3.213 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{2(a+b)^2d} - \frac{\log(1+\cos(c+dx))}{2(a-b)^2d} + \frac{2ab\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^2d}$$

```
[Out] b^2/a/(a^2-b^2)/d/(b+a*cos(d*x+c))+1/2*ln(1-cos(d*x+c))/(a+b)^2/d-1/2*ln(1+
cos(d*x+c))/(a-b)^2/d+2*a*b*ln(b+a*cos(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2916, 12, 1643}

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2}{ad\left(a^2-b^2\right)(a\cos(c+dx)+b)} + \frac{2ab\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^2} + \frac{\log(1-\cos(c+dx))}{2d(a+b)^2} - \frac{\log(\cos(c+dx)+1)}{2d(a-b)^2}$$

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] b^2/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(2*(a +
b)^2*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)^2*d) + (2*a*b*Log[b + a*Cos[c +
d*x]])/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos(c+dx)\cot(c+dx)}{(-b-a\cos(c+dx))^2}\,dx\\
&= \frac{a\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(-b+x)^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{2(a-b)^2(a-x)} + \frac{b^2}{(a-b)(a+b)(b-x)^2} - \frac{2a^2b}{(a-b)^2(a+b)^2(b-x)} + \frac{a}{2(a+b)^2(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{ad}\\
&= \frac{b^2}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{2(a+b)^2d}\\
&\quad - \frac{\log(1+\cos(c+dx))}{2(a-b)^2d} + \frac{2ab\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.51

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{-a^2\cos(c+dx)\left((a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2ab\log(b+a\cos(c+dx))-(a-b)^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{a(a-b)^2(}$$

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-(a^2*Cos[c + d*x]*((a + b)^2*Log[Cos[(c + d*x)/2]] - 2*a*b*Log[b + a*Cos[
c + d*x]] - (a - b)^2*Log[Sin[(c + d*x)/2]])) + b*(-(a*(a + b)^2*Log[Cos[(c
 + d*x)/2]]) + 2*a^2*b*Log[b + a*Cos[c + d*x]] + (a - b)*(b*(a + b) + a*(a
- b)*Log[Sin[(c + d*x)/2]])))/(a*(a - b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])
)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{\ln(\cos(dx+c)+1)}{2(a-b)^2}+\frac{\ln(\cos(dx+c)-1)}{2(a+b)^2}+\frac{b^2}{(a+b)(a-b)a(b+a\cos(dx+c))}+\frac{2ab\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$
default	$\frac{-\frac{\ln(\cos(dx+c)+1)}{2(a-b)^2}+\frac{\ln(\cos(dx+c)-1)}{2(a+b)^2}+\frac{b^2}{(a+b)(a-b)a(b+a\cos(dx+c))}+\frac{2ab\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$
parallelrisch	$\frac{(2a^2b\cos(dx+c)+2a\,b^2)\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)+(a-b)^2(b+a\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b^2(\cos(dx+c)+1)(a-}{d(a-b)^2(a+b)^2(b+a\cos(dx+c))}$
norman	$-\frac{2b^2}{d(a^3-a^2b-a\,b^2+b^3)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a^2+2ab+b^2)}+\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\right.\right.}{d(a^4-2a^2b^2+b^4}$
risch	$\frac{ix}{a^2-2ab+b^2}+\frac{ic}{d(a^2-2ab+b^2)}-\frac{ix}{a^2+2ab+b^2}-\frac{ic}{d(a^2+2ab+b^2)}-\frac{4iabx}{a^4-2a^2b^2+b^4}-\frac{4iabc}{d(a^4-2a^2b^2+b^4)}-\frac{}{ad(-a^2+}$

```
[In] int(csc(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a-b)^2*ln(cos(d*x+c)+1)+1/2/(a+b)^2*ln(cos(d*x+c)-1)+b^2/(a+b)/(
a-b)/a/(b+a*cos(d*x+c))+2*a*b/(a+b)^2/(a-b)^2*ln(b+a*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.93

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{2\,a^2b^2 - 2\,b^4 + 4\,(a^3b\cos(dx+c) + a^2b^2)\log(a\cos(dx+c)+b) - (a^3b + 2\,a^2b^2 + ab^3 + (a^4 + 2\,a^3b + a^2}{2\,((a^6 - 2\,a^4b^2 + a^2b^4}$$

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*b^2 - 2*b^4 + 4*(a^3*b*cos(d*x + c) + a^2*b^2)*log(a*cos(d*x + c
) + b) - (a^3*b + 2*a^2*b^2 + a*b^3 + (a^4 + 2*a^3*b + a^2*b^2)*cos(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) + (a^3*b - 2*a^2*b^2 + a*b^3 + (a^4 - 2*a^3*
b + a^2*b^2)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^6 - 2*a^4*b^2
+ a^2*b^4)*d*cos(d*x + c) + (a^5*b - 2*a^3*b^3 + a*b^5)*d)
```

Sympy [F]

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4\,ab\log(a\cos(dx+c)+b)}{a^4-2\,a^2b^2+b^4} + \frac{2\,b^2}{a^3b-ab^3+(a^4-a^2b^2)\cos(dx+c)} - \frac{\log(\cos(dx+c)+1)}{a^2-2\,ab+b^2} + \frac{\log(\cos(dx+c)-1)}{a^2+2\,ab+b^2}}{2\,d}$$

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*log(a*cos(d*x + c) + b)/(a^4 - 2*a^2*b^2 + b^4) + 2*b^2/(a^3*b -
 a*b^3 + (a^4 - a^2*b^2)*cos(d*x + c)) - log(cos(d*x + c) + 1)/(a^2 - 2*a*b
 + b^2) + log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(105) = 210$.

Time = 0.33 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.95

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{4\,ab\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2}-\frac{4\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{(a^3+a^2b-ab^2-b^3)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}}{2\,d}$$

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - 2*a^2*b^2 + b^4) + log(abs(-cos
(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) - 4*(a*b + b^2 +
a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^3 + a^2*b - a*b^2 - b^3)*(a
+ b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1))))/d
```

Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)}{2\,d\,{\left(a+b\right)}^2}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)}{2\,d\,{\left(a-b\right)}^2}+\frac{b^2}{a\,d\,\left(a^2-b^2\right)\,\left(b+a\,\cos\left(c+d\,x\right)\right)}+\frac{2\,a\,b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{d\,{\left(a^2-b^2\right)}^2}$$

```
[In] int(1/(sin(c + d*x)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] log(cos(c + d*x) - 1)/(2*d*(a + b)^2) - log(cos(c + d*x) + 1)/(2*d*(a - b)^
2) + b^2/(a*d*(a^2 - b^2)*(b + a*cos(c + d*x))) + (2*a*b*log(b + a*cos(c +
d*x)))/(d*(a^2 - b^2)^2)
```

3.214 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 168

$$
\begin{aligned}
\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = {} & \frac{ab^2}{(a^2-b^2)^2 d(b+a\cos(c+dx))} \\
& + \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^2 d} \\
& + \frac{(a-b)\log(1-\cos(c+dx))}{4(a+b)^3 d} - \frac{(a+b)\log(1+\cos(c+dx))}{4(a-b)^3 d} \\
& + \frac{2ab(a^2+b^2)\log(b+a\cos(c+dx))}{(a^2-b^2)^3 d}
\end{aligned}
$$

```
[Out] a*b^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))+1/2*(2*a*b-(a^2+b^2)*cos(d*x+c))*csc(d
*x+c)^2/(a^2-b^2)^2/d+1/4*(a-b)*ln(1-cos(d*x+c))/(a+b)^3/d-1/4*(a+b)*ln(1+c
os(d*x+c))/(a-b)^3/d+2*a*b*(a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used

= {3957, 2916, 12, 1661, 1643}

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{ab^2}{d\left(a^2-b^2\right)^2\left(a\cos(c+dx)+b\right)} + \frac{2ab(a^2+b^2)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3} + \frac{\csc^2(c+dx)\left(2ab-(a^2+b^2)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^2} + \frac{(a-b)\log(1-\cos(c+dx))}{4d(a+b)^3} - \frac{(a+b)\log(\cos(c+dx)+1)}{4d(a-b)^3}$$

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*b^2)/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x])) + ((2*a*b - (a^2 + b^2)*Cos[
c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^2*d) + ((a - b)*Log[1 - Cos[c + d*
x]])/(4*(a + b)^3*d) - ((a + b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^3*d) + (2
*a*b*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cot^2(c+dx)\csc(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \frac{a^3\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a\text{Subst}\left(\int \frac{x^2}{(-b+x)^2(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^2 d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}+\frac{2a^2bx}{a^2-b^2}+\frac{a^2\left(a^2+b^2\right)x^2}{\left(a^2-b^2\right)^2}}{(-b+x)^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{2ad} \\
&= \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^2 d} \\
&\quad + \frac{\text{Subst}\left(\int \left(\frac{a(a+b)}{2(a-b)^3(a-x)}+\frac{2a^2b^2}{(a-b)^2(a+b)^2(b-x)^2}-\frac{4a^2b(a^2+b^2)}{(a-b)^3(a+b)^3(b-x)}+\frac{a(a-b)}{2(a+b)^3(a+x)}\right)\,dx, x, -a\cos(c+dx\right)}{2ad} \\
&= \frac{ab^2}{\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))}+\frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^2 d} \\
&\quad + \frac{(a-b)\log(1-\cos(c+dx))}{4(a+b)^3d}-\frac{(a+b)\log(1+\cos(c+dx))}{4(a-b)^3d} \\
&\quad + \frac{2ab(a^2+b^2)\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^3 d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.33

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{(b+a\cos(c+dx))\left(\frac{8ab^2}{(a-b)^2(a+b)^2}-\frac{(b+a\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}+\frac{4(a+b)(b+a\cos(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(-a+b)^3}+\frac{16ab(a^2+b^2)(\cdots}{\cdots}\right.}{8d(a+b\sec(\cdots}$$

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((8*a*b^2)/((a - b)^2*(a + b)^2) - ((b + a*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/(a + b)^2 + (4*(a + b)*(b + a*Cos[c + d*x])*Log[Cos
[(c + d*x)/2]])/(-a + b)^3 + (16*a*b*(a^2 + b^2)*(b + a*Cos[c + d*x])*Log[b
 + a*Cos[c + d*x]])/(a^2 - b^2)^3 + (4*(a - b)*(b + a*Cos[c + d*x])*Log[Sin
[(c + d*x)/2]])/(a + b)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a -
b)^2)*Sec[c + d*x]^2)/(8*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{1}{4(a-b)^2(\cos(dx+c)+1)}+\frac{(-a-b)\ln(\cos(dx+c)+1)}{4(a-b)^3}+\frac{1}{4(a+b)^2(\cos(dx+c)-1)}+\frac{(a-b)\ln(\cos(dx+c)-1)}{4(a+b)^3}+\frac{b^2a}{(a+b)^2(a-b)^2(b+a\cos(dx+c)}\cdots}{d}$
default	$\frac{\frac{1}{4(a-b)^2(\cos(dx+c)+1)}+\frac{(-a-b)\ln(\cos(dx+c)+1)}{4(a-b)^3}+\frac{1}{4(a+b)^2(\cos(dx+c)-1)}+\frac{(a-b)\ln(\cos(dx+c)-1)}{4(a+b)^3}+\frac{b^2a}{(a+b)^2(a-b)^2(b+a\cos(dx+c)}\cdots}{d}$
norman	$\frac{\frac{1}{8d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8d(a-b)}-\frac{\left(a^4+14a^2b^2+b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4d\left(a^5-a^4b-2a^3b^2+2a^2b^3+a\,b^4-b^5\right)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}+\frac{(a-b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d(a^3+3a^2b+3a\,b^2+b^3)}+\frac{2ab(a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}{d(a^6-3a^4b^2\cdots}$
parallelrisch	$\frac{256ba(a^2+b^2)(b+a\cos(dx+c))\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)+64(a-b)^4(b+a\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(((-8a^5-\cdots\right.}{\cdots}$
risch	$-\frac{iac}{2d(a^3+3a^2b+3a\,b^2+b^3)}-\frac{4ia\,b^3x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{iax}{2a^3-6a^2b+6a\,b^2-2b^3}-\frac{4ia^3bx}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{4i\cdots}{d(a^6-3a^4b\cdots}$

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/(a-b)^2/(cos(d*x+c)+1)+1/4/(a-b)^3*(-a-b)*ln(cos(d*x+c)+1)+1/4/(a+
b)^2/(cos(d*x+c)-1)+1/4*(a-b)/(a+b)^3*ln(cos(d*x+c)-1)+b^2/(a+b)^2*a/(a-b)^
2/(b+a*cos(d*x+c))+2*a*b*(a^2+b^2)/(a-b)^3/(a+b)^3*ln(b+a*cos(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(162) = 324$.

Time = 0.36 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.75

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{8\,a^3b^2 - 8\,ab^4 - 2\,(a^5 + 2\,a^3b^2 - 3\,ab^4)\cos(dx+c)^2 + 2\,(a^4b - 2\,a^2b^3 + b^5)\cos(dx+c) + 8\,(a^3b^2 + ab^4}{}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^3*b^2 - 8*a*b^4 - 2*(a^5 + 2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 +
2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c) + 8*(a^3*b^2 + a*b^4 - (a^4*b + a^
2*b^3)*cos(d*x + c)^3 - (a^3*b^2 + a*b^4)*cos(d*x + c)^2 + (a^4*b + a^2*b^3
)*cos(d*x + c))*log(a*cos(d*x + c) + b) - (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
4*a*b^4 + b^5 - (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(d*x + c
)^3 - (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*cos(d*x + c)^2 + (a^5
 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(d*x + c))*log(1/2*cos(d*x +
 c) + 1/2) + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5 - (a^5 - 4*a^4*
b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (a^4*b - 4*a^3*b^2 + 6*
a^2*b^3 - 4*a*b^4 + b^5)*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^
2*b^3 + a*b^4)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - 3*a^5*b^
2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^3 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
b^7)*d*cos(d*x + c)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c
) - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)
```

Sympy [F]

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\csc^3{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.63

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{8\left(a^3b+ab^3\right)\log(a\cos(dx+c)+b)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{(a+b)\log(\cos(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3} + \frac{(a-b)\log(\cos(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3} + \frac{2\left(4ab^2-\left(a^3+3ab^2\right)\cos(\right.}{a^4b-2a^2b^3+b^5-\left(a^5-2a^3b^2+ab^4\right)\cos(dx+c)^3-(a}}{4d}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(8*(a^3*b + a*b^3)*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) - (a + b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (
a - b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*a*b^2 -
 (a^3 + 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - b^3)*cos(d*x + c))/(a^4*b - 2*a^
2*b^3 + b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - (a^4*b - 2*a^2*b^3
 + b^5)*cos(d*x + c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(162) = 324$.

Time = 0.36 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.71

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{2(a-b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3a^2b+3ab^2+b^3} + \frac{16\left(a^3b+ab^3\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{a^3-a^2b-ab^2+b^3-\frac{8a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8ab^2(\cos}{\cos(d}}{\left(a^4-2a^2b^2+b^4\right)\left(\frac{a(\mathrm{c}}{\mathrm{co}}\right.}}{8d}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 + 3*a
^2*b + 3*a*b^2 + b^3) + 16*(a^3*b + a*b^3)*log(abs(-a - b - a*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6) + (a^3 - a^2*b - a*b^2 + b^3 - 8*a^2*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a^2*b*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^4 - 2*a^2*b^2 +
 b^4)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2)) - (cos(d*x + c) - 1)/((a^2 - 2*a*b + b^2)*
(cos(d*x + c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.04 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.36

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{2\,a\,b^2}{{\left(a^2-b^2\right)}^2}+\frac{b\,\cos\left(c+d\,x\right)}{2\,\left(a^2-b^2\right)}-\frac{{\cos\left(c+d\,x\right)}^2\,\left(a^3+3\,a\,b^2\right)}{2\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(-a\,{\cos\left(c+d\,x\right)}^3-b\,{\cos\left(c+d\,x\right)}^2+a\,\cos\left(c+d\,x\right)+b\right)} - \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{b}{2\,{\left(a+b\right)}^3}-\frac{1}{4\,{\left(a+b\right)}^2}\right)}{d} + \frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(2\,a^3\,b+2\,a\,b^3\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)} - \frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(a+b\right)}{4\,d\,{\left(a-b\right)}^3}$$

```
[In] int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))^2),x)
```

```
[Out] ((2*a*b^2)/(a^2 - b^2)^2 + (b*cos(c + d*x))/(2*(a^2 - b^2)) - (cos(c + d*x)
^2*(3*a*b^2 + a^3))/(2*(a^4 + b^4 - 2*a^2*b^2)))/(d*(b + a*cos(c + d*x) - a
*cos(c + d*x)^3 - b*cos(c + d*x)^2)) - (log(cos(c + d*x) - 1)*(b/(2*(a + b)
^3) - 1/(4*(a + b)^2)))/d + (log(b + a*cos(c + d*x))*(2*a*b^3 + 2*a^3*b))/(
d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (log(cos(c + d*x) + 1)*(a + b))/(4
*d*(a - b)^3)
```

3.215 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 259

$$\begin{aligned}\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{a^3b^2}{(a^2-b^2)^3\,d(b+a\cos(c+dx))}\\ &+\frac{(8ab(a^2+b^2)-(3a^4+12a^2b^2+b^4)\cos(c+dx))\csc^2(c+dx)}{8\,(a^2-b^2)^3\,d}\\ &+\frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^4(c+dx)}{4\,(a^2-b^2)^2\,d}\\ &+\frac{(3a^2-4ab-b^2)\log(1-\cos(c+dx))}{16(a+b)^4d}\\ &-\frac{(3a^2+4ab-b^2)\log(1+\cos(c+dx))}{16(a-b)^4d}\\ &+\frac{2a^3b(a^2+2b^2)\log(b+a\cos(c+dx))}{(a^2-b^2)^4\,d}\end{aligned}$$

```
[Out] a^3*b^2/(a^2-b^2)^3/d/(b+a*cos(d*x+c))+1/8*(8*a*b*(a^2+b^2)-(3*a^4+12*a^2*b
^2+b^4)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^3/d+1/4*(2*a*b-(a^2+b^2)*cos(d*x
+c))*csc(d*x+c)^4/(a^2-b^2)^2/d+1/16*(3*a^2-4*a*b-b^2)*ln(1-cos(d*x+c))/(a+
b)^4/d-1/16*(3*a^2+4*a*b-b^2)*ln(1+cos(d*x+c))/(a-b)^4/d+2*a^3*b*(a^2+2*b^2
)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2916, 12, 1661, 1643}

$$\begin{aligned}\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{(3a^2-4ab-b^2)\log(1-\cos(c+dx))}{16d(a+b)^4} \\ &\quad -\frac{(3a^2+4ab-b^2)\log(\cos(c+dx)+1)}{16d(a-b)^4} \\ &\quad +\frac{\csc^4(c+dx)\left(2ab-(a^2+b^2)\cos(c+dx)\right)}{4d\left(a^2-b^2\right)^2} \\ &\quad +\frac{\csc^2(c+dx)\left(8ab(a^2+b^2)-(3a^4+12a^2b^2+b^4)\cos(c+dx)\right)}{8d\left(a^2-b^2\right)^3} \\ &\quad +\frac{a^3b^2}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)} \\ &\quad +\frac{2a^3b(a^2+2b^2)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}\end{aligned}$$

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^3*b^2)/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((8*a*b*(a^2 + b^2) - (3
*a^4 + 12*a^2*b^2 + b^4)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 - b^2)^3*d)
+ ((2*a*b - (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^4)/(4*(a^2 - b^2)^2*d) +
 ((3*a^2 - 4*a*b - b^2)*Log[1 - Cos[c + d*x]])/(16*(a + b)^4*d) - ((3*a^2 +
 4*a*b - b^2)*Log[1 + Cos[c + d*x]])/(16*(a - b)^4*d) + (2*a^3*b*(a^2 + 2*b
^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
```

```
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot^2(c+dx)\csc^3(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \frac{a^5\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^3\text{Subst}\left(\int \frac{x^2}{(-b+x)^2(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)^2 d} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{-\frac{a^2b^2\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}+\frac{2a^2b\left(a^2-3b^2\right)x}{\left(a^2-b^2\right)^2}+\frac{3a^2\left(a^2+b^2\right)x^2}{\left(a^2-b^2\right)^2}}{(-b+x)^2(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{4d} \\
&= \frac{(8ab(a^2+b^2)-(3a^4+12a^2b^2+b^4)\cos(c+dx))\csc^2(c+dx)}{8\left(a^2-b^2\right)^3 d} \\
&\quad + \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)^2 d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2\left(5a^4+12a^2b^2-b^4\right)}{\left(a^2-b^2\right)^3}+\frac{2a^2b\left(5a^2+b^2\right)x}{\left(a^2-b^2\right)^2}+\frac{a^2\left(3a^4+12a^2b^2+b^4\right)x^2}{\left(a^2-b^2\right)^3}}{(-b+x)^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{8ad}
\end{aligned}
$$

$$= \frac{(8ab(a^2+b^2)-(3a^4+12a^2b^2+b^4)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^3 d} + \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)^2 d} + \frac{\text{Subst}\left(\int\left(\frac{a(3a^2+4ab-b^2)}{2(a-b)^4(a-x)}+\frac{8a^4b^2}{(a^2-b^2)^3(b-x)^2}-\frac{16a^4b(a^2+2b^2)}{(a^2-b^2)^4(b-x)}+\frac{a(3a^2-4ab-b^2)}{2(a+b)^4(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{8ad}$$

$$= \frac{a^3b^2}{(a^2-b^2)^3 d(b+a\cos(c+dx))} + \frac{(8ab(a^2+b^2)-(3a^4+12a^2b^2+b^4)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^3 d} + \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)^2 d} + \frac{(3a^2-4ab-b^2)\log(1-\cos(c+dx))}{16(a+b)^4 d} - \frac{(3a^2+4ab-b^2)\log(1+\cos(c+dx))}{16(a-b)^4 d} + \frac{2a^3b(a^2+2b^2)\log(b+a\cos(c+dx))}{(a^2-b^2)^4 d}$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.24

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{(b+a\cos(c+dx))\left(\frac{64a^3b^2}{(a-b)^3(a+b)^3}+\frac{2(-3a+b)(b+a\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3}-\frac{(b+a\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}+\frac{8(-3a^2-4ab}{}\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((64*a^3*b^2)/((a - b)^3*(a + b)^3) + (2*(-3*a + b)*(
b + a*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/(a + b)^3 - ((b + a*Cos[c + d*x])*C
sc[(c + d*x)/2]^4)/(a + b)^2 + (8*(-3*a^2 - 4*a*b + b^2)*(b + a*Cos[c + d*x
])*Log[Cos[(c + d*x)/2]])/(a - b)^4 + (128*a^3*b*(a^2 + 2*b^2)*(b + a*Cos[c
 + d*x])*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4 + (8*(3*a^2 - 4*a*b - b^2)*
(b + a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]])/(a + b)^4 + (2*(3*a + b)*(b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a - b)^3 + ((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^4)/(a - b)^2)*Sec[c + d*x]^2)/(64*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{a^{3} b^{2}}{(a+b)^{3}(a-b)^{3}(b+a\cos(dx+c))}+\frac{2 a^{3} b\left(a^{2}+2 b^{2}\right)\ln(b+a\cos(dx+c))}{(a+b)^{4}(a-b)^{4}}-\frac{1}{16(a+b)^{2}(\cos(dx+c)-1)^{2}}-\frac{-3a+b}{16(a+b)^{3}(\cos(dx+c)-1)}+\frac{\left(3a^{2}-4ab\right.}{}}{d}$
default	$\frac{\frac{a^{3} b^{2}}{(a+b)^{3}(a-b)^{3}(b+a\cos(dx+c))}+\frac{2 a^{3} b\left(a^{2}+2 b^{2}\right)\ln(b+a\cos(dx+c))}{(a+b)^{4}(a-b)^{4}}-\frac{1}{16(a+b)^{2}(\cos(dx+c)-1)^{2}}-\frac{-3a+b}{16(a+b)^{3}(\cos(dx+c)-1)}+\frac{\left(3a^{2}-4ab\right.}{}}{d}$
norman	$\frac{\frac{1}{64d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{64d(a-b)}+\frac{(7a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{64d\left(a^{2}-2ab+b^{2}\right)}+\frac{(7a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{64d\left(a^{2}+2ab+b^{2}\right)}-\frac{\left(a^{6}+18a^{4}b^{2}+5a^{2}b^{4}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{4d\left(a^{7}-b\,a^{6}-3a^{5}b^{2}+3a^{4}b^{3}+3b^{4}a^{3}-3b^{5}a^{2}-a\,b^{6}+b^{7}\right)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)}-$
parallelrisch	$\frac{8192b\,a^{3}\left(a^{2}+2b^{2}\right)(b+a\cos(dx+c))\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}(a-b)\right)+1536(b+a\cos(dx+c))\left(a^{2}-\frac{4}{3}ab-\frac{1}{3}b^{2}\right)(a-b)^{4}\ln\left(\tan\left(\right.\right.}{}$
risch	Expression too large to display

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*b^2/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+2*a^3*b*(a^2+2*b^2)/(a+b)^4/(
a-b)^4*ln(b+a*cos(d*x+c))-1/16/(a+b)^2/(cos(d*x+c)-1)^2-1/16*(-3*a+b)/(a+b)
^3/(cos(d*x+c)-1)+1/16*(3*a^2-4*a*b-b^2)/(a+b)^4*ln(cos(d*x+c)-1)+1/16/(a-b
)^2/(cos(d*x+c)+1)^2-1/16*(-3*a-b)/(a-b)^3/(cos(d*x+c)+1)+1/16/(a-b)^4*(-3*
a^2-4*a*b+b^2)*ln(cos(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. 2(251) = 502.

Time = 0.49 (sec) , antiderivative size = 1205, normalized size of antiderivative = 4.65

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(40*a^5*b^2 - 32*a^3*b^4 - 8*a*b^6 + 2*(3*a^7 + 17*a^5*b^2 - 19*a^3*b^
4 - a*b^6)*cos(d*x + c)^4 - 2*(5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d
*x + c)^3 - 2*(5*a^7 + 31*a^5*b^2 - 29*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^2 +
2*(7*a^6*b - 15*a^4*b^3 + 9*a^2*b^5 - b^7)*cos(d*x + c) + 32*(a^5*b^2 + 2*a
^3*b^4 + (a^6*b + 2*a^4*b^3)*cos(d*x + c)^5 + (a^5*b^2 + 2*a^3*b^4)*cos(d*x
 + c)^4 - 2*(a^6*b + 2*a^4*b^3)*cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4)*co
s(d*x + c)^2 + (a^6*b + 2*a^4*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b) -
(3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 13*a^2*b^5 - b^7 + (3*a^7
 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c)^5
+ (3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 13*a^2*b^5 - b^7)*cos(d
```

```
*x + c)^4 - 2*(3*a^7 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b^3 + 13*a^3*b^4 - a*
b^6)*cos(d*x + c)^3 - 2*(3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 1
3*a^2*b^5 - b^7)*cos(d*x + c)^2 + (3*a^7 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b
^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (3*a^6
*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*b^5 - b^7 + (3*a^7 - 16*
a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c)^5 + (3*a
^6*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*b^5 - b^7)*cos(d*x + c
)^4 - 2*(3*a^7 - 16*a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*c
os(d*x + c)^3 - 2*(3*a^6*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*
b^5 - b^7)*cos(d*x + c)^2 + (3*a^7 - 16*a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 1
3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^9 - 4*a^
7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^5 + (a^8*b - 4*a^6*b^
3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^4 - 2*(a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 - 2*(a^8*b - 4*a^6*b^3 + 6*a^
4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^2 + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 -
4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*
b^7 + b^9)*d)
```

Sympy [F]

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\csc^5\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
[Out] Integral(csc(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(251) = 502$.

Time = 0.22 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.97

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{32\,(a^5b+2\,a^3b^3)\log(a\cos(dx+c)+b)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} - \frac{(3\,a^2+4\,ab-b^2)\log(\cos(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4} + \frac{(3\,a^2-4\,ab-b^2)\log(\cos(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7+}}{}$$

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] 1/16*(32*(a^5*b + 2*a^3*b^3)*log(a*cos(d*x + c) + b)/(a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8) - (3*a^2 + 4*a*b - b^2)*log(cos(d*x + c) + 1)/(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (3*a^2 - 4*a*b - b^2)*log(cos(d*
```

```
x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(20*a^3*b^2 + 4
*a*b^4 + (3*a^5 + 20*a^3*b^2 + a*b^4)*cos(d*x + c)^4 - (5*a^4*b - 4*a^2*b^3
 - b^5)*cos(d*x + c)^3 - (5*a^5 + 36*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^2 + (7
*a^4*b - 8*a^2*b^3 + b^5)*cos(d*x + c))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^5 + (a^6*b - 3*a^4*b
^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^4 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b
^6)*cos(d*x + c)^3 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^2
 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(251) = 502$.

Time = 0.40 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.74

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{\frac{4\,(3\,a^2-4\,ab-b^2)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{128\,(a^5b+2\,a^3b^3)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} - \frac{\frac{8\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,ab(\cos(dx+c)}{\cos(dx+c)+}}{}}{}$$

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a^2 - 4*a*b - b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 128*(a^5*b + 2*a^3*b^3)*
log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) -
 (8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2*a*b*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b^2*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a^2 + 2*a*b +
b^2 - 8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 18*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2
4*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*b^2*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*(cos(d*x + c) - 1)^2) - 128*(a^6*b + a^4*b^3 + 2*a^3*b^4 + a
^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^5*b^2*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 2*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b
^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4
*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1))))/d
```

Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.73

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3}{16\,{\left(a+b\right)}^2}-\frac{5\,b}{8\,{\left(a+b\right)}^3}+\frac{3\,b^2}{8\,{\left(a+b\right)}^4}\right)}{d} - \frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{3\,b^2}{8\,{\left(a-b\right)}^4}+\frac{5\,b}{8\,{\left(a-b\right)}^3}+\frac{3}{16\,{\left(a-b\right)}^2}\right)}{d} + \frac{\frac{{\cos\left(c+d\,x\right)}^4\,\left(3\,a^5+20\,a^3\,b^2+a\,b^4\right)}{8\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{\cos\left(c+d\,x\right)\,\left(7\,a^2\,b-b^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\cos\left(c+d\,x\right)}^3\,\left(5\,a^2\,b+b^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\cos\left(c+d\,x\right)}^2\,\left(5\,a^5+36\,a^3\,b^2+7\,a\,b^4\right)}{8\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{b}{2\,(a^2-}}{d\,\left(a\cos\left(c+d\,x\right)^5+b\cos\left(c+d\,x\right)^4-2\,a\cos\left(c+d\,x\right)^3-2\,b\cos\left(c+d\,x\right)^2+a\,\cos\left(c+d\,x\right)\right.} + \frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(2\,a^5\,b+4\,a^3\,b^3\right)}{d\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}$$

```
[In] int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))^2),x)
```

```
[Out] (log(cos(c + d*x) - 1)*(3/(16*(a + b)^2) - (5*b)/(8*(a + b)^3) + (3*b^2)/(8
*(a + b)^4)))/d - (log(cos(c + d*x) + 1)*((3*b^2)/(8*(a - b)^4) + (5*b)/(8*
(a - b)^3) + 3/(16*(a - b)^2)))/d + ((cos(c + d*x)^4*(a*b^4 + 3*a^5 + 20*a^
3*b^2))/(8*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (cos(c + d*x)*(7*a^2*b -
b^3))/(8*(a^4 + b^4 - 2*a^2*b^2)) - (cos(c + d*x)^3*(5*a^2*b + b^3))/(8*(a^
4 + b^4 - 2*a^2*b^2)) - (cos(c + d*x)^2*(7*a*b^4 + 5*a^5 + 36*a^3*b^2))/(8*
(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b*(a*b^3 + 5*a^3*b))/(2*(a^2 - b^2)
*(a^4 + b^4 - 2*a^2*b^2)))/(d*(b + a*cos(c + d*x) - 2*a*cos(c + d*x)^3 + a*
cos(c + d*x)^5 - 2*b*cos(c + d*x)^2 + b*cos(c + d*x)^4)) + (log(b + a*cos(c
 + d*x))*(2*a^5*b + 4*a^3*b^3))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a
^6*b^2))
```

3.216 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 473

$$\begin{aligned}\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\,x}{16a^8} \\ &\quad - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\,(2a^2-7b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^8d} \\ &\quad + \frac{b(61a^4-170a^2b^2+105b^4)\sin(c+dx)}{15a^7d} \\ &\quad - \frac{(27a^4-86a^2b^2+56b^4)\cos(c+dx)\sin(c+dx)}{16a^6d} \\ &\quad + \frac{(15a^4-52a^2b^2+35b^4)\cos^2(c+dx)\sin(c+dx)}{15a^5bd} \\ &\quad - \frac{(16a^4-61a^2b^2+42b^4)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d} \\ &\quad - \frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} \\ &\quad + \frac{(5a^4-20a^2b^2+14b^4)\cos^4(c+dx)\sin(c+dx)}{10a^3b^2d(b+a\cos(c+dx))} \\ &\quad + \frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))} - \frac{\cos^6(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))}\end{aligned}$$

```
[Out] 1/16*(5*a^6-90*a^4*b^2+200*a^2*b^4-112*b^6)*x/a^8-2*(a-b)^(3/2)*b*(a+b)^(3/
2)*(2*a^2-7*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^8/d+
1/15*b*(61*a^4-170*a^2*b^2+105*b^4)*sin(d*x+c)/a^7/d-1/16*(27*a^4-86*a^2*b^
2+56*b^4)*cos(d*x+c)*sin(d*x+c)/a^6/d+1/15*(15*a^4-52*a^2*b^2+35*b^4)*cos(d
*x+c)^2*sin(d*x+c)/a^5/b/d-1/24*(16*a^4-61*a^2*b^2+42*b^4)*cos(d*x+c)^3*sin
(d*x+c)/a^4/b^2/d-1/3*cos(d*x+c)^3*sin(d*x+c)/b/d/(b+a*cos(d*x+c))+1/6*a*co
s(d*x+c)^4*sin(d*x+c)/b^2/d/(b+a*cos(d*x+c))+1/10*(5*a^4-20*a^2*b^2+14*b^4)
```

```
*cos(d*x+c)^4*sin(d*x+c)/a^3/b^2/d/(b+a*cos(d*x+c))+7/30*b*cos(d*x+c)^5*sin
(d*x+c)/a^2/d/(b+a*cos(d*x+c))-1/6*cos(d*x+c)^6*sin(d*x+c)/a/d/(b+a*cos(d*x
+c))
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2975, 3126, 3128, 3102, 2814, 2738, 214}

$$\begin{aligned}\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}\\ &-\frac{2b(a-b)^{3/2}(a+b)^{3/2}\left(2a^2-7b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^8d}\\ &-\frac{\left(16a^4-61a^2b^2+42b^4\right)\sin(c+dx)\cos^3(c+dx)}{24a^4b^2d}\\ &+\frac{b\left(61a^4-170a^2b^2+105b^4\right)\sin(c+dx)}{15a^7d}\\ &-\frac{\left(27a^4-86a^2b^2+56b^4\right)\sin(c+dx)\cos(c+dx)}{16a^6d}\\ &+\frac{\left(15a^4-52a^2b^2+35b^4\right)\sin(c+dx)\cos^2(c+dx)}{15a^5bd}\\ &+\frac{\left(5a^4-20a^2b^2+14b^4\right)\sin(c+dx)\cos^4(c+dx)}{10a^3b^2d(a\cos(c+dx)+b)}\\ &+\frac{x\left(5a^6-90a^4b^2+200a^2b^4-112b^6\right)}{16a^8}\\ &+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}\\ &-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}\end{aligned}$$

```
[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*x)/(16*a^8) - (2*(a - b)^(3/2
)*b*(a + b)^(3/2)*(2*a^2 - 7*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(a^8*d) + (b*(61*a^4 - 170*a^2*b^2 + 105*b^4)*Sin[c + d*x])/(15
*a^7*d) - ((27*a^4 - 86*a^2*b^2 + 56*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*a^
6*d) + ((15*a^4 - 52*a^2*b^2 + 35*b^4)*Cos[c + d*x]^2*Sin[c + d*x])/(15*a^5
*b*d) - ((16*a^4 - 61*a^2*b^2 + 42*b^4)*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^
4*b^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(3*b*d*(b + a*Cos[c + d*x])) + (a*
Cos[c + d*x]^4*Sin[c + d*x])/(6*b^2*d*(b + a*Cos[c + d*x])) + ((5*a^4 - 20*
a^2*b^2 + 14*b^4)*Cos[c + d*x]^4*Sin[c + d*x])/(10*a^3*b^2*d*(b + a*Cos[c +
 d*x])) + (7*b*Cos[c + d*x]^5*Sin[c + d*x])/(30*a^2*d*(b + a*Cos[c + d*x]))
 - (Cos[c + d*x]^6*Sin[c + d*x])/(6*a*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^6(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))}+\frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} \\
&\quad+\frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))}-\frac{\cos^6(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))} \\
&\quad+\frac{\int \frac{\cos^4(c+dx)\left(60\left(3a^4-10a^2b^2+7b^4\right)+12ab\left(5a^2-2b^2\right)\cos(c+dx)-12\left(20a^4-65a^2b^2+42b^4\right)\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^2}\,dx}{360a^2b^2}
\end{aligned}
$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{(5a^4-20a^2b^2+14b^4)\cos^4(c+dx)\sin(c+dx)}{10a^3b^2d(b+a\cos(c+dx))} + \frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))} - \frac{\cos^6(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))} - \frac{\int \frac{\cos^3(c+dx)(144(5a^6-25a^4b^2+34a^2b^4-14b^6)+12ab(10a^4-17a^2b^2+7b^4)\cos(c+dx)-60(16a^6-77a^4b^2+103a^2b^4-42b^6)\cos^2(c+d}{-b-a\cos(c+dx)}}{360a^3b^2(a^2-b^2)}$$

$$= -\frac{(16a^4-61a^2b^2+42b^4)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{(5a^4-20a^2b^2+14b^4)\cos^4(c+dx)\sin(c+dx)}{10a^3b^2d(b+a\cos(c+dx))} + \frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))} - \frac{\cos^6(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))} + \frac{\int \frac{\cos^2(c+dx)(180b(16a^6-77a^4b^2+103a^2b^4-42b^6)+36ab^2(15a^4-29a^2b^2+14b^4)\cos(c+dx)-288b(15a^6-67a^4b^2+87a^2b^4-35b^6)\cos}{-b-a\cos(c+dx)}}{1440a^4b^2(a^2-b^2)}$$

$$= \frac{(15a^4-52a^2b^2+35b^4)\cos^2(c+dx)\sin(c+dx)}{15a^5bd} - \frac{(16a^4-61a^2b^2+42b^4)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{(5a^4-20a^2b^2+14b^4)\cos^4(c+dx)\sin(c+dx)}{10a^3b^2d(b+a\cos(c+dx))} + \frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))} - \frac{\cos^6(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)(576b^2(15a^6-67a^4b^2+87a^2b^4-35b^6)+36ab^3(83a^4-153a^2b^2+70b^4)\cos(c+dx)-540b^2(27a^6-113a^4b^2+142a^2b^4-56b^6)}{-b-a\cos(c+dx)}}{4320a^5b^2(a^2-b^2)}$$

$$= -\frac{(27a^4-86a^2b^2+56b^4)\cos(c+dx)\sin(c+dx)}{16a^6d} + \frac{(15a^4-52a^2b^2+35b^4)\cos^2(c+dx)\sin(c+dx)}{15a^5bd} - \frac{(16a^4-61a^2b^2+42b^4)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{(5a^4-20a^2b^2+14b^4)\cos^4(c+dx)\sin(c+dx)}{10a^3b^2d(b+a\cos(c+dx))} + \frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))} - \frac{\cos^6(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))} + \frac{\int \frac{540b^3(27a^6-113a^4b^2+142a^2b^4-56b^6)-36ab^2(75a^6-449a^4b^2+654a^2b^4-280b^6)\cos(c+dx)-576b^3(61a^6-231a^4b^2+275a^2b^4-105}{-b-a\cos(c+dx)}}{8640a^6b^2(a^2-b^2)}$$

$$= \frac{b(61a^4 - 170a^2b^2 + 105b^4)\sin(c+dx)}{15a^7d} - \frac{(27a^4 - 86a^2b^2 + 56b^4)\cos(c+dx)\sin(c+dx)}{16a^6d} + \frac{(15a^4 - 52a^2b^2 + 35b^4)\cos^2(c+dx)\sin(c+dx)}{15a^5bd} - \frac{(16a^4 - 61a^2b^2 + 42b^4)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{(5a^4 - 20a^2b^2 + 14b^4)\cos^4(c+dx)\sin(c+dx)}{10a^3b^2d(b+a\cos(c+dx))} + \frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))} - \frac{\cos^6(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))} - \frac{\int \frac{-540ab^3(27a^6-113a^4b^2+142a^2b^4-56b^6)+540b^2(5a^8-95a^6b^2+290a^4b^4-312a^2b^6+112b^8)\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{8640a^7b^2(a^2-b^2)}$$

$$= \frac{(5a^6 - 90a^4b^2 + 200a^2b^4 - 112b^6)\,x}{16a^8} + \frac{b(61a^4 - 170a^2b^2 + 105b^4)\sin(c+dx)}{15a^7d} - \frac{(27a^4 - 86a^2b^2 + 56b^4)\cos(c+dx)\sin(c+dx)}{16a^6d} + \frac{(15a^4 - 52a^2b^2 + 35b^4)\cos^2(c+dx)\sin(c+dx)}{15a^5bd} - \frac{(16a^4 - 61a^2b^2 + 42b^4)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{(5a^4 - 20a^2b^2 + 14b^4)\cos^4(c+dx)\sin(c+dx)}{10a^3b^2d(b+a\cos(c+dx))} + \frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))} - \frac{\cos^6(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))} + \frac{\left(b(2a^2-7b^2)(a^2-b^2)^2\right)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^8}$$

$$
\begin{aligned}
&= \frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\,x}{16a^8} + \frac{b(61a^4-170a^2b^2+105b^4)\sin(c+dx)}{15a^7d} \\
&\quad - \frac{(27a^4-86a^2b^2+56b^4)\cos(c+dx)\sin(c+dx)}{16a^6d} \\
&\quad + \frac{(15a^4-52a^2b^2+35b^4)\cos^2(c+dx)\sin(c+dx)}{15a^5bd} \\
&\quad - \frac{(16a^4-61a^2b^2+42b^4)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} \\
&\quad + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{(5a^4-20a^2b^2+14b^4)\cos^4(c+dx)\sin(c+dx)}{10a^3b^2d(b+a\cos(c+dx))} \\
&\quad + \frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))} - \frac{\cos^6(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))} \\
&\quad + \frac{\left(2b(2a^2-7b^2)\,(a^2-b^2)^2\right)\mathrm{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^8d} \\
&= \frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\,x}{16a^8} \\
&\quad - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\,(2a^2-7b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^8d} \\
&\quad + \frac{b(61a^4-170a^2b^2+105b^4)\sin(c+dx)}{15a^7d} \\
&\quad - \frac{(27a^4-86a^2b^2+56b^4)\cos(c+dx)\sin(c+dx)}{16a^6d} \\
&\quad + \frac{(15a^4-52a^2b^2+35b^4)\cos^2(c+dx)\sin(c+dx)}{15a^5bd} \\
&\quad - \frac{(16a^4-61a^2b^2+42b^4)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} \\
&\quad + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{(5a^4-20a^2b^2+14b^4)\cos^4(c+dx)\sin(c+dx)}{10a^3b^2d(b+a\cos(c+dx))} \\
&\quad + \frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))} - \frac{\cos^6(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.32 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.85

$$
\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx
$$

$$
= \frac{3840b(2a^2-7b^2)\,(a^2-b^2)^{3/2}\,\mathrm{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + \frac{600a^6bc-10800a^4b^3c+24000a^2b^5c-13440b^7c+600a^6bdx-10800}{}}{}
$$

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3840*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2
])/Sqrt[a^2 - b^2]] + (600*a^6*b*c - 10800*a^4*b^3*c + 24000*a^2*b^5*c - 13
440*b^7*c + 600*a^6*b*d*x - 10800*a^4*b^3*d*x + 24000*a^2*b^5*d*x - 13440*b
^7*d*x + 120*a*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*(c + d*x)*Cos[c
 + d*x] - 15*a*(15*a^6 - 576*a^4*b^2 + 1488*a^2*b^4 - 896*b^6)*Sin[c + d*x]
 + 1910*a^6*b*Sin[2*(c + d*x)] - 5440*a^4*b^3*Sin[2*(c + d*x)] + 3360*a^2*b
^5*Sin[2*(c + d*x)] - 180*a^7*Sin[3*(c + d*x)] + 790*a^5*b^2*Sin[3*(c + d*x
)] - 560*a^3*b^4*Sin[3*(c + d*x)] - 166*a^6*b*Sin[4*(c + d*x)] + 140*a^4*b^
3*Sin[4*(c + d*x)] + 40*a^7*Sin[5*(c + d*x)] - 42*a^5*b^2*Sin[5*(c + d*x)]
+ 14*a^6*b*Sin[6*(c + d*x)] - 5*a^7*Sin[7*(c + d*x)])/(b + a*Cos[c + d*x]))
/(1920*a^8*d)
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{2\left(\left(\frac{5}{16}a^6+2a^5b-\frac{21}{8}a^4b^2-8a^3b^3+\frac{5}{2}a^2b^4+6a\,b^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(\frac{38}{3}a^5b-\frac{87}{8}a^4b^2+\frac{15}{2}a^2b^4+30a\,b^5+\frac{85}{48}a^6-\frac{136}{3}a^3b^3\right)\tan\left(\frac{dx}{2}+\right.\right.}{}$
default	$\frac{2\left(\left(\frac{5}{16}a^6+2a^5b-\frac{21}{8}a^4b^2-8a^3b^3+\frac{5}{2}a^2b^4+6a\,b^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(\frac{38}{3}a^5b-\frac{87}{8}a^4b^2+\frac{15}{2}a^2b^4+30a\,b^5+\frac{85}{48}a^6-\frac{136}{3}a^3b^3\right)\tan\left(\frac{dx}{2}+\right.\right.}{}$
risch	$\frac{2\sqrt{a^2-b^2}\,b\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2-b^2}-b}{a}\right)}{d\,a^4}-\frac{9\sqrt{a^2-b^2}\,b^3\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2-b^2}-b}{a}\right)}{d\,a^6}+\frac{7\sqrt{a^2-b^2}\,b^5\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2}}{}\right.}{d\,a^8}$

```
[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^8*(((5/16*a^6+2*a^5*b-21/8*a^4*b^2-8*a^3*b^3+5/2*a^2*b^4+6*a*b^5)*
tan(1/2*d*x+1/2*c)^11+(38/3*a^5*b-87/8*a^4*b^2+15/2*a^2*b^4+30*a*b^5+85/48*
a^6-136/3*a^3*b^3)*tan(1/2*d*x+1/2*c)^9+(172/5*a^5*b-33/4*a^4*b^2-96*a^3*b^
3+5*a^2*b^4+60*a*b^5+33/8*a^6)*tan(1/2*d*x+1/2*c)^7+(-33/8*a^6+33/4*a^4*b^2
-5*a^2*b^4+172/5*a^5*b-96*a^3*b^3+60*a*b^5)*tan(1/2*d*x+1/2*c)^5+(38/3*a^5*
b+87/8*a^4*b^2-136/3*a^3*b^3-15/2*a^2*b^4+30*a*b^5-85/48*a^6)*tan(1/2*d*x+1
/2*c)^3+(2*a^5*b-8*a^3*b^3+6*a*b^5-5/16*a^6+21/8*a^4*b^2-5/2*a^2*b^4)*tan(1
/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^6+1/16*(5*a^6-90*a^4*b^2+200*a^2*b^
4-112*b^6)*arctan(tan(1/2*d*x+1/2*c)))+2*(a-b)^2*(a+b)^2*b/a^8*(-a*b*tan(1/
2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2-7*b
^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2
))))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 793, normalized size of antiderivative = 1.68

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{15\,(5\,a^7-90\,a^5b^2+200\,a^3b^4-112\,ab^6)dx\cos\left(dx+c\right)+15\,(5\,a^6b-90\,a^4b^3+200\,a^2b^5-112\,b^7)dx+120}{}\right.$$

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*d*x*cos(d*x + c)
+ 15*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*d*x + 120*(2*a^4*b^2 -
9*a^2*b^4 + 7*b^6 + (2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c))*sqrt(a^2
- b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + b^2)) - (40*a^7*cos(d*x + c)^6 - 56*a^6*b*cos(d*x +
 c)^5 - 976*a^5*b^2 + 2720*a^3*b^4 - 1680*a*b^6 - 2*(65*a^7 - 42*a^5*b^2)*c
os(d*x + c)^4 + 2*(111*a^6*b - 70*a^4*b^3)*cos(d*x + c)^3 + (165*a^7 - 458*
a^5*b^2 + 280*a^3*b^4)*cos(d*x + c)^2 - (571*a^6*b - 1430*a^4*b^3 + 840*a^2
*b^5)*cos(d*x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c) + a^8*b*d), 1/240*(15
*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*d*x*cos(d*x + c) + 15*(5*a^
6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*d*x - 240*(2*a^4*b^2 - 9*a^2*b^4
+ 7*b^6 + (2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (
40*a^7*cos(d*x + c)^6 - 56*a^6*b*cos(d*x + c)^5 - 976*a^5*b^2 + 2720*a^3*b^
4 - 1680*a*b^6 - 2*(65*a^7 - 42*a^5*b^2)*cos(d*x + c)^4 + 2*(111*a^6*b - 70
*a^4*b^3)*cos(d*x + c)^3 + (165*a^7 - 458*a^5*b^2 + 280*a^3*b^4)*cos(d*x +
c)^2 - (571*a^6*b - 1430*a^4*b^3 + 840*a^2*b^5)*cos(d*x + c))*sin(d*x + c))
/(a^9*d*cos(d*x + c) + a^8*b*d)]
```

Sympy [F]

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sin^6\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**6/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 870, normalized size of antiderivative = 1.84

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*(d*x + c)/a^8 - 480*
(2*a^6*b - 11*a^4*b^3 + 16*a^2*b^5 - 7*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/
2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*
c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^8) - 480*(a^4*b^2*tan(1/2*d*x +
1/2*c) - 2*a^2*b^4*tan(1/2*d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c))/((a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a^7) + 2*(75*a^5*ta
n(1/2*d*x + 1/2*c)^11 + 480*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 630*a^3*b^2*tan
(1/2*d*x + 1/2*c)^11 - 1920*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 600*a*b^4*tan
(1/2*d*x + 1/2*c)^11 + 1440*b^5*tan(1/2*d*x + 1/2*c)^11 + 425*a^5*tan(1/2*d
*x + 1/2*c)^9 + 3040*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 2610*a^3*b^2*tan(1/2*d*
x + 1/2*c)^9 - 10880*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 + 1800*a*b^4*tan(1/2*d*
x + 1/2*c)^9 + 7200*b^5*tan(1/2*d*x + 1/2*c)^9 + 990*a^5*tan(1/2*d*x + 1/2*
c)^7 + 8256*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 1980*a^3*b^2*tan(1/2*d*x + 1/2*c
)^7 - 23040*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 1200*a*b^4*tan(1/2*d*x + 1/2*c
)^7 + 14400*b^5*tan(1/2*d*x + 1/2*c)^7 - 990*a^5*tan(1/2*d*x + 1/2*c)^5 + 8
256*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 1980*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 23
040*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 14
400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425*a^5*tan(1/2*d*x + 1/2*c)^3 + 3040*a^4*
b*tan(1/2*d*x + 1/2*c)^3 + 2610*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10880*a^2*
b^3*tan(1/2*d*x + 1/2*c)^3 - 1800*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 7200*b^5*t
an(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*d*x + 1/2*c) + 480*a^4*b*tan(1/2*d*x
```

```
 + 1/2*c) + 630*a^3*b^2*tan(1/2*d*x + 1/2*c) - 1920*a^2*b^3*tan(1/2*d*x + 1
/2*c) - 600*a*b^4*tan(1/2*d*x + 1/2*c) + 1440*b^5*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 + 1)^6*a^7))/d
```

Mupad [B] (verification not implemented)

Time = 17.51 (sec) , antiderivative size = 3692, normalized size of antiderivative = 7.81

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int(sin(c + d*x)^6/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (atan(((((((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^
6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 - (tan(c
/2 + (d*x)/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112
i + a^2*b^4*200i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*2
00i - a^4*b^2*90i))/(16*a^8) + (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*
b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^1
1 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 4054
0*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))
/(8*a^14))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i)*1i)/(16*a^8) -
(((((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740
*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 + (tan(c/2 + (d
*x)/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112i + a^2
*b^4*200i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*200i - a
^4*b^2*90i))/(16*a^8) - (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*
a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240
640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b
^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^1
4))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i)*1i)/(16*a^8))/((((((74
*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b
^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 - (tan(c/2 + (d*x)/2)*
(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112i + a^2*b^4*20
0i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*
90i))/(16*a^8) + (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 -
25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5
*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18
136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^14))*(a^
6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i))/(16*a^8) - (32928*a*b^19 - (
25*a^19*b)/2 - 21952*b^20 + 117600*a^2*b^18 - 190120*a^3*b^17 - 257432*a^4*
b^16 + 463764*a^5*b^15 + 290284*a^6*b^14 - 620037*a^7*b^13 - 169030*a^8*b^1
2 + 492572*a^9*b^11 + 35558*a^10*b^10 - (941393*a^11*b^9)/4 + (22469*a^12*b
^8)/2 + (260375*a^13*b^7)/4 - 7490*a^14*b^6 - (37705*a^15*b^5)/4 + (2565*a^
16*b^4)/2 + (2345*a^17*b^3)/4 - 55*a^18*b^2)/a^21 + (((((74*a^23*b - 10*a^2
```

4 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 + (tan(c/2 + (d*x)/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i))/(16*a^8) - (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^14))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i))/(16*a^8)))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i)*1i)/(8*a^8*d) - ((tan(c/2 + (d*x)/2)^11*(336*a*b^5 + 206*a^5*b + 35*a^6 - 1008*b^6 + 1688*a^2*b^4 - 572*a^3*b^3 - 694*a^4*b^2))/(12*a^7) - (tan(c/2 + (d*x)/2)^3*(336*a*b^5 + 206*a^5*b - 35*a^6 + 1008*b^6 - 1688*a^2*b^4 - 572*a^3*b^3 + 694*a^4*b^2))/(12*a^7) - (tan(c/2 + (d*x)/2)^5*(4200*a*b^5 + 3801*a^5*b - 565*a^6 + 25200*b^6 - 40520*a^2*b^4 - 7570*a^3*b^3 + 14266*a^4*b^2))/(120*a^7) + (tan(c/2 + (d*x)/2)^9*(4200*a*b^5 + 3801*a^5*b + 565*a^6 - 25200*b^6 + 40520*a^2*b^4 - 7570*a^3*b^3 - 14266*a^4*b^2))/(120*a^7) - (tan(c/2 + (d*x)/2)^7*(165*a^6 + 2800*b^6 - 4440*a^2*b^4 + 1446*a^4*b^2))/(10*a^7) + (tan(c/2 + (d*x)/2)^13*(a - b)*(56*a*b^4 + 32*a^4*b + 5*a^5 + 112*b^5 - 144*a^2*b^3 - 58*a^3*b^2))/(8*a^7) + (tan(c/2 + (d*x)/2)*(a + b)*(56*a*b^4 - 32*a^4*b + 5*a^5 - 112*b^5 + 144*a^2*b^3 - 58*a^3*b^2))/(8*a^7))/(d*(a + b - tan(c/2 + (d*x)/2)^14*(a - b) + tan(c/2 + (d*x)/2)^2*(5*a + 7*b) - tan(c/2 + (d*x)/2)^12*(5*a - 7*b) + tan(c/2 + (d*x)/2)^4*(9*a + 21*b) - tan(c/2 + (d*x)/2)^10*(9*a - 21*b) + tan(c/2 + (d*x)/2)^6*(5*a + 35*b) - tan(c/2 + (d*x)/2)^8*(5*a - 35*b))) - (b*atan(((b*((tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^14) + (b*((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 - (b*tan(c/2 + (d*x)/2)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2))/(8*a^22))*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2))/a^8)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2)*1i)/a^8 + (b*((tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^14) - (b*((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 + (b*tan(c/2 + (d*x)/2)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2))/(8*a^22))*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2))/a^8)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2)*1i)/a^8)/((32928*a*b^19 - (25*a^19*b)/2 - 21952*b^20 + 117600*a^2*b^18 - 190120*a^3*b^17 - 257432*a^4*b^16 + 463764*a^5*b^15 + 290284*a^6*b^14 - 620037*a^7*b^13 - 169030*a^8*b^12 + 492572*a^9*b^11 + 35558*a^10*b^10 - (941393*a^11*b^9)/4 + (22469*a^12*b^8)/2 + (260375*a^13*b^7)/4 - 7490*a^14*b^6 - (37705*a^15*

b^5)/4 + (2565*a^16*b^4)/2 + (2345*a^17*b^3)/4 - 55*a^18*b^2)/a^21 - (b*((tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^14) + (b*((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 - (b*tan(c/2 + (d*x)/2)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2))/(8*a^22))*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2))/a^8)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2))/a^8 + (b*((tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^14) - (b*((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 + (b*tan(c/2 + (d*x)/2)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2))/(8*a^22))*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2))/a^8)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2))/a^8))*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/(a^8*d)

3.217 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 261

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = {} & \frac{(3a^4-36a^2b^2+40b^4)\,x}{8a^6} \\ & - \frac{2\sqrt{a-b}\,b\sqrt{a+b}(2a^2-5b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d} \\ & + \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d} \\ & - \frac{(13a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^4d} \\ & + \frac{(3a^2-5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3bd} \\ & + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} - \frac{(a^2-b^2)\cos^3(c+dx)\sin(c+dx)}{a^2bd(b+a\cos(c+dx))}\end{aligned}$$

```
[Out] 1/8*(3*a^4-36*a^2*b^2+40*b^4)*x/a^6+1/3*b*(11*a^2-15*b^2)*sin(d*x+c)/a^5/d-
1/8*(13*a^2-20*b^2)*cos(d*x+c)*sin(d*x+c)/a^4/d+1/3*(3*a^2-5*b^2)*cos(d*x+c
)^2*sin(d*x+c)/a^3/b/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a^2/d-(a^2-b^2)*cos(d*x+
c)^3*sin(d*x+c)/a^2/b/d/(b+a*cos(d*x+c))-2*b*(2*a^2-5*b^2)*arctanh((a-b)^(1
/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^6/d
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2971, 3128, 3102, 2814, 2738, 214}

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{(a^2-b^2)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d} - \frac{2b\sqrt{a-b}\sqrt{a+b}(2a^2-5b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d} + \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d} - \frac{(13a^2-20b^2)\sin(c+dx)\cos(c+dx)}{8a^4d} + \frac{(3a^2-5b^2)\sin(c+dx)\cos^2(c+dx)}{3a^3bd} + \frac{x(3a^4-36a^2b^2+40b^4)}{8a^6}$$

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((3*a^4 - 36*a^2*b^2 + 40*b^4)*x)/(8*a^6) - (2*Sqrt[a - b]*b*Sqrt[a + b]*(2
*a^2 - 5*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*d)
+ (b*(11*a^2 - 15*b^2)*Sin[c + d*x])/(3*a^5*d) - ((13*a^2 - 20*b^2)*Cos[c +
 d*x]*Sin[c + d*x])/(8*a^4*d) + ((3*a^2 - 5*b^2)*Cos[c + d*x]^2*Sin[c + d*x
])/(3*a^3*b*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^2*d) - ((a^2 - b^2)*Cos
[c + d*x]^3*Sin[c + d*x])/(a^2*b*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2971

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e +
f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(b^2*d*f*(m + n + 4))), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$
\begin{aligned}
&= \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} - \frac{(a^2-b^2)\cos^3(c+dx)\sin(c+dx)}{a^2bd(b+a\cos(c+dx))} \\
&\quad - \frac{\int \frac{\cos^2(c+dx)\left(-8a^2+15b^2-ab\cos(c+dx)+4(3a^2-5b^2)\cos^2(c+dx)\right)}{-b-a\cos(c+dx)}\,dx}{4a^2b} \\
&= \frac{(3a^2-5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3bd} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} \\
&\quad - \frac{(a^2-b^2)\cos^3(c+dx)\sin(c+dx)}{a^2bd(b+a\cos(c+dx))} \\
&\quad + \frac{\int \frac{\cos(c+dx)\left(-8b(3a^2-5b^2)-5ab^2\cos(c+dx)+3b(13a^2-20b^2)\cos^2(c+dx)\right)}{-b-a\cos(c+dx)}\,dx}{12a^3b} \\
&= -\frac{(13a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^4d} + \frac{(3a^2-5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3bd} \\
&\quad + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} - \frac{(a^2-b^2)\cos^3(c+dx)\sin(c+dx)}{a^2bd(b+a\cos(c+dx))} \\
&\quad - \frac{\int \frac{-3b^2(13a^2-20b^2)+ab(9a^2-20b^2)\cos(c+dx)+8b^2(11a^2-15b^2)\cos^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{24a^4b} \\
&= \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d} - \frac{(13a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^4d} \\
&\quad + \frac{(3a^2-5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3bd} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} \\
&\quad - \frac{(a^2-b^2)\cos^3(c+dx)\sin(c+dx)}{a^2bd(b+a\cos(c+dx))} + \frac{\int \frac{3ab^2(13a^2-20b^2)-3b(3a^4-36a^2b^2+40b^4)\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{24a^5b} \\
&= \frac{(3a^4-36a^2b^2+40b^4)\,x}{8a^6} + \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d} \\
&\quad - \frac{(13a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^4d} + \frac{(3a^2-5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3bd} \\
&\quad + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} - \frac{(a^2-b^2)\cos^3(c+dx)\sin(c+dx)}{a^2bd(b+a\cos(c+dx))} \\
&\quad + \frac{(b(2a^4-7a^2b^2+5b^4))\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^6} \\
&= \frac{(3a^4-36a^2b^2+40b^4)\,x}{8a^6} + \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d} \\
&\quad - \frac{(13a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^4d} + \frac{(3a^2-5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3bd} \\
&\quad + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} - \frac{(a^2-b^2)\cos^3(c+dx)\sin(c+dx)}{a^2bd(b+a\cos(c+dx))} \\
&\quad + \frac{(2b(2a^4-7a^2b^2+5b^4))\,\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^6d}
\end{aligned}
$$

$$= \frac{(3a^4 - 36a^2b^2 + 40b^4)\,x}{8a^6} - \frac{2\sqrt{a-b}\,b\sqrt{a+b}(2a^2 - 5b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6 d}$$
$$+ \frac{b(11a^2 - 15b^2)\sin(c+dx)}{3a^5 d} - \frac{(13a^2 - 20b^2)\cos(c+dx)\sin(c+dx)}{8a^4 d}$$
$$+ \frac{(3a^2 - 5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3 bd}$$
$$+ \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2 d} - \frac{(a^2 - b^2)\cos^3(c+dx)\sin(c+dx)}{a^2 bd(b + a\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 3.06 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.08

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{384b(2a^4-7a^2b^2+5b^4)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{72a^4bc-864a^2b^3c+960b^5c+72a^4bdx-864a^2b^3dx+960b^5dx+24a(3a^4-36a^2b^2+4}{}}{}$$

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((384*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + (72*a^4*b*c - 864*a^2*b^3*c + 960*b^5*c +
72*a^4*b*d*x - 864*a^2*b^3*d*x + 960*b^5*d*x + 24*a*(3*a^4 - 36*a^2*b^2 + 4
0*b^4)*(c + d*x)*Cos[c + d*x] - 24*a*(a^4 - 31*a^2*b^2 + 40*b^4)*Sin[c + d*
x] + 176*a^4*b*Sin[2*(c + d*x)] - 240*a^2*b^3*Sin[2*(c + d*x)] - 21*a^5*Sin
[3*(c + d*x)] + 40*a^3*b^2*Sin[3*(c + d*x)] - 10*a^4*b*Sin[4*(c + d*x)] + 3
*a^5*Sin[5*(c + d*x)])/(b + a*Cos[c + d*x]))/(192*a^6*d)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{2(a-b)(a+b)b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2-5b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^6}+\frac{2\left(\left(\frac{3}{8}a^4+2a^3b-\frac{3}{2}a^2b^2-4a\right.\right.}{}$
default	$\frac{2(a-b)(a+b)b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2-5b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^6}+\frac{2\left(\left(\frac{3}{8}a^4+2a^3b-\frac{3}{2}a^2b^2-4a\right.\right.}{}$
risch	$\frac{3x}{8a^2}-\frac{9x\,b^2}{2a^4}+\frac{5x\,b^4}{a^6}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^2d}-\frac{5ib\,\mathrm{e}^{i(dx+c)}}{4a^3d}-\frac{2ib^3\mathrm{e}^{-i(dx+c)}}{a^5d}+\frac{i\mathrm{e}^{2i(dx+c)}}{8a^2d}+\frac{2ib^3\mathrm{e}^{i(dx+c)}}{a^5d}-\frac{3i\mathrm{e}^{2i(dx+c}}{8a^4d}$

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(a-b)*(a+b)*b/a^6*(-a*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2-5*b^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+2/a^6*(((3/8*a^4+2*a^3*b-3/2*a^2*b
^2-4*a*b^3)*tan(1/2*d*x+1/2*c)^7+(26/3*a^3*b-3/2*a^2*b^2-12*a*b^3+11/8*a^4)
*tan(1/2*d*x+1/2*c)^5+(-11/8*a^4+3/2*a^2*b^2+26/3*a^3*b-12*a*b^3)*tan(1/2*d
*x+1/2*c)^3+(2*a^3*b-4*a*b^3-3/8*a^4+3/2*a^2*b^2)*tan(1/2*d*x+1/2*c))/(1+ta
n(1/2*d*x+1/2*c)^2)^4+1/8*(3*a^4-36*a^2*b^2+40*b^4)*arctan(tan(1/2*d*x+1/2*
c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.23

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$=\left[\frac{3\left(3\,a^5-36\,a^3b^2+40\,ab^4\right)dx\cos\left(dx+c\right)+3\left(3\,a^4b-36\,a^2b^3+40\,b^5\right)dx-12\left(2\,a^2b^2-5\,b^4+\left(2\,a^3b-5\,a\right.\right.}{}\right.$$

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*d*x*cos(d*x + c) + 3*(3*a^4*b - 36
*a^2*b^3 + 40*b^5)*d*x - 12*(2*a^2*b^2 - 5*b^4 + (2*a^3*b - 5*a*b^3)*cos(d*
x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
```

```
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (6*a^5*cos(d*x + c)^4 - 10
*a^4*b*cos(d*x + c)^3 + 88*a^3*b^2 - 120*a*b^4 - 5*(3*a^5 - 4*a^3*b^2)*cos(
d*x + c)^2 + (49*a^4*b - 60*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos
(d*x + c) + a^6*b*d), 1/24*(3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*d*x*cos(d*x +
 c) + 3*(3*a^4*b - 36*a^2*b^3 + 40*b^5)*d*x - 24*(2*a^2*b^2 - 5*b^4 + (2*a^
3*b - 5*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*a^5*cos(d*x + c)^4 - 10*a
^4*b*cos(d*x + c)^3 + 88*a^3*b^2 - 120*a*b^4 - 5*(3*a^5 - 4*a^3*b^2)*cos(d*
x + c)^2 + (49*a^4*b - 60*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(d
*x + c) + a^6*b*d)]
```

Sympy [F]

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sin^4{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

```
[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.85

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{3\,(3\,a^4-36\,a^2b^2+40\,b^4)\,(dx+c)}{a^6} - \frac{48\,(2\,a^4b-7\,a^2b^3+5\,b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^6} - \frac{4}{(a}}{}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 - 36*a^2*b^2 + 40*b^4)*(d*x + c)/a^6 - 48*(2*a^4*b - 7*a^2*b
^3 + 5*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^
2 + b^2)*a^6) - 48*(a^2*b^2*tan(1/2*d*x + 1/2*c) - b^4*tan(1/2*d*x + 1/2*c)
)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a^5) + 2*(
9*a^3*tan(1/2*d*x + 1/2*c)^7 + 48*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 36*a*b^2*t
an(1/2*d*x + 1/2*c)^7 - 96*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^3*tan(1/2*d*x
+ 1/2*c)^5 + 208*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*a*b^2*tan(1/2*d*x + 1/2*
c)^5 - 288*b^3*tan(1/2*d*x + 1/2*c)^5 - 33*a^3*tan(1/2*d*x + 1/2*c)^3 + 208
*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*b^3*t
an(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c) + 48*a^2*b*tan(1/2*d*x +
 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x + 1/2*c))/((ta
n(1/2*d*x + 1/2*c)^2 + 1)^4*a^5))/d
```

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 2804, normalized size of antiderivative = 10.74

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int(sin(c + d*x)^4/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(33*a^4 - 360*b^4 + 244*a^2*b^2))/(6*a^5) - (tan(c/2
 + (d*x)/2)^3*(60*a*b^3 - 59*a^3*b + 12*a^4 + 240*b^4 - 176*a^2*b^2))/(6*a^
5) - (tan(c/2 + (d*x)/2)^7*(59*a^3*b - 60*a*b^3 + 12*a^4 + 240*b^4 - 176*a^
2*b^2))/(6*a^5) + (tan(c/2 + (d*x)/2)^9*(a - b)*(20*a*b^2 - 16*a^2*b - 3*a^
3 + 40*b^3))/(4*a^5) + (tan(c/2 + (d*x)/2)*(a + b)*(20*a*b^2 + 16*a^2*b - 3
*a^3 - 40*b^3))/(4*a^5))/(d*(a + b - tan(c/2 + (d*x)/2)^10*(a - b) + tan(c/
2 + (d*x)/2)^2*(3*a + 5*b) + tan(c/2 + (d*x)/2)^4*(2*a + 10*b) - tan(c/2 +
(d*x)/2)^8*(3*a - 5*b) - tan(c/2 + (d*x)/2)^6*(2*a - 10*b))) + (atan((((((
76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15
*b^3 + 28*a^16*b^2)/a^15 - (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*40i - a^2*b^2*
36i)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(16*a^16))*(a^4*3i + b^4*4
0i - a^2*b^2*36i))/(8*a^6) + (tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b +
 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^
5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10))*(
a^4*3i + b^4*40i - a^2*b^2*36i)*1i)/(8*a^6) - (((((76*a^17*b - 12*a^18 - 16
0*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15
 + (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*40i - a^2*b^2*36i)*(128*a^14*b + 128*a
^12*b^3 - 256*a^13*b^2))/(16*a^16))*(a^4*3i + b^4*40i - a^2*b^2*36i))/(8*a^
6) - (tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 25
60*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 9
04*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10))*(a^4*3i + b^4*40i - a^2*b
```

^2*36i)*1i)/(8*a^6))/((12000*a*b^13 + 18*a^13*b - 8000*b^14 + 21600*a^2*b^1
2 - 37400*a^3*b^11 - 19240*a^4*b^10 + 43960*a^5*b^9 + 4672*a^6*b^8 - 23963*
a^7*b^7 + 1742*a^8*b^6 + 5958*a^9*b^5 - 834*a^10*b^4 - 573*a^11*b^3 + 60*a^
12*b^2)/a^15 + (((((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144
*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15 - (tan(c/2 + (d*x)/2)*(a^4*3i
+ b^4*40i - a^2*b^2*36i)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(16*a^
16))*(a^4*3i + b^4*40i - a^2*b^2*36i))/(8*a^6) + (tan(c/2 + (d*x)/2)*(6400*
a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 26
88*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a
^9*b^2))/(2*a^10))*(a^4*3i + b^4*40i - a^2*b^2*36i))/(8*a^6) + (((((76*a^17
*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 +
28*a^16*b^2)/a^15 + (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*40i - a^2*b^2*36i)*(1
28*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(16*a^16))*(a^4*3i + b^4*40i - a^
2*b^2*36i))/(8*a^6) - (tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b + 9*a^11
 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 -
 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10))*(a^4*3i
+ b^4*40i - a^2*b^2*36i))/(8*a^6)))*(a^4*3i + b^4*40i - a^2*b^2*36i)*1i)/(4
*a^6*d) + (b*atan(((b*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/2 + (d*x)/2
)*(6400*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*
b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^
3 + 67*a^9*b^2))/(2*a^10) + (b*((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a
^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15 - (b*tan(c/2 + (d
x)/2)(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*(128*a^14*b + 128*a^12*b^3 - 256*a
^13*b^2))/(2*a^16))*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2))/a^6)*1i)/a^6 + (b*(a
^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10
*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 614
4*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10
) - (b*((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 -
 316*a^15*b^3 + 28*a^16*b^2)/a^15 + (b*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)
*(2*a^2 - 5*b^2)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(2*a^16))*(a^2
 - b^2)^(1/2)*(2*a^2 - 5*b^2))/a^6)*1i)/a^6)/((12000*a*b^13 + 18*a^13*b - 8
000*b^14 + 21600*a^2*b^12 - 37400*a^3*b^11 - 19240*a^4*b^10 + 43960*a^5*b^9
 + 4672*a^6*b^8 - 23963*a^7*b^7 + 1742*a^8*b^6 + 5958*a^9*b^5 - 834*a^10*b^
4 - 573*a^11*b^3 + 60*a^12*b^2)/a^15 + (b*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)
*((tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*
a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*
a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10) + (b*((76*a^17*b - 12*a^18 -
160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^
15 - (b*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*(128*a^14*b +
128*a^12*b^3 - 256*a^13*b^2))/(2*a^16))*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2))/
a^6))/a^6 - (b*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/2 + (d*x)/2)*(6400
*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2
688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*
a^9*b^2))/(2*a^10) - (b*((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5
 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15 + (b*tan(c/2 + (d*x)/2)*

```
(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2
))/(2*a^16))*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2))/a^6))/a^6))*(a^2 - b^2)^(1/
2)*(2*a^2 - 5*b^2)*2i)/(a^6*d)
```

3.218 $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{(a^2-6b^2)\,x}{2a^4} - \frac{2b(2a^2-3b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))}$$

```
[Out] 1/2*(a^2-6*b^2)*x/a^4+3*b*sin(d*x+c)/a^3/d-3/2*cos(d*x+c)*sin(d*x+c)/a^2/d+
cos(d*x+c)^2*sin(d*x+c)/a/d/(b+a*cos(d*x+c))-2*b*(2*a^2-3*b^2)*arctanh((a-b
)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2968, 3127, 3129, 3102, 2814, 2738, 214}

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{3b\sin(c+dx)}{a^3d} - \frac{3\sin(c+dx)\cos(c+dx)}{2a^2d} - \frac{2b(2a^2-3b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}} + \frac{x(a^2-6b^2)}{2a^4} + \frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a^2 - 6*b^2)*x)/(2*a^4) - (2*b*(2*a^2 - 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + (3*b*Sin[c + d
*x])/(a^3*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + (Cos[c + d*x]^2*Si
n[c + d*x])/(a*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
```

```
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \int \frac{\cos^2(c+dx)\left(1-\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-3\left(a^2-b^2\right)\cos^2(c+dx)\right)}{-b-a\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)} \\
&= -\frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} \\
&\quad + \frac{\int \frac{3b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\cos(c+dx)-6b\left(a^2-b^2\right)\cos^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{2a^2\left(a^2-b^2\right)} \\
&= \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} \\
&\quad + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} - \frac{\int \frac{-3ab\left(a^2-b^2\right)+\left(a^2-6b^2\right)\left(a^2-b^2\right)\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{2a^3\left(a^2-b^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(a^2-6b^2)\,x}{2a^4} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} \\
&\quad + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} + \frac{(b(2a^2-3b^2))\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^4} \\
&= \frac{(a^2-6b^2)\,x}{2a^4} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} \\
&\quad + \frac{(2b(2a^2-3b^2))\,\mathrm{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^4d} \\
&= \frac{(a^2-6b^2)\,x}{2a^4} - \frac{2b(2a^2-3b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} \\
&\quad + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.17

$$
\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx
$$

$$
= \frac{\frac{16b(2a^2-3b^2)\mathrm{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{4a^2bc-24b^3c+4a^2bdx-24b^3dx+4a(a^2-6b^2)(c+dx)\cos(c+dx)-a(a^2-24b^2)\sin(c+dx)+6a}{b+a\cos(c+dx)}}{8a^4d}
$$

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((16*b*(2*a^2 - 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] + (4*a^2*b*c - 24*b^3*c + 4*a^2*b*d*x - 24*b^3*d*x + 4*a*
(a^2 - 6*b^2)*(c + d*x)*Cos[c + d*x] - a*(a^2 - 24*b^2)*Sin[c + d*x] + 6*a^
2*b*Sin[2*(c + d*x)] - a^3*Sin[3*(c + d*x)])/(b + a*Cos[c + d*x]))/(8*a^4*d
)
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2-3b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^4}+\frac{2\left(\frac{\left(\frac{1}{2}a^2+2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(2ab-\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\right.}{}}{d}$
default	$\frac{\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2-3b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^4}+\frac{2\left(\frac{\left(\frac{1}{2}a^2+2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(2ab-\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\right.}{}}{d}$
risch	$\frac{x}{2a^2}-\frac{3xb^2}{a^4}+\frac{ie^{2i(dx+c)}}{8a^2d}-\frac{ib\,e^{i(dx+c)}}{a^3d}+\frac{ib\,e^{-i(dx+c)}}{a^3d}-\frac{ie^{-2i(dx+c)}}{8a^2d}+\frac{2ib^2\left(b\,e^{i(dx+c)}+a\right)}{a^4d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}-$

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/a^4*(-a*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)-(2*a^2-3*b^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+
1/2*c)/((a-b)*(a+b))^(1/2)))+2/a^4*(((1/2*a^2+2*a*b)*tan(1/2*d*x+1/2*c)^3+(
2*a*b-1/2*a^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2-6*b^
2)*arctan(tan(1/2*d*x+1/2*c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 551, normalized size of antiderivative = 3.62

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$=\left[\frac{(a^5-7\,a^3b^2+6\,ab^4)dx\cos{(dx+c)}+(a^4b-7\,a^2b^3+6\,b^5)dx-(2\,a^2b^2-3\,b^4+(2\,a^3b-3\,ab^3)\cos{(dx+}}{}\right.$$

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^5 - 7*a^3*b^2 + 6*a*b^4)*d*x*cos(d*x + c) + (a^4*b - 7*a^2*b^3 + 6
*b^5)*d*x - (2*a^2*b^2 - 3*b^4 + (2*a^3*b - 3*a*b^3)*cos(d*x + c))*sqrt(a^2
 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2
 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + b^2)) + (6*a^3*b^2 - 6*a*b^4 - (a^5 - a^3*b^2)*cos
(d*x + c)^2 + 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b
^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d), 1/2*((a^5 - 7*a^3*b^2 + 6*a*b^4)
*d*x*cos(d*x + c) + (a^4*b - 7*a^2*b^3 + 6*b^5)*d*x - 2*(2*a^2*b^2 - 3*b^4
+ (2*a^3*b - 3*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^
2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*a^3*b^2 - 6*a*b^4
```

```
- (a^5 - a^3*b^2)*cos(d*x + c)^2 + 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*
x + c))/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d)]
```

Sympy [F]

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\int \frac{\sin^2{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.58

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx=$$

$$-\frac{\frac{4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)a^3}-\frac{(a^2-6\,b^2)(dx+c)}{a^4}+\frac{4\,(2\,a^2b-3\,b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\right.}{}\right.\right.}{\sqrt{-a^2+b^2}a^4}}{2\,d}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] -1/2*(4*b^2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x
 + 1/2*c)^2 - a - b)*a^3) - (a^2 - 6*b^2)*(d*x + c)/a^4 + 4*(2*a^2*b - 3*b^
3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*
a^4) - 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2
*d*x + 1/2*c) + 4*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a
^3))/d
```

Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 1655, normalized size of antiderivative = 10.89

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int(sin(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3*(a^2 + 6*b^2))/a^3 + (tan(c/2 + (d*x)/2)*(3*a*b -
a^2 + 6*b^2))/a^3 - (tan(c/2 + (d*x)/2)^5*(3*a*b + a^2 - 6*b^2))/a^3)/(d*(a
 + b + tan(c/2 + (d*x)/2)^2*(a + 3*b) - tan(c/2 + (d*x)/2)^4*(a - 3*b) - ta
n(c/2 + (d*x)/2)^6*(a - b))) + (atan((8*tan(c/2 + (d*x)/2))/((8*b)/a + (24*
b^2)/a^2 - (24*b^3)/a^3 + (144*b^4)/a^4 - (144*b^5)/a^5 - 8) + (8*b*tan(c/2
 + (d*x)/2))/(8*a - 8*b - (24*b^2)/a + (24*b^3)/a^2 - (144*b^4)/a^3 + (144*
b^5)/a^4) - (24*b^2*tan(c/2 + (d*x)/2))/(8*a*b - 8*a^2 + 24*b^2 - (24*b^3)/
a + (144*b^4)/a^2 - (144*b^5)/a^3) + (24*b^3*tan(c/2 + (d*x)/2))/(24*a*b^2
+ 8*a^2*b - 8*a^3 - 24*b^3 + (144*b^4)/a - (144*b^5)/a^2) + (144*b^4*tan(c/
2 + (d*x)/2))/(24*a*b^3 - 8*a^3*b + 8*a^4 - 144*b^4 - 24*a^2*b^2 + (144*b^5
)/a) + (144*b^5*tan(c/2 + (d*x)/2))/(144*a*b^4 + 8*a^4*b - 8*a^5 - 144*b^5
- 24*a^2*b^3 + 24*a^3*b^2))*(a^2*1i - b^2*6i)*1i)/(a^4*d) - (b*atan(((b*((a
 + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144*a*b^6 - 3*
a^6*b + a^7 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*a^5*b^2))/a
^6 + (b*((a + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8*b^4 + 18*a
^9*b^3 + 2*a^10*b^2))/a^9 - (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)
*(2*a^2 - 3*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 - a^4*b^2))
)*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2) + (b*((a + b)*(a -
b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144*a*b^6 - 3*a^6*b + a^7
 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*a^5*b^2))/a^6 - (b*((a
 + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8*b^4 + 18*a^9*b^3 + 2*
a^10*b^2))/a^9 + (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2*a^2 - 3
*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 - a^4*b^2)))*(2*a^2 -
3*b^2))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2))/((16*(162*a*b^7 - 2*a^7*b - 1
08*b^8 + 54*a^2*b^6 - 153*a^3*b^5 + 18*a^4*b^4 + 33*a^5*b^3 - 4*a^6*b^2))/a
^9 + (b*((a + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144
*a*b^6 - 3*a^6*b + a^7 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*
a^5*b^2))/a^6 + (b*((a + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8
*b^4 + 18*a^9*b^3 + 2*a^10*b^2))/a^9 - (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a
- b))^(1/2)*(2*a^2 - 3*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6
- a^4*b^2)))*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2) - (b*((a +
b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144*a*b^6 - 3*a^6
*b + a^7 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*a^5*b^2))/a^6
- (b*((a + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8*b^4 + 18*a^9*
b^3 + 2*a^10*b^2))/a^9 + (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2
*a^2 - 3*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 - a^4*b^2)))*(
```

```
2*a^2 - 3*b^2))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2)))*((a + b)*(a - b))^(1/2)
*(2*a^2 - 3*b^2)*2i)/(d*(a^6 - a^4*b^2))
```

3.219 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 203

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{4a^2b\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2b^3\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{ab^2\sin(c+dx)}{(a^2-b^2)^2\,d(b+a\cos(c+dx))}$$

```
[Out] -4*a^2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a
+b)^(5/2)/d-2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)
^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))+1/2*sin(d*x+c)
/(a-b)^2/d/(1+cos(d*x+c))+a*b^2*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2810, 2727, 2743, 12, 2738, 214}

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{4a^2b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}} + \frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)} - \frac{2b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}} - \frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - C
os[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) + (a*b^2*Si
n[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \int \left(-\frac{1}{2(a+b)^2(-1+\cos(c+dx))}+\frac{1}{2(a-b)^2(1+\cos(c+dx))}\right. \\
&\qquad \left.-\frac{b^2}{\left(-a^2+b^2\right)(b+a\cos(c+dx))^2}-\frac{2a^2b}{\left(a^2-b^2\right)^2(b+a\cos(c+dx))}\right)dx \\
&= \frac{\int \frac{1}{1+\cos(c+dx)}\,dx}{2(a-b)^2}-\frac{\int \frac{1}{-1+\cos(c+dx)}\,dx}{2(a+b)^2}-\frac{\left(2a^2b\right)\int \frac{1}{b+a\cos(c+dx)}\,dx}{\left(a^2-b^2\right)^2}+\frac{b^2\int \frac{1}{(b+a\cos(c+dx))^2}\,dx}{a^2-b^2} \\
&= -\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} \\
&\quad +\frac{ab^2\sin(c+dx)}{\left(a^2-b^2\right)^2d(b+a\cos(c+dx))}-\frac{b^2\int \frac{b}{b+a\cos(c+dx)}\,dx}{\left(a^2-b^2\right)^2} \\
&\quad -\frac{\left(4a^2b\right)\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)^2d}
\end{aligned}
$$

$$= -\frac{4a^2 b \operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2 d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2 d(1+\cos(c+dx))} + \frac{ab^2\sin(c+dx)}{(a^2-b^2)^2 d(b+a\cos(c+dx))} - \frac{b^3\int \frac{1}{b+a\cos(c+dx)}\,dx}{(a^2-b^2)^2}$$

$$= -\frac{4a^2 b \operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2 d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2 d(1+\cos(c+dx))} + \frac{ab^2\sin(c+dx)}{(a^2-b^2)^2 d(b+a\cos(c+dx))} - \frac{(2b^3)\operatorname{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)^2 d}$$

$$= -\frac{4a^2 b \operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2b^3 \operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2 d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2 d(1+\cos(c+dx))} + \frac{ab^2\sin(c+dx)}{(a^2-b^2)^2 d(b+a\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.63

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{4b(2a^2+b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{\frac{2ab^2\sin(c+dx)}{(a+b)^2(b+a\cos(c+dx))} + \tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*b*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a*b^2*Sin[c + d*x])/((a
 + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
risch	$-\frac{2i\left(-2a^2b\,e^{3i(dx+c)}-b^3e^{3i(dx+c)}+a^3e^{2i(dx+c)}-4a\,b^2e^{2i(dx+c)}+3b^3e^{i(dx+c)}+a^3+2a\,b^2\right)}{\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)\left(a^2-b^2\right)^2\left(e^{2i(dx+c)}-1\right)d}-\frac{2b\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2}{\sqrt{a}}\right.}{\sqrt{a^2-b^2}\,(a+b)^2(}$

[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2*tan(1/2*d*x+1/2*c)/(a^2-2*a*b+b^2)+2*b/(a-b)^2/(a+b)^2*(-a*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2+b^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.59

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$=\left[\frac{6\,a^3b^2-6\,ab^4+(2\,a^2b^2+b^4+(2\,a^3b+ab^3)\cos{(dx+c)})\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{}}{a^2\cos(dx+c)^2+2}\right.}{2\,((a^7-3\,a^5b^2+3\,a^3b^4-ab^6)d\cos{(d}}\right.$$

[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + (2*a^2*b^2 + b^4 + (2*a^3*b + a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)*sin(d*x + c)), (3*a^3*b^2 - 3*a*b^4 - (2*a^2*b^2 + b^4 +

```
(2*a^3*b + a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^5 + a^3*b
^2 - 2*a*b^4)*cos(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.42

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{4\,(2\,a^2b+b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2b\tan\left(\frac{1}{2}\,dx}{(a^4-2\,a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}\,dx-\right.\right.}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + a
rctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/
((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*
a*b + b^2) - (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 +
 7*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b
+ a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1
/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

Mupad [B] (verification not implemented)

Time = 14.32 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.21

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^3-3\,a^2\,b+7\,a\,b^2-b^3\right)}{{\left(a+b\right)}^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,{\left(a-b\right)}^2}+\frac{b\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4-2\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^4}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{3/2}}\right)\,\left(2\,a^2+b^2\right)\,2\mathrm{i}}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

```
[In] int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))^2),x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(7*a*b^2 - 3*a^2*b + a
^3 - b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3 -
 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) + tan(c/
2 + (d*x)/2)/(2*d*(a - b)^2) + (b*atan((a^4*tan(c/2 + (d*x)/2)*1i + b^4*tan
(c/2 + (d*x)/2)*1i - a^2*b^2*tan(c/2 + (d*x)/2)*2i)/((a + b)^(5/2)*(a - b)^
(3/2)))*(2*a^2 + b^2)*2i)/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.220 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 343

$$
\begin{aligned}
\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = & -\frac{2a^2b^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} \\
& -\frac{4a^2b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} \\
& -\frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2}-\frac{(a-b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} \\
& -\frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))} \\
& +\frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))^2} \\
& +\frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))}+\frac{(a+b)\sin(c+dx)}{4(a-b)^3d(1+\cos(c+dx))} \\
& +\frac{a^3b^2\sin(c+dx)}{(a^2-b^2)^3\,d(b+a\cos(c+dx))}
\end{aligned}
$$

```
[Out] -2*a^2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/
(a+b)^(7/2)/d-4*a^2*b*(a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/12*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))
^2-1/4*(a-b)*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))-1/12*sin(d*x+c)/(a+b)^2/d/
(1-cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))^2+1/12*sin(d*x+c)/(
a-b)^2/d/(1+cos(d*x+c))+1/4*(a+b)*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))+a^3*b
^2*sin(d*x+c)/(a^2-b^2)^3/d/(b+a*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2976, 2729, 2727, 2743, 12, 2738, 214}

$$\begin{aligned}\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &-\frac{2a^2b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}\\ &-\frac{4a^2b(a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}\\ &+\frac{a^3b^2\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}\\ &-\frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))}-\frac{(a-b)\sin(c+dx)}{4d(a+b)^3(1-\cos(c+dx))}\\ &+\frac{(a+b)\sin(c+dx)}{4d(a-b)^3(\cos(c+dx)+1)}+\frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)}\\ &-\frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))^2}\\ &+\frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)^2}\end{aligned}$$

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(
7/2)*(a + b)^(7/2)*d) - (4*a^2*b*(a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c +
d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(12*(
a + b)^2*d*(1 - Cos[c + d*x])^2) - ((a - b)*Sin[c + d*x])/(4*(a + b)^3*d*(1
 - Cos[c + d*x])) - Sin[c + d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[
c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a - b)^2
*d*(1 + Cos[c + d*x])) + ((a + b)*Sin[c + d*x])/(4*(a - b)^3*d*(1 + Cos[c +
 d*x])) + (a^3*b^2*Sin[c + d*x])/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{1}{4(a-b)^2(-1-\cos(c+dx))^2} + \frac{-a-b}{4(a-b)^3(-1-\cos(c+dx))} + \frac{1}{4(a+b)^2(1-\cos(c+dx))^2} + \frac{a-b}{4(a+b)^3(1-\cos(c+dx))} + \frac{a^2b^2}{(a^2-b^2)^2(-b-a\cos(c+dx))^2} + \frac{2a^2b(a^2+b^2)}{(a^2-b^2)^3(-b-a\cos(c+dx))}\right) dx$$

$$= \frac{\int \frac{1}{(-1-\cos(c+dx))^2}\,dx}{4(a-b)^2} + \frac{(a-b)\int \frac{1}{1-\cos(c+dx)}\,dx}{4(a+b)^3} + \frac{\int \frac{1}{(1-\cos(c+dx))^2}\,dx}{4(a+b)^2} - \frac{(a+b)\int \frac{1}{-1-\cos(c+dx)}\,dx}{4(a-b)^3} + \frac{(a^2b^2)\int \frac{1}{(-b-a\cos(c+dx))^2}\,dx}{(a^2-b^2)^2} + \frac{(2a^2b(a^2+b^2))\int \frac{1}{-b-a\cos(c+dx)}\,dx}{(a^2-b^2)^3}$$

$$= -\frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{(a-b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))^2} + \frac{(a+b)\sin(c+dx)}{4(a-b)^3d(1+\cos(c+dx))} + \frac{a^3b^2\sin(c+dx)}{(a^2-b^2)^3d(b+a\cos(c+dx))} - \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{12(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{12(a+b)^2} + \frac{(a^2b^2)\int \frac{b}{-b-a\cos(c+dx)}\,dx}{(a^2-b^2)^3} + \frac{(4a^2b(a^2+b^2))\operatorname{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)^3d}$$

$$= -\frac{4a^2b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{(a-b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))^2} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))} + \frac{(a+b)\sin(c+dx)}{4(a-b)^3d(1+\cos(c+dx))} + \frac{a^3b^2\sin(c+dx)}{(a^2-b^2)^3d(b+a\cos(c+dx))} + \frac{(a^2b^3)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{(a^2-b^2)^3}$$

$$= -\frac{4a^2b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2}$$
$$- \frac{(a-b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))}$$
$$+ \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))^2} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))}$$
$$+ \frac{(a+b)\sin(c+dx)}{4(a-b)^3d(1+\cos(c+dx))} + \frac{a^3b^2\sin(c+dx)}{(a^2-b^2)^3\, d(b+a\cos(c+dx))}$$
$$+ \frac{(2a^2b^3)\operatorname{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\, dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)^3\, d}$$

$$= -\frac{2a^2b^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4a^2b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}$$
$$- \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{(a-b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))}$$
$$- \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))^2}$$
$$+ \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))} + \frac{(a+b)\sin(c+dx)}{4(a-b)^3d(1+\cos(c+dx))}$$
$$+ \frac{a^3b^2\sin(c+dx)}{(a^2-b^2)^3\, d(b+a\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.82

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\, dx$$
$$= \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(\frac{48a^2b(2a^2+3b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))}{(a^2-b^2)^{7/2}} - \frac{4(2a-b)(b+a\cos(c+dx))\,c}{(a+b)^3}\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((48*a^2*b*(2*a^2 + 3*b^2)*ArcTanh[((-
a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]))/(a^2 - b^2)
^(7/2) - (4*(2*a - b)*(b + a*Cos[c + d*x])*Cot[(c + d*x)/2])/(a + b)^3 - ((
b + a*Cos[c + d*x])*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(a + b)^2 + (24*a^
3*b^2*Sin[c + d*x])/((a - b)^3*(a + b)^3) + (4*(2*a + b)*(b + a*Cos[c + d*x
])*Tan[(c + d*x)/2])/(a - b)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/(a - b)^2))/(24*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}b}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^{2}-2ab+b^{2}\right)(a-b)}-\frac{1}{24(a+b)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{3a-b}{8(a+b)^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2a^{2}b\left(-\frac{}{\tan(}\right.}{}}{d}$
default	$\frac{\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}b}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^{2}-2ab+b^{2}\right)(a-b)}-\frac{1}{24(a+b)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{3a-b}{8(a+b)^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2a^{2}b\left(-\frac{}{\tan(}\right.}{}}{d}$
risch	$2i\left(-6a^{4}b\,e^{7i(dx+c)}-9a^{2}b^{3}e^{7i(dx+c)}-6a^{3}b^{2}e^{6i(dx+c)}-9a\,b^{4}e^{6i(dx+c)}+14a^{4}b\,e^{5i(dx+c)}+25a^{2}b^{3}e^{5i(dx+c)}+6b^{5}e^{5i(dx+c)}\ldots\right.$

[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/8/(a^2-2*a*b+b^2)/(a-b)*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*tan(1/2*d*x+1/2*c)^3*b+3*tan(1/2*d*x+1/2*c)*a+tan(1/2*d*x+1/2*c)*b)-1/24/(a+b)^2/tan(1/2*d*x+1/2*c)^3-1/8*(3*a-b)/(a+b)^3/tan(1/2*d*x+1/2*c)+2*a^2*b/(a-b)^3/(a+b)^3*(-a*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2+3*b^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 1040, normalized size of antiderivative = 3.03

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] [-1/6*(22*a^5*b^2 - 14*a^3*b^4 - 8*a*b^6 + 2*(2*a^7 + 10*a^5*b^2 - 11*a^3*b^4 - a*b^6)*cos(d*x + c)^4 - 2*(4*a^6*b - 7*a^4*b^3 + 2*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*(2*a^4*b^2 + 3*a^2*b^4 - (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^3 - (2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (2*a^5*b + 3*a^3*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 6*(a^7 + 6*a^5*b^2 - 5*a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2 + 10*(a^6*b - 2*a^4*b^3 + a^2*b^5

```
)*cos(d*x + c))/(((a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d
*x + c)^3 + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c
)^2 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) - (a
^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d)*sin(d*x + c)), -1/3*(11*
a^5*b^2 - 7*a^3*b^4 - 4*a*b^6 + (2*a^7 + 10*a^5*b^2 - 11*a^3*b^4 - a*b^6)*c
os(d*x + c)^4 - (4*a^6*b - 7*a^4*b^3 + 2*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*
(2*a^4*b^2 + 3*a^2*b^4 - (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^3 - (2*a^4*b^2
+ 3*a^2*b^4)*cos(d*x + c)^2 + (2*a^5*b + 3*a^3*b^3)*cos(d*x + c))*sqrt(-a^2
 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x
+ c)))*sin(d*x + c) - 3*(a^7 + 6*a^5*b^2 - 5*a^3*b^4 - 2*a*b^6)*cos(d*x + c
)^2 + 5*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))/(((a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^8*b - 4*a^6*b^3 + 6*a^4*
b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*
a^3*b^6 + a*b^8)*d*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^
7 + b^9)*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\int \frac{\csc^4{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.33

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{48\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)} - \frac{48\,(2\,a^4b+3\,a^2b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx-\right.}{}\right.\right.}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}}{}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(48*a^3*b^2*tan(1/2*d*x + 1/2*c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 48*(2*a^4
*b + 3*a^2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((
a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (a^4*tan(1/2*d*x + 1/2*c)
^3 - 4*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*
a*b^3*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d
*x + 1/2*c) - 24*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b^2*tan(1/2*d*x + 1/2*
c) - 3*b^4*tan(1/2*d*x + 1/2*c))/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 +
 15*a^2*b^4 - 6*a*b^5 + b^6) + (9*a*tan(1/2*d*x + 1/2*c)^2 - 3*b*tan(1/2*d*
x + 1/2*c)^2 + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)
^3))/d
```

Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.17

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d\,(a-b)^2} + \frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{3\,(a+b)} + \frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(4\,a^4-13\,a^3\,b+15\,a^2\,b^2-7\,a\,b^3+b^4)}{3\,(a+b)^2} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(3\,a^5-13\,a^4\,b+38\,a^3\,b^2-18\,a^2\,b^3+7\,a\,b^4-}{(a+b)^3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(8\,a^4-32\,a^3\,b+48\,a^2\,b^2-32\,a\,b^3+8\,b^4)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(8\,a^4-16\,a^3\,b+16\,a\,b^3-\right.} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{16\,a^2-16\,b^2}{64\,(a-b)^4}+\frac{1}{8\,(a-b)^2}\right)}{d} + \frac{a^2\,b\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^6-3\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b^2+3\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^4-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^6}{(a+b)^{7/2}\,(a-b)^{5/2}}\right)\,(2\,a^2+3\,b^2)\,2\mathrm{i}}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}$$

```
[In] int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^2) + ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(3
*(a + b)) + (2*tan(c/2 + (d*x)/2)^2*(4*a^4 - 13*a^3*b - 7*a*b^3 + b^4 + 15*
a^2*b^2))/(3*(a + b)^2) - (tan(c/2 + (d*x)/2)^4*(7*a*b^4 - 13*a^4*b + 3*a^5
 - b^5 - 18*a^2*b^3 + 38*a^3*b^2))/(a + b)^3)/(d*(tan(c/2 + (d*x)/2)^5*(8*a
^4 - 32*a^3*b - 32*a*b^3 + 8*b^4 + 48*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(16*a
*b^3 - 16*a^3*b + 8*a^4 - 8*b^4))) + (tan(c/2 + (d*x)/2)*((16*a^2 - 16*b^2)
/(64*(a - b)^4) + 1/(8*(a - b)^2)))/d + (a^2*b*atan((a^6*tan(c/2 + (d*x)/2)
*1i - b^6*tan(c/2 + (d*x)/2)*1i + a^2*b^4*tan(c/2 + (d*x)/2)*3i - a^4*b^2*t
an(c/2 + (d*x)/2)*3i)/((a + b)^(7/2)*(a - b)^(5/2)))*(2*a^2 + 3*b^2)*2i)/(d
*(a + b)^(7/2)*(a - b)^(7/2))
```

3.221 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 329

$$\begin{aligned}\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &-\frac{(a^6-18a^4b^2+45a^2b^4-28b^6)\cos(c+dx)}{a^9d}\\ &-\frac{3b(3a^4-10a^2b^2+7b^4)\cos^2(c+dx)}{2a^8d}\\ &+\frac{(a^4-6a^2b^2+5b^4)\cos^3(c+dx)}{a^7d}+\frac{b(9a^2-10b^2)\cos^4(c+dx)}{4a^6d}\\ &-\frac{3(a^2-2b^2)\cos^5(c+dx)}{5a^5d}-\frac{b\cos^6(c+dx)}{2a^4d}+\frac{\cos^7(c+dx)}{7a^3d}\\ &-\frac{b^3(a^2-b^2)^3}{2a^{10}d(b+a\cos(c+dx))^2}+\frac{3b^2(a^2-3b^2)\,(a^2-b^2)^2}{a^{10}d(b+a\cos(c+dx))}\\ &+\frac{3b(a^2-b^2)\,(a^4-9a^2b^2+12b^4)\log(b+a\cos(c+dx))}{a^{10}d}\end{aligned}$$

```
[Out] -(a^6-18*a^4*b^2+45*a^2*b^4-28*b^6)*cos(d*x+c)/a^9/d-3/2*b*(3*a^4-10*a^2*b^
2+7*b^4)*cos(d*x+c)^2/a^8/d+(a^4-6*a^2*b^2+5*b^4)*cos(d*x+c)^3/a^7/d+1/4*b*
(9*a^2-10*b^2)*cos(d*x+c)^4/a^6/d-3/5*(a^2-2*b^2)*cos(d*x+c)^5/a^5/d-1/2*b*
cos(d*x+c)^6/a^4/d+1/7*cos(d*x+c)^7/a^3/d-1/2*b^3*(a^2-b^2)^3/a^10/d/(b+a*c
os(d*x+c))^2+3*b^2*(a^2-3*b^2)*(a^2-b^2)^2/a^10/d/(b+a*cos(d*x+c))+3*b*(a^2
-b^2)*(a^4-9*a^2*b^2+12*b^4)*ln(b+a*cos(d*x+c))/a^10/d
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2916, 12, 962}

$$\begin{aligned}\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx=&-\frac{b\cos^6(c+dx)}{2a^4d}+\frac{\cos^7(c+dx)}{7a^3d}\\&+\frac{3b^2(a^2-3b^2)\,(a^2-b^2)^2}{a^{10}d(a\cos(c+dx)+b)}-\frac{b^3(a^2-b^2)^3}{2a^{10}d(a\cos(c+dx)+b)^2}\\&+\frac{b(9a^2-10b^2)\cos^4(c+dx)}{4a^6d}-\frac{3(a^2-2b^2)\cos^5(c+dx)}{5a^5d}\\&+\frac{3b(a^2-b^2)\,(a^4-9a^2b^2+12b^4)\log(a\cos(c+dx)+b)}{a^{10}d}\\&-\frac{3b(3a^4-10a^2b^2+7b^4)\cos^2(c+dx)}{2a^8d}\\&+\frac{(a^4-6a^2b^2+5b^4)\cos^3(c+dx)}{a^7d}\\&-\frac{(a^6-18a^4b^2+45a^2b^4-28b^6)\cos(c+dx)}{a^9d}\end{aligned}$$

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^6 - 18*a^4*b^2 + 45*a^2*b^4 - 28*b^6)*Cos[c + d*x])/(a^9*d)) - (3*b*(
3*a^4 - 10*a^2*b^2 + 7*b^4)*Cos[c + d*x]^2)/(2*a^8*d) + ((a^4 - 6*a^2*b^2 +
 5*b^4)*Cos[c + d*x]^3)/(a^7*d) + (b*(9*a^2 - 10*b^2)*Cos[c + d*x]^4)/(4*a^
6*d) - (3*(a^2 - 2*b^2)*Cos[c + d*x]^5)/(5*a^5*d) - (b*Cos[c + d*x]^6)/(2*a
^4*d) + Cos[c + d*x]^7/(7*a^3*d) - (b^3*(a^2 - b^2)^3)/(2*a^10*d*(b + a*Cos
[c + d*x])^2) + (3*b^2*(a^2 - 3*b^2)*(a^2 - b^2)^2)/(a^10*d*(b + a*Cos[c +
d*x])) + (3*b*(a^2 - b^2)*(a^4 - 9*a^2*b^2 + 12*b^4)*Log[b + a*Cos[c + d*x]
])/(a^10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin^7(c+dx)}{(-b-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^3\left(a^2-x^2\right)^3}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^7 d} \\
&= \frac{\text{Subst}\left(\int \frac{x^3\left(a^2-x^2\right)^3}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^{10} d} \\
&= \frac{\text{Subst}\left(\int \left(a^6\left(1+\frac{-18a^4b^2+45a^2b^4-28b^6}{a^6}\right)+\frac{b^3\left(-a^2+b^2\right)^3}{(b-x)^3}+\frac{3b^2\left(a^2-3b^2\right)\left(a^2-b^2\right)^2}{(b-x)^2}+\frac{3b(-a^6+10a^4b^2-21a^2b^4+12b^6)}{b-x}\right.\right.}{} \\
&= -\frac{(a^6-18a^4b^2+45a^2b^4-28b^6)\cos(c+dx)}{a^9 d}-\frac{3b(3a^4-10a^2b^2+7b^4)\cos^2(c+dx)}{2a^8 d} \\
&\quad +\frac{(a^4-6a^2b^2+5b^4)\cos^3(c+dx)}{a^7 d}+\frac{b(9a^2-10b^2)\cos^4(c+dx)}{4a^6 d} \\
&\quad -\frac{3(a^2-2b^2)\cos^5(c+dx)}{5a^5 d}-\frac{b\cos^6(c+dx)}{2a^4 d}+\frac{\cos^7(c+dx)}{7a^3 d} \\
&\quad -\frac{b^3(a^2-b^2)^3}{2a^{10}d(b+a\cos(c+dx))^2}+\frac{3b^2(a^2-3b^2)\left(a^2-b^2\right)^2}{a^{10}d(b+a\cos(c+dx))} \\
&\quad +\frac{3b(a^2-b^2)\left(a^4-9a^2b^2+12b^4\right)\log(b+a\cos(c+dx))}{a^{10}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.50 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.67

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-7945a^8b+164080a^6b^3-502320a^4b^5+425600a^2b^7-76160b^9-784a^9\cos(3(c+dx))+17528a^7b^2\cos(3(c}{}$$

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-7945*a^8*b + 164080*a^6*b^3 - 502320*a^4*b^5 + 425600*a^2*b^7 - 76160*b^9
 - 784*a^9*Cos[3*(c + d*x)] + 17528*a^7*b^2*Cos[3*(c + d*x)] - 43680*a^5*b^
4*Cos[3*(c + d*x)] + 26880*a^3*b^6*Cos[3*(c + d*x)] - 1456*a^8*b*Cos[4*(c +
 d*x)] + 4872*a^6*b^3*Cos[4*(c + d*x)] - 3360*a^4*b^5*Cos[4*(c + d*x)] + 15
2*a^9*Cos[5*(c + d*x)] - 840*a^7*b^2*Cos[5*(c + d*x)] + 672*a^5*b^4*Cos[5*(
c + d*x)] + 174*a^8*b*Cos[6*(c + d*x)] - 168*a^6*b^3*Cos[6*(c + d*x)] - 39*
a^9*Cos[7*(c + d*x)] + 48*a^7*b^2*Cos[7*(c + d*x)] - 15*a^8*b*Cos[8*(c + d*
x)] + 5*a^9*Cos[9*(c + d*x)] + 13440*a^8*b*Log[b + a*Cos[c + d*x]] - 107520
*a^6*b^3*Log[b + a*Cos[c + d*x]] + 13440*a^4*b^5*Log[b + a*Cos[c + d*x]] +
403200*a^2*b^7*Log[b + a*Cos[c + d*x]] - 322560*b^9*Log[b + a*Cos[c + d*x]]
 + 70*a^2*b*Cos[2*(c + d*x)]*(-137*a^6 + 1896*a^4*b^2 - 4656*a^2*b^4 + 2912
*b^6 + 192*(a^6 - 10*a^4*b^2 + 21*a^2*b^4 - 12*b^6)*Log[b + a*Cos[c + d*x]]
) - 70*a*Cos[c + d*x]*(49*a^8 - 1472*a^6*b^2 + 3216*a^4*b^4 + 576*a^2*b^6 -
 2432*b^8 - 768*b^2*(a^6 - 10*a^4*b^2 + 21*a^2*b^4 - 12*b^6)*Log[b + a*Cos[
c + d*x]]))/(8960*a^10*d*(b + a*Cos[c + d*x])^2)
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)^7a^6}{7}-\frac{b\cos(dx+c)^6a^5}{2}-\frac{3a^6\cos(dx+c)^5}{5}+\frac{6a^4b^2\cos(dx+c)^5}{5}+\frac{9\cos(dx+c)^4a^5b}{4}-\frac{5\cos(dx+c)^4a^3b^3}{2}+\cos(dx+c)^3a^6-6\cos(d}{}$
default	$\frac{\frac{\cos(dx+c)^7a^6}{7}-\frac{b\cos(dx+c)^6a^5}{2}-\frac{3a^6\cos(dx+c)^5}{5}+\frac{6a^4b^2\cos(dx+c)^5}{5}+\frac{9\cos(dx+c)^4a^5b}{4}-\frac{5\cos(dx+c)^4a^3b^3}{2}+\cos(dx+c)^3a^6-6\cos(d}{}$
parallelrisch	$\frac{1680b(a-b)(a+b)(a^4-9a^2b^2+12b^4)(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2)\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-1680b(a-}{}$
risch	Expression too large to display

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^9*(1/7*cos(d*x+c)^7*a^6-1/2*b*cos(d*x+c)^6*a^5-3/5*a^6*cos(d*x+c)^
5+6/5*a^4*b^2*cos(d*x+c)^5+9/4*cos(d*x+c)^4*a^5*b-5/2*cos(d*x+c)^4*a^3*b^3+
```

```
cos(d*x+c)^3*a^6-6*cos(d*x+c)^3*a^4*b^2+5*cos(d*x+c)^3*a^2*b^4-9/2*cos(d*x+
c)^2*a^5*b+15*cos(d*x+c)^2*a^3*b^3-21/2*cos(d*x+c)^2*a*b^5-cos(d*x+c)*a^6+1
8*cos(d*x+c)*a^4*b^2-45*cos(d*x+c)*a^2*b^4+28*b^6*cos(d*x+c))+3/a^10*b^2*(a
^6-5*a^4*b^2+7*a^2*b^4-3*b^6)/(b+a*cos(d*x+c))-1/2*b^3*(a^6-3*a^4*b^2+3*a^2
*b^4-b^6)/a^10/(b+a*cos(d*x+c))^2+3/a^10*b*(a^6-10*a^4*b^2+21*a^2*b^4-12*b^
6)*ln(b+a*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.36

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{80\,a^9\cos\left(dx+c\right)^9 - 120\,a^8b\cos\left(dx+c\right)^8 + 2275\,a^6b^3 - 11235\,a^4b^5 + 13860\,a^2b^7 - 4760\,b^9 - 48\,(7\,a^9 - 4}{}$$

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(80*a^9*cos(d*x + c)^9 - 120*a^8*b*cos(d*x + c)^8 + 2275*a^6*b^3 - 11
235*a^4*b^5 + 13860*a^2*b^7 - 4760*b^9 - 48*(7*a^9 - 4*a^7*b^2)*cos(d*x + c
)^7 + 84*(7*a^8*b - 4*a^6*b^3)*cos(d*x + c)^6 + 56*(10*a^9 - 21*a^7*b^2 + 1
2*a^5*b^4)*cos(d*x + c)^5 - 140*(10*a^8*b - 21*a^6*b^3 + 12*a^4*b^5)*cos(d*
x + c)^4 - 560*(a^9 - 10*a^7*b^2 + 21*a^5*b^4 - 12*a^3*b^6)*cos(d*x + c)^3
- 35*(7*a^8*b - 399*a^6*b^3 + 1116*a^4*b^5 - 728*a^2*b^7)*cos(d*x + c)^2 +
70*(41*a^7*b^2 - 81*a^5*b^4 - 108*a^3*b^6 + 152*a*b^8)*cos(d*x + c) + 1680*
(a^6*b^3 - 10*a^4*b^5 + 21*a^2*b^7 - 12*b^9 + (a^8*b - 10*a^6*b^3 + 21*a^4*
b^5 - 12*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 - 10*a^5*b^4 + 21*a^3*b^6 - 1
2*a*b^8)*cos(d*x + c))*log(a*cos(d*x + c) + b))/(a^12*d*cos(d*x + c)^2 + 2*
a^11*b*d*cos(d*x + c) + a^10*b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.99

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{70\left(5a^6b^3-27a^4b^5+39a^2b^7-17b^9+6\left(a^7b^2-5a^5b^4+7a^3b^6-3ab^8\right)\cos(dx+c)\right)}{a^{12}\cos(dx+c)^2+2a^{11}b\cos(dx+c)+a^{10}b^2} + \frac{20a^6\cos(dx+c)^7-70a^5b\cos(dx+c)^6-84\left(a^6-2a^4b^2\right)\cos(dx+}{}}{}$$

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/140*(70*(5*a^6*b^3 - 27*a^4*b^5 + 39*a^2*b^7 - 17*b^9 + 6*(a^7*b^2 - 5*a^
5*b^4 + 7*a^3*b^6 - 3*a*b^8)*cos(d*x + c))/(a^12*cos(d*x + c)^2 + 2*a^11*b*
cos(d*x + c) + a^10*b^2) + (20*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6
 - 84*(a^6 - 2*a^4*b^2)*cos(d*x + c)^5 + 35*(9*a^5*b - 10*a^3*b^3)*cos(d*x
+ c)^4 + 140*(a^6 - 6*a^4*b^2 + 5*a^2*b^4)*cos(d*x + c)^3 - 210*(3*a^5*b -
10*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^2 - 140*(a^6 - 18*a^4*b^2 + 45*a^2*b^4 -
 28*b^6)*cos(d*x + c))/a^9 + 420*(a^6*b - 10*a^4*b^3 + 21*a^2*b^5 - 12*b^7)
*log(a*cos(d*x + c) + b)/a^10)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2150 vs. $2(317) = 634$.

Time = 0.46 (sec) , antiderivative size = 2150, normalized size of antiderivative = 6.53

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/140*(420*(a^7*b - a^6*b^2 - 10*a^5*b^3 + 10*a^4*b^4 + 21*a^3*b^5 - 21*a^2
*b^6 - 12*a*b^7 + 12*b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^11 - a^10*b) - 420*(a
^6*b - 10*a^4*b^3 + 21*a^2*b^5 - 12*b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 1))/a^10 - 70*(9*a^8*b + 6*a^7*b^2 - 105*a^6*b^3 - 148*a^5*b
^4 + 187*a^4*b^5 + 390*a^3*b^6 + 17*a^2*b^7 - 248*a*b^8 - 108*b^9 + 18*a^8*
b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^7*b^2*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 202*a^6*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 56*a^
5*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 566*a^4*b^5*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 76*a^3*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 5
98*a^2*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 32*a*b^8*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 216*b^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*
a^8*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^7*b^2*(cos(d*x + c)
```

- 1)^2/(cos(d*x + c) + 1)^2 - 81*a^6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 180*a^5*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 99*a^4*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 378*a^3*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 81*a^2*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 216*a*b^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 108*b^9*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2*a^10) + (128*a^7 - 1089*a^6*b - 3696*a^5*b^2 + 10890*a^4*b^3 + 11200*a^3*b^4 - 22869*a^2*b^5 - 7840*a*b^6 + 13068*b^7 - 896*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8463*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 24192*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 81830*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 70000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 165963*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 47040*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 91476*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2688*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 64176*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 262290*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 176400*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 509649*a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 117600*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 274428*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 4480*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 80640*a^5*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 453950*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 229600*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 859215*a^2*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 156800*a*b^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 457380*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 56035*a^6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 48720*a^5*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 453950*a^4*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 162400*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 859215*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 117600*a*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 457380*b^7*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 13440*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 262290*a^4*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 58800*a^3*b^4*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 509649*a^2*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 47040*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 274428*b^7*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1680*a^5*b^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 81830*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 8400*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 165963*a^2*b^5*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 7840*a*b^6*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 91476*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*a^6*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 10890*a^4*b^3*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 22869*a^2*b^5*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 13068*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1

```
)^7)/(a^10*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7))/d
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.32

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\cos(c+d\,x)^4\left(\frac{2\,b^3}{a^6}+\frac{3\,b\left(\frac{3}{a^3}-\frac{6\,b^2}{a^5}\right)}{4\,a}\right)}{d}$$

$$-\frac{\frac{-5\,a^6\,b^3+27\,a^4\,b^5-39\,a^2\,b^7+17\,b^9}{2\,a}+\cos(c+d\,x)\,\left(-3\,a^6\,b^2+15\,a^4\,b^4-21\,a^2\,b^6+9\,b^8\right)}{d\,\left(a^{11}\cos(c+d\,x)^2+2\,a^{10}\,b\,\cos(c+d\,x)+a^9\,b^2\right)}$$

$$-\frac{\cos(c+d\,x)^5\left(\frac{3}{5\,a^3}-\frac{6\,b^2}{5\,a^5}\right)}{d}$$

$$-\frac{\cos(c+d\,x)^2\left(\frac{3\,b^2\left(\frac{8\,b^3}{a^6}+\frac{3\,b\left(\frac{3}{a^3}-\frac{6\,b^2}{a^5}\right)}{a}\right)}{2\,a^2}-\frac{b^3\left(\frac{3}{a^3}-\frac{6\,b^2}{a^5}\right)}{2\,a^3}+\frac{3\,b\left(\frac{3}{a^3}+\frac{3\,b^4}{a^7}+\frac{3\,b^2\left(\frac{3}{a^3}-\frac{6\,b^2}{a^5}\right)}{a^2}-\frac{3\,b\left(\frac{8\,b^3}{a^6}+\frac{3\,b\left(\frac{3}{a^3}-\frac{6\,b^2}{a^5}\right)}{a}\right)}{a}\right)}{2\,a}\right)}{d}$$

$$+\frac{\cos(c+d\,x)^7}{7\,a^3\,d}$$

$$-\frac{\cos(c+d\,x)\left(\frac{1}{a^3}+\frac{b^3\left(\frac{8\,b^3}{a^6}+\frac{3\,b\left(\frac{3}{a^3}-\frac{6\,b^2}{a^5}\right)}{a}\right)}{a^3}+\frac{3\,b^2\left(\frac{3}{a^3}+\frac{3\,b^4}{a^7}+\frac{3\,b^2\left(\frac{3}{a^3}-\frac{6\,b^2}{a^5}\right)}{a^2}-\frac{3\,b\left(\frac{8\,b^3}{a^6}+\frac{3\,b\left(\frac{3}{a^3}-\frac{6\,b^2}{a^5}\right)}{a}\right)}{a}\right)}{a^2}-\frac{3\,b\left(\frac{3\,b^2\left(8\ldots\right.}{\ldots}\right.}{\ldots}\right.}{d}$$

$$+\frac{\cos(c+d\,x)^3\left(\frac{1}{a^3}+\frac{b^4}{a^7}+\frac{b^2\left(\frac{3}{a^3}-\frac{6\,b^2}{a^5}\right)}{a^2}-\frac{b\left(\frac{8\,b^3}{a^6}+\frac{3\,b\left(\frac{3}{a^3}-\frac{6\,b^2}{a^5}\right)}{a}\right)}{a}\right)}{d}$$

$$-\frac{b\cos(c+d\,x)^6}{2\,a^4\,d}+\frac{\ln(b+a\,\cos(c+d\,x))\,\left(3\,a^6\,b-30\,a^4\,b^3+63\,a^2\,b^5-36\,b^7\right)}{a^{10}\,d}$$

```
[In] int(sin(c + d*x)^7/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (cos(c + d*x)^4*((2*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/(4*a)))/d - ((17
*b^9 - 39*a^2*b^7 + 27*a^4*b^5 - 5*a^6*b^3)/(2*a) + cos(c + d*x)*(9*b^8 - 2
1*a^2*b^6 + 15*a^4*b^4 - 3*a^6*b^2))/(d*(a^11*cos(c + d*x)^2 + a^9*b^2 + 2*
a^10*b*cos(c + d*x))) - (cos(c + d*x)^5*(3/(5*a^3) - (6*b^2)/(5*a^5)))/d -
(cos(c + d*x)^2*((3*b^2*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/a))/(2*a
^2) - (b^3*(3/a^3 - (6*b^2)/a^5))/(2*a^3) + (3*b*(3/a^3 + (3*b^4)/a^7 + (3*
b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/
a^5))/a))/a))/(2*a)))/d + cos(c + d*x)^7/(7*a^3*d) - (cos(c + d*x)*(1/a^3 +
 (b^3*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/a))/a^3 + (3*b^2*(3/a^3 +
(3*b^4)/a^7 + (3*b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b*
(3/a^3 - (6*b^2)/a^5))/a))/a))/a^2 - (3*b*((3*b^2*((8*b^3)/a^6 + (3*b*(3/a^
3 - (6*b^2)/a^5))/a))/a^2 - (b^3*(3/a^3 - (6*b^2)/a^5))/a^3 + (3*b*(3/a^3 +
 (3*b^4)/a^7 + (3*b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b
*(3/a^3 - (6*b^2)/a^5))/a))/a))/a))/a))/d + (cos(c + d*x)^3*(1/a^3 + b^4/a^
7 + (b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (b*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^
2)/a^5))/a))/a))/d - (b*cos(c + d*x)^6)/(2*a^4*d) + (log(b + a*cos(c + d*x)
)*(3*a^6*b - 36*b^7 + 63*a^2*b^5 - 30*a^4*b^3))/(a^10*d)
```

3.222 $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 239

$$\begin{aligned}\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &-\frac{(a^4-12a^2b^2+15b^4)\cos(c+dx)}{a^7d}-\frac{b(3a^2-5b^2)\cos^2(c+dx)}{a^6d}\\&+\frac{2(a^2-3b^2)\cos^3(c+dx)}{3a^5d}+\frac{3b\cos^4(c+dx)}{4a^4d}-\frac{\cos^5(c+dx)}{5a^3d}\\&-\frac{b^3(a^2-b^2)^2}{2a^8d(b+a\cos(c+dx))^2}+\frac{b^2(3a^4-10a^2b^2+7b^4)}{a^8d(b+a\cos(c+dx))}\\&+\frac{b(3a^4-20a^2b^2+21b^4)\log(b+a\cos(c+dx))}{a^8d}\end{aligned}$$

```
[Out] -(a^4-12*a^2*b^2+15*b^4)*cos(d*x+c)/a^7/d-b*(3*a^2-5*b^2)*cos(d*x+c)^2/a^6/
d+2/3*(a^2-3*b^2)*cos(d*x+c)^3/a^5/d+3/4*b*cos(d*x+c)^4/a^4/d-1/5*cos(d*x+c
)^5/a^3/d-1/2*b^3*(a^2-b^2)^2/a^8/d/(b+a*cos(d*x+c))^2+b^2*(3*a^4-10*a^2*b^
2+7*b^4)/a^8/d/(b+a*cos(d*x+c))+b*(3*a^4-20*a^2*b^2+21*b^4)*ln(b+a*cos(d*x+
c))/a^8/d
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

$= \{3957, 2916, 12, 962\}$

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{3b\cos^4(c+dx)}{4a^4d} - \frac{\cos^5(c+dx)}{5a^3d} - \frac{b^3(a^2-b^2)^2}{2a^8d(a\cos(c+dx)+b)^2} - \frac{b(3a^2-5b^2)\cos^2(c+dx)}{a^6d} + \frac{2(a^2-3b^2)\cos^3(c+dx)}{3a^5d} + \frac{b^2(3a^4-10a^2b^2+7b^4)}{a^8d(a\cos(c+dx)+b)} + \frac{b(3a^4-20a^2b^2+21b^4)\log(a\cos(c+dx)+b)}{a^8d} - \frac{(a^4-12a^2b^2+15b^4)\cos(c+dx)}{a^7d}$$

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^4 - 12*a^2*b^2 + 15*b^4)*Cos[c + d*x])/(a^7*d)) - (b*(3*a^2 - 5*b^2)*
Cos[c + d*x]^2)/(a^6*d) + (2*(a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*a^5*d) + (3*b
*Cos[c + d*x]^4)/(4*a^4*d) - Cos[c + d*x]^5/(5*a^3*d) - (b^3*(a^2 - b^2)^2)
/(2*a^8*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^4 - 10*a^2*b^2 + 7*b^4))/(a^8
*d*(b + a*Cos[c + d*x])) + (b*(3*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c
 + d*x]])/(a^8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos^3(c+dx)\sin^5(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)^2}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)^2}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^8d}$$

$$= \frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{3b^2(-4a^2+5b^2)}{a^4}\right) - \frac{b^3(-a^2+b^2)^2}{(b-x)^3} + \frac{3a^4b^2-10a^2b^4+7b^6}{(b-x)^2} + \frac{-3a^4b+20a^2b^3-21b^5}{b-x} + 2b(-3a^2+5\right.\right.}{a^8d}$$

$$= -\frac{(a^4-12a^2b^2+15b^4)\cos(c+dx)}{a^7d} - \frac{b(3a^2-5b^2)\cos^2(c+dx)}{a^6d} + \frac{2(a^2-3b^2)\cos^3(c+dx)}{3a^5d} + \frac{3b\cos^4(c+dx)}{4a^4d} - \frac{\cos^5(c+dx)}{5a^3d} - \frac{b^3(a^2-b^2)^2}{2a^8d(b+a\cos(c+dx))^2} + \frac{b^2(3a^4-10a^2b^2+7b^4)}{a^8d(b+a\cos(c+dx))} + \frac{b(3a^4-20a^2b^2+21b^4)\log(b+a\cos(c+dx))}{a^8d}$$

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.62

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{-1740a^6b+26160a^4b^3-46080a^2b^5+12480b^7-206a^7\cos(3(c+dx))+2780a^5b^2\cos(3(c+dx))-3360a}{}$$

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
[Out] (-1740*a^6*b + 26160*a^4*b^3 - 46080*a^2*b^5 + 12480*b^7 - 206*a^7*Cos[3*(c
 + d*x)] + 2780*a^5*b^2*Cos[3*(c + d*x)] - 3360*a^3*b^4*Cos[3*(c + d*x)] -
274*a^6*b*Cos[4*(c + d*x)] + 420*a^4*b^3*Cos[4*(c + d*x)] + 38*a^7*Cos[5*(c
 + d*x)] - 84*a^5*b^2*Cos[5*(c + d*x)] + 21*a^6*b*Cos[6*(c + d*x)] - 6*a^7*
Cos[7*(c + d*x)] + 2880*a^6*b*Log[b + a*Cos[c + d*x]] - 13440*a^4*b^3*Log[b
 + a*Cos[c + d*x]] - 18240*a^2*b^5*Log[b + a*Cos[c + d*x]] + 40320*b^7*Log[
b + a*Cos[c + d*x]] + 5*a^2*b*Cos[2*(c + d*x)]*(-407*a^4 + 3888*a^2*b^2 - 4
800*b^4 + 192*(3*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c + d*x]]) - 10*a
*Cos[c + d*x]*(85*a^6 - 1728*a^4*b^2 + 1584*a^2*b^4 + 1536*b^6 - 384*b^2*(3
*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c + d*x]]))/(1920*a^8*d*(b + a*Co
s[c + d*x])^2)
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00

method	result
derivativedivides	$-\frac{\frac{-\frac{\cos(dx+c)^5a^4}{5}-\frac{3b\cos(dx+c)^4a^3}{4}-\frac{2\cos(dx+c)^3a^4}{3}+2\cos(dx+c)^3a^2b^2+3\cos(dx+c)^2a^3b-5\cos(dx+c)^2a\,b^3+\cos(dx+c)a^4-12\cos(dx}{a^7}}{d}$
default	$-\frac{\frac{-\frac{\cos(dx+c)^5a^4}{5}-\frac{3b\cos(dx+c)^4a^3}{4}-\frac{2\cos(dx+c)^3a^4}{3}+2\cos(dx+c)^3a^2b^2+3\cos(dx+c)^2a^3b-5\cos(dx+c)^2a\,b^3+\cos(dx+c)a^4-12\cos(dx}{a^7}}{d}$
parallelrisch	$\frac{2880\left(a^4-\frac{20}{3}a^2b^2+7b^4\right)\left(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2\right)b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-2880\left(a^4-\frac{20}{3}a^2b^2+7}{}$
risch	$\frac{3b\cos(4dx+4c)}{32d\,a^4}-\frac{\cos(5dx+5c)}{80a^3d}-\frac{6ibc}{a^4d}+\frac{40ib^3c}{a^6d}-\frac{42ib^5c}{a^8d}+\frac{2b^2\left(3a^5\mathrm{e}^{3i(dx+c)}-10a^3b^2\mathrm{e}^{3i(dx+c)}+7a\,b^4\mathrm{e}^{3i(dx+c)}+\right.}{}$
norman	$\frac{\frac{\left(6b\,a^6-12a^5b^2-34a^4b^3+80b^4a^3+2b^5a^2-84a\,b^6+42b^7\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{a^7d(a-b)}+\frac{\left(18a^7b-72a^6b^2-30a^5b^3+444a^4b^4-474a^3b^5-264a^2b^6+630\right.}{a^7d\left(a^2-2ab+b^2\right)}}{}$

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^7*(1/5*cos(d*x+c)^5*a^4-3/4*b*cos(d*x+c)^4*a^3-2/3*cos(d*x+c)^3*a
^4+2*cos(d*x+c)^3*a^2*b^2+3*cos(d*x+c)^2*a^3*b-5*cos(d*x+c)^2*a*b^3+cos(d*x
+c)*a^4-12*cos(d*x+c)*a^2*b^2+15*cos(d*x+c)*b^4)-1/2*b^3*(a^4-2*a^2*b^2+b^4
)/a^8/(b+a*cos(d*x+c))^2+1/a^8*b^2*(3*a^4-10*a^2*b^2+7*b^4)/(b+a*cos(d*x+c)
)+1/a^8*b*(3*a^4-20*a^2*b^2+21*b^4)*ln(b+a*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.38

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{96\,a^7\cos\left(dx+c\right)^7-168\,a^6b\cos\left(dx+c\right)^6-1785\,a^4b^3+5520\,a^2b^5-3120\,b^7-16\left(20\,a^7-21\,a^5b^2\right)\cos\left(d\right.}{}$$

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^7*cos(d*x + c)^7 - 168*a^6*b*cos(d*x + c)^6 - 1785*a^4*b^3 + 5
520*a^2*b^5 - 3120*b^7 - 16*(20*a^7 - 21*a^5*b^2)*cos(d*x + c)^5 + 40*(20*a
^6*b - 21*a^4*b^3)*cos(d*x + c)^4 + 160*(3*a^7 - 20*a^5*b^2 + 21*a^3*b^4)*c
os(d*x + c)^3 + 15*(25*a^6*b - 592*a^4*b^3 + 800*a^2*b^5)*cos(d*x + c)^2 -
30*(71*a^5*b^2 - 48*a^3*b^4 - 128*a*b^6)*cos(d*x + c) - 480*(3*a^4*b^3 - 20
```

```
*a^2*b^5 + 21*b^7 + (3*a^6*b - 20*a^4*b^3 + 21*a^2*b^5)*cos(d*x + c)^2 + 2*
(3*a^5*b^2 - 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c))*log(a*cos(d*x + c) + b))/
(a^10*d*cos(d*x + c)^2 + 2*a^9*b*d*cos(d*x + c) + a^8*b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.98

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{30\left(5a^4b^3-18a^2b^5+13b^7+2\left(3a^5b^2-10a^3b^4+7ab^6\right)\cos(dx+c)\right)}{a^{10}\cos(dx+c)^2+2a^9b\cos(dx+c)+a^8b^2} - \frac{12a^4\cos(dx+c)^5-45a^3b\cos(dx+c)^4-40\left(a^4-3a^2b^2\right)\cos(dx+c)^3+60\left(3a\cdots\right.}{a^7}\cdots}{60\,d}$$

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(30*(5*a^4*b^3 - 18*a^2*b^5 + 13*b^7 + 2*(3*a^5*b^2 - 10*a^3*b^4 + 7*a
*b^6)*cos(d*x + c))/(a^10*cos(d*x + c)^2 + 2*a^9*b*cos(d*x + c) + a^8*b^2)
- (12*a^4*cos(d*x + c)^5 - 45*a^3*b*cos(d*x + c)^4 - 40*(a^4 - 3*a^2*b^2)*c
os(d*x + c)^3 + 60*(3*a^3*b - 5*a*b^3)*cos(d*x + c)^2 + 60*(a^4 - 12*a^2*b^
2 + 15*b^4)*cos(d*x + c))/a^7 + 60*(3*a^4*b - 20*a^2*b^3 + 21*b^5)*log(a*co
s(d*x + c) + b)/a^8)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1337 vs. 2(231) = 462.

Time = 0.42 (sec) , antiderivative size = 1337, normalized size of antiderivative = 5.59

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(3*a^5*b - 3*a^4*b^2 - 20*a^3*b^3 + 20*a^2*b^4 + 21*a*b^5 - 21*b^6
)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1)))/(a^9 - a^8*b) - 60*(3*a^4*b - 20*a^2*b^3 + 21*b^
5)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^8 - 30*(9*a^6*b +
 6*a^5*b^2 - 75*a^4*b^3 - 108*a^3*b^4 + 51*a^2*b^5 + 150*a*b^6 + 63*b^7 + 1
8*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^5*b^2*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 142*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 36*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 250*a^2*b^5*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 24*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 126*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^6*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 - 51*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 120*a^3*b^4*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*a^2*b^5*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 - 126*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
+ 63*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a + b + a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2*a^8
) + (64*a^5 - 411*a^4*b - 1200*a^3*b^2 + 2740*a^2*b^3 + 1800*a*b^4 - 2877*b
^5 - 320*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2415*a^4*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 5280*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 14900*a^2*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 7200*a*b^4*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 14385*b^5*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 640*a^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5910*a^4*b*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 7680*a^3*b^2*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 31000*a^2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 + 10800*a*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28770*b^5*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5910*a^4*b*(cos(d*x + c) - 1)^3
/(cos(d*x + c) + 1)^3 + 4320*a^3*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 - 31000*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 7200*a*b^4*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 28770*b^5*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 - 2415*a^4*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 - 720*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 14900*a^2*b^3*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1800*a*b^4*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 - 14385*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 +
 411*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 2740*a^2*b^3*(cos(d*
x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 2877*b^5*(cos(d*x + c) - 1)^5/(cos(d*x
 + c) + 1)^5)/(a^8*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 13.60 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.32

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\cos(c+d\,x)^3\left(\frac{2}{3\,a^3}-\frac{2\,b^2}{a^5}\right)}{d}-\frac{\cos(c+d\,x)^2\left(\frac{4\,b^3}{a^6}+\frac{3\,b\left(\frac{2}{a^3}-\frac{6\,b^2}{a^5}\right)}{2\,a}\right)}{d}+\frac{\cos(c+d\,x)\,\left(3\,a^4\,b^2-10\,a^2\,b^4+7\,b^6\right)+\frac{5\,a^4\,b^3-18\,a^2\,b^5+13\,b^7}{2\,a}}{d\,\left(a^9\cos(c+d\,x)^2+2\,a^8\,b\,\cos(c+d\,x)+a^7\,b^2\right)}-\frac{\cos(c+d\,x)\left(\frac{1}{a^3}+\frac{3\,b^4}{a^7}+\frac{3\,b^2\left(\frac{2}{a^3}-\frac{6\,b^2}{a^5}\right)}{a^2}-\frac{3\,b\left(\frac{8\,b^3}{a^6}+\frac{3\,b\left(\frac{2}{a^3}-\frac{6\,b^2}{a^5}\right)}{a}\right)}{a}\right)}{d}-\frac{\cos(c+d\,x)^5}{5\,a^3\,d}+\frac{3\,b\cos(c+d\,x)^4}{4\,a^4\,d}+\frac{\ln(b+a\,\cos(c+d\,x))\,\left(3\,a^4\,b-20\,a^2\,b^3+21\,b^5\right)}{a^8\,d}$$

```
[In] int(sin(c + d*x)^5/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (cos(c + d*x)^3*(2/(3*a^3) - (2*b^2)/a^5))/d - (cos(c + d*x)^2*((4*b^3)/a^6
 + (3*b*(2/a^3 - (6*b^2)/a^5))/(2*a)))/d + (cos(c + d*x)*(7*b^6 - 10*a^2*b^
4 + 3*a^4*b^2) + (13*b^7 - 18*a^2*b^5 + 5*a^4*b^3)/(2*a))/(d*(a^9*cos(c + d
*x)^2 + a^7*b^2 + 2*a^8*b*cos(c + d*x))) - (cos(c + d*x)*(1/a^3 + (3*b^4)/a
^7 + (3*b^2*(2/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b*(2/a^3 -
(6*b^2)/a^5))/a))/a))/d - cos(c + d*x)^5/(5*a^3*d) + (3*b*cos(c + d*x)^4)/(
4*a^4*d) + (log(b + a*cos(c + d*x))*(3*a^4*b + 21*b^5 - 20*a^2*b^3))/(a^8*d
)
```

3.223 $\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3} dx = -\frac{(a^2-6b^2)\cos(c+dx)}{a^5d} - \frac{3b\cos^2(c+dx)}{2a^4d} + \frac{\cos^3(c+dx)}{3a^3d} - \frac{b^3(a^2-b^2)}{2a^6d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2-5b^2)}{a^6d(b+a\cos(c+dx))} + \frac{b(3a^2-10b^2)\log(b+a\cos(c+dx))}{a^6d}$$

```
[Out] -(a^2-6*b^2)*cos(d*x+c)/a^5/d-3/2*b*cos(d*x+c)^2/a^4/d+1/3*cos(d*x+c)^3/a^3
/d-1/2*b^3*(a^2-b^2)/a^6/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2-5*b^2)/a^6/d/(b+a*
cos(d*x+c))+b*(3*a^2-10*b^2)*ln(b+a*cos(d*x+c))/a^6/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 908}

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3} dx = -\frac{3b\cos^2(c+dx)}{2a^4d} + \frac{\cos^3(c+dx)}{3a^3d} + \frac{b^2(3a^2-5b^2)}{a^6d(a\cos(c+dx)+b)} + \frac{b(3a^2-10b^2)\log(a\cos(c+dx)+b)}{a^6d} - \frac{b^3(a^2-b^2)}{2a^6d(a\cos(c+dx)+b)^2} - \frac{(a^2-6b^2)\cos(c+dx)}{a^5d}$$

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^2 - 6*b^2)*Cos[c + d*x])/(a^5*d)) - (3*b*Cos[c + d*x]^2)/(2*a^4*d) +
Cos[c + d*x]^3/(3*a^3*d) - (b^3*(a^2 - b^2))/(2*a^6*d*(b + a*Cos[c + d*x])^
2) + (b^2*(3*a^2 - 5*b^2))/(a^6*d*(b + a*Cos[c + d*x])) + (b*(3*a^2 - 10*b^
2)*Log[b + a*Cos[c + d*x]])/(a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{(-b-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^3d} \\
&= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^6d} \\
&= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{6b^2}{a^2}\right)+\frac{-a^2b^3+b^5}{(b-x)^3}+\frac{3a^2b^2-5b^4}{(b-x)^2}+\frac{-3a^2b+10b^3}{b-x}-3bx-x^2\right)dx, x, -a\cos(c+dx)\right)}{a^6d}
\end{aligned}
$$

$$= -\frac{(a^2-6b^2)\cos(c+dx)}{a^5d} - \frac{3b\cos^2(c+dx)}{2a^4d} + \frac{\cos^3(c+dx)}{3a^3d} - \frac{b^3(a^2-b^2)}{2a^6d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2-5b^2)}{a^6d(b+a\cos(c+dx))} + \frac{b(3a^2-10b^2)\log(b+a\cos(c+dx))}{a^6d}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.32

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{(b+a\cos(c+dx))\left(9a^4(b+2a\cos(c+dx)) - (b+a\cos(c+dx))^2\left(72a(a^2-8b^2)\cos(c+dx) + \frac{-9a^4b+48a^2\cdots}{(b+a\cos(c\cdots}\right.\right.}{9\cdots}$$

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(9*a^4*(b + 2*a*Cos[c + d*x]) - (b + a*Cos[c + d*x])^
2*(72*a*(a^2 - 8*b^2)*Cos[c + d*x] + (-9*a^4*b + 48*a^2*b^3 - 48*b^5)/(b +
a*Cos[c + d*x])^2 + (6*(3*a^4 - 48*a^2*b^2 + 80*b^4))/(b + a*Cos[c + d*x])
+ 72*a^2*b*Cos[2*(c + d*x)] - 8*a^3*Cos[3*(c + d*x)] + 96*(-3*a^2*b + 10*b^
3)*Log[b + a*Cos[c + d*x]]))*Sec[c + d*x]^3)/(96*a^6*d*(a + b*Sec[c + d*x])
^3)
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\frac{\cos(dx+c)^3a^2}{3}-\frac{3ab\cos(dx+c)^2}{2}-\cos(dx+c)a^2+6\cos(dx+c)b^2}{a^5}-\frac{b^3(a^2-b^2)}{2a^6(b+a\cos(dx+c))^2}+\frac{b^2(3a^2-5b^2)}{a^6(b+a\cos(dx+c))}+\frac{b(3a^2-10b^2)\ln(b+a\cos\cdots}{a^6}}{d}$
default	$\frac{\frac{\frac{\cos(dx+c)^3a^2}{3}-\frac{3ab\cos(dx+c)^2}{2}-\cos(dx+c)a^2+6\cos(dx+c)b^2}{a^5}-\frac{b^3(a^2-b^2)}{2a^6(b+a\cos(dx+c))^2}+\frac{b^2(3a^2-5b^2)}{a^6(b+a\cos(dx+c))}+\frac{b(3a^2-10b^2)\ln(b+a\cos\cdots}{a^6}}{d}$
parallelrisch	$72(a+b)(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2)\left(a^2-\frac{10b^2}{3}\right)b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)-72(a+b)(\cos(2dx+2c)a^2\cdots$
risch	$\frac{20ib^3c}{a^6d}-\frac{3ixb}{a^4}+\frac{e^{3i(dx+c)}}{24a^3d}-\frac{3b\,e^{2i(dx+c)}}{8a^4d}-\frac{3\,e^{i(dx+c)}}{8a^3d}+\frac{3\,e^{i(dx+c)}b^2}{a^5d}-\frac{3\,e^{-i(dx+c)}}{8a^3d}+\frac{3\,e^{-i(dx+c)}b^2}{a^5d}-\frac{3b\,e^{-\cdots}}{8\cdots}$
norman	$\frac{\frac{(6a^4b-12a^3b^2-14a^2b^3+40a\,b^4-20b^5)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{a^5d(a-b)}+\frac{-4a^6+62a^4b^2-118a^2b^4+60b^6}{3a^5d(a^2-2ab+b^2)}-\frac{2(2a^6-5a^5b+18a^4b^2-17a^3b^3-48a^2b^4+90a\cdots}{d\,a^5(a^2-2ab+b^2)}}{\left(1+\tan\left(\frac{d\cdots}{2}\right.\right.}$

[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/a^5*(1/3*cos(d*x+c)^3*a^2-3/2*a*b*cos(d*x+c)^2-cos(d*x+c)*a^2+6*cos(d*x+c)*b^2)-1/2*b^3*(a^2-b^2)/a^6/(b+a*cos(d*x+c))^2+1/a^6*b^2*(3*a^2-5*b^2)/(b+a*cos(d*x+c))+1/a^6*b*(3*a^2-10*b^2)*ln(b+a*cos(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.43

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{4\,a^5\cos\left(dx+c\right)^5 - 10\,a^4b\cos\left(dx+c\right)^4 + 39\,a^2b^3 - 54\,b^5 - 4\,(3\,a^5 - 10\,a^3b^2)\cos\left(dx+c\right)^3 - 3\,(5\,a^4b - 42$$

[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/12*(4*a^5*cos(d*x + c)^5 - 10*a^4*b*cos(d*x + c)^4 + 39*a^2*b^3 - 54*b^5 - 4*(3*a^5 - 10*a^3*b^2)*cos(d*x + c)^3 - 3*(5*a^4*b - 42*a^2*b^3)*cos(d*x + c)^2 + 6*(7*a^3*b^2 + 2*a*b^4)*cos(d*x + c) + 12*(3*a^2*b^3 - 10*b^5 + (3*a^4*b - 10*a^2*b^3)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 10*a*b^4)*cos(d*x + c))*log(a*cos(d*x + c) + b))/(a^8*d*cos(d*x + c)^2 + 2*a^7*b*d*cos(d*x + c) + a^6*b^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c))**3,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{3\,(5\,a^2b^3 - 9\,b^5 + 2\,(3\,a^3b^2 - 5\,ab^4)\cos(dx+c))}{a^8\cos(dx+c)^2 + 2\,a^7b\cos(dx+c) + a^6b^2} + \frac{2\,a^2\cos(dx+c)^3 - 9\,ab\cos(dx+c)^2 - 6\,(a^2 - 6\,b^2)\cos(dx+c)}{a^5} + \frac{6\,(3\,a^2b - 10\,b^3)\log(a\cos(dx+c)-}{a^6}}{6\,d}$$

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*a^2*b^3 - 9*b^5 + 2*(3*a^3*b^2 - 5*a*b^4)*cos(d*x + c))/(a^8*cos(
d*x + c)^2 + 2*a^7*b*cos(d*x + c) + a^6*b^2) + (2*a^2*cos(d*x + c)^3 - 9*a*
b*cos(d*x + c)^2 - 6*(a^2 - 6*b^2)*cos(d*x + c))/a^5 + 6*(3*a^2*b - 10*b^3)
*log(a*cos(d*x + c) + b)/a^6)/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{(3\,a^2b-10\,b^3)\log\left(|-a\cos\left(dx+c\right)-b|\right)}{a^6d} + \frac{5\,a^2b^3-9\,b^5+\frac{2\,(3\,a^3b^2d-5\,ab^4d)\cos(dx+c)}{d}}{2\left(a\cos\left(dx+c\right)+b\right)^2a^6d} + \frac{2\,a^6d^8\cos\left(dx+c\right)^3-9\,a^5bd^8\cos\left(dx+c\right)^2-6\,a^6d^8\cos\left(dx+c\right)+36\,a^4b^2d^8\cos\left(dx+c\right)}{6\,a^9d^9}$$

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*a^2*b - 10*b^3)*log(abs(-a*cos(d*x + c) - b))/(a^6*d) + 1/2*(5*a^2*b^3 -
 9*b^5 + 2*(3*a^3*b^2*d - 5*a*b^4*d)*cos(d*x + c)/d)/((a*cos(d*x + c) + b)^
2*a^6*d) + 1/6*(2*a^6*d^8*cos(d*x + c)^3 - 9*a^5*b*d^8*cos(d*x + c)^2 - 6*a
^6*d^8*cos(d*x + c) + 36*a^4*b^2*d^8*cos(d*x + c))/(a^9*d^9)
```

Mupad [B] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\cos\left(c+d\,x\right)^3}{3\,a^3\,d} - \frac{\cos\left(c+d\,x\right)\,\left(5\,b^4-3\,a^2\,b^2\right)+\frac{9\,b^5-5\,a^2\,b^3}{2\,a}}{d\,\left(a^7\cos\left(c+d\,x\right)^2+2\,a^6\,b\,\cos\left(c+d\,x\right)+a^5\,b^2\right)} - \frac{\cos\left(c+d\,x\right)\,\left(\frac{1}{a^3}-\frac{6\,b^2}{a^5}\right)}{d} - \frac{3\,b\cos\left(c+d\,x\right)^2}{2\,a^4\,d} + \frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(3\,a^2\,b-10\,b^3\right)}{a^6\,d}$$

```
[In] int(sin(c + d*x)^3/(a + b/cos(c + d*x))^3,x)
```

```
[Out] cos(c + d*x)^3/(3*a^3*d) - (cos(c + d*x)*(5*b^4 - 3*a^2*b^2) + (9*b^5 - 5*a
^2*b^3)/(2*a))/(d*(a^7*cos(c + d*x)^2 + a^5*b^2 + 2*a^6*b*cos(c + d*x))) -
(cos(c + d*x)*(1/a^3 - (6*b^2)/a^5))/d - (3*b*cos(c + d*x)^2)/(2*a^4*d) + (
log(b + a*cos(c + d*x))*(3*a^2*b - 10*b^3))/(a^6*d)
```

3.224 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3} dx = -\frac{\cos(c+dx)}{a^3d} - \frac{b^3}{2a^4d(b+a\cos(c+dx))^2} + \frac{3b^2}{a^4d(b+a\cos(c+dx))} + \frac{3b\log(b+a\cos(c+dx))}{a^4d}$$

```
[Out] -cos(d*x+c)/a^3/d-1/2*b^3/a^4/d/(b+a*cos(d*x+c))^2+3*b^2/a^4/d/(b+a*cos(d*x
+c))+3*b*ln(b+a*cos(d*x+c))/a^4/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2912, 12, 45}

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3} dx = -\frac{b^3}{2a^4d(a\cos(c+dx)+b)^2} + \frac{3b^2}{a^4d(a\cos(c+dx)+b)} + \frac{3b\log(a\cos(c+dx)+b)}{a^4d} - \frac{\cos(c+dx)}{a^3d}$$

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]/(a^3*d)) - b^3/(2*a^4*d*(b + a*Cos[c + d*x])^2) + (3*b^2)/(a
^4*d*(b + a*Cos[c + d*x])) + (3*b*Log[b + a*Cos[c + d*x]])/(a^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin(c+dx)}{(-b-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \frac{x^3}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^4d} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{b^3}{(b-x)^3}+\frac{3b^2}{(b-x)^2}-\frac{3b}{b-x}\right)dx, x, -a\cos(c+dx)\right)}{a^4d} \\
&= -\frac{\cos(c+dx)}{a^3d}-\frac{b^3}{2a^4d(b+a\cos(c+dx))^2} \\
&\quad +\frac{3b^2}{a^4d(b+a\cos(c+dx))}+\frac{3b\log(b+a\cos(c+dx))}{a^4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{-2a^3\cos^3(c+dx)+2a^2b\cos^2(c+dx)(-2+3\log(b+a\cos(c+dx)))+4ab^2\cos(c+dx)(1+3\log(b+a\cos \ldots}{2a^4d(b+a\cos(c+dx))^2}$$

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^3 + 2*a^2*b*Cos[c + d*x]^2*(-2 + 3*Log[b + a*Cos[c + d
*x]]) + 4*a*b^2*Cos[c + d*x]*(1 + 3*Log[b + a*Cos[c + d*x]]) + b^3*(5 + 6*L
og[b + a*Cos[c + d*x]]))/(2*a^4*d*(b + a*Cos[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{1}{a^{3}\sec(dx+c)}-\frac{3b\ln(\sec(dx+c))}{a^{4}}-\frac{b}{2a^{2}(a+b\sec(dx+c))^{2}}+\frac{3b\ln(a+b\sec(dx+c))}{a^{4}}-\frac{2b}{a^{3}(a+b\sec(dx+c))}}{d}$
default	$\frac{-\frac{1}{a^{3}\sec(dx+c)}-\frac{3b\ln(\sec(dx+c))}{a^{4}}-\frac{b}{2a^{2}(a+b\sec(dx+c))^{2}}+\frac{3b\ln(a+b\sec(dx+c))}{a^{4}}-\frac{2b}{a^{3}(a+b\sec(dx+c))}}{d}$
risch	$-\frac{3ixb}{a^{4}}-\frac{e^{i(dx+c)}}{2a^{3}d}-\frac{e^{-i(dx+c)}}{2a^{3}d}-\frac{6ibc}{a^{4}d}+\frac{2b^{2}\left(3a\,e^{3i(dx+c)}+5b\,e^{2i(dx+c)}+3\,e^{i(dx+c)}a\right)}{a^{4}\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^{2}d}+\frac{3b\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(\ldots}}{\ldots}\right)}{a^{4}d}$
norman	$\frac{\frac{-2a^{4}+10a^{2}b^{2}-6b^{4}}{d\,a^{3}\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(2a^{3}-6a^{2}b+12a\,b^{2}-6b^{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{d\,a^{3}(a-b)}-\frac{\left(-4a^{4}+8a^{3}b-18a\,b^{3}+12b^{4}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{d\,a^{3}\left(a^{2}-2ab+b^{2}\right)}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b\right)^{2}}-\frac{3b\ln\left(1+\tan\left(\frac{dx}{2}\ldots\right.\right.}{a^{4}d}$
parallelrisch	$\frac{6b(a-b)^{2}\left(\cos(2dx+2c)a^{2}+4\cos(dx+c)ab+a^{2}+2b^{2}\right)\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}(a-b)\right)-6b(a-b)^{2}\left(\cos(2dx+2c)a^{2}+4\cos(\ldots\right.}{2a^{4}d\ldots}$

```
[In] int(sin(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^3/sec(d*x+c)-3/a^4*b*ln(sec(d*x+c))-1/2*b/a^2/(a+b*sec(d*x+c))^2+
3/a^4*b*ln(a+b*sec(d*x+c))-2/a^3*b/(a+b*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.52

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{2\,a^3\cos\left(dx+c\right)^3+4\,a^2b\cos\left(dx+c\right)^2-4\,ab^2\cos\left(dx+c\right)-5\,b^3-6\,\left(a^2b\cos\left(dx+c\right)^2+2\,ab^2\cos\left(dx+\right.\right.}{2\,\left(a^6d\cos\left(dx+c\right)^2+2\,a^5bd\cos\left(dx+c\right)+a^4b^2d\right)}$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 4*a^2*b*cos(d*x + c)^2 - 4*a*b^2*cos(d*x + c)
- 5*b^3 - 6*(a^2*b*cos(d*x + c)^2 + 2*a*b^2*cos(d*x + c) + b^3)*log(a*cos(d
*x + c) + b))/(a^6*d*cos(d*x + c)^2 + 2*a^5*b*d*cos(d*x + c) + a^4*b^2*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sin\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{6\,ab^2\cos(dx+c)+5\,b^3}{a^6\cos(dx+c)^2+2\,a^5b\cos(dx+c)+a^4b^2}-\frac{2\,\cos(dx+c)}{a^3}+\frac{6\,b\log(a\cos(dx+c)+b)}{a^4}}{2\,d}$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((6*a*b^2*cos(d*x + c) + 5*b^3)/(a^6*cos(d*x + c)^2 + 2*a^5*b*cos(d*x +
 c) + a^4*b^2) - 2*cos(d*x + c)/a^3 + 6*b*log(a*cos(d*x + c) + b)/a^4)/d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\cos(dx+c)}{a^3d} + \frac{3\,b\log\left(\left|-a\cos\left(dx+c\right)-b\right|\right)}{a^4d} + \frac{6\,ab^2\cos\left(dx+c\right)+5\,b^3}{2\left(a\cos\left(dx+c\right)+b\right)^2a^4d}$$

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^3*d) + 3*b*log(abs(-a*cos(d*x + c) - b))/(a^4*d) + 1/2*(6*
a*b^2*cos(d*x + c) + 5*b^3)/((a*cos(d*x + c) + b)^2*a^4*d)
```

Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{3\,b^2\,\cos\left(c+d\,x\right)+\frac{5\,b^3}{2\,a}}{d\,\left(a^5\cos\left(c+d\,x\right)^2+2\,a^4\,b\,\cos\left(c+d\,x\right)+a^3\,b^2\right)} - \frac{\cos\left(c+d\,x\right)}{a^3\,d} + \frac{3\,b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{a^4\,d}$$

```
[In] int(sin(c + d*x)/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (3*b^2*cos(c + d*x) + (5*b^3)/(2*a))/(d*(a^5*cos(c + d*x)^2 + a^3*b^2 + 2*a
^4*b*cos(c + d*x))) - cos(c + d*x)/(a^3*d) + (3*b*log(b + a*cos(c + d*x)))/
(a^4*d)
```

3.225 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 163

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3} dx = -\frac{b^3}{2a^2(a^2-b^2)d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2-b^2)}{a^2(a^2-b^2)^2 d(b+a\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{2(a+b)^3 d} - \frac{\log(1+\cos(c+dx))}{2(a-b)^3 d} + \frac{b(3a^2+b^2)\log(b+a\cos(c+dx))}{(a^2-b^2)^3 d}$$

```
[Out] -1/2*b^3/a^2/(a^2-b^2)/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2
/d/(b+a*cos(d*x+c))+1/2*ln(1-cos(d*x+c))/(a+b)^3/d-1/2*ln(1+cos(d*x+c))/(a-
b)^3/d+b*(3*a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3957, 2916, 12, 1643}

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3} dx = \frac{b^2(3a^2-b^2)}{a^2d(a^2-b^2)^2(a\cos(c+dx)+b)} + \frac{b(3a^2+b^2)\log(a\cos(c+dx)+b)}{d(a^2-b^2)^3} - \frac{b^3}{2a^2d(a^2-b^2)(a\cos(c+dx)+b)^2} + \frac{\log(1-\cos(c+dx))}{2d(a+b)^3} - \frac{\log(\cos(c+dx)+1)}{2d(a-b)^3}$$

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*b^3/(a^2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 - b^2))/(
a^2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(2*(a + b
)^3*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)^3*d) + (b*(3*a^2 + b^2)*Log[b + a
*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos^2(c+dx)\cot(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{a\text{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-b+x)^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{2(a-b)^3(a-x)} - \frac{b^3}{(a-b)(a+b)(b-x)^3} + \frac{3a^2b^2-b^4}{(a-b)^2(a+b)^2(b-x)^2} - \frac{a^2b(3a^2+b^2)}{(a-b)^3(a+b)^3(b-x)} + \frac{a^2}{2(a+b)^3(a+x)}\right)\,dx, x,}{a^2d}$$

$$= -\frac{b^3}{2a^2(a^2-b^2)\,d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2-b^2)}{a^2(a^2-b^2)^2\,d(b+a\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{2(a+b)^3d} - \frac{\log(1+\cos(c+dx))}{2(a-b)^3d} + \frac{b(3a^2+b^2)\log(b+a\cos(c+dx))}{(a^2-b^2)^3\,d}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.25

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{(b+a\cos(c+dx))\left(\frac{b^3}{a^2(-a^2+b^2)} - \frac{2b^2(-3a^2+b^2)(b+a\cos(c+dx))}{a^2(a-b)^2(a+b)^2} + \frac{2(b+a\cos(c+dx))^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(-a+b)^3} + \frac{2b(3a^2+b^2)(b+a\cos(\cdots}{(\cdots}\right.}{2d(a+b\sec(c+dx))^3}$$

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(b^3/(a^2*(-a^2 + b^2)) - (2*b^2*(-3*a^2 + b^2)*(b +
a*Cos[c + d*x]))/(a^2*(a - b)^2*(a + b)^2) + (2*(b + a*Cos[c + d*x])^2*Log[
Cos[(c + d*x)/2]])/(-a + b)^3 + (2*b*(3*a^2 + b^2)*(b + a*Cos[c + d*x])^2*L
og[b + a*Cos[c + d*x]])/(a^2 - b^2)^3 + (2*(b + a*Cos[c + d*x])^2*Log[Sin[(
c + d*x)/2]])/(a + b)^3)*Sec[c + d*x]^3)/(2*d*(a + b*Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{b^3}{2a^2(a+b)(a-b)(b+a\cos(dx+c))^2}+\frac{b(3a^2+b^2)\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3}+\frac{b^2(3a^2-b^2)}{(a+b)^2(a-b)^2a^2(b+a\cos(dx+c))}-\frac{\ln(\cos(dx+c)+1)}{2(a-b)^3}+\frac{\ln(\cos(dx\cdots}{2(a+\cdots}}{d}$
default	$\frac{-\frac{b^3}{2a^2(a+b)(a-b)(b+a\cos(dx+c))^2}+\frac{b(3a^2+b^2)\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3}+\frac{b^2(3a^2-b^2)}{(a+b)^2(a-b)^2a^2(b+a\cos(dx+c))}-\frac{\ln(\cos(dx+c)+1)}{2(a-b)^3}+\frac{\ln(\cos(dx\cdots}{2(a+\cdots}}{d}$
parallelrisch	$\frac{3\left(a^2+\frac{b^2}{3}\right)(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2)b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)+(a-b)^3(\cos(2dx+2c)a^2+4\cos(dx\cdots}{(a-b)^3(a+b)^3d(\cos(2dx+2c)a^2+4\cdots}$
norman	$\frac{\frac{6ab^2}{d(a^4-2a^3b+2ab^3-b^4)}-\frac{2(3ab^2+b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d(a^4-2a^2b^2+b^4)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a^3+3a^2b+3ab^2+b^3)}+\frac{b(3a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}\cdots\right.\right.}{d(a^6-3a^4b^2+3a^2b^4-b^6)}$
risch	$\frac{ix}{a^3-3a^2b+3ab^2-b^3}+\frac{ic}{d(a^3-3a^2b+3ab^2-b^3)}-\frac{ix}{a^3+3a^2b+3ab^2+b^3}-\frac{ic}{d(a^3+3a^2b+3ab^2+b^3)}-\frac{6iba^2x}{a^6-3a^4b^2+3a^2b^4\cdots}$

```
[In] int(csc(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^2*b^3/(a+b)/(a-b)/(b+a*cos(d*x+c))^2+b*(3*a^2+b^2)/(a+b)^3/(a-b
)^3*ln(b+a*cos(d*x+c))+b^2*(3*a^2-b^2)/(a+b)^2/(a-b)^2/a^2/(b+a*cos(d*x+c))
-1/2/(a-b)^3*ln(cos(d*x+c)+1)+1/2/(a+b)^3*ln(cos(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(157) = 314$.

Time = 0.34 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.91

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{5\,a^4b^3 - 6\,a^2b^5 + b^7 + 2\,(3\,a^5b^2 - 4\,a^3b^4 + ab^6)\cos\left(dx+c\right) + 2\,\left(3\,a^4b^3 + a^2b^5 + \left(3\,a^6b + a^4b^3\right)\cos\left(dx+c\right)\right.}{}$$

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*a^4*b^3 - 6*a^2*b^5 + b^7 + 2*(3*a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*
x + c) + 2*(3*a^4*b^3 + a^2*b^5 + (3*a^6*b + a^4*b^3)*cos(d*x + c)^2 + 2*(3
*a^5*b^2 + a^3*b^4)*cos(d*x + c))*log(a*cos(d*x + c) + b) - (a^5*b^2 + 3*a^
4*b^3 + 3*a^3*b^4 + a^2*b^5 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*cos(d*x
 + c)^2 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*cos(d*x + c))*log(1/2
*cos(d*x + c) + 1/2) + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5 + (a^7 -
3*a^6*b + 3*a^5*b^2 - a^4*b^3)*cos(d*x + c)^2 + 2*(a^6*b - 3*a^5*b^2 + 3*a^
4*b^3 - a^3*b^4)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 3*a^8
*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c)^2 + 2*(a^9*b - 3*a^7*b^3 + 3*a^5
*b^5 - a^3*b^7)*d*cos(d*x + c) + (a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8
)*d)
```

Sympy [F]

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\csc\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.48

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{2\left(3a^2b+b^3\right)\log(a\cos(dx+c)+b)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{5a^2b^3-b^5+2\left(3a^3b^2-ab^4\right)\cos(dx+c)}{a^6b^2-2a^4b^4+a^2b^6+\left(a^8-2a^6b^2+a^4b^4\right)\cos(dx+c)^2+2\left(a^7b-2a^5b^3+a^3b^5\right)\cos(dx+c)} - \frac{\log(\cos(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3} \cdots}{2d}$$

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(3*a^2*b + b^3)*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) + (5*a^2*b^3 - b^5 + 2*(3*a^3*b^2 - a*b^4)*cos(d*x + c))/(a^6*b^2 -
 2*a^4*b^4 + a^2*b^6 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x + c)^2 + 2*(a^7*
b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)) - log(cos(d*x + c) + 1)/(a^3 - 3*a^2
*b + 3*a*b^2 - b^3) + log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(157) = 314.

Time = 0.39 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.77

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{2\left(3a^2b+b^3\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3a^2b+3ab^2+b^3} - \frac{9a^3b+15a^2b^2+3ab^3-3b^4+\frac{18a^3b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\cdots}{\cdots}}{2d}$$

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(3*a^2*b + b^3)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) + log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 + 3*a^2*
b + 3*a*b^2 + b^3) - (9*a^3*b + 15*a^2*b^2 + 3*a*b^3 - 3*b^4 + 18*a^3*b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^2*b^2*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 10*a*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^4*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 9*a^3*b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 9*a^2*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*a*b^3*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*(a + b +
 a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1))^2))/d
```

Mupad [B] (verification not implemented)

Time = 14.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)}{2\,d\,{\left(a+b\right)}^3} - \frac{\frac{\cos(c+d\,x)\,\left(b^4-3\,a^2\,b^2\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{b\,\left(b^4-5\,a^2\,b^2\right)}{2\,a^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2\cos\left(c+d\,x\right)^2+2\,a\,b\,\cos\left(c+d\,x\right)+b^2\right)} - \frac{\ln\left(\cos\left(c+d\,x\right)+1\right)}{2\,d\,{\left(a-b\right)}^3} - \frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(\frac{1}{2\,{\left(a+b\right)}^3}-\frac{1}{2\,{\left(a-b\right)}^3}\right)}{d}$$

```
[In] int(1/(sin(c + d*x)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] log(cos(c + d*x) - 1)/(2*d*(a + b)^3) - ((cos(c + d*x)*(b^4 - 3*a^2*b^2))/(
a*(a^4 + b^4 - 2*a^2*b^2)) + (b*(b^4 - 5*a^2*b^2))/(2*a^2*(a^4 + b^4 - 2*a^
2*b^2)))/(d*(b^2 + a^2*cos(c + d*x)^2 + 2*a*b*cos(c + d*x))) - log(cos(c +
d*x) + 1)/(2*d*(a - b)^3) - (log(b + a*cos(c + d*x))*(1/(2*(a + b)^3) - 1/(
2*(a - b)^3)))/d
```

3.226 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 229

$$\begin{aligned}\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx =& -\frac{b^3}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2}\\ &+\frac{b^2(3a^2+b^2)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}\\ &+\frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d}\\ &+\frac{(a-2b)\log(1-\cos(c+dx))}{4(a+b)^4 d}-\frac{(a+2b)\log(1+\cos(c+dx))}{4(a-b)^4 d}\\ &+\frac{b(3a^4+8a^2b^2+b^4)\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^4 d}\end{aligned}$$

```
[Out] -1/2*b^3/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2+b^2)/(a^2-b^2)^3/d/(b+
a*cos(d*x+c))+1/2*(b*(3*a^2+b^2)-a*(a^2+3*b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^
2-b^2)^3/d+1/4*(a-2*b)*ln(1-cos(d*x+c))/(a+b)^4/d-1/4*(a+2*b)*ln(1+cos(d*x+
c))/(a-b)^4/d+b*(3*a^4+8*a^2*b^2+b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

$= \{3957, 2800, 1661, 1643\}$

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{b^2(3a^2+b^2)}{d\,(a^2-b^2)^3\,(a\cos(c+dx)+b)} + \frac{\csc^2(c+dx)\,(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))}{2d\,(a^2-b^2)^3} - \frac{b^3}{2d\,(a^2-b^2)^2\,(a\cos(c+dx)+b)^2} + \frac{b(3a^4+8a^2b^2+b^4)\log(a\cos(c+dx)+b)}{d\,(a^2-b^2)^4} + \frac{(a-2b)\log(1-\cos(c+dx))}{4d(a+b)^4} - \frac{(a+2b)\log(\cos(c+dx)+1)}{4d(a-b)^4}$$

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*b^3/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 + b^2))/((a
^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*C
os[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((a - 2*b)*Log[1 - Cos[c
 + d*x]])/(4*(a + b)^4*d) - ((a + 2*b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*
d) + (b*(3*a^4 + 8*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d
)
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot^3(c+dx)}{(-b-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^3}{(-b+x)^3(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^3 d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\frac{a^4b^3(a^2+3b^2)}{(a^2-b^2)^3} - \frac{a^2b^2(3a^4+3a^2b^2-2b^4)x}{(a^2-b^2)^3} + \frac{a^4b(3a^2-7b^2)x^2}{(a^2-b^2)^3} + \frac{a^4(a^2+3b^2)x^3}{(a^2-b^2)^3}}{(-b+x)^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{2a^2d} \\
&= \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^3 d} \\
&\quad + \frac{\text{Subst}\left(\int \left(\frac{a^2(a+2b)}{2(a-b)^4(a-x)} - \frac{2a^2b^3}{(a^2-b^2)^2(b-x)^3} + \frac{2a^2b^2(3a^2+b^2)}{(a^2-b^2)^3(b-x)^2} - \frac{2a^2b(3a^4+8a^2b^2+b^4)}{(a^2-b^2)^4(b-x)} + \frac{a^2(a-2b)}{2(a+b)^4(a+x)}\right)\,dx, x, -\right.}{2a^2d} \\
&= -\frac{b^3}{2(a^2-b^2)^2 d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2+b^2)}{(a^2-b^2)^3 d(b+a\cos(c+dx))} \\
&\quad + \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^3 d} \\
&\quad + \frac{(a-2b)\log(1-\cos(c+dx))}{4(a+b)^4 d} - \frac{(a+2b)\log(1+\cos(c+dx))}{4(a-b)^4 d} \\
&\quad + \frac{b(3a^4+8a^2b^2+b^4)\log(b+a\cos(c+dx))}{(a^2-b^2)^4 d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.49 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx =& -\frac{2i(3a^4b+8a^2b^3+b^5)\,(c+dx)}{(a-b)^4(a+b)^4d} \\ &- \frac{i(-a-2b)\arctan(\tan(c+dx))}{2(-a+b)^4d} \\ &- \frac{i(a-2b)\arctan(\tan(c+dx))}{2(a+b)^4d} \\ &- \frac{b^3}{2(-a+b)^2(a+b)^2d(b+a\cos(c+dx))^2} \\ &- \frac{b^2(3a^2+b^2)}{(-a+b)^3(a+b)^3d(b+a\cos(c+dx))} \\ &- \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8(a+b)^3d} + \frac{(-a-2b)\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)}{4(-a+b)^4d} \\ &+ \frac{(3a^4b+8a^2b^3+b^5)\log(b+a\cos(c+dx))}{(-a^2+b^2)^4\,d} \\ &+ \frac{(a-2b)\log\left(\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}{4(a+b)^4d} - \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8(-a+b)^3d}\end{aligned}$$

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*I)*(3*a^4*b + 8*a^2*b^3 + b^5)*(c + d*x))/((a - b)^4*(a + b)^4*d) - ((
I/2)*(-a - 2*b)*ArcTan[Tan[c + d*x]])/((-a + b)^4*d) - ((I/2)*(a - 2*b)*Arc
Tan[Tan[c + d*x]])/((a + b)^4*d) - b^3/(2*(-a + b)^2*(a + b)^2*d*(b + a*Cos
[c + d*x])^2) - (b^2*(3*a^2 + b^2))/((-a + b)^3*(a + b)^3*d*(b + a*Cos[c +
d*x])) - Csc[(c + d*x)/2]^2/(8*(a + b)^3*d) + ((-a - 2*b)*Log[Cos[(c + d*x)
/2]^2])/(4*(-a + b)^4*d) + ((3*a^4*b + 8*a^2*b^3 + b^5)*Log[b + a*Cos[c + d
*x]])/((-a^2 + b^2)^4*d) + ((a - 2*b)*Log[Sin[(c + d*x)/2]^2])/(4*(a + b)^4
*d) - Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(a-2b)\ln(\cos(dx+c)-1)}{4(a+b)^4}-\frac{b^3}{2(a+b)^2(a-b)^2(b+a\cos(dx+c))^2}+\frac{b\left(3a^4+8a^2b^2+b^4\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}+\frac{}{(a+b)}}{d}$
default	$\frac{\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(a-2b)\ln(\cos(dx+c)-1)}{4(a+b)^4}-\frac{b^3}{2(a+b)^2(a-b)^2(b+a\cos(dx+c))^2}+\frac{b\left(3a^4+8a^2b^2+b^4\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}+\frac{}{(a+b)}}{d}$
norman	$\frac{-\frac{1}{8d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8d(a-b)}+\frac{\left(a^5+22a^3b^2+13a\,b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d\left(a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2a\,b^5+b^6\right)}-\frac{\left(2a^5+5a^4b+44a^3b^2+18a^2b^3+26a\,b^4+b^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4d\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}$
parallelrisch	$192(a^4+\frac{8}{3}a^2b^2+\frac{1}{3}b^4)\left(\cos(2dx+2c)a^2+4\cos(dx+c)ab+a^2+2b^2\right)b\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a-b)\right)+32(a-2b)(a-b)^4(\cos(2$
risch	Expression too large to display

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/(a+b)^3/(cos(d*x+c)-1)+1/4*(a-2*b)/(a+b)^4*ln(cos(d*x+c)-1)-1/2*b^
3/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2+b*(3*a^4+8*a^2*b^2+b^4)/(a+b)^4/(a-b)^
4*ln(b+a*cos(d*x+c))+b^2*(3*a^2+b^2)/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+1/4/(
a-b)^3/(cos(d*x+c)+1)+1/4/(a-b)^4*(-a-2*b)*ln(cos(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(221) = 442$.

Time = 0.45 (sec) , antiderivative size = 1071, normalized size of antiderivative = 4.68

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(16*a^4*b^3 - 8*a^2*b^5 - 8*b^7 - 2*(a^7 + 8*a^5*b^2 - 7*a^3*b^4 - 2*a
*b^6)*cos(d*x + c)^3 + 2*(a^6*b - 11*a^4*b^3 + 7*a^2*b^5 + 3*b^7)*cos(d*x +
 c)^2 + 2*(11*a^5*b^2 - 10*a^3*b^4 - a*b^6)*cos(d*x + c) + 4*(3*a^4*b^3 + 8
*a^2*b^5 + b^7 - (3*a^6*b + 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(3*a^5*
b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (3*a^6*b + 5*a^4*b^3 - 7*a^2*b^5
- b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c))*log
(a*cos(d*x + c) + b) - (a^5*b^2 + 6*a^4*b^3 + 14*a^3*b^4 + 16*a^2*b^5 + 9*a
*b^6 + 2*b^7 - (a^7 + 6*a^6*b + 14*a^5*b^2 + 16*a^4*b^3 + 9*a^3*b^4 + 2*a^2
*b^5)*cos(d*x + c)^4 - 2*(a^6*b + 6*a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a
^2*b^5 + 2*a*b^6)*cos(d*x + c)^3 + (a^7 + 6*a^6*b + 13*a^5*b^2 + 10*a^4*b^3
 - 5*a^3*b^4 - 14*a^2*b^5 - 9*a*b^6 - 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b + 6*
a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a^2*b^5 + 2*a*b^6)*cos(d*x + c))*log(
1/2*cos(d*x + c) + 1/2) + (a^5*b^2 - 6*a^4*b^3 + 14*a^3*b^4 - 16*a^2*b^5 +
9*a*b^6 - 2*b^7 - (a^7 - 6*a^6*b + 14*a^5*b^2 - 16*a^4*b^3 + 9*a^3*b^4 - 2*
```

```
a^2*b^5)*cos(d*x + c)^4 - 2*(a^6*b - 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 +
9*a^2*b^5 - 2*a*b^6)*cos(d*x + c)^3 + (a^7 - 6*a^6*b + 13*a^5*b^2 - 10*a^4*
b^3 - 5*a^3*b^4 + 14*a^2*b^5 - 9*a*b^6 + 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b -
 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 + 9*a^2*b^5 - 2*a*b^6)*cos(d*x + c))*l
og(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a
^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a
*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^
2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b
^7 + a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
 b^10)*d)
```

Sympy [F]

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\csc^3\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.90

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{4\left(3a^4b+8a^2b^3+b^5\right)\log(a\cos(dx+c)+b)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8} - \frac{(a+2b)\log(\cos(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4} + \frac{(a-2b)\log(\cos(dx+c)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} + \frac{}{a^6b^2-3a^4b^4+3a^2b^6-b^8-(a^8-3}}{}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(3*a^4*b + 8*a^2*b^3 + b^5)*log(a*cos(d*x + c) + b)/(a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + 2*b)*log(cos(d*x + c) + 1)/(a^4 - 4*
a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (a - 2*b)*log(cos(d*x + c) - 1)/(a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(8*a^2*b^3 + 4*b^5 - (a^5 + 9*a^3
*b^2 + 2*a*b^4)*cos(d*x + c)^3 + (a^4*b - 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^
2 + (11*a^3*b^2 + a*b^4)*cos(d*x + c))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b
^8 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^4 - 2*(a^7*b - 3*
a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4
- 4*a^2*b^6 + b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^
7)*cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(221) = 442$.

Time = 0.44 (sec) , antiderivative size = 800, normalized size of antiderivative = 3.49

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(abs
(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b
 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*(cos(d*x + c) + 1)) - 4*(9*a^6*b + 6*a^5*b^2 + 9*a^4*b^3 + 28*a^3*b^4 +
11*a^2*b^5 - 2*a*b^6 + 3*b^7 + 18*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 12*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 26*a^4*b^3*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 4*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 38*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^6*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 6*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 48*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 27*a^2
*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.65

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(\frac{3\,b}{4\,{\left(a+b\right)}^4}-\frac{1}{4\,{\left(a+b\right)}^3}+\frac{3\,b}{4\,{\left(a-b\right)}^4}+\frac{1}{4\,{\left(a-b\right)}^3}\right)}{d}$$
$$-\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3\,b}{4\,{\left(a+b\right)}^4}-\frac{1}{4\,{\left(a+b\right)}^3}\right)}{d}$$
$$-\frac{\frac{{\cos\left(c+d\,x\right)}^3\,\left(a^5+9\,a^3\,b^2+2\,a\,b^4\right)}{2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{{\cos\left(c+d\,x\right)}^2\,\left(-a^4\,b+10\,a^2\,b^3+3\,b^5\right)}{2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{2\,b\,\left(2\,a^2\,b^2+b^4\right)}{\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{a\,\cos\left(c+d\,x\right)\,\left(11\,a^2\,b^2+b^4\right)}{2\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\cos\left(c+d\,x\right)}^2\,\left(a^2-b^2\right)+b^2-a^2\,{\cos\left(c+d\,x\right)}^4+2\,a\,b\,\cos\left(c+d\,x\right)-2\,a\,b\,{\cos\left(c+d\,x\right)}^3\right)}$$
$$-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(a+2\,b\right)}{4\,d\,{\left(a-b\right)}^4}$$

```
[In] int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))^3),x)
```

```
[Out] (log(b + a*cos(c + d*x))*((3*b)/(4*(a + b)^4) - 1/(4*(a + b)^3) + (3*b)/(4*
(a - b)^4) + 1/(4*(a - b)^3)))/d - (log(cos(c + d*x) - 1)*((3*b)/(4*(a + b)
^4) - 1/(4*(a + b)^3)))/d - ((cos(c + d*x)^3*(2*a*b^4 + a^5 + 9*a^3*b^2))/(
2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (cos(c + d*x)^2*(3*b^5 - a^4*b + 1
0*a^2*b^3))/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*b*(b^4 + 2*a^2*b^2
))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (a*cos(c + d*x)*(b^4 + 11*a^2*b^
2))/(2*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c + d*x)^2*(a^2 - b^2)
 + b^2 - a^2*cos(c + d*x)^4 + 2*a*b*cos(c + d*x) - 2*a*b*cos(c + d*x)^3)) -
 (log(cos(c + d*x) + 1)*(a + 2*b))/(4*d*(a - b)^4)
```

3.227 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 313

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2b^3}{2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))^2}+\frac{3a^2b^2(a^2+b^2)}{\left(a^2-b^2\right)^4 d(b+a\cos(c+dx))} + \frac{(4b(3a^4+8a^2b^2+b^4)-3a(a^4+10a^2b^2+5b^4)\cos(c+dx))\csc^2(c+dx)}{8\left(a^2-b^2\right)^4 d} + \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)^3 d}+\frac{3a(a-3b)\log(1-\cos(c+dx))}{16(a+b)^5 d} - \frac{3a(a+3b)\log(1+\cos(c+dx))}{16(a-b)^5 d}+\frac{3a^2b(a^4+5a^2b^2+2b^4)\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^5 d}$$

```
[Out] -1/2*a^2*b^3/(a^2-b^2)^3/d/(b+a*cos(d*x+c))^2+3*a^2*b^2*(a^2+b^2)/(a^2-b^2)
^4/d/(b+a*cos(d*x+c))+1/8*(4*b*(3*a^4+8*a^2*b^2+b^4)-3*a*(a^4+10*a^2*b^2+5*
b^4)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^4/d+1/4*(b*(3*a^2+b^2)-a*(a^2+3*b^2
)*cos(d*x+c))*csc(d*x+c)^4/(a^2-b^2)^3/d+3/16*a*(a-3*b)*ln(1-cos(d*x+c))/(a
+b)^5/d-3/16*a*(a+3*b)*ln(1+cos(d*x+c))/(a-b)^5/d+3*a^2*b*(a^4+5*a^2*b^2+2*
b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^5/d
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2916, 12, 1661, 1643}

$$
\begin{aligned}
&\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx \\
&= \frac{3a^2b^2(a^2+b^2)}{d\,(a^2-b^2)^4\,(a\cos(c+dx)+b)}+\frac{\csc^4(c+dx)\,(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))}{4d\,(a^2-b^2)^3} \\
&\quad -\frac{a^2b^3}{2d\,(a^2-b^2)^3\,(a\cos(c+dx)+b)^2}+\frac{3a^2b(a^4+5a^2b^2+2b^4)\log(a\cos(c+dx)+b)}{d\,(a^2-b^2)^5} \\
&\quad +\frac{\csc^2(c+dx)\,(4b(3a^4+8a^2b^2+b^4)-3a(a^4+10a^2b^2+5b^4)\cos(c+dx))}{8d\,(a^2-b^2)^4} \\
&\quad +\frac{3a(a-3b)\log(1-\cos(c+dx))}{16d(a+b)^5}-\frac{3a(a+3b)\log(\cos(c+dx)+1)}{16d(a-b)^5}
\end{aligned}
$$

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*(a^2*b^3)/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x])^2) + (3*a^2*b^2*(a^2 +
 b^2))/((a^2 - b^2)^4*d*(b + a*Cos[c + d*x])) + ((4*b*(3*a^4 + 8*a^2*b^2 +
b^4) - 3*a*(a^4 + 10*a^2*b^2 + 5*b^4)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2
 - b^2)^4*d) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos[c + d*x])*Csc[c + d*
x]^4)/(4*(a^2 - b^2)^3*d) + (3*a*(a - 3*b)*Log[1 - Cos[c + d*x]])/(16*(a +
b)^5*d) - (3*a*(a + 3*b)*Log[1 + Cos[c + d*x]])/(16*(a - b)^5*d) + (3*a^2*b
*(a^4 + 5*a^2*b^2 + 2*b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
```

```
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx \\
&= \frac{a^5\text{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^2\text{Subst}\left(\int \frac{x^3}{(-b+x)^3(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)^3 d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\frac{a^4b^3\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3}-\frac{a^2b^2\left(3a^4-3a^2b^2-4b^4\right)x}{\left(a^2-b^2\right)^3}+\frac{a^4b\left(3a^2-23b^2\right)x^2}{\left(a^2-b^2\right)^3}+\frac{3a^4\left(a^2+3b^2\right)x^3}{\left(a^2-b^2\right)^3}}{(-b+x)^3(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{4d} \\
&= \frac{(4b(3a^4+8a^2b^2+b^4)-3a(a^4+10a^2b^2+5b^4)\cos(c+dx))\csc^2(c+dx)}{8\left(a^2-b^2\right)^4 d} \\
&\quad + \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)^3 d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\frac{a^4b^3\left(5a^4+34a^2b^2+9b^4\right)}{\left(a^2-b^2\right)^4}-\frac{3a^4b^2\left(5a^4+18a^2b^2-7b^4\right)x}{\left(a^2-b^2\right)^4}+\frac{a^4b\left(15a^4-26a^2b^2-37b^4\right)x^2}{\left(a^2-b^2\right)^4}+\frac{3a^4\left(a^4+10a^2b^2+5b^4\right)x^3}{\left(a^2-b^2\right)^4}}{(-b+x)^3(a^2-x^2)}\,dx, x, -a\,\mathrm{c}\right.}{8a^2d}
\end{aligned}
$$

$$= \frac{(4b(3a^4+8a^2b^2+b^4)-3a(a^4+10a^2b^2+5b^4)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^4 d}$$
$$+ \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)^3 d}$$
$$+ \frac{\text{Subst}\left(\int\left(\frac{3a^3(a+3b)}{2(a-b)^5(a-x)}-\frac{8a^4b^3}{(a^2-b^2)^3(b-x)^3}+\frac{24a^4b^2(a^2+b^2)}{(a^2-b^2)^4(b-x)^2}-\frac{24a^4b(a^4+5a^2b^2+2b^4)}{(a^2-b^2)^5(b-x)}+\frac{3a^3(a-3b)}{2(a+b)^5(a+x)}\right)dx, x,}{8a^2d}$$
$$= -\frac{a^2b^3}{2(a^2-b^2)^3 d(b+a\cos(c+dx))^2}+\frac{3a^2b^2(a^2+b^2)}{(a^2-b^2)^4 d(b+a\cos(c+dx))}$$
$$+ \frac{(4b(3a^4+8a^2b^2+b^4)-3a(a^4+10a^2b^2+5b^4)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^4 d}$$
$$+ \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)^3 d}$$
$$+ \frac{3a(a-3b)\log(1-\cos(c+dx))}{16(a+b)^5 d}-\frac{3a(a+3b)\log(1+\cos(c+dx))}{16(a-b)^5 d}$$
$$+ \frac{3a^2b(a^4+5a^2b^2+2b^4)\log(b+a\cos(c+dx))}{(a^2-b^2)^5 d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.51 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.58

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{(b+a\cos(c+dx))\left(\frac{32a^2b^3}{(-a+b)^3(a+b)^3}+\frac{192a^2(a-ib)(a+ib)b^2(b+a\cos(c+dx))}{(a-b)^4(a+b)^4}-\frac{384ia^2b(a^4+5a^2b^2+2b^4)(c+dx)(b+a\cos(c+dx))^2}{(a-b)^5(a+b)^5}\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((32*a^2*b^3)/((-a + b)^3*(a + b)^3) + (192*a^2*(a -
I*b)*(a + I*b)*b^2*(b + a*Cos[c + d*x]))/((a - b)^4*(a + b)^4) - ((384*I)*a
^2*b*(a^4 + 5*a^2*b^2 + 2*b^4)*(c + d*x)*(b + a*Cos[c + d*x])^2)/((a - b)^5
*(a + b)^5) - ((24*I)*a*(a - 3*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])
^2)/(a + b)^5 + ((24*I)*a*(a + 3*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x
])^2)/(a - b)^5 + (6*(-a + b)*(b + a*Cos[c + d*x])^2*Csc[(c + d*x)/2]^2)/(a
 + b)^4 - ((b + a*Cos[c + d*x])^2*Csc[(c + d*x)/2]^4)/(a + b)^3 - (12*a*(a
+ 3*b)*(b + a*Cos[c + d*x])^2*Log[Cos[(c + d*x)/2]^2])/(a - b)^5 + (192*a^2
*b*(a^4 + 5*a^2*b^2 + 2*b^4)*(b + a*Cos[c + d*x])^2*Log[b + a*Cos[c + d*x]]
)/(a^2 - b^2)^5 + (12*a*(a - 3*b)*(b + a*Cos[c + d*x])^2*Log[Sin[(c + d*x)/
2]^2])/(a + b)^5 + (6*(a + b)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2)/(a
 - b)^4 + ((b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^4)/(a - b)^3)*Sec[c + d*
x]^3)/(64*d*(a + b*Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-\frac{1}{16(a+b)^{3}(\cos(dx+c)-1)^{2}}-\frac{-3a+3b}{16(a+b)^{4}(\cos(dx+c)-1)}+\frac{3a(a-3b)\ln(\cos(dx+c)-1)}{16(a+b)^{5}}-\frac{b^{3}a^{2}}{2(a+b)^{3}(a-b)^{3}(b+a\cos(dx+c))^{2}}+\frac{3a^{2}b\left(a^{4}+5a\right.}{}}{}$
default	$\frac{-\frac{1}{16(a+b)^{3}(\cos(dx+c)-1)^{2}}-\frac{-3a+3b}{16(a+b)^{4}(\cos(dx+c)-1)}+\frac{3a(a-3b)\ln(\cos(dx+c)-1)}{16(a+b)^{5}}-\frac{b^{3}a^{2}}{2(a+b)^{3}(a-b)^{3}(b+a\cos(dx+c))^{2}}+\frac{3a^{2}b\left(a^{4}+5a\right.}{}}{}$
norman	$\frac{-\frac{1}{64d(a+b)}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{64d(a-b)}-\frac{(3a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{32d\left(a^{2}+2ab+b^{2}\right)}+\frac{(3a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{32d\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(15a^{7}+459a^{5}b^{2}+621b^{4}a^{3}+57a\,b^{6}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{32\left(a^{8}-2a^{7}b-2a^{6}b^{2}+6a^{5}b^{3}-6a^{3}b^{5}+2a^{2}b^{6}+2a\,b^{7}-b\right.}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-\right.}$
parallelrisch	$\frac{3b\,a^{2}\left(a^{4}+5a^{2}b^{2}+2b^{4}\right)\left(\cos(2dx+2c)a^{2}+4\cos(dx+c)ab+a^{2}+2b^{2}\right)\ln\left(-2a+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}(a-b)\right)+\frac{3a(a-3b)(a-b)^{5}\left(\cos(2dx\right.}{}}{}$
risch	Expression too large to display

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/16/(a+b)^3/(cos(d*x+c)-1)^2-1/16*(-3*a+3*b)/(a+b)^4/(cos(d*x+c)-1)+
3/16*a*(a-3*b)/(a+b)^5*ln(cos(d*x+c)-1)-1/2*b^3/(a+b)^3*a^2/(a-b)^3/(b+a*co
s(d*x+c))^2+3*a^2*b*(a^4+5*a^2*b^2+2*b^4)/(a-b)^5/(a+b)^5*ln(b+a*cos(d*x+c)
)+3*a^2*b^2*(a^2+b^2)/(a+b)^4/(a-b)^4/(b+a*cos(d*x+c))+1/16/(a-b)^3/(cos(d*
x+c)+1)^2-1/16*(-3*a-3*b)/(a-b)^4/(cos(d*x+c)+1)-3/16*a*(a+3*b)/(a-b)^5*ln(
cos(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1803 vs. $2(303) = 606$.

Time = 0.56 (sec) , antiderivative size = 1803, normalized size of antiderivative = 5.76

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(76*a^6*b^3 + 36*a^4*b^5 - 108*a^2*b^7 - 4*b^9 + 6*(a^9 + 17*a^7*b^2 -
 5*a^5*b^4 - 13*a^3*b^6)*cos(d*x + c)^5 - 12*(a^8*b - 9*a^6*b^3 - a^4*b^5 +
 9*a^2*b^7)*cos(d*x + c)^4 - 2*(5*a^9 + 98*a^7*b^2 - 12*a^5*b^4 - 98*a^3*b^
6 + 7*a*b^8)*cos(d*x + c)^3 + 8*(2*a^8*b - 25*a^6*b^3 - 3*a^4*b^5 + 25*a^2*
b^7 + b^9)*cos(d*x + c)^2 + 2*(55*a^7*b^2 - 9*a^5*b^4 - 51*a^3*b^6 + 5*a*b^
8)*cos(d*x + c) + 48*(a^6*b^3 + 5*a^4*b^5 + 2*a^2*b^7 + (a^8*b + 5*a^6*b^3
+ 2*a^4*b^5)*cos(d*x + c)^6 + 2*(a^7*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*x +
 c)^5 - (2*a^8*b + 9*a^6*b^3 - a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^4 - 4*(a^7
```

```
*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*x + c)^3 + (a^8*b + 3*a^6*b^3 - 8*a^4*b
^5 - 4*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*
x + c))*log(a*cos(d*x + c) + b) - 3*(a^7*b^2 + 8*a^6*b^3 + 25*a^5*b^4 + 40*
a^4*b^5 + 35*a^3*b^6 + 16*a^2*b^7 + 3*a*b^8 + (a^9 + 8*a^8*b + 25*a^7*b^2 +
 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5 + 3*a^3*b^6)*cos(d*x + c)^6 + 2*(a^8*
b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*b
^7)*cos(d*x + c)^5 - (2*a^9 + 16*a^8*b + 49*a^7*b^2 + 72*a^6*b^3 + 45*a^5*b
^4 - 8*a^4*b^5 - 29*a^3*b^6 - 16*a^2*b^7 - 3*a*b^8)*cos(d*x + c)^4 - 4*(a^8
*b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*
b^7)*cos(d*x + c)^3 + (a^9 + 8*a^8*b + 23*a^7*b^2 + 24*a^6*b^3 - 15*a^5*b^4
 - 64*a^4*b^5 - 67*a^3*b^6 - 32*a^2*b^7 - 6*a*b^8)*cos(d*x + c)^2 + 2*(a^8*
b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*b
^7)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(a^7*b^2 - 8*a^6*b^3 + 25
*a^5*b^4 - 40*a^4*b^5 + 35*a^3*b^6 - 16*a^2*b^7 + 3*a*b^8 + (a^9 - 8*a^8*b
+ 25*a^7*b^2 - 40*a^6*b^3 + 35*a^5*b^4 - 16*a^4*b^5 + 3*a^3*b^6)*cos(d*x +
c)^6 + 2*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^3
*b^6 + 3*a^2*b^7)*cos(d*x + c)^5 - (2*a^9 - 16*a^8*b + 49*a^7*b^2 - 72*a^6*
b^3 + 45*a^5*b^4 + 8*a^4*b^5 - 29*a^3*b^6 + 16*a^2*b^7 - 3*a*b^8)*cos(d*x +
 c)^4 - 4*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^
3*b^6 + 3*a^2*b^7)*cos(d*x + c)^3 + (a^9 - 8*a^8*b + 23*a^7*b^2 - 24*a^6*b^
3 - 15*a^5*b^4 + 64*a^4*b^5 - 67*a^3*b^6 + 32*a^2*b^7 - 6*a*b^8)*cos(d*x +
c)^2 + 2*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^3
*b^6 + 3*a^2*b^7)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^12 - 5*a^
10*b^2 + 10*a^8*b^4 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^6 +
 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*co
s(d*x + c)^5 - (2*a^12 - 11*a^10*b^2 + 25*a^8*b^4 - 30*a^6*b^6 + 20*a^4*b^8
 - 7*a^2*b^10 + b^12)*d*cos(d*x + c)^4 - 4*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5
 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3 + (a^12 - 7*a^10*b^2 +
 20*a^8*b^4 - 30*a^6*b^6 + 25*a^4*b^8 - 11*a^2*b^10 + 2*b^12)*d*cos(d*x + c
)^2 + 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)
*d*cos(d*x + c) + (a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b
^10 - b^12)*d)
```

Sympy [F]

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\csc^5{(c+dx)}}{(a+b\sec{(c+dx)})^3}\,dx$$

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**5/(a + b*sec(c + d*x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(303) = 606$.

Time = 0.23 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.26

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{48\,(a^6b+5\,a^4b^3+2\,a^2b^5)\log(a\cos(dx+c)+b)}{a^{10}-5\,a^8b^2+10\,a^6b^4-10\,a^4b^6+5\,a^2b^8-b^{10}} - \frac{3\,(a^2+3\,ab)\log(\cos(dx+c)+1)}{a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5} + \frac{3\,(a^2-3\,ab)\log(\cos(dx+c)-1)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5} + \frac{\ldots}{a^8b^2-4\,a^6b\ldots}}{\ldots}$$

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(48*(a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*log(a*cos(d*x + c) + b)/(a^10 - 5*
a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - 3*(a^2 + 3*a*b)*log
(cos(d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
) + 3*(a^2 - 3*a*b)*log(cos(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*
a^2*b^3 + 5*a*b^4 + b^5) + 2*(38*a^4*b^3 + 56*a^2*b^5 + 2*b^7 + 3*(a^7 + 18
*a^5*b^2 + 13*a^3*b^4)*cos(d*x + c)^5 - 6*(a^6*b - 8*a^4*b^3 - 9*a^2*b^5)*c
os(d*x + c)^4 - (5*a^7 + 103*a^5*b^2 + 91*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^3
 + 4*(2*a^6*b - 23*a^4*b^3 - 26*a^2*b^5 - b^7)*cos(d*x + c)^2 + (55*a^5*b^2
 + 46*a^3*b^4 - 5*a*b^6)*cos(d*x + c))/(a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4
*a^2*b^8 + b^10 + (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*cos(
d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x
+ c)^5 - (2*a^10 - 9*a^8*b^2 + 16*a^6*b^4 - 14*a^4*b^6 + 6*a^2*b^8 - b^10)*
cos(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(
d*x + c)^3 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4 - 16*a^4*b^6 + 9*a^2*b^8 - 2*b^
10)*cos(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*
cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(303) = 606$.

Time = 0.48 (sec) , antiderivative size = 1551, normalized size of antiderivative = 4.96

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 3*a*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) + 192*(a^6*b + 5*a^
4*b^3 + 2*a^2*b^5)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^10 - 5*a^8*b^2 + 10*a^6*b^4
```

```
- 10*a^4*b^6 + 5*a^2*b^8 - b^10) - (8*a^3*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 12*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b^3*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 3*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
)/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6) -
(a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 -
b^8 - 6*a^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a^7*b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 12*a^6*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 28*a^5*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 40*a^4*b^4*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 20*a^2*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a*b^7*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 2*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
6*a^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 163*a^7*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 - 257*a^6*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 339*a^5*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 203*a^4*b
^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 223*a^3*b^5*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 309*a^2*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 23*a*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 7*b^8*(cos(d*
x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 10*a^8*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 + 186*a^7*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 274*a^6*
b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 890*a^5*b^3*(cos(d*x + c) -
 1)^3/(cos(d*x + c) + 1)^3 - 894*a^4*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c)
 + 1)^3 + 478*a^3*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 374*a^2*b
^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 18*a*b^7*(cos(d*x + c) - 1)^
3/(cos(d*x + c) + 1)^3 - 4*b^8*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
9*a^8*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 45*a^7*b*(cos(d*x + c) -
1)^4/(cos(d*x + c) + 1)^4 + 45*a^6*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
 1)^4 - 63*a^5*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 117*a^4*b^4*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 9*a^3*b^5*(cos(d*x + c) - 1)^4/
(cos(d*x + c) + 1)^4 + 63*a^2*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 + 27*a*b^7*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((a^9 - a^8*b - 4*a^
7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 -
 b^9)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2)^2))/d
```

Mupad [B] (verification not implemented)

Time = 14.98 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.15

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3}{16\,{\left(a+b\right)}^3}-\frac{15\,b}{16\,{\left(a+b\right)}^4}+\frac{3\,b^2}{4\,{\left(a+b\right)}^5}\right)}{d}$$
$$+\frac{\frac{19\,a^4\,b^3+28\,a^2\,b^5+b^7}{4\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}-\frac{\cos(c+d\,x)^2\,\left(-2\,a^6\,b+23\,a^4\,b^3+26\,a^2\,b^5+b^7\right)}{2\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}+\frac{3\cos(c+d\,x)^4\,\left(-a^6\,b+8\,a^4\,b^3+9\,a^2\,b^5\right)}{4\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}+\frac{3\cos(c+d\,x}{8\,(a^8-4\,a}}{d\,\left({\cos\left(c+d\,x\right)}^2\,\left(a^2-2\,b^2\right)-{\cos\left(c+d\,x\right)}^4\,\left(2\,a^2-b^2\right)+b^2+a^2\,{\cos\left(c+d\,x\right)}^6}$$
$$-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{3\,b^2}{4\,{\left(a-b\right)}^5}+\frac{15\,b}{16\,{\left(a-b\right)}^4}+\frac{3}{16\,{\left(a-b\right)}^3}\right)}{d}$$
$$+\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(3\,a^6\,b+15\,a^4\,b^3+6\,a^2\,b^5\right)}{d\,\left(a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}\right)}$$

```
[In] int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))^3),x)
```

```
[Out] (log(cos(c + d*x) - 1)*(3/(16*(a + b)^3) - (15*b)/(16*(a + b)^4) + (3*b^2)/
(4*(a + b)^5)))/d + ((b^7 + 28*a^2*b^5 + 19*a^4*b^3)/(4*(a^8 + b^8 - 4*a^2*
b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (cos(c + d*x)^2*(b^7 - 2*a^6*b + 26*a^2*b^5
 + 23*a^4*b^3))/(2*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (3*co
s(c + d*x)^4*(9*a^2*b^5 - a^6*b + 8*a^4*b^3))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6
*a^4*b^4 - 4*a^6*b^2)) + (3*cos(c + d*x)^5*(a^7 + 13*a^3*b^4 + 18*a^5*b^2))
/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (a*cos(c + d*x)*(46*
a^2*b^4 - 5*b^6 + 55*a^4*b^2))/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^
6*b^2)) - (a*cos(c + d*x)^3*(5*a^6 - 7*b^6 + 91*a^2*b^4 + 103*a^4*b^2))/(8*
(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)))/(d*(cos(c + d*x)^2*(a^2 -
 2*b^2) - cos(c + d*x)^4*(2*a^2 - b^2) + b^2 + a^2*cos(c + d*x)^6 + 2*a*b*c
os(c + d*x) - 4*a*b*cos(c + d*x)^3 + 2*a*b*cos(c + d*x)^5)) - (log(cos(c +
d*x) + 1)*((3*b^2)/(4*(a - b)^5) + (15*b)/(16*(a - b)^4) + 3/(16*(a - b)^3)
))/d + (log(b + a*cos(c + d*x))*(3*a^6*b + 6*a^2*b^5 + 15*a^4*b^3))/(d*(a^1
0 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2))
```

3.228 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 539

$$
\begin{aligned}
\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx
&= \frac{(5a^6-180a^4b^2+600a^2b^4-448b^6)\,x}{16a^9} \\
&\quad - \frac{\sqrt{a-b}\,b\sqrt{a+b}(6a^4-47a^2b^2+56b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^9d} \\
&\quad + \frac{b(213a^4-985a^2b^2+840b^4)\sin(c+dx)}{30a^8d} \\
&\quad - \frac{(43a^4-244a^2b^2+224b^4)\cos(c+dx)\sin(c+dx)}{16a^7d} \\
&\quad + \frac{(45a^4-291a^2b^2+280b^4)\cos^2(c+dx)\sin(c+dx)}{30a^6bd} \\
&\quad - \frac{(24a^4-169a^2b^2+168b^4)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d} - \frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} \\
&\quad + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{(9a^4-60a^2b^2+56b^4)\cos^5(c+dx)\sin(c+dx)}{60a^3b^2d(b+a\cos(c+dx))^2} \\
&\quad + \frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2} - \frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2} \\
&\quad + \frac{(15a^4-110a^2b^2+112b^4)\cos^4(c+dx)\sin(c+dx)}{20a^4b^2d(b+a\cos(c+dx))}
\end{aligned}
$$

```
[Out] 1/16*(5*a^6-180*a^4*b^2+600*a^2*b^4-448*b^6)*x/a^9+1/30*b*(213*a^4-985*a^2*
b^2+840*b^4)*sin(d*x+c)/a^8/d-1/16*(43*a^4-244*a^2*b^2+224*b^4)*cos(d*x+c)*
sin(d*x+c)/a^7/d+1/30*(45*a^4-291*a^2*b^2+280*b^4)*cos(d*x+c)^2*sin(d*x+c)/
a^6/b/d-1/24*(24*a^4-169*a^2*b^2+168*b^4)*cos(d*x+c)^3*sin(d*x+c)/a^5/b^2/d
```

```
-1/4*cos(d*x+c)^4*sin(d*x+c)/b/d/(b+a*cos(d*x+c))^2+1/10*a*cos(d*x+c)^5*sin
(d*x+c)/b^2/d/(b+a*cos(d*x+c))^2+1/60*(9*a^4-60*a^2*b^2+56*b^4)*cos(d*x+c)^
5*sin(d*x+c)/a^3/b^2/d/(b+a*cos(d*x+c))^2+4/15*b*cos(d*x+c)^6*sin(d*x+c)/a^
2/d/(b+a*cos(d*x+c))^2-1/6*cos(d*x+c)^7*sin(d*x+c)/a/d/(b+a*cos(d*x+c))^2+1
/20*(15*a^4-110*a^2*b^2+112*b^4)*cos(d*x+c)^4*sin(d*x+c)/a^4/b^2/d/(b+a*cos
(d*x+c))-b*(6*a^4-47*a^2*b^2+56*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)
/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^9/d
```

Rubi [A] (verified)

Time = 2.75 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2975, 3126, 3128, 3102, 2814, 2738, 214}

$$\begin{aligned}
&\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx\\
&=\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{(15a^4-110a^2b^2+112b^4)\sin(c+dx)\cos^4(c+dx)}{20a^4b^2d(a\cos(c+dx)+b)}\\
&\quad-\frac{b\sqrt{a-b}\sqrt{a+b}(6a^4-47a^2b^2+56b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^9d}\\
&\quad+\frac{b(213a^4-985a^2b^2+840b^4)\sin(c+dx)}{30a^8d}\\
&\quad-\frac{(43a^4-244a^2b^2+224b^4)\sin(c+dx)\cos(c+dx)}{16a^7d}\\
&\quad+\frac{(45a^4-291a^2b^2+280b^4)\sin(c+dx)\cos^2(c+dx)}{30a^6bd}\\
&\quad-\frac{(24a^4-169a^2b^2+168b^4)\sin(c+dx)\cos^3(c+dx)}{24a^5b^2d}\\
&\quad+\frac{(9a^4-60a^2b^2+56b^4)\sin(c+dx)\cos^5(c+dx)}{60a^3b^2d(a\cos(c+dx)+b)^2}+\frac{x(5a^6-180a^4b^2+600a^2b^4-448b^6)}{16a^9}\\
&\quad+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}
\end{aligned}$$

```
[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b^6)*x)/(16*a^9) - (Sqrt[a - b]*b
*Sqrt[a + b]*(6*a^4 - 47*a^2*b^2 + 56*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^9*d) + (b*(213*a^4 - 985*a^2*b^2 + 840*b^4)*Sin[c +
 d*x])/(30*a^8*d) - ((43*a^4 - 244*a^2*b^2 + 224*b^4)*Cos[c + d*x]*Sin[c +
d*x])/(16*a^7*d) + ((45*a^4 - 291*a^2*b^2 + 280*b^4)*Cos[c + d*x]^2*Sin[c +
 d*x])/(30*a^6*b*d) - ((24*a^4 - 169*a^2*b^2 + 168*b^4)*Cos[c + d*x]^3*Sin[
c + d*x])/(24*a^5*b^2*d) - (Cos[c + d*x]^4*Sin[c + d*x])/(4*b*d*(b + a*Cos[
c + d*x])^2) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(10*b^2*d*(b + a*Cos[c + d*x
])^2) + ((9*a^4 - 60*a^2*b^2 + 56*b^4)*Cos[c + d*x]^5*Sin[c + d*x])/(60*a^3
```

```
*b^2*d*(b + a*Cos[c + d*x])^2) + (4*b*Cos[c + d*x]^6*Sin[c + d*x])/(15*a^2*
d*(b + a*Cos[c + d*x])^2) - (Cos[c + d*x]^7*Sin[c + d*x])/(6*a*d*(b + a*Cos
[c + d*x])^2) + ((15*a^4 - 110*a^2*b^2 + 112*b^4)*Cos[c + d*x]^4*Sin[c + d*
x])/(20*a^4*b^2*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos^3(c+dx)\sin^6(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2} - \frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2} - \frac{\int \frac{\cos^5(c+dx)(30(6a^4-35a^2b^2+32b^4)+30ab(3a^2-2b^2)\cos(c+dx)-20(12a^4-65a^2b^2+56b^4)\cos^2(c+dx))}{(-b-a\cos(c+dx))^3}\,dx}{600a^2b^2}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{(9a^4-60a^2b^2+56b^4)\cos^5(c+dx)\sin(c+dx)}{60a^3b^2d(b+a\cos(c+dx))^2} + \frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2} - \frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2} + \frac{\int \frac{\cos^4(c+dx)(100(9a^6-69a^4b^2+116a^2b^4-56b^6)+20ab(15a^4-31a^2b^2+16b^4)\cos(c+dx)-40(30a^6-215a^4b^2+353a^2b^4-168b^6)\cos}{(-b-a\cos(c+dx))^2}}{1200a^3b^2(a^2-b^2)}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{(9a^4-60a^2b^2+56b^4)\cos^5(c+dx)\sin(c+dx)}{60a^3b^2d(b+a\cos(c+dx))^2} + \frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2} - \frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2} + \frac{(15a^4-110a^2b^2+112b^4)\cos^4(c+dx)\sin(c+dx)}{20a^4b^2d(b+a\cos(c+dx))} - \frac{\int \frac{\cos^3(c+dx)\Big(240(a^2-b^2)^2(15a^4-110a^2b^2+112b^4)+40ab(15a^2-28b^2)(a^2-b^2)^2\cos(c+dx)-200(a^2-b^2)^2(24a^4-169a^2b^2+168b}{-b-a\cos(c+dx)}}{1200a^4b^2(a^2-b^2)^2}$$

$$= -\frac{(24a^4-169a^2b^2+168b^4)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d} - \frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{(9a^4-60a^2b^2+56b^4)\cos^5(c+dx)\sin(c+dx)}{60a^3b^2d(b+a\cos(c+dx))^2} + \frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2} - \frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2} + \frac{(15a^4-110a^2b^2+112b^4)\cos^4(c+dx)\sin(c+dx)}{20a^4b^2d(b+a\cos(c+dx))} + \frac{\int \frac{\cos^2(c+dx)\Big(600b(a^2-b^2)^2(24a^4-169a^2b^2+168b^4)+840ab^2(5a^2-8b^2)(a^2-b^2)^2\cos(c+dx)-480b(a^2-b^2)^2(45a^4-291a^2b^2+28}{-b-a\cos(c+dx)}}{4800a^5b^2(a^2-b^2)^2}$$

$$
\begin{aligned}
&= \frac{(45a^4-291a^2b^2+280b^4)\cos^2(c+dx)\sin(c+dx)}{30a^6bd} \\
&\quad - \frac{(24a^4-169a^2b^2+168b^4)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d} \\
&\quad - \frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} \\
&\quad + \frac{(9a^4-60a^2b^2+56b^4)\cos^5(c+dx)\sin(c+dx)}{60a^3b^2d(b+a\cos(c+dx))^2} + \frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2} \\
&\quad - \frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2} + \frac{(15a^4-110a^2b^2+112b^4)\cos^4(c+dx)\sin(c+dx)}{20a^4b^2d(b+a\cos(c+dx))} \\
&\quad - \frac{\int \frac{\cos(c+dx)\big(960b^2(a^2-b^2)^2(45a^4-291a^2b^2+280b^4)+120ab^3(207a^2-280b^2)(a^2-b^2)^2\cos(c+dx)-1800b^2(a^2-b^2)^2(43a^4-244a^2b}{-b-a\cos(c+dx)}}{14400a^6b^2(a^2-b^2)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(43a^4-244a^2b^2+224b^4)\cos(c+dx)\sin(c+dx)}{16a^7d} \\
&\quad + \frac{(45a^4-291a^2b^2+280b^4)\cos^2(c+dx)\sin(c+dx)}{30a^6bd} \\
&\quad - \frac{(24a^4-169a^2b^2+168b^4)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d} \\
&\quad - \frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} \\
&\quad + \frac{(9a^4-60a^2b^2+56b^4)\cos^5(c+dx)\sin(c+dx)}{60a^3b^2d(b+a\cos(c+dx))^2} + \frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2} \\
&\quad - \frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2} + \frac{(15a^4-110a^2b^2+112b^4)\cos^4(c+dx)\sin(c+dx)}{20a^4b^2d(b+a\cos(c+dx))} \\
&\quad + \frac{\int \frac{1800b^3(a^2-b^2)^2(43a^4-244a^2b^2+224b^4)-120ab^2(a^2-b^2)^2(75a^4-996a^2b^2+1120b^4)\cos(c+dx)-960b^3(a^2-b^2)^2(213a^4-985a^2b^2}{-b-a\cos(c+dx)}}{28800a^7b^2(a^2-b^2)^2}
\end{aligned}
$$

$$= \frac{b(213a^4 - 985a^2b^2 + 840b^4)\sin(c+dx)}{30a^8d} - \frac{(43a^4 - 244a^2b^2 + 224b^4)\cos(c+dx)\sin(c+dx)}{16a^7d} + \frac{(45a^4 - 291a^2b^2 + 280b^4)\cos^2(c+dx)\sin(c+dx)}{30a^6bd} - \frac{(24a^4 - 169a^2b^2 + 168b^4)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d} - \frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{(9a^4 - 60a^2b^2 + 56b^4)\cos^5(c+dx)\sin(c+dx)}{60a^3b^2d(b+a\cos(c+dx))^2} + \frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2} - \frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2} + \frac{(15a^4 - 110a^2b^2 + 112b^4)\cos^4(c+dx)\sin(c+dx)}{20a^4b^2d(b+a\cos(c+dx))} - \frac{\int \frac{-1800ab^3(a^2-b^2)^2(43a^4-244a^2b^2+224b^4)+1800b^2(a^2-b^2)^2(5a^6-180a^4b^2+600a^2b^4-448b^6)\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{28800a^8b^2(a^2-b^2)^2}$$

$$= \frac{(5a^6 - 180a^4b^2 + 600a^2b^4 - 448b^6)\,x}{16a^9} + \frac{b(213a^4 - 985a^2b^2 + 840b^4)\sin(c+dx)}{30a^8d} - \frac{(43a^4 - 244a^2b^2 + 224b^4)\cos(c+dx)\sin(c+dx)}{16a^7d} + \frac{(45a^4 - 291a^2b^2 + 280b^4)\cos^2(c+dx)\sin(c+dx)}{30a^6bd} - \frac{(24a^4 - 169a^2b^2 + 168b^4)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d} - \frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{(9a^4 - 60a^2b^2 + 56b^4)\cos^5(c+dx)\sin(c+dx)}{60a^3b^2d(b+a\cos(c+dx))^2} + \frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2} - \frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2} + \frac{(15a^4 - 110a^2b^2 + 112b^4)\cos^4(c+dx)\sin(c+dx)}{20a^4b^2d(b+a\cos(c+dx))} + \frac{(b(a^2-b^2)(6a^4 - 47a^2b^2 + 56b^4))\int \frac{1}{-b-a\cos(c+dx)}\,dx}{2a^9}$$

$$= \frac{(5a^6-180a^4b^2+600a^2b^4-448b^6)\,x}{16a^9}+\frac{b(213a^4-985a^2b^2+840b^4)\sin(c+dx)}{30a^8d}$$
$$-\frac{(43a^4-244a^2b^2+224b^4)\cos(c+dx)\sin(c+dx)}{16a^7d}$$
$$+\frac{(45a^4-291a^2b^2+280b^4)\cos^2(c+dx)\sin(c+dx)}{30a^6bd}$$
$$-\frac{(24a^4-169a^2b^2+168b^4)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d}-\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2}$$
$$+\frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2}+\frac{(9a^4-60a^2b^2+56b^4)\cos^5(c+dx)\sin(c+dx)}{60a^3b^2d(b+a\cos(c+dx))^2}$$
$$+\frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2}-\frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2}$$
$$+\frac{(15a^4-110a^2b^2+112b^4)\cos^4(c+dx)\sin(c+dx)}{20a^4b^2d(b+a\cos(c+dx))}$$
$$+\frac{(b(a^2-b^2)\,(6a^4-47a^2b^2+56b^4))\,\mathrm{Subst}\left(\int\frac{1}{-a-b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^9d}$$

$$= \frac{(5a^6-180a^4b^2+600a^2b^4-448b^6)\,x}{16a^9}$$
$$-\frac{\sqrt{a-b}b\sqrt{a+b}(6a^4-47a^2b^2+56b^4)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^9d}$$
$$+\frac{b(213a^4-985a^2b^2+840b^4)\sin(c+dx)}{30a^8d}$$
$$-\frac{(43a^4-244a^2b^2+224b^4)\cos(c+dx)\sin(c+dx)}{16a^7d}$$
$$+\frac{(45a^4-291a^2b^2+280b^4)\cos^2(c+dx)\sin(c+dx)}{30a^6bd}$$
$$-\frac{(24a^4-169a^2b^2+168b^4)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d}-\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2}$$
$$+\frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2}+\frac{(9a^4-60a^2b^2+56b^4)\cos^5(c+dx)\sin(c+dx)}{60a^3b^2d(b+a\cos(c+dx))^2}$$
$$+\frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2}-\frac{\cos^7(c+dx)\sin(c+dx)}{6ad(b+a\cos(c+dx))^2}$$
$$+\frac{(15a^4-110a^2b^2+112b^4)\cos^4(c+dx)\sin(c+dx)}{20a^4b^2d(b+a\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 9.15 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.11

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-7680b(-a^2+b^2)^3\,(6a^4-47a^2b^2+56b^4)\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2+2(a^2-b^2)^{5/2}}{}$$

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-7680*b*(-a^2 + b^2)^3*(6*a^4 - 47*a^2*b^2 + 56*b^4)*ArcTanh[((-a + b)*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2 + 2*(a^2 - b^2)^(5/2
)*(600*a^8*c - 20400*a^6*b^2*c + 28800*a^4*b^4*c + 90240*a^2*b^6*c - 107520
*b^8*c + 600*a^8*d*x - 20400*a^6*b^2*d*x + 28800*a^4*b^4*d*x + 90240*a^2*b^
6*d*x - 107520*b^8*d*x + 480*a*b*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b
^6)*(c + d*x)*Cos[c + d*x] + 120*a^2*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 4
48*b^6)*(c + d*x)*Cos[2*(c + d*x)] + 2640*a^7*b*Sin[c + d*x] + 16160*a^5*b^
3*Sin[c + d*x] - 117120*a^3*b^5*Sin[c + d*x] + 107520*a*b^7*Sin[c + d*x] -
405*a^8*Sin[2*(c + d*x)] + 24600*a^6*b^2*Sin[2*(c + d*x)] - 99040*a^4*b^4*S
in[2*(c + d*x)] + 80640*a^2*b^6*Sin[2*(c + d*x)] + 2436*a^7*b*Sin[3*(c + d*
x)] - 10880*a^5*b^3*Sin[3*(c + d*x)] + 8960*a^3*b^5*Sin[3*(c + d*x)] - 140*
a^8*Sin[4*(c + d*x)] + 1164*a^6*b^2*Sin[4*(c + d*x)] - 1120*a^4*b^4*Sin[4*(
c + d*x)] - 188*a^7*b*Sin[5*(c + d*x)] + 224*a^5*b^3*Sin[5*(c + d*x)] + 35*
a^8*Sin[6*(c + d*x)] - 56*a^6*b^2*Sin[6*(c + d*x)] + 16*a^7*b*Sin[7*(c + d*
x)] - 5*a^8*Sin[8*(c + d*x)]))/(7680*a^9*(a - b)^2*(a + b)^2*Sqrt[a^2 - b^2
]*d*(b + a*Cos[c + d*x])^2)
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.08

method	result
derivativedivides	$2\left(\left(\frac{5}{16}a^6+3a^5b-\frac{21}{4}a^4b^2-20a^3b^3+\frac{15}{2}a^2b^4+21ab^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(19a^5b-\frac{87}{4}a^4b^2+\frac{45}{2}a^2b^4+105ab^5+\frac{85}{48}a^6-\frac{340}{3}a^3b^3\right)\tan\left(\frac{dx}{2}\right.\right.$
default	$2\left(\left(\frac{5}{16}a^6+3a^5b-\frac{21}{4}a^4b^2-20a^3b^3+\frac{15}{2}a^2b^4+21ab^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+\left(19a^5b-\frac{87}{4}a^4b^2+\frac{45}{2}a^2b^4+105ab^5+\frac{85}{48}a^6-\frac{340}{3}a^3b^3\right)\tan\left(\frac{dx}{2}\right.\right.$
risch	$\frac{5x}{16a^3}-\frac{45x\,b^2}{4a^5}+\frac{75x\,b^4}{2a^7}-\frac{28x\,b^6}{a^9}-\frac{\sin(6dx+6c)}{192a^3d}+\frac{3\sin(4dx+4c)}{64d\,a^3}+\frac{3b\sin(5dx+5c)}{80a^4d}-\frac{3\sin(4dx+4c)b^2}{16d\,a^5}+15$

[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(2/a^9*(((5/16*a^6+3*a^5*b-21/4*a^4*b^2-20*a^3*b^3+15/2*a^2*b^4+21*a*b^5)*tan(1/2*d*x+1/2*c)^11+(19*a^5*b-87/4*a^4*b^2+45/2*a^2*b^4+105*a*b^5+85/48*a^6-340/3*a^3*b^3)*tan(1/2*d*x+1/2*c)^9+(258/5*a^5*b-33/2*a^4*b^2-240*a^3*b^3+15*a^2*b^4+210*a*b^5+33/8*a^6)*tan(1/2*d*x+1/2*c)^7+(-33/8*a^6+33/2*a^4*b^2-15*a^2*b^4+258/5*a^5*b-240*a^3*b^3+210*a*b^5)*tan(1/2*d*x+1/2*c)^5+(19*a^5*b+87/4*a^4*b^2-340/3*a^3*b^3-45/2*a^2*b^4+105*a*b^5-85/48*a^6)*tan(1/2*d*x+1/2*c)^3+(3*a^5*b-20*a^3*b^3+21*a*b^5-5/16*a^6+21/4*a^4*b^2-15/2*a^2*b^4)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^6+1/16*(5*a^6-180*a^4*b^2+600*a^2*b^4-448*b^6)*arctan(tan(1/2*d*x+1/2*c)))+2*b*(a+b)*(a-b)/a^9*(((5/2*a^3*b^2-7*a*b^4-3*a^4*b+15/2*a^2*b^3)*tan(1/2*d*x+1/2*c)^3+(5/2*a^3*b^2-7*a*b^4+3*a^4*b-15/2*a^2*b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-1/2*(6*a^4-47*a^2*b^2+56*b^4)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 1057, normalized size of antiderivative = 1.96

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] [1/240*(15*(5*a^8 - 180*a^6*b^2 + 600*a^4*b^4 - 448*a^2*b^6)*d*x*cos(d*x + c)^2 + 30*(5*a^7*b - 180*a^5*b^3 + 600*a^3*b^5 - 448*a*b^7)*d*x*cos(d*x + c) + 15*(5*a^6*b^2 - 180*a^4*b^4 + 600*a^2*b^6 - 448*b^8)*d*x + 60*(6*a^4*b^3 - 47*a^2*b^5 + 56*b^7 + (6*a^6*b - 47*a^4*b^3 + 56*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 47*a^3*b^4 + 56*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log

```
((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*
cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*c
os(d*x + c) + b^2)) - (40*a^8*cos(d*x + c)^7 - 64*a^7*b*cos(d*x + c)^6 - 17
04*a^5*b^3 + 7880*a^3*b^5 - 6720*a*b^7 - 2*(65*a^8 - 56*a^6*b^2)*cos(d*x +
c)^5 + 4*(67*a^7*b - 56*a^5*b^3)*cos(d*x + c)^4 + (165*a^8 - 694*a^6*b^2 +
560*a^4*b^4)*cos(d*x + c)^3 - 2*(387*a^7*b - 1444*a^5*b^3 + 1120*a^3*b^5)*c
os(d*x + c)^2 - (2763*a^6*b^2 - 12100*a^4*b^4 + 10080*a^2*b^6)*cos(d*x + c)
)*sin(d*x + c))/(a^11*d*cos(d*x + c)^2 + 2*a^10*b*d*cos(d*x + c) + a^9*b^2*
d), 1/240*(15*(5*a^8 - 180*a^6*b^2 + 600*a^4*b^4 - 448*a^2*b^6)*d*x*cos(d*x
 + c)^2 + 30*(5*a^7*b - 180*a^5*b^3 + 600*a^3*b^5 - 448*a*b^7)*d*x*cos(d*x
+ c) + 15*(5*a^6*b^2 - 180*a^4*b^4 + 600*a^2*b^6 - 448*b^8)*d*x - 120*(6*a^
4*b^3 - 47*a^2*b^5 + 56*b^7 + (6*a^6*b - 47*a^4*b^3 + 56*a^2*b^5)*cos(d*x +
 c)^2 + 2*(6*a^5*b^2 - 47*a^3*b^4 + 56*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2
)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))
 - (40*a^8*cos(d*x + c)^7 - 64*a^7*b*cos(d*x + c)^6 - 1704*a^5*b^3 + 7880*a
^3*b^5 - 6720*a*b^7 - 2*(65*a^8 - 56*a^6*b^2)*cos(d*x + c)^5 + 4*(67*a^7*b
- 56*a^5*b^3)*cos(d*x + c)^4 + (165*a^8 - 694*a^6*b^2 + 560*a^4*b^4)*cos(d*
x + c)^3 - 2*(387*a^7*b - 1444*a^5*b^3 + 1120*a^3*b^5)*cos(d*x + c)^2 - (27
63*a^6*b^2 - 12100*a^4*b^4 + 10080*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/(a^
11*d*cos(d*x + c)^2 + 2*a^10*b*d*cos(d*x + c) + a^9*b^2*d)]
```

Sympy [F]

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**6/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(508) = 1016$.

Time = 0.49 (sec) , antiderivative size = 1030, normalized size of antiderivative = 1.91

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b^6)*(d*x + c)/a^9 - 240
*(6*a^6*b - 53*a^4*b^3 + 103*a^2*b^5 - 56*b^7)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^9) - 240*(6*a^5*b^2*tan(1/2*d
*x + 1/2*c)^3 - 5*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 21*a^3*b^4*tan(1/2*d*x +
 1/2*c)^3 + 19*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 15*a*b^6*tan(1/2*d*x + 1/2*
c)^3 - 14*b^7*tan(1/2*d*x + 1/2*c)^3 - 6*a^5*b^2*tan(1/2*d*x + 1/2*c) - 5*a
^4*b^3*tan(1/2*d*x + 1/2*c) + 21*a^3*b^4*tan(1/2*d*x + 1/2*c) + 19*a^2*b^5*
tan(1/2*d*x + 1/2*c) - 15*a*b^6*tan(1/2*d*x + 1/2*c) - 14*b^7*tan(1/2*d*x +
 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2*a
^8) + 2*(75*a^5*tan(1/2*d*x + 1/2*c)^11 + 720*a^4*b*tan(1/2*d*x + 1/2*c)^11
 - 1260*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 - 4800*a^2*b^3*tan(1/2*d*x + 1/2*c)
^11 + 1800*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 5040*b^5*tan(1/2*d*x + 1/2*c)^11
 + 425*a^5*tan(1/2*d*x + 1/2*c)^9 + 4560*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 522
0*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 27200*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 + 5
400*a*b^4*tan(1/2*d*x + 1/2*c)^9 + 25200*b^5*tan(1/2*d*x + 1/2*c)^9 + 990*a
^5*tan(1/2*d*x + 1/2*c)^7 + 12384*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 3960*a^3*b
^2*tan(1/2*d*x + 1/2*c)^7 - 57600*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 3600*a*b
^4*tan(1/2*d*x + 1/2*c)^7 + 50400*b^5*tan(1/2*d*x + 1/2*c)^7 - 990*a^5*tan(
1/2*d*x + 1/2*c)^5 + 12384*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 3960*a^3*b^2*tan(
1/2*d*x + 1/2*c)^5 - 57600*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 3600*a*b^4*tan(
1/2*d*x + 1/2*c)^5 + 50400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425*a^5*tan(1/2*d*x
 + 1/2*c)^3 + 4560*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 5220*a^3*b^2*tan(1/2*d*x
+ 1/2*c)^3 - 27200*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 5400*a*b^4*tan(1/2*d*x
+ 1/2*c)^3 + 25200*b^5*tan(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*d*x + 1/2*c)
 + 720*a^4*b*tan(1/2*d*x + 1/2*c) + 1260*a^3*b^2*tan(1/2*d*x + 1/2*c) - 480
0*a^2*b^3*tan(1/2*d*x + 1/2*c) - 1800*a*b^4*tan(1/2*d*x + 1/2*c) + 5040*b^5
*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^8))/d
```

Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 3975, normalized size of antiderivative = 7.37

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int(sin(c + d*x)^6/(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(10080*a*b^6 + 454*a^6*b - 55*a^7 + 9408*b^7 - 9688*
a^2*b^5 - 12212*a^3*b^4 + 608*a^4*b^3 + 2969*a^5*b^2))/(24*a^8) + (tan(c/2
+ (d*x)/2)^13*(454*a^6*b - 10080*a*b^6 + 55*a^7 + 9408*b^7 - 9688*a^2*b^5 +
 12212*a^3*b^4 + 608*a^4*b^3 - 2969*a^5*b^2))/(24*a^8) + (tan(c/2 + (d*x)/2
)^5*(90720*a*b^6 + 2154*a^6*b - 215*a^7 + 141120*b^7 - 163240*a^2*b^5 - 107
220*a^3*b^4 + 32224*a^4*b^3 + 22673*a^5*b^2))/(120*a^8) + (tan(c/2 + (d*x)/
2)^11*(2154*a^6*b - 90720*a*b^6 + 215*a^7 + 141120*b^7 - 163240*a^2*b^5 + 1
07220*a^3*b^4 + 32224*a^4*b^3 - 22673*a^5*b^2))/(120*a^8) + (tan(c/2 + (d*x
)/2)^7*(50400*a*b^6 - 4994*a^6*b + 2545*a^7 + 235200*b^7 - 287000*a^2*b^5 -
 58820*a^3*b^4 + 74752*a^4*b^3 + 11173*a^5*b^2))/(120*a^8) - (tan(c/2 + (d*
x)/2)^9*(50400*a*b^6 + 4994*a^6*b + 2545*a^7 - 235200*b^7 + 287000*a^2*b^5
- 58820*a^3*b^4 - 74752*a^4*b^3 + 11173*a^5*b^2))/(120*a^8) + (tan(c/2 + (d
*x)/2)^15*(a - b)*(224*a*b^5 + 43*a^5*b + 5*a^6 - 448*b^6 + 600*a^2*b^4 - 2
44*a^3*b^3 - 180*a^4*b^2))/(8*a^8) - (tan(c/2 + (d*x)/2)*(2*a*b + a^2 + b^2
)*(224*a*b^4 - 48*a^4*b + 5*a^5 - 448*b^5 + 376*a^2*b^3 - 132*a^3*b^2))/(8*
a^8))/(d*(2*a*b - tan(c/2 + (d*x)/2)^8*(10*a^2 - 70*b^2) + tan(c/2 + (d*x)/
2)^2*(12*a*b + 4*a^2 + 8*b^2) + tan(c/2 + (d*x)/2)^14*(4*a^2 - 12*a*b + 8*b
^2) + tan(c/2 + (d*x)/2)^4*(28*a*b + 4*a^2 + 28*b^2) + tan(c/2 + (d*x)/2)^1
2*(4*a^2 - 28*a*b + 28*b^2) + tan(c/2 + (d*x)/2)^6*(28*a*b - 4*a^2 + 56*b^2
) - tan(c/2 + (d*x)/2)^10*(28*a*b + 4*a^2 - 56*b^2) + tan(c/2 + (d*x)/2)^16
*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (atan(((((((106*a^25*b - 10*a^26 + 896
*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 -
1122*a^23*b^3 + 178*a^24*b^2)/a^24 - (tan(c/2 + (d*x)/2)*(512*a^20*b + 512*
a^18*b^3 - 1024*a^19*b^2)*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i)
)/(128*a^25))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(16*a^9) +
 (tan(c/2 + (d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 6
73792*a^2*b^13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601
600*a^6*b^9 - 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10
*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16))*(a^6*5i - b
^6*448i + a^2*b^4*600i - a^4*b^2*180i)*1i)/(16*a^9) - (((((106*a^25*b - 10*
a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*a
^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 + (tan(c/2 + (d*x)/2)*(512*a^2
0*b + 512*a^18*b^3 - 1024*a^19*b^2)*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4
*b^2*180i))/(128*a^25))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/
(16*a^9) - (tan(c/2 + (d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 40140
8*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*
```

b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i)*1i)/(16*a^9))/(((75*a^19*b)/4 - 2107392*a*b^19 + 1404928*b^20 - 5644800*a^2*b^18 + 9345280*a^3*b^17 + 8902208*a^4*b^16 - 17144736*a^5*b^15 - 6722456*a^6*b^14 + 16804748*a^7*b^13 + 2126380*a^8*b^12 - 9486373*a^9*b^11 + 163573*a^10*b^10 + 3099308*a^11*b^9 - 297558*a^12*b^8 - (4466945*a^13*b^7)/8 + (296845*a^14*b^6)/4 + (196765*a^15*b^5)/4 - (26515*a^16*b^4)/4 - (13415*a^17*b^3)/8 + (285*a^18*b^2)/2)/a^24 + (((((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 - (tan(c/2 + (d*x)/2)*(512*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2)*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(128*a^25))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(16*a^9) + (tan(c/2 + (d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(16*a^9) + (((((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 + (tan(c/2 + (d*x)/2)*(512*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2)*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(128*a^25))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(16*a^9) - (tan(c/2 + (d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(16*a^9)))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i)*1i)/(8*a^9*d) + (b*atan(((b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16) + (b*((a + b)*(a - b))^(1/2)*((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 - (b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(6*a^4 + 56*b^4 - 47*a^2*b^2)*(512*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2))/(16*a^25))*(6*a^4 + 56*b^4 - 47*a^2*b^2))/(2*a^9))*(6*a^4 + 56*b^4 - 47*a^2*b^2)*1i)/(2*a^9) + (b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16) - (b*((a + b)*(a - b))^(1/2)*((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 + (b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(6*a^4 + 56*b^4 - 47*a^2*b^2)*(512*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2))/(16*a^25))*(6*a^4 + 56*b

```
^4 - 47*a^2*b^2))/(2*a^9))*(6*a^4 + 56*b^4 - 47*a^2*b^2)*1i)/(2*a^9))/(((75
*a^19*b)/4 - 2107392*a*b^19 + 1404928*b^20 - 5644800*a^2*b^18 + 9345280*a^3
*b^17 + 8902208*a^4*b^16 - 17144736*a^5*b^15 - 6722456*a^6*b^14 + 16804748*
a^7*b^13 + 2126380*a^8*b^12 - 9486373*a^9*b^11 + 163573*a^10*b^10 + 3099308
*a^11*b^9 - 297558*a^12*b^8 - (4466945*a^13*b^7)/8 + (296845*a^14*b^6)/4 +
(196765*a^15*b^5)/4 - (26515*a^16*b^4)/4 - (13415*a^17*b^3)/8 + (285*a^18*b
^2)/2)/a^24 + (b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*(802816*a*b^1
4 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12
+ 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364
160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b
^3 + 579*a^13*b^2))/(8*a^16) + (b*((a + b)*(a - b))^(1/2)*((106*a^25*b - 10
*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*
a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 - (b*tan(c/2 + (d*x)/2)*((a +
 b)*(a - b))^(1/2)*(6*a^4 + 56*b^4 - 47*a^2*b^2)*(512*a^20*b + 512*a^18*b^3
 - 1024*a^19*b^2))/(16*a^25))*(6*a^4 + 56*b^4 - 47*a^2*b^2))/(2*a^9))*(6*a^
4 + 56*b^4 - 47*a^2*b^2))/(2*a^9) - (b*((a + b)*(a - b))^(1/2)*((tan(c/2 +
(d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^
13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9
- 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*
a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16) - (b*((a + b)*(a - b))^(
1/2)*((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6
+ 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 + (b*ta
n(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(6*a^4 + 56*b^4 - 47*a^2*b^2)*(512
*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2))/(16*a^25))*(6*a^4 + 56*b^4 - 47*a^
2*b^2))/(2*a^9))*(6*a^4 + 56*b^4 - 47*a^2*b^2))/(2*a^9)))*((a + b)*(a - b))
^(1/2)*(6*a^4 + 56*b^4 - 47*a^2*b^2)*1i)/(a^9*d)
```

3.229 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 333

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= \frac{3(a^4-24a^2b^2+40b^4)\,x}{8a^7} \\ &\quad - \frac{3b(2a^4-11a^2b^2+10b^4)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7\sqrt{a-b}\sqrt{a+b}d} \\ &\quad + \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} \\ &\quad - \frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^5d} \\ &\quad + \frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd} \\ &\quad - \frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} \\ &\quad - \frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2} \\ &\quad + \frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))}\end{aligned}$$

```
[Out] 3/8*(a^4-24*a^2*b^2+40*b^4)*x/a^7+1/2*b*(13*a^2-30*b^2)*sin(d*x+c)/a^6/d-3/
8*(7*a^2-20*b^2)*cos(d*x+c)*sin(d*x+c)/a^5/d+1/2*(3*a^2-10*b^2)*cos(d*x+c)^
2*sin(d*x+c)/a^4/b/d-1/4*(4*a^2-15*b^2)*cos(d*x+c)^3*sin(d*x+c)/a^3/b^2/d-1
/2*(a^2-b^2)*cos(d*x+c)^4*sin(d*x+c)/a^2/b/d/(b+a*cos(d*x+c))^2+1/2*(2*a^2-
7*b^2)*cos(d*x+c)^4*sin(d*x+c)/a^2/b^2/d/(b+a*cos(d*x+c))-3*b*(2*a^4-11*a^2
*b^2+10*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^7/d/(a-b
)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2970, 3128, 3102, 2814, 2738, 214}

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = & \frac{(2a^2-7b^2)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)} \\ & -\frac{(a^2-b^2)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2} \\ & +\frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} \\ & -\frac{3(7a^2-20b^2)\sin(c+dx)\cos(c+dx)}{8a^5d} \\ & +\frac{(3a^2-10b^2)\sin(c+dx)\cos^2(c+dx)}{2a^4bd} \\ & -\frac{(4a^2-15b^2)\sin(c+dx)\cos^3(c+dx)}{4a^3b^2d} \\ & -\frac{3b(2a^4-11a^2b^2+10b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7d\sqrt{a-b}\sqrt{a+b}} \\ & +\frac{3x(a^4-24a^2b^2+40b^4)}{8a^7}\end{aligned}$$

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*(a^4 - 24*a^2*b^2 + 40*b^4)*x)/(8*a^7) - (3*b*(2*a^4 - 11*a^2*b^2 + 10*b
^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^7*Sqrt[a - b]*S
qrt[a + b]*d) + (b*(13*a^2 - 30*b^2)*Sin[c + d*x])/(2*a^6*d) - (3*(7*a^2 -
20*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*a^5*d) + ((3*a^2 - 10*b^2)*Cos[c + d*
x]^2*Sin[c + d*x])/(2*a^4*b*d) - ((4*a^2 - 15*b^2)*Cos[c + d*x]^3*Sin[c + d
*x])/(4*a^3*b^2*d) - ((a^2 - b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(2*a^2*b*d*(
b + a*Cos[c + d*x])^2) + ((2*a^2 - 7*b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(2*a
^2*b^2*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2970

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[(a^2*(n - m + 1) - b^2*(m + n
+ 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(
a^2*b^2*d*f*(m + 1)*(m + 2))), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)\sin^4(c+dx)}{(-b-a\cos(c+dx))^3}\,dx \\
&= -\frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2}+\frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))} \\
&\quad+\frac{\int \frac{\cos^3(c+dx)\left(-6(a^2-4b^2)-ab\cos(c+dx)+2(4a^2-15b^2)\cos^2(c+dx)\right)}{-b-a\cos(c+dx)}\,dx}{2a^2b^2} \\
&= -\frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} \\
&\quad-\frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2}+\frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))} \\
&\quad-\frac{\int \frac{\cos^2(c+dx)\left(-6b(4a^2-15b^2)-6ab^2\cos(c+dx)+12b(3a^2-10b^2)\cos^2(c+dx)\right)}{-b-a\cos(c+dx)}\,dx}{8a^3b^2} \\
&= \frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd}-\frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} \\
&\quad-\frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2}+\frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))} \\
&\quad+\frac{\int \frac{\cos(c+dx)\left(-24b^2(3a^2-10b^2)-30ab^3\cos(c+dx)+18b^2(7a^2-20b^2)\cos^2(c+dx)\right)}{-b-a\cos(c+dx)}\,dx}{24a^4b^2} \\
&= -\frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^5d}+\frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd} \\
&\quad-\frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} \\
&\quad-\frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2}+\frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))} \\
&\quad-\frac{\int \frac{-18b^3(7a^2-20b^2)+6ab^2(3a^2-20b^2)\cos(c+dx)+24b^3(13a^2-30b^2)\cos^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{48a^5b^2} \\
&= \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d}-\frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^5d} \\
&\quad+\frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd}-\frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} \\
&\quad-\frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2}+\frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))} \\
&\quad+\frac{\int \frac{18ab^3(7a^2-20b^2)-18b^2(a^4-24a^2b^2+40b^4)\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{48a^6b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3(a^4-24a^2b^2+40b^4)\,x}{8a^7} + \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} \\
&\quad - \frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^5d} \\
&\quad + \frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd} - \frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} \\
&\quad - \frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2} + \frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))} \\
&\quad + \frac{(3b(2a^4-11a^2b^2+10b^4))\int \frac{1}{-b-a\cos(c+dx)}\,dx}{2a^7}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3(a^4-24a^2b^2+40b^4)\,x}{8a^7} + \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} \\
&\quad - \frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^5d} \\
&\quad + \frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd} - \frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} \\
&\quad - \frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2} + \frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))} \\
&\quad + \frac{(3b(2a^4-11a^2b^2+10b^4))\operatorname{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^7d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3(a^4-24a^2b^2+40b^4)\,x}{8a^7} - \frac{3b(2a^4-11a^2b^2+10b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7\sqrt{a-b}\sqrt{a+b}d} \\
&\quad + \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} - \frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^5d} \\
&\quad + \frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd} - \frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} \\
&\quad - \frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2} + \frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1178 vs. $2(333) = 666$.

Time = 6.86 (sec) , antiderivative size = 1178, normalized size of antiderivative = 3.54

$$
\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx
$$

$$
= \frac{-\frac{6\left(8(c+dx)+\frac{2b\left(15a^4-20a^2b^2+8b^4\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{ab\left(3a^2-4b^2\right)\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))^2}-\frac{3a\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))}\right)}{a^3} + \frac{6\left(\frac{6ab\ldots}{\ldots}\right.}{\ldots}}{\ldots}
$$

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-6*(8*(c + d*x) + (2*b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[((-a + b)*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(3*a^2 - 4*b^2)*
Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])^2) - (3*a*(2*a^4 - 7*a^
2*b^2 + 4*b^4)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x]))))/a
^3 + (6*((6*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[
a^2 - b^2] + ((b*(a^2 + 2*b^2) + a*(2*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x]
)/(b + a*Cos[c + d*x])^2))/((a - b)^2*(a + b)^2) - (2*(-24*(a^2 - 8*b^2)*(c
 + d*x) + (6*b*(-35*a^6 + 140*a^4*b^2 - 168*a^2*b^4 + 64*b^6)*ArcTanh[((-a
+ b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 96*a*b*Sin[c +
 d*x] + (a*b*(-5*a^4 + 20*a^2*b^2 - 16*b^4)*Sin[c + d*x])/((a - b)*(a + b)*
(b + a*Cos[c + d*x])^2) + (a*(10*a^6 - 115*a^4*b^2 + 220*a^2*b^4 - 112*b^6)
*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])) + 8*a^2*Sin[2*(c
+ d*x)]))/a^5 + ((12*b*(105*a^8 - 840*a^6*b^2 + 2016*a^4*b^4 - 1920*a^2*b^6
 + 640*b^8)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(5/2) + (48*a^10*c - 960*a^8*b^2*c + 1776*a^6*b^4*c + 2976*a^4*b^6*c - 7
680*a^2*b^8*c + 3840*b^10*c + 48*a^10*d*x - 960*a^8*b^2*d*x + 1776*a^6*b^4*
d*x + 2976*a^4*b^6*d*x - 7680*a^2*b^8*d*x + 3840*b^10*d*x + 192*a*b*(a^2 -
b^2)^2*(a^4 - 20*a^2*b^2 + 40*b^4)*(c + d*x)*Cos[c + d*x] + 48*(a^3 - a*b^2
)^2*(a^4 - 20*a^2*b^2 + 40*b^4)*(c + d*x)*Cos[2*(c + d*x)] + 114*a^9*b*Sin[
c + d*x] + 788*a^7*b^3*Sin[c + d*x] - 5696*a^5*b^5*Sin[c + d*x] + 8640*a^3*
b^7*Sin[c + d*x] - 3840*a*b^9*Sin[c + d*x] - 36*a^10*Sin[2*(c + d*x)] + 122
1*a^8*b^2*Sin[2*(c + d*x)] - 5182*a^6*b^4*Sin[2*(c + d*x)] + 6880*a^4*b^6*S
in[2*(c + d*x)] - 2880*a^2*b^8*Sin[2*(c + d*x)] + 120*a^9*b*Sin[3*(c + d*x)
] - 560*a^7*b^3*Sin[3*(c + d*x)] + 760*a^5*b^5*Sin[3*(c + d*x)] - 320*a^3*b
^7*Sin[3*(c + d*x)] - 8*a^10*Sin[4*(c + d*x)] + 56*a^8*b^2*Sin[4*(c + d*x)]
 - 88*a^6*b^4*Sin[4*(c + d*x)] + 40*a^4*b^6*Sin[4*(c + d*x)] - 8*a^9*b*Sin[
5*(c + d*x)] + 16*a^7*b^3*Sin[5*(c + d*x)] - 8*a^5*b^5*Sin[5*(c + d*x)] + 2
*a^10*Sin[6*(c + d*x)] - 4*a^8*b^2*Sin[6*(c + d*x)] + 2*a^6*b^4*Sin[6*(c +
d*x)])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/a^7)/(256*d)
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{2\left(\left(\frac{3}{8}a^4+3a^3b-3a^2b^2-10a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(13a^3b-3a^2b^2-30a\,b^3+\frac{11}{8}a^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{11}{8}a^4+3a^2b^2+13a^3b-30a\,b^3\right)\tan(}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}}{a^7}$
default	$\frac{\frac{2\left(\left(\frac{3}{8}a^4+3a^3b-3a^2b^2-10a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(13a^3b-3a^2b^2-30a\,b^3+\frac{11}{8}a^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{11}{8}a^4+3a^2b^2+13a^3b-30a\,b^3\right)\tan(}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}}{a^7}$
risch	$\frac{3x}{8a^3}-\frac{9x\,b^2}{a^5}+\frac{15x\,b^4}{a^7}-\frac{ib\,\mathrm{e}^{-3i(dx+c)}}{8a^4d}-\frac{3i\mathrm{e}^{2i(dx+c)}b^2}{4a^5d}-\frac{15ib\,\mathrm{e}^{i(dx+c)}}{8a^4d}-\frac{ib^2\left(-7a^3b\,\mathrm{e}^{3i(dx+c)}+12a\,b^3\mathrm{e}^{3i(dx+c)}-\right.}{}$

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^7*(((3/8*a^4+3*a^3*b-3*a^2*b^2-10*a*b^3)*tan(1/2*d*x+1/2*c)^7+(13*
a^3*b-3*a^2*b^2-30*a*b^3+11/8*a^4)*tan(1/2*d*x+1/2*c)^5+(-11/8*a^4+3*a^2*b^
2+13*a^3*b-30*a*b^3)*tan(1/2*d*x+1/2*c)^3+(3*a^3*b-10*a*b^3-3/8*a^4+3*a^2*b
^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4+3/8*(a^4-24*a^2*b^2+40*b
^4)*arctan(tan(1/2*d*x+1/2*c)))+2*b/a^7*(((5/2*a^3*b^2-5*a*b^4-3*a^4*b+11/2
*a^2*b^3)*tan(1/2*d*x+1/2*c)^3+(5/2*a^3*b^2-5*a*b^4+3*a^4*b-11/2*a^2*b^3)*t
an(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-3/
2*(2*a^4-11*a^2*b^2+10*b^4)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1
/2*c)/((a-b)*(a+b))^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 1041, normalized size of antiderivative = 3.13

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a^8 - 25*a^6*b^2 + 64*a^4*b^4 - 40*a^2*b^6)*d*x*cos(d*x + c)^2 + 6
*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*d*x*cos(d*x + c) + 3*(a^6*b^2
 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*d*x + 6*(2*a^4*b^3 - 11*a^2*b^5 + 10*b
^7 + (2*a^6*b - 11*a^4*b^3 + 10*a^2*b^5)*cos(d*x + c)^2 + 2*(2*a^5*b^2 - 11
*a^3*b^4 + 10*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c)
- (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin
(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) +
 (52*a^5*b^3 - 172*a^3*b^5 + 120*a*b^7 + 2*(a^8 - a^6*b^2)*cos(d*x + c)^5 -
 4*(a^7*b - a^5*b^3)*cos(d*x + c)^4 - 5*(a^8 - 3*a^6*b^2 + 2*a^4*b^4)*cos(d
```

```
*x + c)^3 + 2*(11*a^7*b - 31*a^5*b^3 + 20*a^3*b^5)*cos(d*x + c)^2 + (83*a^6
*b^2 - 263*a^4*b^4 + 180*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - a^9*
b^2)*d*cos(d*x + c)^2 + 2*(a^10*b - a^8*b^3)*d*cos(d*x + c) + (a^9*b^2 - a^
7*b^4)*d), 1/8*(3*(a^8 - 25*a^6*b^2 + 64*a^4*b^4 - 40*a^2*b^6)*d*x*cos(d*x
+ c)^2 + 6*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*d*x*cos(d*x + c) +
3*(a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*d*x - 12*(2*a^4*b^3 - 11*a^2
*b^5 + 10*b^7 + (2*a^6*b - 11*a^4*b^3 + 10*a^2*b^5)*cos(d*x + c)^2 + 2*(2*a
^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (52*a^5*b^3
 - 172*a^3*b^5 + 120*a*b^7 + 2*(a^8 - a^6*b^2)*cos(d*x + c)^5 - 4*(a^7*b -
a^5*b^3)*cos(d*x + c)^4 - 5*(a^8 - 3*a^6*b^2 + 2*a^4*b^4)*cos(d*x + c)^3 +
2*(11*a^7*b - 31*a^5*b^3 + 20*a^3*b^5)*cos(d*x + c)^2 + (83*a^6*b^2 - 263*a
^4*b^4 + 180*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - a^9*b^2)*d*cos(d
*x + c)^2 + 2*(a^10*b - a^8*b^3)*d*cos(d*x + c) + (a^9*b^2 - a^7*b^4)*d)]
```

Sympy [F]

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sin^4{(c+dx)}}{(a+b\sec{(c+dx)})^3}\,dx$$

[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c))**3,x)

[Out] Integral(sin(c + d*x)**4/(a + b*sec(c + d*x))**3, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.75

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{3\left(a^4-24\,a^2b^2+40\,b^4\right)(dx+c)}{a^7} - \frac{24\left(2\,a^4b-11\,a^2b^3+10\,b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}a^7} - \frac{8\,(6}{}}{}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^4 - 24*a^2*b^2 + 40*b^4)*(d*x + c)/a^7 - 24*(2*a^4*b - 11*a^2*b^3
 + 10*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2
 + b^2)*a^7) - 8*(6*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x
+ 1/2*c)^3 - 11*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 10*b^5*tan(1/2*d*x + 1/2*c)^
3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*a^2*b^3*tan(1/2*d*x + 1/2*c) + 11*a*
b^4*tan(1/2*d*x + 1/2*c) + 10*b^5*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1
/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2*a^6) + 2*(3*a^3*tan(1/2*d*x +
 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 24*a*b^2*tan(1/2*d*x + 1/2*c)
^7 - 80*b^3*tan(1/2*d*x + 1/2*c)^7 + 11*a^3*tan(1/2*d*x + 1/2*c)^5 + 104*a^
2*b*tan(1/2*d*x + 1/2*c)^5 - 24*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 240*b^3*tan(
1/2*d*x + 1/2*c)^5 - 11*a^3*tan(1/2*d*x + 1/2*c)^3 + 104*a^2*b*tan(1/2*d*x
+ 1/2*c)^3 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*b^3*tan(1/2*d*x + 1/2*c)
^3 - 3*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/2*c) + 24*a*b^2*
tan(1/2*d*x + 1/2*c) - 80*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^
2 + 1)^4*a^6))/d
```

Mupad [B] (verification not implemented)

Time = 18.29 (sec) , antiderivative size = 3255, normalized size of antiderivative = 9.77

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int(sin(c + d*x)^4/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (atan(((((3*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16
*b^4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 - (3*tan(c/2 + (d*x)/2)*(a^4*1i +
b^4*40i - a^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19
))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) + (tan(c/2 + (d*x)/2)*(57600*a
*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22
```

752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i)*3i)/(8*a^7) - (((3*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 + (3*tan(c/2 + (d*x)/2)*(a^4*1i + b^4*40i - a^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) - (tan(c/2 + (d*x)/2)*(57600*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i)*3i)/(8*a^7))/((324000*a*b^13 + 27*a^13*b - 216000*b^14 + 388800*a^2*b^12 - 718200*a^3*b^11 - 195480*a^4*b^10 + 576720*a^5*b^9 - 4104*a^6*b^8 - 205119*a^7*b^7 + 24408*a^8*b^6 + (62181*a^9*b^5)/2 - 4671*a^10*b^4 - (3267*a^11*b^3)/2 + 162*a^12*b^2)/a^18 + (3*((3*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 - (3*tan(c/2 + (d*x)/2)*(a^4*1i + b^4*40i - a^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) + (tan(c/2 + (d*x)/2)*(57600*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) + (3*((3*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 + (3*tan(c/2 + (d*x)/2)*(a^4*1i + b^4*40i - a^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) - (tan(c/2 + (d*x)/2)*(57600*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7)))*(a^4*1i + b^4*40i - a^2*b^2*24i)*3i)/(4*a^7*d) - ((tan(c/2 + (d*x)/2)^5*(180*a*b^4 + 26*a^4*b - 15*a^5 + 600*b^5 - 300*a^2*b^3 - 73*a^3*b^2))/(2*a^6) - (3*tan(c/2 + (d*x)/2)^11*(60*a*b^4 + 6*a^4*b + a^5 - 40*b^5 + 4*a^2*b^3 - 31*a^3*b^2))/(4*a^6) + (tan(c/2 + (d*x)/2)^7*(26*a^4*b - 180*a*b^4 + 15*a^5 + 600*b^5 - 300*a^2*b^3 + 73*a^3*b^2))/(2*a^6) + (tan(c/2 + (d*x)/2)^3*(540*a*b^4 - 34*a^4*b + 5*a^5 + 600*b^5 - 220*a^2*b^3 - 239*a^3*b^2))/(4*a^6) - (tan(c/2 + (d*x)/2)^9*(540*a*b^4 + 34*a^4*b + 5*a^5 - 600*b^5 + 220*a^2*b^3 - 239*a^3*b^2))/(4*a^6) + (3*tan(c/2 + (d*x)/2)*(a + b)*(20*a*b^3 - 7*a^3*b + a^4 + 40*b^4 - 24*a^2*b^2))/(4*a^6))/(d*(2*a*b - tan(c/2 + (d*x)/2)^6*(4*a^2 - 20*b^2) + tan(c/2 + (d*x)/2)^2*(8*a*b + 2*a^2 + 6*b^2) + tan(c/2 + (d*x)/2)^10*(2*a^2 - 8*a*b + 6*b^2) + tan(c/2 + (d*x)/2)^4*(10*a*b - a^2 + 15*b^2) + tan(c/2 + (d*x)/2)^12*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 + (d*x)/2)^8*(10*a*b + a^2 - 15*b^2))) + (b*atan(((b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*(57600*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2))/(2*a^12) + (3*b*((a + b)*(a - b))^(1/2)*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 - (3*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2*a^4 + 10*b^4 - 11*a^2*b^2)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(4*a^12*(a

```
^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4
+ 10*b^4 - 11*a^2*b^2)*3i)/(2*(a^9 - a^7*b^2)) + (b*((a + b)*(a - b))^(1/2)
*((tan(c/2 + (d*x)/2)*(57600*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 576
0*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 -
 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2))/(2*a^12) - (3*b*((a + b)*(a - b
))^(1/2)*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^
4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 + (3*b*tan(c/2 + (d*x)/2)*((a + b)*(a
 - b))^(1/2)*(2*a^4 + 10*b^4 - 11*a^2*b^2)*(128*a^16*b + 128*a^14*b^3 - 256
*a^15*b^2))/(4*a^12*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^
9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^2)*3i)/(2*(a^9 - a^7*b^2)))/((324
000*a*b^13 + 27*a^13*b - 216000*b^14 + 388800*a^2*b^12 - 718200*a^3*b^11 -
195480*a^4*b^10 + 576720*a^5*b^9 - 4104*a^6*b^8 - 205119*a^7*b^7 + 24408*a^
8*b^6 + (62181*a^9*b^5)/2 - 4671*a^10*b^4 - (3267*a^11*b^3)/2 + 162*a^12*b^
2)/a^18 + (3*b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*(57600*a*b^10 -
 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4
*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9
*b^2))/(2*a^12) + (3*b*((a + b)*(a - b))^(1/2)*((108*a^19*b - 12*a^20 - 480
*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18
 - (3*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2*a^4 + 10*b^4 - 11*a^2
*b^2)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(4*a^12*(a^9 - a^7*b^2)))
*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a
^2*b^2))/(2*(a^9 - a^7*b^2)) - (3*b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*
x)/2)*(57600*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 6912
0*a^3*b^8 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 +
711*a^8*b^3 + 171*a^9*b^2))/(2*a^12) - (3*b*((a + b)*(a - b))^(1/2)*((108*a
^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^3
 + 108*a^18*b^2)/a^18 + (3*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2*
a^4 + 10*b^4 - 11*a^2*b^2)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(4*a
^12*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(
2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2))))*((a + b)*(a - b))^(1/2)
*(2*a^4 + 10*b^4 - 11*a^2*b^2)*3i)/(d*(a^9 - a^7*b^2))
```

3.230 $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 267

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= \frac{(a^2-12b^2)\,x}{2a^5} \\ &\quad - \frac{b(6a^4-19a^2b^2+12b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} \\ &\quad + \frac{b(11a^2-12b^2)\sin(c+dx)}{2a^4\,(a^2-b^2)\,d} \\ &\quad - \frac{(5a^2-6b^2)\cos(c+dx)\sin(c+dx)}{2a^3\,(a^2-b^2)\,d} \\ &\quad + \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} \\ &\quad + \frac{(3a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{2a^2\,(a^2-b^2)\,d(b+a\cos(c+dx))}\end{aligned}$$

```
[Out] 1/2*(a^2-12*b^2)*x/a^5-b*(6*a^4-19*a^2*b^2+12*b^4)*arctanh((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*b*(11*a^2-12*
b^2)*sin(d*x+c)/a^4/(a^2-b^2)/d-1/2*(5*a^2-6*b^2)*cos(d*x+c)*sin(d*x+c)/a^3
/(a^2-b^2)/d+1/2*cos(d*x+c)^3*sin(d*x+c)/a/d/(b+a*cos(d*x+c))^2+1/2*(3*a^2-
4*b^2)*cos(d*x+c)^2*sin(d*x+c)/a^2/(a^2-b^2)/d/(b+a*cos(d*x+c))
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2968, 3127, 3128, 3102, 2814, 2738, 214}

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &\frac{(3a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{2a^2d\,(a^2-b^2)\,(a\cos(c+dx)+b)} \\ &+\frac{x(a^2-12b^2)}{2a^5}+\frac{b(11a^2-12b^2)\sin(c+dx)}{2a^4d\,(a^2-b^2)} \\ &-\frac{(5a^2-6b^2)\sin(c+dx)\cos(c+dx)}{2a^3d\,(a^2-b^2)} \\ &-\frac{b(6a^4-19a^2b^2+12b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}} \\ &+\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}\end{aligned}$$

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2 - 12*b^2)*x)/(2*a^5) - (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTanh[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d)
 + (b*(11*a^2 - 12*b^2)*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*d) - ((5*a^2 - 6*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*d) + (Cos[c + d*x]^3*Sin[
c + d*x])/(2*a*d*(b + a*Cos[c + d*x])^2) + ((3*a^2 - 4*b^2)*Cos[c + d*x]^2*
Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$
$$= -\int \frac{\cos^3(c+dx)\left(1-\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^3}\,dx$$
$$= \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)\left(3(a^2-b^2)-4(a^2-b^2)\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^2}\,dx}{2a(a^2-b^2)}$$
$$= \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{(3a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)d(b+a\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(2(3a^4-7a^2b^2+4b^4)+ab(a^2-b^2)\cos(c+dx)-2(5a^2-6b^2)(a^2-b^2)\cos^2(c+dx)\right)}{-b-a\cos(c+dx)}\,dx}{2a^2(a^2-b^2)^2}$$
$$= -\frac{(5a^2-6b^2)\cos(c+dx)\sin(c+dx)}{2a^3(a^2-b^2)d} + \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{(3a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)d(b+a\cos(c+dx))} + \frac{\int \frac{2b(5a^4-11a^2b^2+6b^4)-2a(a^4-3a^2b^2+2b^4)\cos(c+dx)-2b(11a^2-12b^2)(a^2-b^2)\cos^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{4a^3(a^2-b^2)^2}$$
$$= \frac{b(11a^2-12b^2)\sin(c+dx)}{2a^4(a^2-b^2)d} - \frac{(5a^2-6b^2)\cos(c+dx)\sin(c+dx)}{2a^3(a^2-b^2)d} + \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{(3a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)d(b+a\cos(c+dx))} - \frac{\int \frac{-2ab(5a^4-11a^2b^2+6b^4)+2(a^2-12b^2)(a^2-b^2)^2\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{4a^4(a^2-b^2)^2}$$
$$= \frac{(a^2-12b^2)x}{2a^5} + \frac{b(11a^2-12b^2)\sin(c+dx)}{2a^4(a^2-b^2)d} - \frac{(5a^2-6b^2)\cos(c+dx)\sin(c+dx)}{2a^3(a^2-b^2)d} + \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{(3a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)d(b+a\cos(c+dx))} + \frac{(b(6a^4-19a^2b^2+12b^4))\int \frac{1}{-b-a\cos(c+dx)}\,dx}{2a^5(a^2-b^2)}$$
$$= \frac{(a^2-12b^2)x}{2a^5} + \frac{b(11a^2-12b^2)\sin(c+dx)}{2a^4(a^2-b^2)d} - \frac{(5a^2-6b^2)\cos(c+dx)\sin(c+dx)}{2a^3(a^2-b^2)d} + \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{(3a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{2a^2(a^2-b^2)d(b+a\cos(c+dx))} + \frac{(b(6a^4-19a^2b^2+12b^4))\,\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^5(a^2-b^2)d}$$

$$= \frac{(a^2-12b^2)\,x}{2a^5} - \frac{b(6a^4-19a^2b^2+12b^4)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d}$$
$$+ \frac{b(11a^2-12b^2)\sin(c+dx)}{2a^4\,(a^2-b^2)\,d} - \frac{(5a^2-6b^2)\cos(c+dx)\sin(c+dx)}{2a^3\,(a^2-b^2)\,d}$$
$$+ \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{(3a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{2a^2\,(a^2-b^2)\,d(b+a\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 3.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.06

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{4b(6a^4-19a^2b^2+12b^4)\mathrm{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{4ab(a^4-13a^2b^2+12b^4)(c+dx)\cos(c+dx)-2a^4(a^2-b^2)\cos^3(c+dx)\sin(c+dx)+2a}{}}{4a^5(a-}$$

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + (4*a*b*(a^4 - 13*a^2*b^2 + 12*b^4)*(c + d*
x)*Cos[c + d*x] - 2*a^4*(a^2 - b^2)*Cos[c + d*x]^3*Sin[c + d*x] + 2*a^2*(a^
2 - b^2)*Cos[c + d*x]^2*((a^2 - 12*b^2)*(c + d*x) + 4*a*b*Sin[c + d*x]) + b
^2*(2*(a^4 - 13*a^2*b^2 + 12*b^4)*(c + d*x) + (22*a^3*b - 24*a*b^3)*Sin[c +
 d*x] + (17*a^4 - 18*a^2*b^2)*Sin[2*(c + d*x)]))/(b + a*Cos[c + d*x])^2)/(4
*a^5*(a - b)*(a + b)*d)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{2b\left(\frac{-\frac{(6a^2+ab-6b^2)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a+b)}+\frac{(6a^2-ab-6b^2)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{(6a^4-19a^2b^2+12b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2(a^2-b^2)\sqrt{(a-b)(a+b)}}\right)}{a^5}+}{d}$
default	$\frac{\frac{2b\left(\frac{-\frac{(6a^2+ab-6b^2)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a+b)}+\frac{(6a^2-ab-6b^2)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{(6a^4-19a^2b^2+12b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2(a^2-b^2)\sqrt{(a-b)(a+b)}}\right)}{a^5}+}{d}$
risch	$\frac{x}{2a^3}-\frac{6xb^2}{a^5}+\frac{i e^{2i(dx+c)}}{8a^3d}-\frac{3ib\,e^{i(dx+c)}}{2a^4d}+\frac{3ib\,e^{-i(dx+c)}}{2a^4d}-\frac{i e^{-2i(dx+c)}}{8a^3d}+\frac{ib^2\left(-7a^3b\,e^{3i(dx+c)}+8a\,b^3e^{3i(dx+c)}-\right.}{}$

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/a^5*((-1/2*(6*a^2+a*b-6*b^2)*a*b/(a+b)*tan(1/2*d*x+1/2*c)^3+1/2*(6
*a^2-a*b-6*b^2)*a*b/(a-b)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^2-1/2*(6*a^4-19*a^2*b^2+12*b^4)/(a^2-b^2)/((a-b)*(a+
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+2/a^5*(((1
/2*a^2+3*a*b)*tan(1/2*d*x+1/2*c)^3+(3*a*b-1/2*a^2)*tan(1/2*d*x+1/2*c))/(1+t
an(1/2*d*x+1/2*c)^2)^2+1/2*(a^2-12*b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 984, normalized size of antiderivative = 3.69

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$=\left[\frac{2\,(a^8-14\,a^6b^2+25\,a^4b^4-12\,a^2b^6)dx\cos{(dx+c)}^2+4\,(a^7b-14\,a^5b^3+25\,a^3b^5-12\,ab^7)dx\cos{(dx+c)}+}{}\right.$$

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12*a^2*b^6)*d*x*cos(d*x + c)^2 + 4
*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*d*x*cos(d*x + c) + 2*(a^6*b^2
 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*d*x - (6*a^4*b^3 - 19*a^2*b^5 + 12*b^7
 + (6*a^6*b - 19*a^4*b^3 + 12*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 19*a
^3*b^4 + 12*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) -
```

```
(a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d
*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2
*(11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7 - (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x
+ c)^3 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + (17*a^6*b^2 - 35*
a^4*b^4 + 18*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 2*a^9*b^2 + a^7*
b^4)*d*cos(d*x + c)^2 + 2*(a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c) + (
a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d), 1/2*((a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12
*a^2*b^6)*d*x*cos(d*x + c)^2 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^
7)*d*x*cos(d*x + c) + (a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*d*x - (6
*a^4*b^3 - 19*a^2*b^5 + 12*b^7 + (6*a^6*b - 19*a^4*b^3 + 12*a^2*b^5)*cos(d*
x + c)^2 + 2*(6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6)*cos(d*x + c))*sqrt(-a^2 +
b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c
))) + (11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7 - (a^8 - 2*a^6*b^2 + a^4*b^4)*cos
(d*x + c)^3 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + (17*a^6*b^2
- 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 2*a^9*b^2 +
 a^7*b^4)*d*cos(d*x + c)^2 + 2*(a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c
) + (a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d)]
```

Sympy [F]

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sin^2\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. $2(248) = 496$.

Time = 0.49 (sec) , antiderivative size = 1193, normalized size of antiderivative = 4.47

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^11 - 7*a^10*b - 14*a^9*b^2 + 39*a^8*b^3 + 25*a^7*b^4 - 56*a^6*b^5 -
 12*a^5*b^6 + 24*a^4*b^7 - a^4*abs(-a^7 + a^5*b^2) - 5*a^3*b*abs(-a^7 + a^5
*b^2) + 13*a^2*b^2*abs(-a^7 + a^5*b^2) + 6*a*b^3*abs(-a^7 + a^5*b^2) - 12*b
^4*abs(-a^7 + a^5*b^2))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*
d*x + 1/2*c)/sqrt(-(a^6*b - a^4*b^3 + sqrt((a^7 + a^6*b - a^5*b^2 - a^4*b^3
)*(a^7 - a^6*b - a^5*b^2 + a^4*b^3) + (a^6*b - a^4*b^3)^2))/(a^7 - a^6*b -
a^5*b^2 + a^4*b^3))))/(a^6*b*abs(-a^7 + a^5*b^2) - a^4*b^3*abs(-a^7 + a^5*b
^2) + (a^7 - a^5*b^2)^2) + ((a^4 + 5*a^3*b - 13*a^2*b^2 - 6*a*b^3 + 12*b^4)
*sqrt(-a^2 + b^2)*abs(-a^7 + a^5*b^2)*abs(-a + b) + (a^11 - 7*a^10*b - 14*a
^9*b^2 + 39*a^8*b^3 + 25*a^7*b^4 - 56*a^6*b^5 - 12*a^5*b^6 + 24*a^4*b^7)*sq
rt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(
1/2*d*x + 1/2*c)/sqrt(-(a^6*b - a^4*b^3 - sqrt((a^7 + a^6*b - a^5*b^2 - a^4
*b^3)*(a^7 - a^6*b - a^5*b^2 + a^4*b^3) + (a^6*b - a^4*b^3)^2))/(a^7 - a^6*
b - a^5*b^2 + a^4*b^3))))/((a^7 - a^5*b^2)^2*(a^2 - 2*a*b + b^2) - (a^8*b -
 2*a^7*b^2 + 2*a^5*b^4 - a^4*b^5)*abs(-a^7 + a^5*b^2)) + 2*(a^5*tan(1/2*d*x
 + 1/2*c)^7 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 18*a^3*b^2*tan(1/2*d*x + 1/2
*c)^7 + 7*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^7
- 12*b^5*tan(1/2*d*x + 1/2*c)^7 - 3*a^5*tan(1/2*d*x + 1/2*c)^5 - 4*a^4*b*ta
n(1/2*d*x + 1/2*c)^5 - 14*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 37*a^2*b^3*tan(1
/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 36*b^5*tan(1/2*d*x +
1/2*c)^5 + 3*a^5*tan(1/2*d*x + 1/2*c)^3 - 4*a^4*b*tan(1/2*d*x + 1/2*c)^3 +
14*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 37*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 18*
a*b^4*tan(1/2*d*x + 1/2*c)^3 - 36*b^5*tan(1/2*d*x + 1/2*c)^3 - a^5*tan(1/2*
d*x + 1/2*c) + 4*a^4*b*tan(1/2*d*x + 1/2*c) + 18*a^3*b^2*tan(1/2*d*x + 1/2*
c) + 7*a^2*b^3*tan(1/2*d*x + 1/2*c) - 18*a*b^4*tan(1/2*d*x + 1/2*c) - 12*b^
5*tan(1/2*d*x + 1/2*c))/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(
1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 21.74 (sec) , antiderivative size = 4026, normalized size of antiderivative = 15.08

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int(sin(c + d*x)^2/(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(6*a*b^3 - 5*a^3*b + a^4 + 12*b^4 - 13*a^2*b^2))/(a^4*
b - a^5) + (tan(c/2 + (d*x)/2)^3*(18*a*b^4 + 4*a^4*b - 3*a^5 + 36*b^5 - 37*
a^2*b^3 - 14*a^3*b^2))/((a^4*b - a^5)*(a + b)) + (tan(c/2 + (d*x)/2)^5*(4*a
^4*b - 18*a*b^4 + 3*a^5 + 36*b^5 - 37*a^2*b^3 + 14*a^3*b^2))/((a^4*b - a^5)
*(a + b)) + (tan(c/2 + (d*x)/2)^7*(5*a^3*b - 6*a*b^3 + a^4 + 12*b^4 - 13*a^
2*b^2))/(a^4*(a + b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) + t
an(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2) - tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2)
+ tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (atan((((a^2*1i
- b^2*12i)*((((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12
*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3
 - a^13*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*
b^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b +
a^11 - a^8*b^3 - a^9*b^2)))*(a^2*1i - b^2*12i))/(2*a^5) + (8*tan(c/2 + (d*x
)/2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 3
86*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b +
 a^11 - a^8*b^3 - a^9*b^2))*1i)/(2*a^5) - ((a^2*1i - b^2*12i)*((((4*(24*a^1
6*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100
*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (4*tan(c/
2 + (d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^
12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2
)))*(a^2*1i - b^2*12i))/(2*a^5) - (8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 2
88*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386*a^4*b^6 - 386*a^5*b^5
 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^
2))*1i)/(2*a^5))/((8*(864*a*b^10 + 6*a^10*b - 1728*b^11 + 4752*a^2*b^9 - 21
60*a^3*b^8 - 4356*a^4*b^7 + 1746*a^5*b^6 + 1495*a^6*b^5 - 491*a^7*b^4 - 169
*a^8*b^3 + 30*a^9*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + ((a^2*1i -
b^2*12i)*((((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12*b
^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3 -
 a^13*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*b^
6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b + a^
11 - a^8*b^3 - a^9*b^2)))*(a^2*1i - b^2*12i))/(2*a^5) + (8*tan(c/2 + (d*x)/
2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386
*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a
^11 - a^8*b^3 - a^9*b^2)))/(2*a^5) + ((a^2*1i - b^2*12i)*((((4*(24*a^16*b -
 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100*a^14
*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (4*tan(c/2 + (
```

d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^
4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*(
a^2*1i - b^2*12i))/(2*a^5) - (8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 288*a*
b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386*a^4*b^6 - 386*a^5*b^5 - 61
*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))/
(2*a^5)))*(a^2*1i - b^2*12i)*1i)/(a^5*d) + (b*atan(((b*((8*tan(c/2 + (d*x)/
2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386
*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a
^11 - a^8*b^3 - a^9*b^2) + (b*((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^
11*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b
+ a^15 - a^12*b^3 - a^13*b^2) - (4*b*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^
3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5
+ 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/((a^10*b + a^11 - a^8*b^3 - a^9*
b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(a - b)^3)^(1/2)
*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)
))*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*1i)/(2*(a^11 -
a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)) + (b*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^
9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386*a^4*b^6 - 386*
a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a^11 - a^8*b^3 -
a^9*b^2) - (b*((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^
12*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b
^3 - a^13*b^2) + (4*b*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4
+ 12*b^4 - 19*a^2*b^2)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^4 -
16*a^13*b^3 - 8*a^14*b^2))/((a^10*b + a^11 - a^8*b^3 - a^9*b^2)*(a^11 - a^
5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^
4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(
a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*1i)/(2*(a^11 - a^5*b^6 + 3*a^
7*b^4 - 3*a^9*b^2)))/((8*(864*a*b^10 + 6*a^10*b - 1728*b^11 + 4752*a^2*b^9
- 2160*a^3*b^8 - 4356*a^4*b^7 + 1746*a^5*b^6 + 1495*a^6*b^5 - 491*a^7*b^4 -
169*a^8*b^3 + 30*a^9*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (b*((8*
tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 6
24*a^3*b^7 + 386*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b
^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) + (b*((4*(24*a^16*b - 4*a^17 - 48*
a^10*b^7 + 24*a^11*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^1
5*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (4*b*tan(c/2 + (d*x)/2)*((a
+ b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*(8*a^15*b - 8*a^10*b
^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/((a^10*b + a^11
- a^8*b^3 - a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*
(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*
b^4 - 3*a^9*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2
))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)) - (b*((8*tan(c/2 + (d*x)/2)
*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386*a
^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a^1
1 - a^8*b^3 - a^9*b^2) - (b*((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11
*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b +

```
a^15 - a^12*b^3 - a^13*b^2) + (4*b*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)
^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 +
16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/((a^10*b + a^11 - a^8*b^3 - a^9*b^
2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(
6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))
*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*
b^6 + 3*a^7*b^4 - 3*a^9*b^2))))*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4
 - 19*a^2*b^2)*1i)/(d*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))
```

3.231 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 376

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx =& -\frac{2b^3(3a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d} \\ &-\frac{2ab(3a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} \\ &-\frac{b^3(a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d} \\ &-\frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2(a-b)^3d(1+\cos(c+dx))} \\ &-\frac{b^3\sin(c+dx)}{2\left(a^2-b^2\right)^2d(b+a\cos(c+dx))^2} \\ &+\frac{3b^4\sin(c+dx)}{2\left(a^2-b^2\right)^3d(b+a\cos(c+dx))} \\ &+\frac{b^2(3a^2-b^2)\sin(c+dx)}{\left(a^2-b^2\right)^3d(b+a\cos(c+dx))}\end{aligned}$$

```
[Out] -2*b^3*(3*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a
-b)^(7/2)/(a+b)^(7/2)/d-2*a*b*(3*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-b^3*(a^2+2*b^2)*arctanh((a-b)^
(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a-b)^(7/2)/(a+b)^(7/2)/d-1/2*sin(d
*x+c)/(a+b)^3/d/(1-cos(d*x+c))+1/2*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))-1/2*
b^3*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+3/2*b^4*sin(d*x+c)/(a^2-b^2
)^3/d/(b+a*cos(d*x+c))+b^2*(3*a^2-b^2)*sin(d*x+c)/(a^2-b^2)^3/d/(b+a*cos(d*
x+c))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2976, 2727, 2743, 12, 2738, 214, 2833}

$$
\begin{aligned}
\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = & -\frac{2ab(3a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} \\
& -\frac{2b^3(3a^2-b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{7/2}(a+b)^{7/2}} \\
& -\frac{b^3(a^2+2b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{7/2}(a+b)^{7/2}} \\
& +\frac{b^2(3a^2-b^2)\sin(c+dx)}{d\left(a^2-b^2\right)^3\left(a\cos(c+dx)+b\right)} \\
& +\frac{3b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^3\left(a\cos(c+dx)+b\right)} \\
& -\frac{b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^2\left(a\cos(c+dx)+b\right)^2} \\
& -\frac{\sin(c+dx)}{2d(a+b)^3(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2d(a-b)^3(\cos(c+dx)+1)}
\end{aligned}
$$

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*b^3*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a*(a - b)^(7/2)*(a + b)^(7/2)*d) - (2*a*b*(3*a^2 + b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (b^3*(
a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b
)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(2*(a + b)^3*d*(1 - Cos[c + d*x]))
+ Sin[c + d*x]/(2*(a - b)^3*d*(1 + Cos[c + d*x])) - (b^3*Sin[c + d*x])/(2*(
a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (3*b^4*Sin[c + d*x])/(2*(a^2 - b^2
)^3*d*(b + a*Cos[c + d*x])) + (b^2*(3*a^2 - b^2)*Sin[c + d*x])/((a^2 - b^2)
^3*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\cot^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx\\
&= \int \left(-\frac{1}{2(a-b)^3(-1-\cos(c+dx))}+\frac{1}{2(a+b)^3(1-\cos(c+dx))}\right.\\
&\qquad +\frac{3a^2b^2-b^4}{a\left(a^2-b^2\right)^2(-b-a\cos(c+dx))^2}+\frac{a(3a^2b+b^3)}{\left(a^2-b^2\right)^3(-b-a\cos(c+dx))}\\
&\qquad \left.+\frac{b^3}{a\left(-a^2+b^2\right)(b+a\cos(c+dx))^3}\right)dx\\
&= -\frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^3}+\frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^3}-\frac{b^3\int \frac{1}{(b+a\cos(c+dx))^3}\,dx}{a\left(a^2-b^2\right)}\\
&\quad +\frac{\left(b^2(3a^2-b^2)\right)\int \frac{1}{(-b-a\cos(c+dx))^2}\,dx}{a\left(a^2-b^2\right)^2}+\frac{\left(ab(3a^2+b^2)\right)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{\left(a^2-b^2\right)^3}\\
&= -\frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2(a-b)^3d(1+\cos(c+dx))}\\
&\quad -\frac{b^3\sin(c+dx)}{2\left(a^2-b^2\right)^2d(b+a\cos(c+dx))^2}+\frac{b^2(3a^2-b^2)\sin(c+dx)}{\left(a^2-b^2\right)^3d(b+a\cos(c+dx))}\\
&\quad -\frac{b^3\int \frac{-2b+a\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)^2}+\frac{\left(b^2(3a^2-b^2)\right)\int \frac{b}{-b-a\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)^3}\\
&\quad +\frac{\left(2ab(3a^2+b^2)\right)\operatorname{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)^3d}\\
&= -\frac{2ab(3a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))}\\
&\quad +\frac{\sin(c+dx)}{2(a-b)^3d(1+\cos(c+dx))}-\frac{b^3\sin(c+dx)}{2\left(a^2-b^2\right)^2d(b+a\cos(c+dx))^2}\\
&\quad +\frac{3b^4\sin(c+dx)}{2\left(a^2-b^2\right)^3d(b+a\cos(c+dx))}+\frac{b^2(3a^2-b^2)\sin(c+dx)}{\left(a^2-b^2\right)^3d(b+a\cos(c+dx))}\\
&\quad -\frac{b^3\int \frac{a^2+2b^2}{b+a\cos(c+dx)}\,dx}{2a\left(a^2-b^2\right)^3}+\frac{\left(b^3(3a^2-b^2)\right)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)^3}
\end{aligned}
$$

$$= -\frac{2ab(3a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^3d(1+\cos(c+dx))} - \frac{b^3\sin(c+dx)}{2\left(a^2-b^2\right)^2d(b+a\cos(c+dx))^2} + \frac{3b^4\sin(c+dx)}{2\left(a^2-b^2\right)^3d(b+a\cos(c+dx))} + \frac{b^2(3a^2-b^2)\sin(c+dx)}{\left(a^2-b^2\right)^3d(b+a\cos(c+dx))} - \frac{(b^3(a^2+2b^2))\int\frac{1}{b+a\cos(c+dx)}\,dx}{2a\left(a^2-b^2\right)^3} + \frac{(2b^3(3a^2-b^2))\operatorname{Subst}\left(\int\frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a\left(a^2-b^2\right)^3d}$$

$$= -\frac{2b^3(3a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d} - \frac{2ab(3a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^3d(1+\cos(c+dx))} - \frac{b^3\sin(c+dx)}{2\left(a^2-b^2\right)^2d(b+a\cos(c+dx))^2} + \frac{3b^4\sin(c+dx)}{2\left(a^2-b^2\right)^3d(b+a\cos(c+dx))} + \frac{b^2(3a^2-b^2)\sin(c+dx)}{\left(a^2-b^2\right)^3d(b+a\cos(c+dx))} - \frac{(b^3(a^2+2b^2))\operatorname{Subst}\left(\int\frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a\left(a^2-b^2\right)^3d}$$

$$= -\frac{2b^3(3a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d} - \frac{2ab(3a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b^3(a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^3d(1+\cos(c+dx))} - \frac{b^3\sin(c+dx)}{2\left(a^2-b^2\right)^2d(b+a\cos(c+dx))^2} + \frac{3b^4\sin(c+dx)}{2\left(a^2-b^2\right)^3d(b+a\cos(c+dx))} + \frac{b^2(3a^2-b^2)\sin(c+dx)}{\left(a^2-b^2\right)^3d(b+a\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.61

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{(b+a\cos(c+dx))\sec^3(c+dx)\left(\frac{6ab(2a^2+3b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2}{(a^2-b^2)^{7/2}} - \frac{(b+a\cos(c+dx))^2\cot\left(\frac{1}{2}(\right.}{(a+b)^3}\right.}{2d(a+b\sec(c+dx))^3}$$

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*((6*a*b*(2*a^2 + 3*b^2)*ArcTanh[((-a +
 b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^
(7/2) - ((b + a*Cos[c + d*x])^2*Cot[(c + d*x)/2])/(a + b)^3 - (b^3*Sin[c +
d*x])/((a - b)^2*(a + b)^2) + (b^2*(6*a^2 + b^2)*(b + a*Cos[c + d*x])*Sin[c
 + d*x])/((a - b)^3*(a + b)^3) + ((b + a*Cos[c + d*x])^2*Tan[(c + d*x)/2])/
(a - b)^3))/(2*d*(a + b*Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3-6a^2b+6ab^2-2b^3}-\frac{1}{2(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\frac{\left(-3a^3b+\frac{5}{2}a^2b^2-\frac{1}{2}ab^3+b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(3a^3b+\frac{5}{2}a^2b^2+\frac{1}{2}ab^3+b^4\right)\tan\left(\frac{d}{}\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}\right.}{(a-b)^3(a+b)^3}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3-6a^2b+6ab^2-2b^3}-\frac{1}{2(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\frac{\left(-3a^3b+\frac{5}{2}a^2b^2-\frac{1}{2}ab^3+b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(3a^3b+\frac{5}{2}a^2b^2+\frac{1}{2}ab^3+b^4\right)\tan\left(\frac{d}{}\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}\right.}{(a-b)^3(a+b)^3}}{d}$
risch	$-\frac{i\left(6a^5b\,e^{5i(dx+c)}+9a^3b^3e^{5i(dx+c)}-2a^6e^{4i(dx+c)}+24a^4b^2e^{4i(dx+c)}+21a^2b^4e^{4i(dx+c)}+2b^6e^{4i(dx+c)}+4a^5b\,e^{3i(dx+c)}+\right.}{a(a}$

```
[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*tan(1/2*d*x+1/2*c)/(a^3-3*a^2*b+3*a*b^2-b^3)-1/2/(a+b)^3/tan(1/2*d
*x+1/2*c)+2*b/(a-b)^3/(a+b)^3*(((-3*a^3*b+5/2*a^2*b^2-1/2*a*b^3+b^4)*tan(1/
2*d*x+1/2*c)^3+(3*a^3*b+5/2*a^2*b^2+1/2*a*b^3+b^4)*tan(1/2*d*x+1/2*c))/(tan
```

```
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-3/2*(2*a^2+3*b^2)*a/((a-b
)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 841, normalized size of antiderivative = 2.24

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \left[\frac{22\,a^4b^3 - 14\,a^2b^5 - 8\,b^7 - 2\,(2\,a^7 + 10\,a^5b^2 - 11\,a^3b^4 - ab^6)\cos{(dx+c)}^3 - 3\,\left(2\,a^3b^3 + 3\,ab^5 + (2\,a^5b + 3\,a\right.}{4\,\left((a^{10} - 4\,a^8b^2 + \right.} \right.$$

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(22*a^4*b^3 - 14*a^2*b^5 - 8*b^7 - 2*(2*a^7 + 10*a^5*b^2 - 11*a^3*b^4
- a*b^6)*cos(d*x + c)^3 - 3*(2*a^3*b^3 + 3*a*b^5 + (2*a^5*b + 3*a^3*b^3)*co
s(d*x + c)^2 + 2*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*log(
(2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*c
os(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*co
s(d*x + c) + b^2))*sin(d*x + c) + 2*(2*a^6*b - 17*a^4*b^3 + 13*a^2*b^5 + 2*
b^7)*cos(d*x + c)^2 + 2*(16*a^5*b^2 - 17*a^3*b^4 + a*b^6)*cos(d*x + c))/(((
a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d*cos(d*x + c)^2 + 2*(a
^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c) + (a^8*b^2
 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d)*sin(d*x + c)), 1/2*(11*a^4*
b^3 - 7*a^2*b^5 - 4*b^7 - (2*a^7 + 10*a^5*b^2 - 11*a^3*b^4 - a*b^6)*cos(d*x
 + c)^3 - 3*(2*a^3*b^3 + 3*a*b^5 + (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 2
*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
 b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) + (2*a^
6*b - 17*a^4*b^3 + 13*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + (16*a^5*b^2 - 17*a^
3*b^4 + a*b^6)*cos(d*x + c))/(((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 +
a^2*b^8)*d*cos(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
a*b^9)*d*cos(d*x + c) + (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10
)*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\csc^2{(c+dx)}}{(a+b\sec{(c+dx)})^3}\,dx$$

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.03

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{6\,(2\,a^3b+3\,ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{2\left(6\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] 1/2*(6*(2*a^3*b + 3*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b)
 + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2
)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + tan(1/2*d*x +
1/2*c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 2*(6*a^3*b^2*tan(1/2*d*x + 1/2*c)^
3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + a*b^4*tan(1/2*d*x + 1/2*c)^3 - 2*b^5
*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*a^2*b^3*tan(1/
2*d*x + 1/2*c) - a*b^4*tan(1/2*d*x + 1/2*c) - 2*b^5*tan(1/2*d*x + 1/2*c))/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d
*x + 1/2*c)^2 - a - b)^2) - 1/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x
+ 1/2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 15.35 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d\,(a-b)^3} - \frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{a+b}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(a^5-5\,a^4\,b+22\,a^3\,b^2-20\,a^2\,b^3+7\,a}{(a+b)^3}}{d\,\left((2\,a^5-10\,a^4\,b+20\,a^3\,b^2-20\,a^2\,b^3+10\,a\,b^4-2\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+(-4\,a^5+12\,a^4\,b-8\,a^3\,b^2-8\,a^2\right.} + \frac{a\,b\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^6-3\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,b^2+3\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b^4-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^6}{(a+b)^{7/2}\,(a-b)^{5/2}}\right)\,(2\,a^2+3\,b^2)\,3\mathrm{i}}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}$$

```
[In] int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(a +
b) + (tan(c/2 + (d*x)/2)^4*(7*a*b^4 - 5*a^4*b + a^5 - 5*b^5 - 20*a^2*b^3 +
22*a^3*b^2))/(a + b)^3 - (2*tan(c/2 + (d*x)/2)^2*(a^4 - 4*a^3*b - 5*a*b^3 +
 3*b^4 + 12*a^2*b^2))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)*(2*a*b^4 - 2*a^4*b
+ 2*a^5 - 2*b^5 + 4*a^2*b^3 - 4*a^3*b^2) - tan(c/2 + (d*x)/2)^3*(4*a^5 - 12
*a^4*b - 12*a*b^4 + 4*b^5 + 8*a^2*b^3 + 8*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(
10*a*b^4 - 10*a^4*b + 2*a^5 - 2*b^5 - 20*a^2*b^3 + 20*a^3*b^2))) + (a*b*ata
n((a^6*tan(c/2 + (d*x)/2)*1i - b^6*tan(c/2 + (d*x)/2)*1i + a^2*b^4*tan(c/2
+ (d*x)/2)*3i - a^4*b^2*tan(c/2 + (d*x)/2)*3i)/((a + b)^(7/2)*(a - b)^(5/2)
))*(2*a^2 + 3*b^2)*3i)/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.232 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 515

$$\begin{aligned}\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = & -\frac{2ab^3(3a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} \\ & - \frac{ab^3(a^2+2b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} \\ & - \frac{2ab(3a^4+8a^2b^2+b^4)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} \\ & - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2} - \frac{(a-2b)\sin(c+dx)}{4(a+b)^4d(1-\cos(c+dx))} \\ & - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))} \\ & + \frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))^2} \\ & + \frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))} + \frac{(a+2b)\sin(c+dx)}{4(a-b)^4d(1+\cos(c+dx))} \\ & - \frac{a^2b^3\sin(c+dx)}{2(a^2-b^2)^3d(b+a\cos(c+dx))^2} \\ & + \frac{3a^2b^4\sin(c+dx)}{2(a^2-b^2)^4d(b+a\cos(c+dx))} \\ & + \frac{a^2b^2(3a^2+b^2)\sin(c+dx)}{(a^2-b^2)^4d(b+a\cos(c+dx))}\end{aligned}$$

```
[Out] -2*a*b^3*(3*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a
-b)^(9/2)/(a+b)^(9/2)/d-a*b^3*(a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
```

```
/2*c)/(a+b)^(1/2))/(a-b)^(9/2)/(a+b)^(9/2)/d-2*a*b*(3*a^4+8*a^2*b^2+b^4)*ar
ctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(9/2)/(a+b)^(9/2)/d
-1/12*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))^2-1/4*(a-2*b)*sin(d*x+c)/(a+b)^4/
d/(1-cos(d*x+c))-1/12*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))+1/12*sin(d*x+c)/(
a-b)^3/d/(1+cos(d*x+c))^2+1/12*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))+1/4*(a+2
*b)*sin(d*x+c)/(a-b)^4/d/(1+cos(d*x+c))-1/2*a^2*b^3*sin(d*x+c)/(a^2-b^2)^3/
d/(b+a*cos(d*x+c))^2+3/2*a^2*b^4*sin(d*x+c)/(a^2-b^2)^4/d/(b+a*cos(d*x+c))+
a^2*b^2*(3*a^2+b^2)*sin(d*x+c)/(a^2-b^2)^4/d/(b+a*cos(d*x+c))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2976, 2729, 2727, 2743, 2833, 12, 2738, 214}

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = & -\frac{2ab^3(3a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{9/2}(a+b)^{9/2}} \\
& -\frac{ab^3(a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{9/2}(a+b)^{9/2}} \\
& +\frac{a^2b^2(3a^2+b^2)\sin(c+dx)}{d\left(a^2-b^2\right)^4\left(a\cos(c+dx)+b\right)} \\
& +\frac{3a^2b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^4\left(a\cos(c+dx)+b\right)} \\
& -\frac{a^2b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^3\left(a\cos(c+dx)+b\right)^2} \\
& -\frac{2ab(3a^4+8a^2b^2+b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{9/2}(a+b)^{9/2}} \\
& -\frac{\sin(c+dx)}{12d(a+b)^3(1-\cos(c+dx))}-\frac{(a-2b)\sin(c+dx)}{4d(a+b)^4(1-\cos(c+dx))} \\
& +\frac{(a+2b)\sin(c+dx)}{4d(a-b)^4(\cos(c+dx)+1)}+\frac{\sin(c+dx)}{12d(a-b)^3(\cos(c+dx)+1)} \\
& -\frac{\sin(c+dx)}{12d(a+b)^3(1-\cos(c+dx))^2} \\
& +\frac{\sin(c+dx)}{12d(a-b)^3(\cos(c+dx)+1)^2}
\end{aligned}$$

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a*b^3*(3*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/((a - b)^(9/2)*(a + b)^(9/2)*d) - (a*b^3*(a^2 + 2*b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - (2*a*b
```

```
*(3*a^4 + 8*a^2*b^2 + b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - Sin[c + d*x]/(12*(a + b)^3*d*(1 - Co
s[c + d*x])^2) - ((a - 2*b)*Sin[c + d*x])/(4*(a + b)^4*d*(1 - Cos[c + d*x])
) - Sin[c + d*x]/(12*(a + b)^3*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(12*(a
- b)^3*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a - b)^3*d*(1 + Cos[c +
d*x])) + ((a + 2*b)*Sin[c + d*x])/(4*(a - b)^4*d*(1 + Cos[c + d*x])) - (a^2
*b^3*Sin[c + d*x])/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])^2) + (3*a^2*b^4*
Sin[c + d*x])/(2*(a^2 - b^2)^4*d*(b + a*Cos[c + d*x])) + (a^2*b^2*(3*a^2 +
b^2)*Sin[c + d*x])/((a^2 - b^2)^4*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cot^3(c+dx)\csc(c+dx)}{(-b-a\cos(c+dx))^3}\,dx\\
&= \int \left(\frac{1}{4(a-b)^3(-1-\cos(c+dx))^2}+\frac{-a-2b}{4(a-b)^4(-1-\cos(c+dx))}\right.\\
&\quad+\frac{1}{4(a+b)^3(1-\cos(c+dx))^2}+\frac{a-2b}{4(a+b)^4(1-\cos(c+dx))}\\
&\quad+\frac{ab^3}{(a^2-b^2)^2(-b-a\cos(c+dx))^3}+\frac{ab^2(3a^2+b^2)}{(a^2-b^2)^3(-b-a\cos(c+dx))^2}\\
&\quad\left.+\frac{ab(3a^4+8a^2b^2+b^4)}{(a^2-b^2)^4(-b-a\cos(c+dx))}\right)dx\\
&= \frac{\int \frac{1}{(-1-\cos(c+dx))^2}\,dx}{4(a-b)^3}+\frac{(a-2b)\int \frac{1}{1-\cos(c+dx)}\,dx}{4(a+b)^4}+\frac{\int \frac{1}{(1-\cos(c+dx))^2}\,dx}{4(a+b)^3}\\
&\quad-\frac{(a+2b)\int \frac{1}{-1-\cos(c+dx)}\,dx}{4(a-b)^4}+\frac{(ab^3)\int \frac{1}{(-b-a\cos(c+dx))^3}\,dx}{(a^2-b^2)^2}\\
&\quad+\frac{(ab^2(3a^2+b^2))\int \frac{1}{(-b-a\cos(c+dx))^2}\,dx}{(a^2-b^2)^3}+\frac{(ab(3a^4+8a^2b^2+b^4))\int \frac{1}{-b-a\cos(c+dx)}\,dx}{(a^2-b^2)^4}
\end{aligned}
$$

$$= -\frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2} - \frac{(a-2b)\sin(c+dx)}{4(a+b)^4d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))^2} + \frac{(a+2b)\sin(c+dx)}{4(a-b)^4d(1+\cos(c+dx))} - \frac{a^2b^3\sin(c+dx)}{2\left(a^2-b^2\right)^3d(b+a\cos(c+dx))^2} + \frac{a^2b^2(3a^2+b^2)\sin(c+dx)}{\left(a^2-b^2\right)^4d(b+a\cos(c+dx))} - \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{12(a-b)^3} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{12(a+b)^3} + \frac{(ab^3)\int \frac{2b-a\cos(c+dx)}{(-b-a\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)^3} + \frac{(ab^2(3a^2+b^2))\int \frac{b}{-b-a\cos(c+dx)}\,dx}{\left(a^2-b^2\right)^4} + \frac{(2ab(3a^4+8a^2b^2+b^4))\,\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)^4 d}$$

$$= -\frac{2ab(3a^4+8a^2b^2+b^4)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2} - \frac{(a-2b)\sin(c+dx)}{4(a+b)^4d(1-\cos(c+dx))} - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))^2} + \frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))} + \frac{(a+2b)\sin(c+dx)}{4(a-b)^4d(1+\cos(c+dx))} - \frac{a^2b^3\sin(c+dx)}{2\left(a^2-b^2\right)^3d(b+a\cos(c+dx))^2} + \frac{3a^2b^4\sin(c+dx)}{2\left(a^2-b^2\right)^4d(b+a\cos(c+dx))} + \frac{a^2b^2(3a^2+b^2)\sin(c+dx)}{\left(a^2-b^2\right)^4d(b+a\cos(c+dx))} + \frac{(ab^3)\int \frac{a^2+2b^2}{-b-a\cos(c+dx)}\,dx}{2\left(a^2-b^2\right)^4} + \frac{(ab^3(3a^2+b^2))\int \frac{1}{-b-a\cos(c+dx)}\,dx}{\left(a^2-b^2\right)^4}$$

$$= -\frac{2ab(3a^4+8a^2b^2+b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d}$$
$$-\frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2}-\frac{(a-2b)\sin(c+dx)}{4(a+b)^4d(1-\cos(c+dx))}$$
$$-\frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))}+\frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))^2}$$
$$+\frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))}+\frac{(a+2b)\sin(c+dx)}{4(a-b)^4d(1+\cos(c+dx))}$$
$$-\frac{a^2b^3\sin(c+dx)}{2(a^2-b^2)^3d(b+a\cos(c+dx))^2}+\frac{3a^2b^4\sin(c+dx)}{2(a^2-b^2)^4d(b+a\cos(c+dx))}$$
$$+\frac{a^2b^2(3a^2+b^2)\sin(c+dx)}{(a^2-b^2)^4d(b+a\cos(c+dx))}+\frac{(ab^3(a^2+2b^2))\int\frac{1}{-b-a\cos(c+dx)}\,dx}{2(a^2-b^2)^4}$$
$$+\frac{(2ab^3(3a^2+b^2))\operatorname{Subst}\left(\int\frac{1}{-a-b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)^4d}$$

$$= -\frac{2ab^3(3a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d}$$
$$-\frac{2ab(3a^4+8a^2b^2+b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d}$$
$$-\frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2}-\frac{(a-2b)\sin(c+dx)}{4(a+b)^4d(1-\cos(c+dx))}$$
$$-\frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))}+\frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))^2}$$
$$+\frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))}+\frac{(a+2b)\sin(c+dx)}{4(a-b)^4d(1+\cos(c+dx))}$$
$$-\frac{a^2b^3\sin(c+dx)}{2(a^2-b^2)^3d(b+a\cos(c+dx))^2}$$
$$+\frac{3a^2b^4\sin(c+dx)}{2(a^2-b^2)^4d(b+a\cos(c+dx))}+\frac{a^2b^2(3a^2+b^2)\sin(c+dx)}{(a^2-b^2)^4d(b+a\cos(c+dx))}$$
$$+\frac{(ab^3(a^2+2b^2))\operatorname{Subst}\left(\int\frac{1}{-a-b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)^4d}$$

$$
\begin{aligned}
&= -\frac{2ab^3(3a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} \\
&\quad - \frac{ab^3(a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} \\
&\quad - \frac{2ab(3a^4+8a^2b^2+b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} \\
&\quad - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2} - \frac{(a-2b)\sin(c+dx)}{4(a+b)^4d(1-\cos(c+dx))} \\
&\quad - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))^2} \\
&\quad + \frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx))} + \frac{(a+2b)\sin(c+dx)}{4(a-b)^4d(1+\cos(c+dx))} \\
&\quad - \frac{a^2b^3\sin(c+dx)}{2\left(a^2-b^2\right)^3d(b+a\cos(c+dx))^2} \\
&\quad + \frac{3a^2b^4\sin(c+dx)}{2\left(a^2-b^2\right)^4d(b+a\cos(c+dx))} + \frac{a^2b^2(3a^2+b^2)\sin(c+dx)}{\left(a^2-b^2\right)^4d(b+a\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.75

$$
\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx
$$

$$
= \frac{(b+a\cos(c+dx))\left(\frac{96ab(6a^4+23a^2b^2+6b^4)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2}{\sqrt{a^2-b^2}} + (36a^6b+154a^4b^3+424a\ldots\right.}{}
$$

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((96*a*b*(6*a^4 + 23*a^2*b^2 + 6*b^4)*ArcTanh[((-a +
b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/Sqrt[a^2 - b^
2] + (36*a^6*b + 154*a^4*b^3 + 424*a^2*b^5 + 16*b^7 - 2*a*(16*a^6 - 94*a^4*
b^2 - 35*a^2*b^4 + 8*b^6)*Cos[c + d*x] + 8*(2*a^6*b - 45*a^4*b^3 - 56*a^2*b
^5 - 6*b^7)*Cos[2*(c + d*x)] - 4*a^7*Cos[3*(c + d*x)] - 154*a^5*b^2*Cos[3*(
c + d*x)] - 205*a^3*b^4*Cos[3*(c + d*x)] + 48*a*b^6*Cos[3*(c + d*x)] - 20*a
^6*b*Cos[4*(c + d*x)] + 110*a^4*b^3*Cos[4*(c + d*x)] + 120*a^2*b^5*Cos[4*(c
 + d*x)] + 4*a^7*Cos[5*(c + d*x)] + 62*a^5*b^2*Cos[5*(c + d*x)] + 39*a^3*b^
4*Cos[5*(c + d*x)])*Csc[c + d*x]^3)*Sec[c + d*x]^3)/(96*(a^2 - b^2)^4*d*(a
+ b*Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3} a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3} b}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)(a-b)}-\frac{1}{24(a+b)^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{3a-3b}{8(a+b)^{4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+ 2ab\left(\frac{\left(\frac{5}{2}a^{3}b^{2}\right.}{}\right.$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3} a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3} b}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)(a-b)}-\frac{1}{24(a+b)^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{3a-3b}{8(a+b)^{4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+ 2ab\left(\frac{\left(\frac{5}{2}a^{3}b^{2}\right.}{}\right.$
risch	Expression too large to display

```
[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*ta
n(1/2*d*x+1/2*c)^3*b+3*tan(1/2*d*x+1/2*c)*a+3*tan(1/2*d*x+1/2*c)*b)-1/24/(a
+b)^3/tan(1/2*d*x+1/2*c)^3-1/8/(a+b)^4*(3*a-3*b)/tan(1/2*d*x+1/2*c)+2*a*b/(
a-b)^4/(a+b)^4*(((5/2*a^3*b^2+3*a*b^4-3*a^4*b-5/2*a^2*b^3)*tan(1/2*d*x+1/2*
c)^3+(5/2*a^3*b^2+3*a*b^4+3*a^4*b+5/2*a^2*b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-1/2*(6*a^4+23*a^2*b^2+6*b^4)/
((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 1550, normalized size of antiderivative = 3.01

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(78*a^6*b^3 + 46*a^4*b^5 - 116*a^2*b^7 - 8*b^9 + 2*(4*a^9 + 58*a^7*b
^2 - 23*a^5*b^4 - 39*a^3*b^6)*cos(d*x + c)^5 - 10*(2*a^8*b - 13*a^6*b^3 - a
^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 - 4*(3*a^9 + 55*a^7*b^2 - 8*a^5*b^4 - 5
6*a^3*b^6 + 6*a*b^8)*cos(d*x + c)^3 + 3*(6*a^5*b^3 + 23*a^3*b^5 + 6*a*b^7 -
 (6*a^7*b + 23*a^5*b^3 + 6*a^3*b^5)*cos(d*x + c)^4 - 2*(6*a^6*b^2 + 23*a^4*
b^4 + 6*a^2*b^6)*cos(d*x + c)^3 + (6*a^7*b + 17*a^5*b^3 - 17*a^3*b^5 - 6*a*
b^7)*cos(d*x + c)^2 + 2*(6*a^6*b^2 + 23*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c))*
sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*
```

```
sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d
*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) + 4*(6*a^8*b - 56*a^6*b
^3 - 8*a^4*b^5 + 55*a^2*b^7 + 3*b^9)*cos(d*x + c)^2 + 10*(12*a^7*b^2 - a^5*
b^4 - 13*a^3*b^6 + 2*a*b^8)*cos(d*x + c))/(((a^12 - 5*a^10*b^2 + 10*a^8*b^4
 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^4 + 2*(a^11*b - 5*a^9*
b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3 - (a^1
2 - 6*a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*b^10 + b^12)*
d*cos(d*x + c)^2 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*
b^9 - a*b^11)*d*cos(d*x + c) - (a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*
b^8 + 5*a^2*b^10 - b^12)*d)*sin(d*x + c)), -1/6*(39*a^6*b^3 + 23*a^4*b^5 -
58*a^2*b^7 - 4*b^9 + (4*a^9 + 58*a^7*b^2 - 23*a^5*b^4 - 39*a^3*b^6)*cos(d*x
 + c)^5 - 5*(2*a^8*b - 13*a^6*b^3 - a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 -
2*(3*a^9 + 55*a^7*b^2 - 8*a^5*b^4 - 56*a^3*b^6 + 6*a*b^8)*cos(d*x + c)^3 -
3*(6*a^5*b^3 + 23*a^3*b^5 + 6*a*b^7 - (6*a^7*b + 23*a^5*b^3 + 6*a^3*b^5)*co
s(d*x + c)^4 - 2*(6*a^6*b^2 + 23*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c)^3 + (6*a
^7*b + 17*a^5*b^3 - 17*a^3*b^5 - 6*a*b^7)*cos(d*x + c)^2 + 2*(6*a^6*b^2 + 2
3*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) + 2*(6*a^
8*b - 56*a^6*b^3 - 8*a^4*b^5 + 55*a^2*b^7 + 3*b^9)*cos(d*x + c)^2 + 5*(12*a
^7*b^2 - a^5*b^4 - 13*a^3*b^6 + 2*a*b^8)*cos(d*x + c))/(((a^12 - 5*a^10*b^2
 + 10*a^8*b^4 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^4 + 2*(a^
11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x
+ c)^3 - (a^12 - 6*a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*
b^10 + b^12)*d*cos(d*x + c)^2 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5
*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c) - (a^10*b^2 - 5*a^8*b^4 + 10*a^6*
b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\csc^4{(c+dx)}}{(a+b\sec{(c+dx)})^3}\,dx$$

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.38

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{24\,(6\,a^5b+23\,a^3b^3+6\,ab^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\sqrt{-a^2+b^2}}+\frac{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{}}{}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(24*(6*a^5*b + 23*a^3*b^3 + 6*a*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(-a^2 + b^2)))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt(-a
^2 + b^2)) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 6*a^5*b*tan(1/2*d*x + 1/2*c)^3 +
 15*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 20*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 15
*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^5*tan(1/2*d*x + 1/2*c)^3 + b^6*tan(
1/2*d*x + 1/2*c)^3 + 9*a^6*tan(1/2*d*x + 1/2*c) - 36*a^5*b*tan(1/2*d*x + 1/
2*c) + 45*a^4*b^2*tan(1/2*d*x + 1/2*c) - 45*a^2*b^4*tan(1/2*d*x + 1/2*c) +
36*a*b^5*tan(1/2*d*x + 1/2*c) - 9*b^6*tan(1/2*d*x + 1/2*c))/(a^9 - 9*a^8*b
+ 36*a^7*b^2 - 84*a^6*b^3 + 126*a^5*b^4 - 126*a^4*b^5 + 84*a^3*b^6 - 36*a^2
*b^7 + 9*a*b^8 - b^9) - 24*(6*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^4*b^3*ta
n(1/2*d*x + 1/2*c)^3 + 5*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^2*b^5*tan(1/2
*d*x + 1/2*c)^3 - 6*a^5*b^2*tan(1/2*d*x + 1/2*c) - 5*a^4*b^3*tan(1/2*d*x +
1/2*c) - 5*a^3*b^4*tan(1/2*d*x + 1/2*c) - 6*a^2*b^5*tan(1/2*d*x + 1/2*c))/(
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (9*a*tan(1/2*d*x + 1/2*c)^2 - 9*b*t
an(1/2*d*x + 1/2*c)^2 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B] (verification not implemented)

Time = 14.27 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.14

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d\,(a-b)^3}$$
$$+\frac{\frac{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}{3\,(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,\left(3\,a^7-21\,a^6\,b+111\,a^5\,b^2-145\,a^4\,b^3+145\,a^3\,b^4-111\,a^2\,b^5+21\,a\right.}{(a+b)^4}}{d\,\left(\left(-8\,a^6+48\,a^5\,b-120\,a^4\,b^2+160\,a^3\,b^3-120\,a^2\,b^4+48\,a\,b^5-8\,b^6\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(16\,a^6-64\,a^5\,b\right.\right.}$$
$$+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a+b)}{8\,d\,(a-b)^4}$$
$$+\frac{a\,b\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^8-4\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^6\,b^2+6\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^4\,b^4-4\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2\,b^6+1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)b^8}{(a+b)^{9/2}\,(a-b)^{7/2}}\right)\,\left(6\,a^4+23\,a^2\,b^2\right.}{d\,(a+b)^{9/2}\,(a-b)^{9/2}}$$

```
[In] int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^3) + ((a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6
*a^2*b^2)/(3*(a + b)) + (tan(c/2 + (d*x)/2)^6*(21*a*b^6 - 21*a^6*b + 3*a^7
- 3*b^7 - 111*a^2*b^5 + 145*a^3*b^4 - 145*a^4*b^3 + 111*a^5*b^2))/(a + b)^4
 - (tan(c/2 + (d*x)/2)^4*(17*a^6 - 102*a^5*b - 102*a*b^5 + 17*b^6 + 399*a^2
*b^4 - 364*a^3*b^3 + 399*a^4*b^2))/(3*(a + b)^3) + (7*tan(c/2 + (d*x)/2)^2*
(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))/(3*(a + b)^2))/(d*(tan
(c/2 + (d*x)/2)^3*(16*a*b^5 + 16*a^5*b - 8*a^6 - 8*b^6 + 8*a^2*b^4 - 32*a^3
*b^3 + 8*a^4*b^2) - tan(c/2 + (d*x)/2)^7*(8*a^6 - 48*a^5*b - 48*a*b^5 + 8*b
^6 + 120*a^2*b^4 - 160*a^3*b^3 + 120*a^4*b^2) + tan(c/2 + (d*x)/2)^5*(64*a*
b^5 - 64*a^5*b + 16*a^6 - 16*b^6 - 80*a^2*b^4 + 80*a^4*b^2))) + (3*tan(c/2
+ (d*x)/2)*(a + b))/(8*d*(a - b)^4) + (a*b*atan((a^8*tan(c/2 + (d*x)/2)*1i
+ b^8*tan(c/2 + (d*x)/2)*1i - a^2*b^6*tan(c/2 + (d*x)/2)*4i + a^4*b^4*tan(c
/2 + (d*x)/2)*6i - a^6*b^2*tan(c/2 + (d*x)/2)*4i)/((a + b)^(9/2)*(a - b)^(7
/2)))*(6*a^4 + 6*b^4 + 23*a^2*b^2)*1i)/(d*(a + b)^(9/2)*(a - b)^(9/2))
```

3.233 $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 516

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx &= -\frac{b(a^2-b^2)^{5/4}\,e^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d} \\
&\quad - \frac{b(a^2-b^2)^{5/4}\,e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d} \\
&\quad + \frac{2(5a^4-28a^2b^2+21b^4)\,e^4\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^5d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{b^2(a^2-b^2)^2\,e^4\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^5\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{b^2(a^2-b^2)^2\,e^4\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^5\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2e^3(21b(a^2-b^2)-a(5a^2-7b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{21a^4d} \\
&\quad + \frac{2e(7b-5a\cos(c+dx))(e\sin(c+dx))^{5/2}}{35a^2d}
\end{aligned}
$$

```
[Out] -b*(a^2-b^2)^(5/4)*e^(7/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1
/4)/e^(1/2))/a^(9/2)/d-b*(a^2-b^2)^(5/4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d*x
+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+2/35*e*(7*b-5*a*cos(d*x+c))*(
e*sin(d*x+c))^(5/2)/a^2/d-2/21*(5*a^4-28*a^2*b^2+21*b^4)*e^4*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+
1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^5/d/(e*sin(d*x+c))^(1/2)-b^2*(a^2-b^2)
^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
```

```
icPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/a^5/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-b^2*(a^2-b^2)^
2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(
1/2)/a^5/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2/21*e^3*(21*b*
(a^2-b^2)-a*(5*a^2-7*b^2)*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/a^4/d
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3957, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx &= \frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d}\\ &-\frac{be^{7/2}(a^2-b^2)^{5/4}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{9/2}d}\\ &-\frac{be^{7/2}(a^2-b^2)^{5/4}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{9/2}d}\\ &+\frac{b^2e^4(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^5d\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}\\ &+\frac{b^2e^4(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^5d\left(a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}\\ &+\frac{2e^3\sqrt{e\sin(c+dx)}(21b(a^2-b^2)-a(5a^2-7b^2)\cos(c+dx))}{21a^4d}\\ &+\frac{2e^4(5a^4-28a^2b^2+21b^4)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21a^5d\sqrt{e\sin(c+dx)}}\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*(a^2 - b^2)^(5/4)*e^(7/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2
- b^2)^(1/4)*Sqrt[e])])/(a^(9/2)*d)) - (b*(a^2 - b^2)^(5/4)*e^(7/2)*ArcTanh
[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(9/2)*d) +
 (2*(5*a^4 - 28*a^2*b^2 + 21*b^4)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/(21*a^5*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)^2*e^4*El
lipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(a^5*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(
a^2 - b^2)^2*e^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2
, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[
c + d*x]]) + (2*e^3*(21*b*(a^2 - b^2) - a*(5*a^2 - 7*b^2)*Cos[c + d*x])*Sqr
```

```
t[e*Sin[c + d*x]])/(21*a^4*d) + (2*e*(7*b - 5*a*Cos[c + d*x])*(e*Sin[c + d*
x])^(5/2))/(35*a^2*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{-b-a\cos(c+dx)}\,dx$$

$$=\frac{2e(7b-5a\cos(c+dx))(e\sin(c+dx))^{5/2}}{35a^2d}-\frac{(2e^2)\int\frac{\left(-ab+\frac{1}{2}(5a^2-7b^2)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{-b-a\cos(c+dx)}\,dx}{7a^2}$$

$$=\frac{2e^3(21b(a^2-b^2)-a(5a^2-7b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{21a^4d}+\frac{2e(7b-5a\cos(c+dx))(e\sin(c+dx))^{5/2}}{35a^2d}-\frac{(4e^4)\int\frac{-\frac{1}{2}ab(8a^2-7b^2)+\frac{1}{4}(5a^4-28a^2b^2+21b^4)\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{21a^4}$$

$$=\frac{2e^3(21b(a^2-b^2)-a(5a^2-7b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{21a^4d}+\frac{2e(7b-5a\cos(c+dx))(e\sin(c+dx))^{5/2}}{35a^2d}+\frac{\left(b(a^2-b^2)^2e^4\right)\int\frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a^5}+\frac{((5a^4-28a^2b^2+21b^4)e^4)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{21a^5}$$

$$=\frac{2e^3(21b(a^2-b^2)-a(5a^2-7b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{21a^4d}+\frac{2e(7b-5a\cos(c+dx))(e\sin(c+dx))^{5/2}}{35a^2d}+\frac{\left(b^2(a^2-b^2)^{3/2}e^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a^5}+\frac{\left(b^2(a^2-b^2)^{3/2}e^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a^5}+\frac{\left(b(a^2-b^2)^2e^5\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}((-a^2+b^2)e^2+a^2x^2)}\,dx,x,e\sin(c+dx)\right)}{a^4d}+\frac{\left((5a^4-28a^2b^2+21b^4)e^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21a^5\sqrt{e\sin(c+dx)}}$$

$$= \frac{2(5a^4 - 28a^2b^2 + 21b^4)\, e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right) \sqrt{\sin(c + dx)}}{21a^5 d\sqrt{e\sin(c + dx)}}$$
$$+ \frac{2e^3(21b(a^2 - b^2) - a(5a^2 - 7b^2)\cos(c + dx))\sqrt{e\sin(c + dx)}}{21a^4 d}$$
$$+ \frac{2e(7b - 5a\cos(c + dx))(e\sin(c + dx))^{5/2}}{35a^2 d}$$
$$+ \frac{\left(2b(a^2 - b^2)^2 e^5\right) \operatorname{Subst}\left(\int \frac{1}{(-a^2+b^2)e^2+a^2x^4}\, dx, x, \sqrt{e\sin(c + dx)}\right)}{a^4 d}$$
$$+ \frac{\left(b^2(a^2 - b^2)^{3/2} e^4 \sqrt{\sin(c + dx)}\right) \int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\, dx}{2a^5\sqrt{e\sin(c + dx)}}$$
$$+ \frac{\left(b^2(a^2 - b^2)^{3/2} e^4 \sqrt{\sin(c + dx)}\right) \int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\, dx}{2a^5\sqrt{e\sin(c + dx)}}$$

$$= \frac{2(5a^4 - 28a^2b^2 + 21b^4)\, e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right) \sqrt{\sin(c + dx)}}{21a^5 d\sqrt{e\sin(c + dx)}}$$
$$- \frac{b^2(a^2 - b^2)^{3/2} e^4 \operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right) \sqrt{\sin(c + dx)}}{a^5\left(a - \sqrt{a^2 - b^2}\right) d\sqrt{e\sin(c + dx)}}$$
$$+ \frac{b^2(a^2 - b^2)^{3/2} e^4 \operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right) \sqrt{\sin(c + dx)}}{a^5\left(a + \sqrt{a^2 - b^2}\right) d\sqrt{e\sin(c + dx)}}$$
$$+ \frac{2e^3(21b(a^2 - b^2) - a(5a^2 - 7b^2)\cos(c + dx))\sqrt{e\sin(c + dx)}}{21a^4 d}$$
$$+ \frac{2e(7b - 5a\cos(c + dx))(e\sin(c + dx))^{5/2}}{35a^2 d}$$
$$- \frac{\left(b(a^2 - b^2)^{3/2} e^4\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e-ax^2}\, dx, x, \sqrt{e\sin(c + dx)}\right)}{a^4 d}$$
$$- \frac{\left(b(a^2 - b^2)^{3/2} e^4\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e+ax^2}\, dx, x, \sqrt{e\sin(c + dx)}\right)}{a^4 d}$$

$$= -\frac{b(a^2-b^2)^{5/4}\,e^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d} - \frac{b(a^2-b^2)^{5/4}\,e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d} + \frac{2(5a^4-28a^2b^2+21b^4)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^5d\sqrt{e\sin(c+dx)}} - \frac{b^2(a^2-b^2)^{3/2}\,e^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{b^2(a^2-b^2)^{3/2}\,e^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{2e^3(21b(a^2-b^2)-a(5a^2-7b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{21a^4d} + \frac{2e(7b-5a\cos(c+dx))(e\sin(c+dx))^{5/2}}{35a^2d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 47.84 (sec) , antiderivative size = 2049, normalized size of antiderivative = 3.97

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(-1/42*((23*a^2 - 28*b^2)*Cos[c + d*x])/a^3 - (b*Cos[
2*(c + d*x)])/(5*a^2) + Cos[3*(c + d*x)]/(14*a))*Csc[c + d*x]^3*Sec[c + d*x
]*(e*Sin[c + d*x])^(7/2))/(d*(a + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*
Sec[c + d*x]*(e*Sin[c + d*x])^(7/2)*((2*(-100*a^3 + 98*a*b^2)*Cos[c + d*x]^
2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Si
n[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(
-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^
2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]
]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4,
 -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c
 + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5
/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2
 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
```

```
2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos
[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(89*a^2*b - 70*b^3)*Cos[c + d*x]*(b +
 a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*
Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt
[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Si
n[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/
4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(S
qrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d
*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9
/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF
1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[
c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sqrt
[1 - Sin[c + d*x]^2]) + ((-231*a^2*b + 210*b^3)*Cos[c + d*x]*Cos[2*(c + d*x
)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 -
((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^
2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I/4)*
(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[
Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4 - I/
4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*Sqrt
[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*Appe
llF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a
^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2
)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^2]
)))/(420*a^3*d*(a + b*Sec[c + d*x])*Sin[c + d*x]^(7/2))
```

Maple [A] (verified)

Time = 16.19 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.50

method	result
default	$2eb\left(-\frac{\sqrt{e\sin(dx+c)}\,e^{2}\left(\cos(dx+c)^{2}a^{2}-6a^{2}+5b^{2}\right)}{5a^{4}}+\frac{e^{4}\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\ldots\right.\right.}{4a^{4}\left(-a^{2}e^{2}+b^{2}e^{2}\right)}\right.$

```
[In] int((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (2*e*b*(-1/5/a^4*(e*sin(d*x+c))^(1/2)*e^2*(cos(d*x+c)^2*a^2-6*a^2+5*b^2)+1/
4*e^4*(a^4-2*a^2*b^2+b^4)/a^4*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*
(ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-
(e^2*(a^2-b^2)/a^2)^(1/4)))+2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^
2)^(1/4))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a*e^4*(-1/21/a^6/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)*(-6*a^4*cos(d*x+c)^4*sin(d*x+c)+5*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))*a^4-28*a^2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+21*b^4*(-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c
)+1)^(1/2),1/2*2^(1/2))+16*a^4*cos(d*x+c)^2*sin(d*x+c)-14*a^2*b^2*cos(d*x+c
)^2*sin(d*x+c))-b^2*(a^4-2*a^2*b^2+b^4)/a^6*(-1/2/(a^2-b^2)^(1/2)/a*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(
a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/
a),1/2*2^(1/2)))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\,\sin(c+d\,x))^{7/2}}{b+a\,\cos(c+d\,x)}\,dx$$

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(7/2))/(b + a*cos(c + d*x)), x)
```

3.234 $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 430

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx &= \frac{b(a^2-b^2)^{3/4}\,e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} \\ &- \frac{b(a^2-b^2)^{3/4}\,e^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} \\ &- \frac{b^2(a^2-b^2)\,e^3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{b^2(a^2-b^2)\,e^3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{2(3a^2-5b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^3d\sqrt{\sin(c+dx)}} \\ &+ \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d}\end{aligned}$$

```
[Out] b*(a^2-b^2)^(3/4)*e^(5/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/
4)/e^(1/2))/a^(7/2)/d-b*(a^2-b^2)^(3/4)*e^(5/2)*arctanh(a^(1/2)*(e*sin(d*x+
c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/d+2/15*e*(5*b-3*a*cos(d*x+c))*(e
*sin(d*x+c))^(3/2)/a^2/d+b^2*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(
a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a-(a^2-b^2)^(1/2))/(e*sin(
d*x+c))^(1/2)+b^2*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1
/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/
```

```
2)-2/5*(3*a^2-5*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/
2)/a^3/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3957, 2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d} + \frac{be^{5/2}(a^2-b^2)^{3/4}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{7/2}d} - \frac{be^{5/2}(a^2-b^2)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{7/2}d} - \frac{b^2e^3(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^4d\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}} - \frac{b^2e^3(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^4d\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}} + \frac{2e^2(3a^2-5b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^3d\sqrt{\sin(c+dx)}}$$

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*(a^2 - b^2)^(3/4)*e^(5/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 -
b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d) - (b*(a^2 - b^2)^(3/4)*e^(5/2)*ArcTanh[(S
qrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d) - (b
^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]
]) - (b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2
 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (2*(3*a^2 - 5*b^2)*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*
Sin[c + d*x]])/(5*a^3*d*Sqrt[Sin[c + d*x]]) + (2*e*(5*b - 3*a*Cos[c + d*x])
*(e*Sin[c + d*x])^(3/2))/(15*a^2*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
```

```
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} - \frac{(2e^2)\int \frac{\left(-ab+\frac{1}{2}(3a^2-5b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{5a^2}$$

$$= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} + \frac{((3a^2-5b^2)\,e^2)\int \sqrt{e\sin(c+dx)}\,dx}{5a^3} + \frac{(b(a^2-b^2)\,e^2)\int \frac{\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{a^3}$$

$$
\begin{aligned}
&= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} \\
&\quad + \frac{\left(b^2(a^2-b^2)\,e^3\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a^4} \\
&\quad - \frac{\left(b^2(a^2-b^2)\,e^3\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a^4} \\
&\quad + \frac{\left(b(a^2-b^2)\,e^3\right)\operatorname{Subst}\left(\int \frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}\,dx, x, e\sin(c+dx)\right)}{a^2d} \\
&\quad + \frac{\left((3a^2-5b^2)\,e^2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{5a^3\sqrt{\sin(c+dx)}} \\
&= \frac{2(3a^2-5b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5a^3d\sqrt{\sin(c+dx)}} \\
&\quad + \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} \\
&\quad + \frac{\left(2b(a^2-b^2)\,e^3\right)\operatorname{Subst}\left(\int \frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^2d} \\
&\quad + \frac{\left(b^2(a^2-b^2)\,e^3\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a^4\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{\left(b^2(a^2-b^2)\,e^3\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a^4\sqrt{e\sin(c+dx)}} \\
&= -\frac{b^2(a^2-b^2)\,e^3\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^4\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{b^2(a^2-b^2)\,e^3\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^4\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2(3a^2-5b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5a^3d\sqrt{\sin(c+dx)}} \\
&\quad + \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} \\
&\quad - \frac{\left(b(a^2-b^2)\,e^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^3d} \\
&\quad + \frac{\left(b(a^2-b^2)\,e^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^3d}
\end{aligned}
$$

$$= \frac{b(a^2-b^2)^{3/4} e^{5/2} \arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} - \frac{b(a^2-b^2)^{3/4} e^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} - \frac{b^2(a^2-b^2)e^3 \operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^4\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b^2(a^2-b^2)e^3 \operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^4\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{2(3a^2-5b^2)e^2 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^3d\sqrt{\sin(c+dx)}} + \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 36.32 (sec) , antiderivative size = 853, normalized size of antiderivative = 1.98

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx =$$

$$-\frac{(b+a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{(-3a^2+5b^2)\cos^2(c+dx)\left(3\sqrt{2}b(-a^2+b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right.\right.\right.}{}\right.}{} + \frac{(b+a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{2b\sin(c+dx)}{3a^2}-\frac{\sin(2(c+dx))}{5a}\right)}{d(a+b\sec(c+dx))}$$

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/5*((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*(((-3*a^2 +
5*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2
]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*S
qrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqr
t[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[
Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
```

```
a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a
^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c +
 d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2
)) + (4*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[
Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[
c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] +
 (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))
/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2
, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b^2)))*(
b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x
]^2])))/(a^2*d*(a + b*Sec[c + d*x])*Sin[c + d*x]^(5/2)) + ((b + a*Cos[c + d
*x])*Csc[c + d*x]^2*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*((2*b*Sin[c + d*x])
/(3*a^2) - Sin[2*(c + d*x)]/(5*a)))/(d*(a + b*Sec[c + d*x]))
```

Maple [A] (verified)

Time = 13.67 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.60

method	result
default	$\frac{2eb\left(\frac{(e\sin(dx+c))^{\frac{3}{2}}}{3a^2}+\frac{e^2\left(a^2-b^2\right)\left(2\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)\right)}{4a^4\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)+\frac{\sqrt{\cos(dx+c)^2 e\sin(dx+c)}\,a\,e^3\left(-\right.}{}}{}$

```
[In] int((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (2*e*b*(1/3*(e*sin(d*x+c))^(3/2)/a^2+1/4*e^2*(a^2-b^2)/a^4/(e^2*(a^2-b^2)/a
^2)^(1/4)*(2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((e*
sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^
2-b^2)/a^2)^(1/4)))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a*e^3*(-1/5/a^2/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-3*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^4+2*cos(d*x+c)^2)+b^2/a^4*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((-sin(d
*x+c)+1)^(1/2),1/2*2^(1/2)))-b^2*(a^2-b^2)/a^4*(-1/2/a^2*(-sin(d*x+c)+1)^(1
```

```
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1
/2)/a),1/2*2^(1/2))-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x
+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\,\sin(c+d\,x))^{5/2}}{b+a\,\cos(c+d\,x)}\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(5/2))/(b + a*cos(c + d*x)), x)
```

3.235 $\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 444

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx &= -\frac{b\sqrt[4]{a^2-b^2}\,e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}d} \\
&- \frac{b\sqrt[4]{a^2-b^2}\,e^{3/2}\mathrm{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}d} \\
&+ \frac{2(a^2-3b^2)\,e^2\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^3d\sqrt{e\sin(c+dx)}} \\
&+ \frac{b^2(a^2-b^2)\,e^2\,\mathrm{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{b^2(a^2-b^2)\,e^2\,\mathrm{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d}
\end{aligned}
$$

```
[Out] -b*(a^2-b^2)^(1/4)*e^(3/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1
/4)/e^(1/2))/a^(5/2)/d-b*(a^2-b^2)^(1/4)*e^(3/2)*arctanh(a^(1/2)*(e*sin(d*x
+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(5/2)/d-2/3*(a^2-3*b^2)*e^2*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/
4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^3/d/(e*sin(d*x+c))^(1/2)-b^2*(a^2
-b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+
c)^(1/2)/a^3/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-b^2*(a^2-b^
2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
```

```
icPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/a^3/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2/3*e*(3*b-a*c
os(d*x+c))*(e*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3957, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx &= \frac{2e\sqrt{e\sin(c+dx)}(3b-a\cos(c+dx))}{3a^2d}\\ &\quad-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{5/2}d}-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{5/2}d}\\ &\quad+\frac{2e^2(a^2-3b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3a^3d\sqrt{e\sin(c+dx)}}\\ &\quad+\frac{b^2e^2(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^3d\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}\\ &\quad+\frac{b^2e^2(a^2-b^2)\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^3d\left(a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}\end{aligned}$$

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2
- b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*d)) - (b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcTanh
[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*d) +
 (2*(a^2 - 3*b^2)*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/
(3*a^3*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)*e^2*EllipticPi[(2*a)/(a -
 Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a^2 - b
^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)*e^2*Elli
pticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x
]])/(a^3*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*e*(3*
b - a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*a^2*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
```

```
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{-b-a\cos(c+dx)}\,dx\\
&= \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d}-\frac{(2e^2)\int \frac{-ab+\frac{1}{2}(a^2-3b^2)\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3a^2}\\
&= \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d}+\frac{((a^2-3b^2)\,e^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^3}\\
&\quad+\frac{(b(a^2-b^2)\,e^2)\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a^3}
\end{aligned}
$$

$$= \frac{2e(3b - a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{\left(b^2\sqrt{a^2-b^2}e^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a^3} + \frac{\left(b^2\sqrt{a^2-b^2}e^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a^3} + \frac{(b(a^2-b^2)e^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}((-a^2+b^2)e^2+a^2x^2)}\,dx, x, e\sin(c+dx)\right)}{a^2d} + \frac{\left((a^2-3b^2)e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a^3\sqrt{e\sin(c+dx)}}$$

$$= \frac{2(a^2-3b^2)e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^3d\sqrt{e\sin(c+dx)}} + \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{(2b(a^2-b^2)e^3)\,\text{Subst}\left(\int \frac{1}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^2d} + \frac{\left(b^2\sqrt{a^2-b^2}e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a^3\sqrt{e\sin(c+dx)}} + \frac{\left(b^2\sqrt{a^2-b^2}e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a^3\sqrt{e\sin(c+dx)}}$$

$$= \frac{2(a^2-3b^2)e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^3d\sqrt{e\sin(c+dx)}} - \frac{b^2\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{b^2\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} - \frac{\left(b\sqrt{a^2-b^2}e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^2d} - \frac{\left(b\sqrt{a^2-b^2}e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^2d}$$

$$= -\frac{b\sqrt[4]{a^2-b^2}e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}d} - \frac{b\sqrt[4]{a^2-b^2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}d}$$
$$+ \frac{2(a^2-3b^2)\,e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^3d\sqrt{e\sin(c+dx)}}$$
$$- \frac{b^2\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{b^2\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 24.66 (sec) , antiderivative size = 1959, normalized size of antiderivative = 4.41

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = -\frac{2(b+a\cos(c+dx))\csc(c+dx)(e\sin(c+dx))^{3/2}}{3ad(a+b\sec(c+dx))}$$
$$+ \frac{(b+a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^{3/2}\left(\frac{4a\cos^2(c+dx)\left(b+a\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{b\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)+2a\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Csc[c + d*x]*(e*Sin[c + d*x])^(3/2))/(3*a*d*(a + b
*Sec[c + d*x])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Sin[c + d*x])^(3/2)
*((4*a*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (
Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqr
t[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
 - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2
- b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*Appe
llF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/
```

```
(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2
*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x
]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) - (2*b*Cos[c + d*x]*(b
 + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I
)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I
)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sq
rt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*
Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/
(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c +
 d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2,
 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*Appel
lF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Si
n[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sq
rt[1 - Sin[c + d*x]^2]) + (3*b*Cos[c + d*x]*Cos[2*(c + d*x)]*(b + a*Sqrt[1
- Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 - ((1 + I)*Sqrt[a]*
Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/2
 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2
- b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I/4)*(a^2 - 2*b^2)*Log
[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I
*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4 - I/4)*(a^2 - 2*b^2)*
Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*Sqrt[Sin[c + d*x]])/a
 + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*(a^2 - b^2)*AppellF1[
1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Si
n[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1
[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2
 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos
[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^2])))/(6*a*d*(a + b
*Sec[c + d*x])*Sin[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 10.10 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.43

method	result
default	$\frac{eb\left(-\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)-2\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{a^2e^2-b^2e^2}{a^2}\right)^{\frac{1}{4}}}\right)+4\sqrt{e\sin(dx+c)}\right)}{2a^2}+\frac{\sqrt{\cos(dx+c)^2 e}}{}$

[In] int((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] (1/2*e*b*(-(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-2*(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/((a^2*e^2-b^2*e^2)/a^2)^(1/4))+4*(e*sin(d*x+c))^(1/2))/a^2+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a*e^2*(-1/3/a^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))+b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-b^2*(a^2-b^2)/a^4*(-1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(c+dx))^{\frac{3}{2}}}{a+b\sec(c+dx)}\,dx$$

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*sec(d*x+c)),x)
[Out] Integral((e*sin(c + d*x))**(3/2)/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^{3/2}}{b+a\,\cos(c+d\,x)}\,dx$$

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(3/2))/(b + a*cos(c + d*x)), x)
```

3.236 $\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 356

$$
\begin{aligned}
\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx &= \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}d} - \frac{b\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}d} \\
&\quad - \frac{b^2e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{b^2e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}
\end{aligned}
$$

```
[Out] b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(3
/2)/(a^2-b^2)^(1/4)/d-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4
)/e^(1/2))*e^(1/2)/a^(3/2)/(a^2-b^2)^(1/4)/d+b^2*e*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a-(a^2-b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+b^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2))
,2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3957, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx = -\frac{b^2e\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^2d\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}} - \frac{b^2e\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^2d\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{3/2}d\sqrt[4]{a^2-b^2}} - \frac{b\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{3/2}d\sqrt[4]{a^2-b^2}} + \frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/(a^(3/2)*(a^2 - b^2)^(1/4)*d) - (b*Sqrt[e]*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(1/4)*d) - (b
^2*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Si
n[c + d*x]])/(a^2*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (b^2*e*El
lipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(a^2*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*EllipticE[(c
- Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a*d*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
```

```
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx \\
&= \frac{\int \sqrt{e\sin(c+dx)}\,dx}{a} + \frac{b\int \frac{\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{a} \\
&= \frac{(b^2e)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a^2} - \frac{(b^2e)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a^2} \\
&\quad + \frac{(be)\text{Subst}\left(\int \frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}\,dx, x, e\sin(c+dx)\right)}{d} + \frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}\,dx}{a\sqrt{\sin(c+dx)}} \\
&= \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} \\
&\quad + \frac{(2be)\text{Subst}\left(\int \frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} \\
&\quad + \frac{\left(b^2e\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a^2\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{\left(b^2e\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a^2\sqrt{e\sin(c+dx)}}
\end{aligned}$$

$$= -\frac{b^2 e \operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^2\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b^2 e \operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^2\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} - \frac{(be)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{ad} + \frac{(be)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{ad}$$

$$= \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}d} - \frac{b\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}d} - \frac{b^2 e \operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^2\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b^2 e \operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^2\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx$$

$$= \frac{\left(b+a\sqrt{\cos^2(c+dx)}\right)\sqrt{e\sin(c+dx)}\left(3\sqrt{2}b(-a^2+b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\arctan\left(1\right.\right.\right.}{}$$

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Sqrt[Cos[c + d*x]^2])*Sqrt[e*Sin[c + d*x]]*(3*Sqrt[2]*b*(-a^2 + b^2
)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d
```

```
*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2
)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/
2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(
3/2)))/(12*a^(3/2)*(a^2 - b^2)*d*(b + a*Cos[c + d*x])*Sqrt[Sin[c + d*x]])
```

Maple [A] (verified)

Time = 6.26 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.23

method	result
default	$\frac{eb\left(2\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)\right)}{2a^2\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}-\frac{e\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\,b^2\left(\mathrm{EllipticPi}\left(\sqrt{-\right.\right.}{}$

```
[In] int((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*e*b/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*(2*arctan((e*sin(d*x+c))^(1/2)/(e^2*
(a^2-b^2)/a^2)^(1/4))-ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/(
(e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4))))-1/2*e*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*b^2*(EllipticPi((-sin(d*x+c)+1)^
(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))*(a^2-b^2)^(1/2)-EllipticPi((-sin(
d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)-4*Ellipt
icE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a+2*EllipticF((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))*a+EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*
2^(1/2))*a+EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/
2))*a)/a^2/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{e\sin\left(c+dx\right)}}{a+b\sec\left(c+dx\right)}\,dx$$

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*sec(d*x+c)),x)
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{e\sin\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{e\sin\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\sqrt{e\,\sin\left(c+d\,x\right)}}{b+a\,\cos\left(c+d\,x\right)}\,dx$$

```
[In] int((e*sin(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(1/2))/(b + a*cos(c + d*x)), x)
```

3.237 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 370

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = -\frac{b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\,(a^2-b^2)^{3/4}\,d\sqrt{e}} - \frac{b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\,(a^2-b^2)^{3/4}\,d\sqrt{e}} + \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}} + \frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$

```
[Out] -b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(
3/4)/d/a^(1/2)/e^(1/2)-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/
4)/e^(1/2))/(a^2-b^2)^(3/4)/d/a^(1/2)/e^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1
/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(a^2-b^2-a*(a^2-b^2)^
(1/2))/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1
/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c
))^(1/2)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3957, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$
$$= -\frac{b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}d\sqrt{e}\,(a^2-b^2)^{3/4}} - \frac{b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}d\sqrt{e}\,(a^2-b^2)^{3/4}}$$
$$+ \frac{b^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}$$
$$+ \frac{b^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\left(a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}$$
$$+ \frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + b*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])]])/(S
qrt[a]*(a^2 - b^2)^(3/4)*d*Sqrt[e])) - (b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d
*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(3/4)*d*Sqrt[e]) +
 (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*d*Sqrt[e*Sin[c
+ d*x]]) + (b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
 d*x]]) + (b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{a} + \frac{b\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a} \\
&= \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a\sqrt{a^2-b^2}} + \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a\sqrt{a^2-b^2}} \\
&\quad + \frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left((-a^2+b^2)e^2+a^2x^2\right)}\,dx, x, e\sin(c+dx)\right)}{d} \\
&\quad + \frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{a\sqrt{e\sin(c+dx)}} \\
&= \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{(2be)\text{Subst}\left(\int \frac{1}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} \\
&\quad + \frac{\left(b^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(b^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}}
\end{aligned}$$

$$= \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\sqrt{a^2-b^2}\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{\sqrt{a^2-b^2}d}$$
$$-\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{\sqrt{a^2-b^2}d}$$

$$= -\frac{b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}-\frac{b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}$$
$$+\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\sqrt{a^2-b^2}\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.37 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.48

$$\int\frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{2\left(b+a\sqrt{\cos^2(c+dx)}\right)\sqrt{\sin(c+dx)}\left(\frac{b\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-\log\left(\sqrt{-a^2+b^2}-\cdots\right.\right.}{}\right.}{}$$

```
[In] Integrate[1/((a + b*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (2*(b + a*Sqrt[Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]]*((b*(-2*ArcTan[1 - (Sqrt
[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
```

*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]])/((-a^2 + b^2 + a^2*Sin[c + d*x]^2)*(5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)))/(d*(b + a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])

Maple [A] (verified)

Time = 7.54 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.34

method	result
default	$\frac{b e \left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}} \left(\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}}}\right)\right)}{-2a^{2}e^{2}+2b^{2}e^{2}} - \sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\sqrt{\sin(dx+c)}\left(2\left(\right.\right.$

[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] (1/2*b*e*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*(ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4)))-1/2/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*(2*(a^2-b^2)^(3/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*(a^2-b^2)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+(a^2-b^2)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))*b^2+(a^2-b^2)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*b^2+EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*a*b^2)/(a^2-b^2)^(1/2)/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sin(c+dx)}\,(a+b\sec(c+dx))}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*sin(c + d*x))*(a + b*sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)}{\sqrt{e\,\sin(c+d\,x)}\,(b+a\,\cos(c+d\,x))}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(1/2)*(b + a*cos(c + d*x))), x)
```

3.238 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 430

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{5/4}de^{3/2}} - \frac{\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{5/4}de^{3/2}} + \frac{2(b-a\cos(c+dx))}{(a^2-b^2)de\sqrt{e\sin(c+dx)}} - \frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}} - \frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)de^2\sqrt{\sin(c+dx)}}$$

```
[Out] b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2
-b^2)^(5/4)/d/e^(3/2)-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4
)/e^(1/2))*a^(1/2)/(a^2-b^2)^(5/4)/d/e^(3/2)+2*(b-a*cos(d*x+c))/(a^2-b^2)/d
/e/(e*sin(d*x+c))^(1/2)+b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2))
,2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))
^(1/2)+b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x
+c)^(1/2)/(a^2-b^2)/d/e/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*a*(sin(1
/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c
+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)/d/e^2/sin(d*x+c)^(
1/2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3957, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\left(a^2-b^2\right)^{5/4}} - \frac{\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\left(a^2-b^2\right)^{5/4}} - \frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\left(a^2-b^2\right)\sqrt{\sin(c+dx)}} + \frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}} - \frac{b^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}} - \frac{b^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de\left(a^2-b^2\right)\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[a]*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/((a^2 - b^2)^(5/4)*d*e^(3/2)) - (Sqrt[a]*b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(5/4)*d*e^(3/2)) + (2
*(b - a*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - (b^2*Ellipt
icPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/((a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (b^2*Ellip
ticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/((a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*Elli
pticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e^2*Sqrt[
Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx \\
&= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} + \frac{2\int \frac{\left(ab+\frac{1}{2}a^2\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{(a^2-b^2)\,e^2} \\
&= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{a\int \sqrt{e\sin(c+dx)}\,dx}{(a^2-b^2)\,e^2} + \frac{(ab)\int \frac{\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{(a^2-b^2)\,e^2}
\end{aligned}
$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} + \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2\,(a^2-b^2)\,e}$$
$$- \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2\,(a^2-b^2)\,e}$$
$$+ \frac{(a^2b)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}\,dx, x, e\sin(c+dx)\right)}{(a^2-b^2)\,de}$$
$$- \frac{\left(a\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{(a^2-b^2)\,e^2\sqrt{\sin(c+dx)}}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}$$
$$+ \frac{(2a^2b)\,\mathrm{Subst}\left(\int \frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)\,de}$$
$$+ \frac{\left(b^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2\,(a^2-b^2)\,e\sqrt{e\sin(c+dx)}}$$
$$- \frac{\left(b^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2\,(a^2-b^2)\,e\sqrt{e\sin(c+dx)}}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}$$
$$- \frac{b^2\,\mathrm{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$- \frac{b^2\,\mathrm{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$
$$- \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}$$
$$- \frac{(ab)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)\,de}$$
$$+ \frac{(ab)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)\,de}$$

$$= \frac{\sqrt{a}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{5/4}\,de^{3/2}} - \frac{\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{5/4}\,de^{3/2}} + \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}} - \frac{b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.80 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.65

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))\left(-\frac{\sqrt{a}\left(b+a\sqrt{\cos^2(c+dx)}\right)\sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\left(\cos(c\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(-((Sqrt[a]*(b + a*Sqrt[Cos[c + d*x]^2])*Sec[c + d*x]
^2*Sin[c + d*x]^(3/2)*(Cos[c + d*x]*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcT
an[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[
1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^
2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c
+ d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2)) + (2 + 2*
I)*b*Sqrt[Cos[c + d*x]^2]*(3*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - ((1 + I)*Sqrt[
a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*S
qrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[
a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2
- b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c +
 d*x]]) - (4 - 4*I)*Sqrt[a]*b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a
^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))))/((a - b)*(a + b)*(b +
 a*Cos[c + d*x]))) + 24*(b - a*Cos[c + d*x])*Tan[c + d*x]))/(12*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 8.16 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.54

method	result
default	$\frac{be\left(\frac{2\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{2e^2(a-b)(a+b)\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}+\frac{2}{e^2(a^2-b^2)\sqrt{e\sin(dx+c)}}\right)-\frac{b^2\left(\sqrt{a^2-b^2}\,\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(}\right.}{}}{}$

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*e*(1/2/e^2/(a-b)/(a+b)/(e^2*(a^2-b^2)/a^2)^(1/4)*(2*arctan((e*sin(d*x+c)
)^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/
a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4))))+2/e^2/(a^2-b
^2)/(e*sin(d*x+c))^(1/2))-1/2*b^2*((a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/(
(a^2-b^2)^(1/2)-a),1/2*2^(1/2))-(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*si
n(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a
^2-b^2)^(1/2)),1/2*2^(1/2))+4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a-2*(-sin(d*x
+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)
+1)^(1/2),1/2*2^(1/2))*a+(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)*a*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2
^(1/2))+(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*a*Ell
ipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))-4*cos(d*x+
c)^2*a)/e/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/(a+b)/(a-b)/cos(d*x+c)/(e
*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\sin{(c+dx)})^{\frac{3}{2}}\,(a+b\sec{(c+dx)})}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
[Out] Integral(1/((e*sin(c + d*x))**(3/2)*(a + b*sec(c + d*x))), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)\,(e\sin{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos{(c+d\,x)}}{{(e\,\sin{(c+d\,x)})}^{3/2}\,(b+a\,\cos{(c+d\,x)})}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(3/2)*(b + a*cos(c + d*x))), x)
```

3.239 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 452

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\frac{a^{3/2}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{7/4}\,de^{5/2}} - \frac{a^{3/2}b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{7/4}\,de^{5/2}} + \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3(a^2-b^2)\,de^2\sqrt{e\sin(c+dx)}} + \frac{ab^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a^2-b^2-a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}} + \frac{ab^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a^2-b^2+a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$

```
[Out] -a^(3/2)*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^
2-b^2)^(7/4)/d/e^(5/2)-a^(3/2)*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-
b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(7/4)/d/e^(5/2)+2/3*(b-a*cos(d*x+c))/(a^2-b^2
)/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)
^(1/2)/(a^2-b^2)/d/e^2/(e*sin(d*x+c))^(1/2)-a*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2
*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b^2-a
*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-a*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a
+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b^2+a*(a^2
-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3957, 2945, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}$$
$$+\frac{ab^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de^2\left(a^2-b^2\right)\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}$$
$$+\frac{ab^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de^2\left(a^2-b^2\right)\left(a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$
$$-\frac{a^{3/2}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{5/2}\left(a^2-b^2\right)^{7/4}}-\frac{a^{3/2}b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{5/2}\left(a^2-b^2\right)^{7/4}}$$

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -((a^(3/2)*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/((a^2 - b^2)^(7/4)*d*e^(5/2))) - (a^(3/2)*b*ArcTanh[(Sqrt[a]*Sqrt[e*S
in[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4)*d*e^(5/2)) +
 (2*(b - a*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a
*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*(a^2 - b^2)*d*e^2*
Sqrt[e*Sin[c + d*x]]) + (a*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2
 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (a*b^2*EllipticPi[(2*a)/(a + Sqrt[a^
2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b
^2 + a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx \\
&= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}}+\frac{2\int \frac{ab-\frac{1}{2}a^2\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2} \\
&= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}}+\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2}+\frac{(ab)\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{\left(a^2-b^2\right)e^2} \\
&= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}}+\frac{\left(ab^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2\left(a^2-b^2\right)^{3/2}e^2} \\
&\quad+\frac{\left(ab^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2\left(a^2-b^2\right)^{3/2}e^2} \\
&\quad+\frac{\left(a^2b\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(\left(-a^2+b^2\right)e^2+a^2x^2\right)}\,dx,x,e\sin(c+dx)\right)}{\left(a^2-b^2\right)de} \\
&\quad+\frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)de(e\sin(c+dx))^{3/2}}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3(a^2-b^2)de^2\sqrt{e\sin(c+dx)}} \\
&+\frac{(2a^2b)\,\text{Subst}\left(\int\frac{1}{(-a^2+b^2)e^2+a^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)\,de} \\
&+\frac{\left(ab^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2(a^2-b^2)^{3/2}e^2\sqrt{e\sin(c+dx)}} \\
&+\frac{\left(ab^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2(a^2-b^2)^{3/2}e^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)de(e\sin(c+dx))^{3/2}}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3(a^2-b^2)de^2\sqrt{e\sin(c+dx)}} \\
&-\frac{ab^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^{3/2}\left(a-\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}} \\
&+\frac{ab^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^{3/2}\left(a+\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}} \\
&-\frac{(a^2b)\,\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^{3/2}de^2} \\
&-\frac{(a^2b)\,\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^{3/2}de^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a^{3/2}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{7/4}de^{5/2}}-\frac{a^{3/2}b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{7/4}de^{5/2}} \\
&+\frac{2(b-a\cos(c+dx))}{3(a^2-b^2)de(e\sin(c+dx))^{3/2}}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3(a^2-b^2)de^2\sqrt{e\sin(c+dx)}} \\
&-\frac{ab^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^{3/2}\left(a-\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}} \\
&+\frac{ab^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^{3/2}\left(a+\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.09 (sec) , antiderivative size = 1233, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx =$$

$$-\frac{a(b+a\cos(c+dx))\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\left(-\frac{2a\cos^2(c+dx)\left(b+a\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{b\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)+2\,\mathrm{ar}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$-\frac{2(b-a\cos(c+dx))(b+a\cos(c+dx))\tan(c+dx)}{3\left(-a^2+b^2\right)d(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}$$

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -1/3*(a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[c + d*x]^(5/2)*((-2*a*Cos[c +
 d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a
]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqr
t[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^
2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c
+ d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF
1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt
[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2
, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*Appe
llF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
 (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^
2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b +
 a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*b*Cos[c + d*x]*(b + a*Sqrt[1 -
Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt[a^2 - b^2]
+ (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])
)/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d
*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[
c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*
Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c +
d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2,
1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*
```

```
(b^2 + a^2*(-1 + Sin[c + d*x]^2))))))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c +
 d*x]^2])))/((a - b)*(a + b)*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2))
 - (2*(b - a*Cos[c + d*x])*(b + a*Cos[c + d*x])*Tan[c + d*x])/(3*(-a^2 + b^
2)*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 12.04 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.34

method	result
default	$\dfrac{2be\left(\frac{1}{3e^2\left(a^2-b^2\right)\left(e\sin(dx+c)\right)^{\frac{3}{2}}}+\frac{a^2\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)\right)}{4e^2(a-b)(a+b)\left(-a^2e^2+b^2e^2\right)}\right)+\frac{\sqrt{\cos(dx\cdots}}{\cdots}}{\cdots}$

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*b*e*(1/3/e^2/(a^2-b^2)/(e*sin(d*x+c))^(3/2)+1/4/e^2/(a-b)/(a+b)*a^2*(e^2
*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*(ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^
2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2*arct
an((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))))+(cos(d*x+c)^2*e*sin(d*
x+c))^(1/2)*a/e^2*(1/3/(a^2-b^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x
+c)^2-1)*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*Ell
ipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))-1/(a-b
)/(a+b)*b^2*(-1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)
/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1
/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi(
(-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)))))/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\,(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx=\int \frac{\cos(c+d\,x)}{(e\,\sin(c+d\,x))^{5/2}\,(b+a\,\cos(c+d\,x))}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(5/2)*(b + a*cos(c + d*x))), x)
```

3.240 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 511

$$\begin{aligned}
\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx &= \frac{a^{5/2}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{9/4}\,de^{7/2}} \\
&- \frac{a^{5/2}b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{9/4}\,de^{7/2}} + \frac{2(b-a\cos(c+dx))}{5\,(a^2-b^2)\,de(e\sin(c+dx))^{5/2}} \\
&+ \frac{2(5a^2b-a(3a^2+2b^2)\cos(c+dx))}{5\,(a^2-b^2)^2\,de^3\sqrt{e\sin(c+dx)}} \\
&- \frac{a^2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(a-\sqrt{a^2-b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&- \frac{a^2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(a+\sqrt{a^2-b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&- \frac{2a(3a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5\,(a^2-b^2)^2\,de^4\sqrt{\sin(c+dx)}}
\end{aligned}$$

```
[Out] a^(5/2)*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2
-b^2)^(9/4)/d/e^(7/2)-a^(5/2)*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b
^2)^(1/4)/e^(1/2))/(a^2-b^2)^(9/4)/d/e^(7/2)+2/5*(b-a*cos(d*x+c))/(a^2-b^2)
/d/e/(e*sin(d*x+c))^(5/2)+2/5*(5*a^2*b-a*(3*a^2+2*b^2)*cos(d*x+c))/(a^2-b^2
)^2/d/e^3/(e*sin(d*x+c))^(1/2)+a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-
b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(a-(a^2-b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
```

```
+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2
)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(a+(a^2-b^2)^(1/2))/(e*sin(d
*x+c))^(1/2)+2/5*a*(3*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x
+c))^(1/2)/(a^2-b^2)^2/d/e^4/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3957, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\begin{aligned}
&\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \\
&-\frac{2a(3a^2+2b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\left(a^2-b^2\right)^2\sqrt{\sin(c+dx)}} \\
&+\frac{2(5a^2b-a(3a^2+2b^2)\cos(c+dx))}{5de^3\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}} \\
&-\frac{a^2b^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de^3\left(a^2-b^2\right)^2\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}} \\
&-\frac{a^2b^2\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{de^3\left(a^2-b^2\right)^2\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}} \\
&+\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}+\frac{a^{5/2}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{7/2}\left(a^2-b^2\right)^{9/4}} \\
&-\frac{a^{5/2}b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{7/2}\left(a^2-b^2\right)^{9/4}}
\end{aligned}$$

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (a^(5/2)*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/((a^2 - b^2)^(9/4)*d*e^(7/2)) - (a^(5/2)*b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(9/4)*d*e^(7/2)) + (2
*(b - a*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(5/2)) + (2*(5*a
^2*b - a*(3*a^2 + 2*b^2)*Cos[c + d*x]))/(5*(a^2 - b^2)^2*d*e^3*Sqrt[e*Sin[c
 + d*x]]) - (a^2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(a - Sqrt[a^2 - b^2])*d*e^3*Sqr
t[e*Sin[c + d*x]]) - (a^2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(a + Sqrt[a^2 - b^2])*
d*e^3*Sqrt[e*Sin[c + d*x]]) - (2*a*(3*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*
x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*(a^2 - b^2)^2*d*e^4*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\int \frac{ab-\frac{3}{2}a^2\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{5\left(a^2-b^2\right)e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{5(a^2-b^2)de(e\sin(c+dx))^{5/2}} + \frac{2(5a^2b-a(3a^2+2b^2)\cos(c+dx))}{5(a^2-b^2)^2de^3\sqrt{e\sin(c+dx)}} + \frac{4\int \frac{\left(\frac{1}{2}ab(4a^2+b^2)+\frac{1}{4}a^2(3a^2+2b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}dx}{5(a^2-b^2)^2e^4}$$

$$= \frac{2(b-a\cos(c+dx))}{5(a^2-b^2)de(e\sin(c+dx))^{5/2}} + \frac{2(5a^2b-a(3a^2+2b^2)\cos(c+dx))}{5(a^2-b^2)^2de^3\sqrt{e\sin(c+dx)}} + \frac{(a^3b)\int \frac{\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}dx}{(a^2-b^2)^2e^4} - \frac{(a(3a^2+2b^2))\int \sqrt{e\sin(c+dx)}\,dx}{5(a^2-b^2)^2e^4}$$

$$= \frac{2(b-a\cos(c+dx))}{5(a^2-b^2)de(e\sin(c+dx))^{5/2}} + \frac{2(5a^2b-a(3a^2+2b^2)\cos(c+dx))}{5(a^2-b^2)^2de^3\sqrt{e\sin(c+dx)}} + \frac{(a^2b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2(a^2-b^2)^2e^3} - \frac{(a^2b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}dx}{2(a^2-b^2)^2e^3} + \frac{(a^4b)\operatorname{Subst}\left(\int \frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}dx,x,e\sin(c+dx)\right)}{(a^2-b^2)^2de^3} - \frac{\left(a(3a^2+2b^2)\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{5(a^2-b^2)^2e^4\sqrt{\sin(c+dx)}}$$

$$= \frac{2(b-a\cos(c+dx))}{5(a^2-b^2)de(e\sin(c+dx))^{5/2}} + \frac{2(5a^2b-a(3a^2+2b^2)\cos(c+dx))}{5(a^2-b^2)^2de^3\sqrt{e\sin(c+dx)}} - \frac{2a(3a^2+2b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5(a^2-b^2)^2de^4\sqrt{\sin(c+dx)}} + \frac{(2a^4b)\operatorname{Subst}\left(\int \frac{x^2}{(-a^2+b^2)e^2+a^2x^4}dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^2de^3} + \frac{\left(a^2b^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}dx}{2(a^2-b^2)^2e^3\sqrt{e\sin(c+dx)}} - \frac{\left(a^2b^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}dx}{2(a^2-b^2)^2e^3\sqrt{e\sin(c+dx)}}$$

$$= \frac{2(b-a\cos(c+dx))}{5(a^2-b^2)de(e\sin(c+dx))^{5/2}} + \frac{2(5a^2b-a(3a^2+2b^2)\cos(c+dx))}{5(a^2-b^2)^2de^3\sqrt{e\sin(c+dx)}} - \frac{a^2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(a-\sqrt{a^2-b^2}\right)de^3\sqrt{e\sin(c+dx)}} - \frac{a^2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(a+\sqrt{a^2-b^2}\right)de^3\sqrt{e\sin(c+dx)}} - \frac{2a(3a^2+2b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5(a^2-b^2)^2de^4\sqrt{\sin(c+dx)}} - \frac{(a^3b)\,\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^2de^3} + \frac{(a^3b)\,\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^2de^3}$$

$$= \frac{a^{5/2}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{9/4}de^{7/2}} - \frac{a^{5/2}b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{9/4}de^{7/2}} + \frac{2(b-a\cos(c+dx))}{5(a^2-b^2)de(e\sin(c+dx))^{5/2}} + \frac{2(5a^2b-a(3a^2+2b^2)\cos(c+dx))}{5(a^2-b^2)^2de^3\sqrt{e\sin(c+dx)}} - \frac{a^2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(a-\sqrt{a^2-b^2}\right)de^3\sqrt{e\sin(c+dx)}} - \frac{a^2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(a+\sqrt{a^2-b^2}\right)de^3\sqrt{e\sin(c+dx)}} - \frac{2a(3a^2+2b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5(a^2-b^2)^2de^4\sqrt{\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.40 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.56

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \frac{(b+a\cos(c+dx))\sin^3(c+dx)\left(-\frac{2((a^2-b^2)(-b+a\cos(c+dx))\csc^2(c\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sin[c + d*x]^3*((-2*((a^2 - b^2)*(-b + a*Cos[c + d*x]
)*Csc[c + d*x]^2*Sec[c + d*x] + a*(3*a^2 + 2*b^2 - 5*a*b*Sec[c + d*x])))/(a
^2 - b^2)^2 - ((b + a*Sqrt[Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x
]]*((3*a^3 + 2*a*b^2)*Cos[c + d*x]*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTa
n[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1
 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2
 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c +
 d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin
[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c
 + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2)) + (2 + 2*I
)*a*b*(4*a^2 + b^2)*Sqrt[Cos[c + d*x]^2]*(3*(a^2 - b^2)^(3/4)*(2*ArcTan[1 -
 ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1
 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
- (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]
+ Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]
] + I*a*Sin[c + d*x]]) - (4 - 4*I)*Sqrt[a]*b*AppellF1[3/4, 1/2, 1, 7/4, Sin
[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))))/(12*Sq
rt[a]*(a - b)^2*(a + b)^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(5*d*(a + b*S
ec[c + d*x])*(e*Sin[c + d*x])^(7/2))
```

Maple [A] (verified)

Time = 11.09 (sec) , antiderivative size = 892, normalized size of antiderivative = 1.75

method	result	size
default	Expression too large to display	892

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*b*e*(1/5/e^2/(a+b)/(a-b)/(e*sin(d*x+c))^(5/2)+1/e^4/(a-b)^2/(a+b)^2*a^2/
(e*sin(d*x+c))^(1/2)+1/4*a^2/e^4/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*
(2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((e*sin(d*x+c)
)^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2
)^(1/4)))))-1/10/e^3*b^2*(12*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+8*(-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c
)+1)^(1/2),1/2*2^(1/2))*b^2-6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-4*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))*b^2+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(7/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1
/2*2^(1/2))*(a^2-b^2)^(1/2)*a+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(7/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),
1/2*2^(1/2))*a^2-5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(7/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*(
```

```
a^2-b^2)^(1/2)*a+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(7/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*a
^2+12*a^2*cos(d*x+c)^4*sin(d*x+c)+8*b^2*cos(d*x+c)^4*sin(d*x+c)-16*a^2*cos(
d*x+c)^2*sin(d*x+c)-4*b^2*cos(d*x+c)^2*sin(d*x+c))*a/(a+(a^2-b^2)^(1/2))/((
a^2-b^2)^(1/2)-a)/(a+b)^2/(a-b)^2/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1
/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \int \frac{\cos(c+d\,x)}{(e\,\sin(c+d\,x))^{7/2}\,(b+a\,\cos(c+d\,x))}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(7/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(7/2)*(b + a*cos(c + d*x))), x)
```

3.241 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 1070

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = &-\frac{7b^3(a^2-b^2)^{3/4}\,e^{9/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{13/2}d} \\
&+\frac{2b(a^2-b^2)^{7/4}\,e^{9/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{13/2}d} \\
&+\frac{7b^3(a^2-b^2)^{3/4}\,e^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{13/2}d} \\
&-\frac{2b(a^2-b^2)^{7/4}\,e^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{13/2}d} \\
&+\frac{7b^4(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&-\frac{2b^2(a^2-b^2)^2\,e^5\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+\frac{7b^4(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&-\frac{2b^2(a^2-b^2)^2\,e^5\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+\frac{14e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2d\sqrt{\sin(c+dx)}} \\
&-\frac{7b^2(3a^2-5b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^6d\sqrt{\sin(c+dx)}} \\
&-\frac{4b^2(8a^2-5b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^6d\sqrt{\sin(c+dx)}} \\
&-\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} \\
&-\frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d} \\
&+\frac{4be^3(5(a^2-b^2)+3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15a^5d} \\
&+\frac{4be(e\sin(c+dx))^{7/2}}{7a^3d}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2d} \\
&+\frac{b^2e(e\sin(c+dx))^{7/2}}{a^3d(b+a\cos(c+dx))}
\end{aligned}
$$

```
[Out] -7/2*b^3*(a^2-b^2)^(3/4)*e^(9/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b
^2)^(1/4)/e^(1/2))/a^(13/2)/d+2*b*(a^2-b^2)^(7/4)*e^(9/2)*arctan(a^(1/2)*(e
*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(13/2)/d+7/2*b^3*(a^2-b^2)^(3
/4)*e^(9/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a
^(13/2)/d-2*b*(a^2-b^2)^(7/4)*e^(9/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/
(a^2-b^2)^(1/4)/e^(1/2))/a^(13/2)/d-14/45*e^3*cos(d*x+c)*(e*sin(d*x+c))^(3/
2)/a^2/d-7/15*b^2*e^3*(5*b-3*a*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/a^5/d+4/15*
b*e^3*(5*a^2-5*b^2+3*a*b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/a^5/d+4/7*b*e*(e*
sin(d*x+c))^(7/2)/a^3/d-2/9*e*cos(d*x+c)*(e*sin(d*x+c))^(7/2)/a^2/d+b^2*e*(
e*sin(d*x+c))^(7/2)/a^3/d/(b+a*cos(d*x+c))-7/2*b^4*(a^2-b^2)*e^5*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/
4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a-(a
^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)^2*e^5*(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/
2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a-(a^2-b^2)
^(1/2))/(e*sin(d*x+c))^(1/2)-7/2*b^4*(a^2-b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a+(a^2-b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)^2*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+
(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a+(a^2-b^2)^(1/2))/(e*sin
(d*x+c))^(1/2)-14/15*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/
2)/a^2/d/sin(d*x+c)^(1/2)+7/5*b^2*(3*a^2-5*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),
2^(1/2))*(e*sin(d*x+c))^(1/2)/a^6/d/sin(d*x+c)^(1/2)+4/5*b^2*(8*a^2-5*b^2)*
e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^6/d/sin(d*x+c)^(
1/2)
```

Rubi [A] (verified)

Time = 3.47 (sec) , antiderivative size = 1070, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3957, 2991, 2715, 2721, 2719, 2772, 2944, 2946, 2780, 2886, 2884, 335, 304, 211,

214, 2774}

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx =$$

$$-\frac{2b^2(a^2-b^2)^2\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^5}{a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{7b^4(a^2-b^2)\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^5}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2(a^2-b^2)^2\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^5}{a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{7b^4(a^2-b^2)\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^5}{2a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2b(a^2-b^2)^{7/4}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{9/2}}{a^{13/2}d}$$
$$-\frac{7b^3(a^2-b^2)^{3/4}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{9/2}}{2a^{13/2}d}$$
$$-\frac{2b(a^2-b^2)^{7/4}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{9/2}}{a^{13/2}d}$$
$$+\frac{7b^3(a^2-b^2)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{9/2}}{2a^{13/2}d}$$
$$-\frac{7b^2(3a^2-5b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}e^4}{5a^6d\sqrt{\sin(c+dx)}}$$
$$-\frac{4b^2(8a^2-5b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}e^4}{5a^6d\sqrt{\sin(c+dx)}}$$
$$+\frac{14E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}e^4}{15a^2d\sqrt{\sin(c+dx)}}$$
$$-\frac{14\cos(c+dx)(e\sin(c+dx))^{3/2}e^3}{45a^2d}$$
$$-\frac{7b^2(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}e^3}{15a^5d}$$
$$+\frac{4b(5(a^2-b^2)+3ab\cos(c+dx))(e\sin(c+dx))^{3/2}e^3}{15a^5d}$$
$$-\frac{2\cos(c+dx)(e\sin(c+dx))^{7/2}e}{9a^2d}$$
$$+\frac{4b(e\sin(c+dx))^{7/2}e}{7a^3d}+\frac{b^2(e\sin(c+dx))^{7/2}e}{a^3d(b+a\cos(c+dx))}$$

```
[In] Int[(e*Sin[c + d*x])^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a
^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) + (2*b*(a^2 - b^2)^(7/4)*e^(9/2)*
ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(13/
2)*d) + (7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*
x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) - (2*b*(a^2 - b^2)^(7/4)*
e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])]
)/(a^(13/2)*d) + (7*b^4*(a^2 - b^2)*e^5*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^
2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^7*(a - Sqrt[a^2 - b^2]
)*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)^2*e^5*EllipticPi[(2*a)/(a -
Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^7*(a - Sqrt
[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (7*b^4*(a^2 - b^2)*e^5*EllipticPi[(2
*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^
7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)^2*e^5*
EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(a^7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (14*e^4*Ellipt
icE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(15*a^2*d*Sqrt[Sin[c + d*x
]]) - (7*b^2*(3*a^2 - 5*b^2)*e^4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Si
n[c + d*x]])/(5*a^6*d*Sqrt[Sin[c + d*x]]) - (4*b^2*(8*a^2 - 5*b^2)*e^4*Elli
pticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^6*d*Sqrt[Sin[c + d*
x]]) - (14*e^3*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(45*a^2*d) - (7*b^2*e^3
*(5*b - 3*a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*a^5*d) + (4*b*e^3*(5*
(a^2 - b^2) + 3*a*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*a^5*d) + (4*b
*e*(e*Sin[c + d*x])^(7/2))/(7*a^3*d) - (2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(
7/2))/(9*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(7/2))/(a^3*d*(b + a*Cos[c + d*x]
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
```

```
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
```

```
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{9/2}}{(-b-a\cos(c+dx))^2}\,dx\\
&= \int \left(\frac{(e\sin(c+dx))^{9/2}}{a^2}+\frac{b^2(e\sin(c+dx))^{9/2}}{a^2(b+a\cos(c+dx))^2}-\frac{2b(e\sin(c+dx))^{9/2}}{a^2(b+a\cos(c+dx))}\right)dx\\
&= \frac{\int (e\sin(c+dx))^{9/2}\,dx}{a^2}-\frac{(2b)\int \frac{(e\sin(c+dx))^{9/2}}{b+a\cos(c+dx)}\,dx}{a^2}+\frac{b^2\int \frac{(e\sin(c+dx))^{9/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}\\
&= \frac{4be(e\sin(c+dx))^{7/2}}{7a^3d}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2d}\\
&\quad+\frac{b^2e(e\sin(c+dx))^{7/2}}{a^3d(b+a\cos(c+dx))}+\frac{(7e^2)\int (e\sin(c+dx))^{5/2}\,dx}{9a^2}\\
&\quad+\frac{(2be^2)\int \frac{(-a-b\cos(c+dx))(e\sin(c+dx))^{5/2}}{b+a\cos(c+dx)}\,dx}{a^3}-\frac{(7b^2e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{b+a\cos(c+dx)}\,dx}{2a^3}\\
&= -\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d}-\frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d}\\
&\quad+\frac{4be^3\left(5(a^2-b^2)+3ab\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{15a^5d}\\
&\quad+\frac{4be(e\sin(c+dx))^{7/2}}{7a^3d}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2d}\\
&\quad+\frac{b^2e(e\sin(c+dx))^{7/2}}{a^3d(b+a\cos(c+dx))}+\frac{(7e^4)\int \sqrt{e\sin(c+dx)}\,dx}{15a^2}\\
&\quad+\frac{(4be^4)\int \frac{\left(-\frac{1}{2}a(5a^2-2b^2)-\frac{1}{2}b(8a^2-5b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{5a^5}\\
&\quad-\frac{(7b^2e^4)\int \frac{\left(-ab+\frac{1}{2}(3a^2-5b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{5a^5}
\end{aligned}
$$

$$= -\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} - \frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d} + \frac{4be^3(5(a^2-b^2)+3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15a^5d} + \frac{4be(e\sin(c+dx))^{7/2}}{7a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2d} + \frac{b^2e(e\sin(c+dx))^{7/2}}{a^3d(b+a\cos(c+dx))} - \frac{(7b^2(3a^2-5b^2)\,e^4)\int\sqrt{e\sin(c+dx)}\,dx}{10a^6} - \frac{(2b^2(8a^2-5b^2)\,e^4)\int\sqrt{e\sin(c+dx)}\,dx}{5a^6} + \frac{(7b^3(a^2-b^2)\,e^4)\int\frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{2a^6} - \frac{\left(2b(a^2-b^2)^2\,e^4\right)\int\frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{a^6} + \frac{\left(7e^4\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{15a^2\sqrt{\sin(c+dx)}}$$

$$= \frac{14e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{15a^2d\sqrt{\sin(c+dx)}} - \frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} - \frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d} + \frac{4be^3(5(a^2-b^2)+3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15a^5d} + \frac{4be(e\sin(c+dx))^{7/2}}{7a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2d} + \frac{b^2e(e\sin(c+dx))^{7/2}}{a^3d(b+a\cos(c+dx))} - \frac{(7b^4(a^2-b^2)\,e^5)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^7} + \frac{(7b^4(a^2-b^2)\,e^5)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^7} + \frac{\left(b^2(a^2-b^2)^2\,e^5\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^7} - \frac{\left(b^2(a^2-b^2)^2\,e^5\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^7} - \frac{(7b^3(a^2-b^2)\,e^5)\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}\,dx,x,e\sin(c+dx)\right)}{2a^5d} + \frac{\left(2b(a^2-b^2)^2\,e^5\right)\mathrm{Subst}\left(\int\frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}\,dx,x,e\sin(c+dx)\right)}{a^5d} - \frac{\left(7b^2(3a^2-5b^2)\,e^4\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{10a^6\sqrt{\sin(c+dx)}} - \frac{\left(2b^2(8a^2-5b^2)\,e^4\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{5a^6\sqrt{\sin(c+dx)}}$$

$$= \frac{14e^4 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2 d\sqrt{\sin(c+dx)}}$$
$$-\frac{7b^2(3a^2-5b^2)\,e^4 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^6 d\sqrt{\sin(c+dx)}}$$
$$-\frac{4b^2(8a^2-5b^2)\,e^4 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^6 d\sqrt{\sin(c+dx)}}$$
$$-\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2 d}-\frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5 d}$$
$$+\frac{4be^3(5(a^2-b^2)+3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15a^5 d}+\frac{4be(e\sin(c+dx))^{7/2}}{7a^3 d}$$
$$-\frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2 d}+\frac{b^2e(e\sin(c+dx))^{7/2}}{a^3d(b+a\cos(c+dx))}$$
$$-\frac{(7b^3(a^2-b^2)\,e^5)\,\mathrm{Subst}\left(\int\frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a^5 d}$$
$$+\frac{\left(4b(a^2-b^2)^2\,e^5\right)\mathrm{Subst}\left(\int\frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a^5 d}$$
$$-\frac{\left(7b^4(a^2-b^2)\,e^5\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^7\sqrt{e\sin(c+dx)}}$$
$$+\frac{\left(7b^4(a^2-b^2)\,e^5\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^7\sqrt{e\sin(c+dx)}}$$
$$+\frac{\left(b^2(a^2-b^2)^2\,e^5\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^7\sqrt{e\sin(c+dx)}}$$
$$-\frac{\left(b^2(a^2-b^2)^2\,e^5\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^7\sqrt{e\sin(c+dx)}}$$

$$= \frac{7b^4(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2(a^2-b^2)^2\,e^5\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{7b^4(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2(a^2-b^2)^2\,e^5\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{14e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2d\sqrt{\sin(c+dx)}}$$
$$-\frac{7b^2(3a^2-5b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^6d\sqrt{\sin(c+dx)}}$$
$$-\frac{4b^2(8a^2-5b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^6d\sqrt{\sin(c+dx)}}$$
$$-\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d}-\frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d}$$
$$+\frac{4be^3(5(a^2-b^2)+3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15a^5d}+\frac{4be(e\sin(c+dx))^{7/2}}{7a^3d}$$
$$-\frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2d}+\frac{b^2e(e\sin(c+dx))^{7/2}}{a^3d(b+a\cos(c+dx))}$$
$$+\frac{(7b^3(a^2-b^2)\,e^5)\,\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2a^6d}$$
$$-\frac{(7b^3(a^2-b^2)\,e^5)\,\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2a^6d}$$
$$-\frac{\left(2b(a^2-b^2)^2\,e^5\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a^6d}$$
$$+\frac{\left(2b(a^2-b^2)^2\,e^5\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a^6d}$$

$$
\begin{aligned}
&= -\frac{7b^3(a^2-b^2)^{3/4}\, e^{9/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{13/2}d} \\
&+ \frac{2b(a^2-b^2)^{7/4}\, e^{9/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{13/2}d} \\
&+ \frac{7b^3(a^2-b^2)^{3/4}\, e^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{13/2}d} \\
&- \frac{2b(a^2-b^2)^{7/4}\, e^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{13/2}d} \\
&+ \frac{7b^4(a^2-b^2)\, e^5\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{2b^2(a^2-b^2)^2\, e^5\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{7b^4(a^2-b^2)\, e^5\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{2b^2(a^2-b^2)^2\, e^5\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^7\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{14e^4 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle| 2\right)\sqrt{e\sin(c+dx)}}{15a^2d\sqrt{\sin(c+dx)}} \\
&- \frac{7b^2(3a^2-5b^2)\, e^4 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle| 2\right)\sqrt{e\sin(c+dx)}}{5a^6d\sqrt{\sin(c+dx)}} \\
&- \frac{4b^2(8a^2-5b^2)\, e^4 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle| 2\right)\sqrt{e\sin(c+dx)}}{5a^6d\sqrt{\sin(c+dx)}} \\
&- \frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} - \frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d} \\
&+ \frac{4be^3(5(a^2-b^2)+3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15a^5d} + \frac{4be(e\sin(c+dx))^{7/2}}{7a^3d} \\
&- \frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2d} + \frac{b^2e(e\sin(c+dx))^{7/2}}{a^3d(b+a\cos(c+dx))}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.32 (sec) , antiderivative size = 974, normalized size of antiderivative = 0.91

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^{9/2}\left(\frac{(14a^4-159a^2b^2+165b^4)\cos^2(c+dx)}{}\right.}{} + \frac{(b+a\cos(c+dx))^2\csc^4(c+dx)\sec^2(c+dx)(e\sin(c+dx))^{9/2}\left(-\frac{b(-37a^2+56b^2)\sin(c+dx)}{21a^5}+\frac{a^2b^2\sin(c+dx)-b^4\sin(}{a^5(b+a\cos(c+dx)}\right.}{d(a+b\sec(c+dx))^2}$$

```
[In] Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(9/2)*(((14*a^4 - 1
59*a^2*b^2 + 165*b^4)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*Arc
Tan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan
[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a
^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c
 + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[S
in[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin
[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*S
qrt[1 - Sin[c + d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 -
 Sin[c + d*x]^2)) + (2*(-46*a^3*b + 66*a*b^3)*Cos[c + d*x]*(((1/8 + I/8)*(2
*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[
a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + I*a*Sin[c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF
1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c
 + d*x]^(3/2))/(3*(-a^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*
Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(30*a^5*d*(a + b*Sec[c + d*x])^2*
Sin[c + d*x]^(9/2)) + ((b + a*Cos[c + d*x])^2*Csc[c + d*x]^4*Sec[c + d*x]^2
*(e*Sin[c + d*x])^(9/2)*(-1/21*(b*(-37*a^2 + 56*b^2)*Sin[c + d*x])/a^5 + (a
^2*b^2*Sin[c + d*x] - b^4*Sin[c + d*x])/(a^5*(b + a*Cos[c + d*x])) - ((19*a
^2 - 54*b^2)*Sin[2*(c + d*x)])/(90*a^4) - (b*Sin[3*(c + d*x)])/(7*a^3) + Si
n[4*(c + d*x)]/(36*a^2)))/(d*(a + b*Sec[c + d*x])^2)
```

Maple [A] (warning: unable to verify)

Time = 44.55 (sec) , antiderivative size = 1725, normalized size of antiderivative = 1.61

method	result	size
default	Expression too large to display	1725

```
[In] int((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (4*e*a*b*(-1/21/a^6*(e*sin(d*x+c))^(3/2)*e^2*(3*cos(d*x+c)^2*a^2-10*a^2+14*
b^2)+e^4/a^6*((1/4*a^2*b^2-1/4*b^4)*(e*sin(d*x+c))^(3/2)/(-a^2*e^2*cos(d*x+
c)^2+b^2*e^2)+1/4*(a^4-15/4*a^2*b^2+11/4*b^4)/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)
*(2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((e*sin(d*x+c
))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^
2)^(1/4))))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^5*(-1/45/a^6/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)*(10*a^4*cos(d*x+c)^6+42*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))*a^4-432*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*El
lipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2*b^2+450*(-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))*b^4-21*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^4+216*a^2*b^2*(-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c
)+1)^(1/2),1/2*2^(1/2))-225*b^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-34*cos(d*x+
c)^4*a^4+54*cos(d*x+c)^4*a^2*b^2+24*cos(d*x+c)^2*a^4-54*cos(d*x+c)^2*a^2*b^
2)+2*b^4*(a^4-2*a^2*b^2+b^4)/a^6*(-1/2*a^2/e/b^2/(a^2-b^2)*sin(d*x+c)*(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)+1/2/b^2/(a^2-b^2)*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-
b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/
b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d
*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a^2-b^2)/a^2*(-sin
(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*si
n(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1
-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+
(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a)
,1/2*2^(1/2))+3/8/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*
EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)))-b^2*
(3*a^4-10*a^2*b^2+7*b^4)/a^6*(-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1
```

```
/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2)
)-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))
^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(9/2)/(a+b*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx=\int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(9/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx=\int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^{9/2}}{(b+a\,\cos(c+d\,x))^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(9/2))/(b + a*cos(c + d*x))^2, x)
```

3.242 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 1101

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \frac{5b^3\sqrt[4]{a^2-b^2}e^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{11/2}d}$$
$$-\frac{2b(a^2-b^2)^{5/4}e^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{11/2}d}$$
$$+\frac{5b^3\sqrt[4]{a^2-b^2}e^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{11/2}d}$$
$$-\frac{2b(a^2-b^2)^{5/4}e^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{11/2}d}$$
$$+\frac{10e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5b^2(a^2-3b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^6d\sqrt{e\sin(c+dx)}}$$
$$-\frac{4b^2(4a^2-3b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^6d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5b^4(a^2-b^2)\,e^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^6\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2b^2(a^2-b^2)^2\,e^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^6\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{5b^4(a^2-b^2)\,e^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^6\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2b^2(a^2-b^2)^2\,e^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^6\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d}$$
$$-\frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d}$$
$$+\frac{4be^3(3(a^2-b^2)+ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d}+\frac{4be(e\sin(c+dx))^{5/2}}{5a^3d}$$
$$-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d}+\frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx))}$$

```
[Out] 5/2*b^3*(a^2-b^2)^(1/4)*e^(7/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^
2)^(1/4)/e^(1/2))/a^(11/2)/d-2*b*(a^2-b^2)^(5/4)*e^(7/2)*arctan(a^(1/2)*(e*
sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(11/2)/d+5/2*b^3*(a^2-b^2)^(1/
4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^
(11/2)/d-2*b*(a^2-b^2)^(5/4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(
a^2-b^2)^(1/4)/e^(1/2))/a^(11/2)/d+4/5*b*e*(e*sin(d*x+c))^(5/2)/a^3/d-2/7*e
*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(5/2)/a^3/d/(b+
a*cos(d*x+c))-10/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^
2/d/(e*sin(d*x+c))^(1/2)+5/3*b^2*(a^2-3*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(
1/2))*sin(d*x+c)^(1/2)/a^6/d/(e*sin(d*x+c))^(1/2)+4/3*b^2*(4*a^2-3*b^2)*e^4
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(e*sin(d*x+c))^(1/2
)+5/2*b^4*(a^2-b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(
1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/
2)-2*b^2*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^
(1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1
/2)+5/2*b^4*(a^2-b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2
^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(
1/2)-2*b^2*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/
4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),
2^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^
(1/2)-10/21*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d-5/3*b^2*e^3*(3*b-a*co
s(d*x+c))*(e*sin(d*x+c))^(1/2)/a^5/d+4/3*b*e^3*(3*a^2-3*b^2+a*b*cos(d*x+c))
*(e*sin(d*x+c))^(1/2)/a^5/d
```

Rubi [A] (verified)

Time = 3.52 (sec) , antiderivative size = 1101, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3957, 2991, 2715, 2721, 2720, 2772, 2944, 2946, 2781, 2886, 2884, 335, 218, 214,

211, 2774}

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx =$$

$$-\frac{5b^2(a^2-3b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{3a^6d\sqrt{e\sin(c+dx)}}$$

$$-\frac{4b^2(4a^2-3b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{3a^6d\sqrt{e\sin(c+dx)}}$$

$$+\frac{10\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{21a^2d\sqrt{e\sin(c+dx)}}$$

$$+\frac{2b^2(a^2-b^2)^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{a^6\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}$$

$$-\frac{5b^4(a^2-b^2)\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{2a^6\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}$$

$$+\frac{2b^2(a^2-b^2)^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{a^6\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}$$

$$-\frac{5b^4(a^2-b^2)\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}e^4}{2a^6\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}$$

$$-\frac{2b(a^2-b^2)^{5/4}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{7/2}}{a^{11/2}d}$$

$$+\frac{5b^3\sqrt[4]{a^2-b^2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{7/2}}{2a^{11/2}d}$$

$$-\frac{2b(a^2-b^2)^{5/4}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{7/2}}{a^{11/2}d}$$

$$+\frac{5b^3\sqrt[4]{a^2-b^2}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)e^{7/2}}{2a^{11/2}d}$$

$$-\frac{10\cos(c+dx)\sqrt{e\sin(c+dx)}e^3}{21a^2d}$$

$$-\frac{5b^2(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}e^3}{3a^5d}$$

$$+\frac{4b(3(a^2-b^2)+ab\cos(c+dx))\sqrt{e\sin(c+dx)}e^3}{3a^5d}$$

$$-\frac{2\cos(c+dx)(e\sin(c+dx))^{5/2}e}{7a^2d}$$

$$+\frac{4b(e\sin(c+dx))^{5/2}e}{5a^3d}+\frac{b^2(e\sin(c+dx))^{5/2}e}{a^3d(b+a\cos(c+dx))}$$

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^
2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5/4)*e^(7/2)*A
rcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(11/2
)*d) + (5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x
]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5/4)*e
^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])
/(a^(11/2)*d) + (10*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(21*a^2*d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*(a^2 - 3*b^2)*e^4*EllipticF[(c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^6*d*Sqrt[e*Sin[c + d*x]]) - (4*
b^2*(4*a^2 - 3*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(3*a^6*d*Sqrt[e*Sin[c + d*x]]) - (5*b^4*(a^2 - b^2)*e^4*EllipticPi[(2*a)/
(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^6*(a
^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)^
2*e^4*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(a^6*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
- (5*b^4*(a^2 - b^2)*e^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2
+ d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^6*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*S
qrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)^2*e^4*EllipticPi[(2*a)/(a + Sqrt[
a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^6*(a^2 - b^2 + a
*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (10*e^3*Cos[c + d*x]*Sqrt[e*Sin
[c + d*x]])/(21*a^2*d) - (5*b^2*e^3*(3*b - a*Cos[c + d*x])*Sqrt[e*Sin[c + d
*x]])/(3*a^5*d) + (4*b*e^3*(3*(a^2 - b^2) + a*b*Cos[c + d*x])*Sqrt[e*Sin[c
+ d*x]])/(3*a^5*d) + (4*b*e*(e*Sin[c + d*x])^(5/2))/(5*a^3*d) - (2*e*Cos[c
+ d*x]*(e*Sin[c + d*x])^(5/2))/(7*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(5/2))/(
a^3*d*(b + a*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
```

```
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
```

```
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{7/2}}{(-b-a\cos(c+dx))^2}\,dx\\
&= \int \left(\frac{(e\sin(c+dx))^{7/2}}{a^2}+\frac{b^2(e\sin(c+dx))^{7/2}}{a^2(b+a\cos(c+dx))^2}-\frac{2b(e\sin(c+dx))^{7/2}}{a^2(b+a\cos(c+dx))}\right)dx\\
&= \frac{\int (e\sin(c+dx))^{7/2}\,dx}{a^2}-\frac{(2b)\int \frac{(e\sin(c+dx))^{7/2}}{b+a\cos(c+dx)}\,dx}{a^2}+\frac{b^2\int \frac{(e\sin(c+dx))^{7/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}\\
&= \frac{4be(e\sin(c+dx))^{5/2}}{5a^3d}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d}\\
&\quad+\frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx))}+\frac{(5e^2)\int (e\sin(c+dx))^{3/2}\,dx}{7a^2}\\
&\quad+\frac{(2be^2)\int \frac{(-a-b\cos(c+dx))(e\sin(c+dx))^{3/2}}{b+a\cos(c+dx)}\,dx}{a^3}-\frac{(5b^2e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{b+a\cos(c+dx)}\,dx}{2a^3}\\
&= -\frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d}-\frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d}\\
&\quad+\frac{4be^3(3(a^2-b^2)+ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d}+\frac{4be(e\sin(c+dx))^{5/2}}{5a^3d}\\
&\quad-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d}+\frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx))}+\frac{(5e^4)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{21a^2}\\
&\quad+\frac{(4be^4)\int \frac{-\frac{1}{2}a(3a^2-2b^2)-\frac{1}{2}b(4a^2-3b^2)\cos(c+dx)}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3a^5}-\frac{(5b^2e^4)\int \frac{-ab+\frac{1}{2}(a^2-3b^2)\cos(c+dx)}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3a^5}
\end{aligned}
$$

$$= -\frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d}$$
$$+ \frac{4be^3(3(a^2-b^2)+ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} + \frac{4be(e\sin(c+dx))^{5/2}}{5a^3d}$$
$$- \frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d} + \frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx))}$$
$$- \frac{(5b^2(a^2-3b^2)\,e^4)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{6a^6} - \frac{(2b^2(4a^2-3b^2)\,e^4)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^6}$$
$$+ \frac{(5b^3(a^2-b^2)\,e^4)\int\frac{1}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2a^6}$$
$$- \frac{\left(2b(a^2-b^2)^2\,e^4\right)\int\frac{1}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a^6}$$
$$+ \frac{\left(5e^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21a^2\sqrt{e\sin(c+dx)}}$$

$$
\begin{aligned}
&= \frac{10e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} \\
&\quad + \frac{4be^3(3(a^2-b^2)+ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} + \frac{4be(e\sin(c+dx))^{5/2}}{5a^3d} \\
&\quad - \frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d} + \frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx))} \\
&\quad - \frac{\left(5b^4\sqrt{a^2-b^2}e^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^6} \\
&\quad - \frac{\left(5b^4\sqrt{a^2-b^2}e^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^6} \\
&\quad + \frac{\left(b^2(a^2-b^2)^{3/2}e^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^6} \\
&\quad + \frac{\left(b^2(a^2-b^2)^{3/2}e^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^6} \\
&\quad - \frac{\left(5b^3(a^2-b^2)e^5\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{x}((-a^2+b^2)e^2+a^2x^2)}\,dx,x,e\sin(c+dx)\right)}{2a^5d} \\
&\quad + \frac{\left(2b(a^2-b^2)^2e^5\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{x}((-a^2+b^2)e^2+a^2x^2)}\,dx,x,e\sin(c+dx)\right)}{a^5d} \\
&\quad - \frac{\left(5b^2(a^2-3b^2)e^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{6a^6\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{\left(2b^2(4a^2-3b^2)e^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a^6\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{10e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{5b^2(a^2-3b^2)\,e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^6d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{4b^2(4a^2-3b^2)\,e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^6d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} \\
&\quad + \frac{4be^3(3(a^2-b^2)+ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} + \frac{4be(e\sin(c+dx))^{5/2}}{5a^3d} \\
&\quad - \frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d} + \frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx))} \\
&\quad - \frac{(5b^3(a^2-b^2)\,e^5)\operatorname{Subst}\left(\int \frac{1}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^5d} \\
&\quad + \frac{\left(4b(a^2-b^2)^2\,e^5\right)\operatorname{Subst}\left(\int \frac{1}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^5d} \\
&\quad - \frac{\left(5b^4\sqrt{a^2-b^2}e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^6\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{\left(5b^4\sqrt{a^2-b^2}e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^6\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(b^2(a^2-b^2)^{3/2}\,e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^6\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(b^2(a^2-b^2)^{3/2}\,e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^6\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{10e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} \\
&- \frac{5b^2(a^2-3b^2)\,e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^6d\sqrt{e\sin(c+dx)}} \\
&- \frac{4b^2(4a^2-3b^2)\,e^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^6d\sqrt{e\sin(c+dx)}} \\
&+ \frac{5b^4\sqrt{a^2-b^2}e^4 \operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^6\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{2b^2(a^2-b^2)^{3/2}\,e^4 \operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^6\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{5b^4\sqrt{a^2-b^2}e^4 \operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^6\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{2b^2(a^2-b^2)^{3/2}\,e^4 \operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^6\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} \\
&+ \frac{4be^3(3(a^2-b^2)+ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} + \frac{4be(e\sin(c+dx))^{5/2}}{5a^3d} \\
&- \frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d} + \frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx))} \\
&+ \frac{\left(5b^3\sqrt{a^2-b^2}e^4\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2a^5d} \\
&+ \frac{\left(5b^3\sqrt{a^2-b^2}e^4\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2a^5d} \\
&- \frac{\left(2b(a^2-b^2)^{3/2}\,e^4\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a^5d} \\
&- \frac{\left(2b(a^2-b^2)^{3/2}\,e^4\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a^5d}
\end{aligned}
$$

$$
\begin{aligned}
= & \frac{5b^3\sqrt[4]{a^2-b^2}e^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{11/2}d} \\
& -\frac{2b(a^2-b^2)^{5/4}e^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{11/2}d} \\
& +\frac{5b^3\sqrt[4]{a^2-b^2}e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{11/2}d} \\
& -\frac{2b(a^2-b^2)^{5/4}e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{11/2}d} \\
& +\frac{10e^4\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} \\
& -\frac{5b^2(a^2-3b^2)e^4\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^6d\sqrt{e\sin(c+dx)}} \\
& -\frac{4b^2(4a^2-3b^2)e^4\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^6d\sqrt{e\sin(c+dx)}} \\
& +\frac{5b^4\sqrt{a^2-b^2}e^4\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^6\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
& -\frac{2b^2(a^2-b^2)^{3/2}e^4\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^6\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
& -\frac{5b^4\sqrt{a^2-b^2}e^4\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^6\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
& +\frac{2b^2(a^2-b^2)^{3/2}e^4\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^6\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
& -\frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d}-\frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} \\
& +\frac{4be^3(3(a^2-b^2)+ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d}+\frac{4be(e\sin(c+dx))^{5/2}}{5a^3d} \\
& -\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d}+\frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx))}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 16.73 (sec) , antiderivative size = 2095, normalized size of antiderivative = 1.90

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(-1/42*((23*a^2 - 84*b^2)*Cos[c + d*x])/a^4 - (b^2*
(-a^2 + b^2))/(a^5*(b + a*Cos[c + d*x])) - (2*b*Cos[2*(c + d*x)])/(5*a^3) +
 Cos[3*(c + d*x)]/(14*a^2))*Csc[c + d*x]^3*Sec[c + d*x]^2*(e*Sin[c + d*x])^
(7/2))/(d*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*
(e*Sin[c + d*x])^(7/2)*((2*(50*a^4 - 273*a^2*b^2 + 105*b^4)*Cos[c + d*x]^2*
(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[S
in[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[
c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a
^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2]
 + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]])
)/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c +
 d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4
, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 +
 b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c
 + d*x])*(1 - Sin[c + d*x]^2)) + (2*(-139*a^3*b + 210*a*b^3)*Cos[c + d*x]*(
b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 +
I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*S
qrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 +
I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[S
qrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a
*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])
/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c
+ d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2
, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*Appe
llF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*S
in[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*S
qrt[1 - Sin[c + d*x]^2]) + ((231*a^3*b - 420*a*b^3)*Cos[c + d*x]*Cos[2*(c +
 d*x)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[
1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2
- b^2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I
```

```
/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4
- I/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*
Sqrt[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)
] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2
)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*
x]^2)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x
]^2])))/(210*a^5*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(7/2))
```

Maple [A] (warning: unable to verify)

Time = 45.23 (sec) , antiderivative size = 1494, normalized size of antiderivative = 1.36

method	result	size
default	Expression too large to display	1494

```
[In] int((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (4*e*a*b*(-1/5/a^6*(e*sin(d*x+c))^(1/2)*e^2*(cos(d*x+c)^2*a^2-6*a^2+10*b^2)
+e^4/a^6*((1/4*a^2*b^2-1/4*b^4)*(e*sin(d*x+c))^(1/2)/(-a^2*e^2*cos(d*x+c)^2
+b^2*e^2)+1/16*(4*a^4-13*a^2*b^2+9*b^4)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2
+b^2*e^2)*(ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+
c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a
^2-b^2)/a^2)^(1/4)))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^4*(-1/21/a^6/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)*(-6*a^4*cos(d*x+c)^4*sin(d*x+c)+5*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))*a^4-84*a^2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+105*b^
4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+16*a^4*cos(d*x+c)^2*sin(d*x+c)-42*a^2*b^2
*cos(d*x+c)^2*sin(d*x+c))+2*b^4*(a^4-2*a^2*b^2+b^4)/a^6*(-1/2*a^2/e/b^2/(a^
2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)-1/4/b^2/(a
^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1
/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*Ellipti
cPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/8/(a^2-b^2
)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c
```

```
)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/4/b^2/(a^2-b^2)^(3/2)*a*(
-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*
e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),
1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^
(1/2)/a),1/2*2^(1/2)))-1/a^6*b^2*(3*a^4-10*a^2*b^2+7*b^4)*(-1/2/(a^2-b^2)^(
1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1
)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a
^2-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^{7/2}}{(b+a\,\cos(c+d\,x))^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(7/2))/(b + a*cos(c + d*x))^2, x)
```

3.243 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 850

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = -\frac{3b^3e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{9/2}\sqrt[4]{a^2-b^2}d}$$
$$+\frac{2b(a^2-b^2)^{3/4}e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d}+\frac{3b^3e^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{9/2}\sqrt[4]{a^2-b^2}d}$$
$$-\frac{2b(a^2-b^2)^{3/4}e^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d}$$
$$+\frac{3b^4e^3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2(a^2-b^2)e^3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{3b^4e^3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2(a^2-b^2)e^3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}$$
$$-\frac{7b^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^4d\sqrt{\sin(c+dx)}}+\frac{4be(e\sin(c+dx))^{3/2}}{3a^3d}$$
$$-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d}+\frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))}$$

```
[Out] -3/2*b^3*e^(5/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2
))/a^(9/2)/(a^2-b^2)^(1/4)/d+2*b*(a^2-b^2)^(3/4)*e^(5/2)*arctan(a^(1/2)*(e*
sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+3/2*b^3*e^(5/2)*arctan
h(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/(a^2-b^2)^(
1/4)/d-2*b*(a^2-b^2)^(3/4)*e^(5/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^
2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+4/3*b*e*(e*sin(d*x+c))^(3/2)/a^3/d-2/5*e*co
s(d*x+c)*(e*sin(d*x+c))^(3/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(3/2)/a^3/d/(b+a*c
os(d*x+c))-3/2*b^4*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1
/2))*sin(d*x+c)^(1/2)/a^5/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*
(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
```

```
*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(
d*x+c)^(1/2)/a^5/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/2*b^4*e^3*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1
/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^5/
d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)*e^3*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*
Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^5/d/(a+(a^2
-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-6/5*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(
e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)+7*b^2*e^2*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x)
,2^(1/2))*(e*sin(d*x+c))^(1/2)/a^4/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 3.06 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3957, 2991, 2715, 2721, 2719, 2772, 2946, 2780, 2886, 2884, 335, 304, 211, 214, 2774}

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \frac{3e^3\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{3e^3\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{3e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{9/2}\sqrt[4]{a^2-b^2}d}+\frac{3e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{9/2}\sqrt[4]{a^2-b^2}d}$$
$$+\frac{e(e\sin(c+dx))^{3/2}b^2}{a^3d(b+a\cos(c+dx))}-\frac{7e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}b^2}{a^4d\sqrt{\sin(c+dx)}}$$
$$-\frac{2(a^2-b^2)e^3\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2(a^2-b^2)e^3\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{4e(e\sin(c+dx))^{3/2}b}{3a^3d}+\frac{2(a^2-b^2)^{3/4}e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{9/2}d}$$
$$-\frac{2(a^2-b^2)^{3/4}e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{9/2}d}$$
$$-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d}+\frac{6e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}$$

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-3*b^3*e^(5/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sq
rt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) + (2*b*(a^2 - b^2)^(3/4)*e^(5/2)*A
rcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(9/2)
*d) + (3*b^3*e^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1
/4)*Sqrt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) - (2*b*(a^2 - b^2)^(3/4)*e^(
5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(
a^(9/2)*d) + (3*b^4*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a - Sqrt[a^2 - b^2])*d*Sqr
t[e*Sin[c + d*x]]) + (3*b^4*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c
- Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a + Sqrt[a^2 - b^2])*d*Sqrt
[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 -
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a + Sqrt[a^2 - b^2
])*d*Sqrt[e*Sin[c + d*x]]) + (6*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e
*Sin[c + d*x]])/(5*a^2*d*Sqrt[Sin[c + d*x]]) - (7*b^2*e^2*EllipticE[(c - Pi
/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^4*d*Sqrt[Sin[c + d*x]]) + (4*b*e*(
e*Sin[c + d*x])^(3/2))/(3*a^3*d) - (2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2)
)/(5*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(3/2))/(a^3*d*(b + a*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{5/2}}{(-b-a\cos(c+dx))^2}\,dx \\
&= \int \left(\frac{(e\sin(c+dx))^{5/2}}{a^2}+\frac{b^2(e\sin(c+dx))^{5/2}}{a^2(b+a\cos(c+dx))^2}-\frac{2b(e\sin(c+dx))^{5/2}}{a^2(b+a\cos(c+dx))}\right)dx \\
&= \frac{\int (e\sin(c+dx))^{5/2}\,dx}{a^2}-\frac{(2b)\int \frac{(e\sin(c+dx))^{5/2}}{b+a\cos(c+dx)}\,dx}{a^2}+\frac{b^2\int \frac{(e\sin(c+dx))^{5/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} \\
&\quad + \frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))} + \frac{(3e^2)\int\sqrt{e\sin(c+dx)}\,dx}{5a^2} \\
&\quad + \frac{(2be^2)\int\frac{(-a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{a^3} - \frac{(3b^2e^2)\int\frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{2a^3} \\
&= \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} + \frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))} \\
&\quad - \frac{(3b^2e^2)\int\sqrt{e\sin(c+dx)}\,dx}{2a^4} - \frac{(2b^2e^2)\int\sqrt{e\sin(c+dx)}\,dx}{a^4} + \frac{(3b^3e^2)\int\frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{2a^4} \\
&\quad - \frac{(2b(a^2-b^2)e^2)\int\frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{a^4} + \frac{\left(3e^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{5a^2\sqrt{\sin(c+dx)}} \\
&= \frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}} + \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} \\
&\quad - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} + \frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))} \\
&\quad - \frac{(3b^4e^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^5} \\
&\quad + \frac{(3b^4e^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^5} \\
&\quad + \frac{(b^2(a^2-b^2)e^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^5} \\
&\quad - \frac{(b^2(a^2-b^2)e^3)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^5} \\
&\quad - \frac{(3b^3e^3)\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}\,dx, x, e\sin(c+dx)\right)}{2a^3d} \\
&\quad + \frac{(2b(a^2-b^2)e^3)\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}\,dx, x, e\sin(c+dx)\right)}{a^3d} \\
&\quad - \frac{\left(3b^2e^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{2a^4\sqrt{\sin(c+dx)}} \\
&\quad - \frac{\left(2b^2e^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{a^4\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}} \\
&\quad - \frac{7b^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^4d\sqrt{\sin(c+dx)}} + \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} \\
&\quad - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} + \frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))} \\
&\quad - \frac{(3b^3e^3)\operatorname{Subst}\left(\int\frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a^3d} \\
&\quad + \frac{(4b(a^2-b^2)\,e^3)\operatorname{Subst}\left(\int\frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a^3d} \\
&\quad - \frac{\left(3b^4e^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^5\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(3b^4e^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^5\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(b^2(a^2-b^2)\,e^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^5\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{\left(b^2(a^2-b^2)\,e^3\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^5\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3b^4e^3 \operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b^2(a^2-b^2)\,e^3 \operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{3b^4e^3 \operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b^2(a^2-b^2)\,e^3 \operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\, 2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}} \\
&\quad - \frac{7b^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\, 2\right)\sqrt{e\sin(c+dx)}}{a^4d\sqrt{\sin(c+dx)}} + \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} \\
&\quad - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} + \frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))} \\
&\quad + \frac{(3b^3e^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2a^4d} \\
&\quad - \frac{(3b^3e^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2a^4d} \\
&\quad - \frac{(2b(a^2-b^2)\,e^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^4d} \\
&\quad + \frac{(2b(a^2-b^2)\,e^3)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^4d}
\end{aligned}
$$

$$= -\frac{3b^3e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{9/2}\sqrt[4]{a^2-b^2}d} + \frac{2b(a^2-b^2)^{3/4}e^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d}$$
$$+ \frac{3b^3e^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{9/2}\sqrt[4]{a^2-b^2}d} - \frac{2b(a^2-b^2)^{3/4}e^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d}$$
$$+ \frac{3b^4e^3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{2b^2(a^2-b^2)e^3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{3b^4e^3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{2b^2(a^2-b^2)e^3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}$$
$$- \frac{7b^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^4d\sqrt{\sin(c+dx)}} + \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d}$$
$$- \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} + \frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.51 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.04

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx =$$

$$-\frac{(b+a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{(-6a^2+35b^2)\cos^2(c+dx)\left(3\sqrt{2}b(-a^2+b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c}}{\sqrt[4]{-a^2-}}\right.\right.\right.}{}\right.}{}$$

$$+\frac{(b+a\cos(c+dx))^2\csc^2(c+dx)\sec^2(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{4b\sin(c+dx)}{3a^3}+\frac{b^2\sin(c+dx)}{a^3(b+a\cos(c+dx))}-\frac{\sin(2(c+dx))}{5a^2}\right)}{d(a+b\sec(c+dx))^2}$$

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/10*((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(((-6*a
^2 + 35*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (
Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqr
t[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
 - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]
^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - S
in[c + d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c +
d*x]^2)) + (28*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sq
rt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a
]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 -
 b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c +
d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c +
 d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b
^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[
c + d*x]^2])))/(a^3*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(5/2)) + ((b + a*
Cos[c + d*x])^2*Csc[c + d*x]^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*((4*b*
Sin[c + d*x])/(3*a^3) + (b^2*Sin[c + d*x])/(a^3*(b + a*Cos[c + d*x])) - Sin
[2*(c + d*x)]/(5*a^2)))/(d*(a + b*Sec[c + d*x])^2)
```

Maple [A] (warning: unable to verify)

Time = 34.74 (sec) , antiderivative size = 1482, normalized size of antiderivative = 1.74

method	result	size
default	Expression too large to display	1482

```
[In] int((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (4*e*a*b*(1/3*(e*sin(d*x+c))^(3/2)/a^4+e^2/a^4*(1/4*(e*sin(d*x+c))^(3/2)*b^
2/(-a^2*e^2*cos(d*x+c)^2+b^2*e^2)+1/4*(a^2-7/4*b^2)/a^2/(e^2*(a^2-b^2)/a^2)
^(1/4)*(2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((e*sin
(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b
^2)/a^2)^(1/4))))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^3*(-1/5/a^2/(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1
)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^4+2*cos(d*x+c)^2)+3*b^2/a^4*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((-sin(d*x
+c)+1)^(1/2),1/2*2^(1/2)))-b^2*(3*a^2-5*b^2)/a^4*(-1/2/a^2*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
```

```
1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^
(1/2)/a),1/2*2^(1/2))-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*El
lipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)))+2*b^4*
(a^2-b^2)/a^4*(-1/2*a^2/e/b^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)+1/2/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*El
lipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin
(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*si
n(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1
-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-
(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a)
,1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*
EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(
a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/
(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*
x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+
c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^{5/2}}{(b+a\,\cos(c+d\,x))^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(5/2))/(b + a*cos(c + d*x))^2, x)
```

3.244 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 882

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \frac{b^3e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d} - \frac{2b\sqrt[4]{a^2-b^2}e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} + \frac{b^3e^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d} - \frac{2b\sqrt[4]{a^2-b^2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} + \frac{2e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4d\sqrt{e\sin(c+dx)}} - \frac{b^4e^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^4\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{2b^2(a^2-b^2)\,e^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b^4e^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^4\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{2b^2(a^2-b^2)\,e^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))}$$

```
[Out] 1/2*b^3*e^(3/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2)
)/a^(7/2)/(a^2-b^2)^(3/4)/d-2*b*(a^2-b^2)^(1/4)*e^(3/2)*arctan(a^(1/2)*(e*s
in(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/d+1/2*b^3*e^(3/2)*arctanh
(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/(a^2-b^2)^(3
/4)/d-2*b*(a^2-b^2)^(1/4)*e^(3/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2
-b^2)^(1/4)/e^(1/2))/a^(7/2)/d-2/3*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(
d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)+5*b^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^
(1/2))*sin(d*x+c)^(1/2)/a^4/d/(e*sin(d*x+c))^(1/2)+1/2*b^4*e^2*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*
```

```
Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^
2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(a^2-b^2)*e^2*(sin(1/2*c+1/
4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*P
i+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^2
-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+1/2*b^4*e^2*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^2+a*(a^2-b
^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(a^2-b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x)
,2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^2+a*(a^2-b^
2)^(1/2))/(e*sin(d*x+c))^(1/2)+4*b*e*(e*sin(d*x+c))^(1/2)/a^3/d-2/3*e*cos(d
*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(1/2)/a^3/d/(b+a*cos(
d*x+c))
```

Rubi [A] (verified)

Time = 2.68 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used

$= \{3957, 2991, 2715, 2721, 2720, 2772, 2946, 2781, 2886, 2884, 335, 218, 214, 211, 2774\}$

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = -\frac{e^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^4\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}} - \frac{e^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^4\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d} + \frac{e^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d} + \frac{e\sqrt{e\sin(c+dx)}b^2}{a^3d(b+a\cos(c+dx))} - \frac{5e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^4d\sqrt{e\sin(c+dx)}} + \frac{2(a^2-b^2)\,e^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^4\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}} + \frac{2(a^2-b^2)\,e^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^4\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}} - \frac{2\sqrt[4]{a^2-b^2}e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{7/2}d} - \frac{2\sqrt[4]{a^2-b^2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{7/2}d} + \frac{4e\sqrt{e\sin(c+dx)}b}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{2e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}}$$

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (b^3*e^(3/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcT
an[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d)
 + (b^3*e^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(3/2)*
ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/
2)*d) + (2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*
d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(a^4*d*Sqrt[e*Sin[c + d*x]]) - (b^4*e^2*EllipticPi[(2*a)/(a
```

```
- Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^4*(a^2
- b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)*e^2
*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(a^4*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (b^
4*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(2*a^4*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]
) + (2*b^2*(a^2 - b^2)*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*S
qrt[e*Sin[c + d*x]]) + (4*b*e*Sqrt[e*Sin[c + d*x]])/(a^3*d) - (2*e*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(3*a^2*d) + (b^2*e*Sqrt[e*Sin[c + d*x]])/(a^3*d*
(b + a*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
```

```
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}}{(-b-a\cos(c+dx))^2}\,dx \\
&= \int \left(\frac{(e\sin(c+dx))^{3/2}}{a^2}+\frac{b^2(e\sin(c+dx))^{3/2}}{a^2(b+a\cos(c+dx))^2}-\frac{2b(e\sin(c+dx))^{3/2}}{a^2(b+a\cos(c+dx))}\right)dx \\
&= \frac{\int (e\sin(c+dx))^{3/2}\,dx}{a^2}-\frac{(2b)\int \frac{(e\sin(c+dx))^{3/2}}{b+a\cos(c+dx)}\,dx}{a^2}+\frac{b^2\int \frac{(e\sin(c+dx))^{3/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2} \\
&= \frac{4be\sqrt{e\sin(c+dx)}}{a^3d}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} \\
&\quad +\frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))}+\frac{e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^2} \\
&\quad +\frac{(2be^2)\int \frac{-a-b\cos(c+dx)}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a^3}-\frac{(b^2e^2)\int \frac{\cos(c+dx)}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2a^3}
\end{aligned}
$$

$$= \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))} - \frac{(b^2e^2)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{2a^4} - \frac{(2b^2e^2)\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{a^4} + \frac{(b^3e^2)\int\frac{1}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2a^4} - \frac{(2b(a^2-b^2)e^2)\int\frac{1}{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a^4} + \frac{\left(e^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a^2\sqrt{e\sin(c+dx)}}$$

$$= \frac{2e^2\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} + \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))} - \frac{(b^4e^2)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^4\sqrt{a^2-b^2}} - \frac{(b^4e^2)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^4\sqrt{a^2-b^2}} + \frac{\left(b^2\sqrt{a^2-b^2}e^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^4} + \frac{\left(b^2\sqrt{a^2-b^2}e^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^4} - \frac{(b^3e^3)\operatorname{Subst}\left(\int\frac{1}{\sqrt{x}((-a^2+b^2)e^2+a^2x^2)}\,dx,x,e\sin(c+dx)\right)}{2a^3d} + \frac{(2b(a^2-b^2)e^3)\operatorname{Subst}\left(\int\frac{1}{\sqrt{x}((-a^2+b^2)e^2+a^2x^2)}\,dx,x,e\sin(c+dx)\right)}{a^3d} - \frac{\left(b^2e^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{2a^4\sqrt{e\sin(c+dx)}} - \frac{\left(2b^2e^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{a^4\sqrt{e\sin(c+dx)}}$$

$$= \frac{2e^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2e^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4d\sqrt{e\sin(c+dx)}} + \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))} - \frac{(b^3e^3)\operatorname{Subst}\left(\int \frac{1}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^3d} + \frac{(4b(a^2-b^2)\,e^3)\operatorname{Subst}\left(\int \frac{1}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^3d} - \frac{\left(b^4e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^4\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}} - \frac{\left(b^4e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^4\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}} + \frac{\left(b^2\sqrt{a^2-b^2}e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^4\sqrt{e\sin(c+dx)}} + \frac{\left(b^2\sqrt{a^2-b^2}e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^4\sqrt{e\sin(c+dx)}}$$

$$
\begin{aligned}
&= \frac{2e^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{5b^2e^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b^2\sqrt{a^2-b^2}e^2 \operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{b^4e^2 \operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^4\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{b^4e^2 \operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^4\sqrt{a^2-b^2}\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2b^2\sqrt{a^2-b^2}e^2 \operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} \\
&\quad + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))} + \frac{(b^3e^2)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2a^3\sqrt{a^2-b^2}d} \\
&\quad + \frac{(b^3e^2)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2a^3\sqrt{a^2-b^2}d} \\
&\quad - \frac{\left(2b\sqrt{a^2-b^2}e^2\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a^3d} \\
&\quad - \frac{\left(2b\sqrt{a^2-b^2}e^2\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a^3d}
\end{aligned}
$$

$$= \frac{b^3 e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d} - \frac{2b\sqrt[4]{a^2-b^2}e^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d}$$
$$+ \frac{b^3 e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d} - \frac{2b\sqrt[4]{a^2-b^2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d}$$
$$+ \frac{2e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}}$$
$$- \frac{5b^2e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4d\sqrt{e\sin(c+dx)}}$$
$$- \frac{2b^2\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{b^4e^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^4\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{b^4e^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^4\sqrt{a^2-b^2}\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{2b^2\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^4\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 16.51 (sec) , antiderivative size = 2012, normalized size of antiderivative = 2.28

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*((-2*Cos[c + d*x])/(3*a^2) + b^2/(a^3*(b + a*Cos[c
+ d*x])))*Csc[c + d*x]*Sec[c + d*x]^2*(e*Sin[c + d*x])^(3/2))/(d*(a + b*Sec
[c + d*x])^2) - ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(3/
2)*((2*(-2*a^2 + 3*b^2)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b
*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
```

```
 a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3
/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5
*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^
2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2
*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin
[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*
(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (8*
a*b*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2
*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[
a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + I*a*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*S
qrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1
/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*Ap
pellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]
 + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^
2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b +
 a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]) - (6*a*b*Cos[c + d*x]*Cos[2*(c +
 d*x)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[
1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2
- b^2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I
/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4
- I/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*
Sqrt[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)
] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2
)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*
x]^2)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x
]^2])))/(6*a^3*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(3/2))
```

Maple [A] (warning: unable to verify)

Time = 26.56 (sec) , antiderivative size = 1350, normalized size of antiderivative = 1.53

method	result	size
default	Expression too large to display	1350

```
[In] int((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (4*e*a*b*((e*sin(d*x+c))^(1/2)/a^4+e^2/a^4*(1/4*(e*sin(d*x+c))^(1/2)*b^2/(-
a^2*e^2*cos(d*x+c)^2+b^2*e^2)+1/16*(4*a^2-5*b^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/
(-a^2*e^2+b^2*e^2)*(ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e
*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2*arctan((e*sin(d*x+c))^(1/2
)/(e^2*(a^2-b^2)/a^2)^(1/4)))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^2*(-1/3
/a^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*c
os(d*x+c)^2*sin(d*x+c))+3*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))-1/a^4*b^2*(3*a^2-5*b^2)*(-1/2/(a^2-b^2)^(1/2)/a*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(
1/2)/a),1/2*2^(1/2)))+2*b^4*(a^2-b^2)/a^4*(-1/2*a^2/e/b^2/(a^2-b^2)*(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)-1/4/b^2/(a^2-b^2)*(-sin(
d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^
2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+
c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/8/(a^2-b^2)^(3/2)/a*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(
1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)
^(1/2)/a),1/2*2^(1/2))-5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b
^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2
^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^{3/2}}{(b+a\,\cos(c+d\,x))^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(3/2))/(b + a*cos(c + d*x))^2, x)
```

3.245 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 809

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \frac{b^3\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d}+\frac{2b\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d}$$
$$-\frac{b^3\sqrt{e}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d}$$
$$-\frac{2b\sqrt{e}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d}$$
$$-\frac{2b^2e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{b^4e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^3\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{b^4e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^3\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}}$$
$$-\frac{b^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\left(a^2-b^2\right)d\sqrt{\sin(c+dx)}}$$
$$+\frac{b^2(e\sin(c+dx))^{3/2}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}$$

```
[Out] b^2*(e*sin(d*x+c))^(3/2)/a/(a^2-b^2)/d/e/(b+a*cos(d*x+c))+1/2*b^3*arctan(a^
(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2)/(a^2-b^
2)^(5/4)/d+2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))
*e^(1/2)/a^(5/2)/(a^2-b^2)^(1/4)/d-1/2*b^3*arctanh(a^(1/2)*(e*sin(d*x+c))^(
1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2)/(a^2-b^2)^(5/4)/d-2*b*arctanh
(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2)/(a^2
-b^2)^(1/4)/d+2*b^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+
1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/
2))*sin(d*x+c)^(1/2)/a^3/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+1/2*b^4
*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi
(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2
```

```
)/a^3/(a^2-b^2)/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*e*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^3/d/(a+
(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+1/2*b^4*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a
/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^3/(a^2-b^2)/d/(a+(a^2-b^2)
^(1/2))/(e*sin(d*x+c))^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c
))^(1/2)/a^2/d/sin(d*x+c)^(1/2)+b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(
d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules

used = {3957, 2991, 2721, 2719, 2773, 2946, 2780, 2886, 2884, 335, 304, 211, 214}

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = -\frac{e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^3(a^2-b^2)\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^3(a^2-b^2)\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{5/2}(a^2-b^2)^{5/4}d} - \frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{5/2}(a^2-b^2)^{5/4}d} + \frac{(e\sin(c+dx))^{3/2}b^2}{a(a^2-b^2)de(b+a\cos(c+dx))} - \frac{E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}b^2}{a^2(a^2-b^2)d\sqrt{\sin(c+dx)}} - \frac{2e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{2e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{2\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{5/2}\sqrt[4]{a^2-b^2}d} - \frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{5/2}\sqrt[4]{a^2-b^2}d} + \frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}}$$

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (b^3*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) + (2*b*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e
*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^2)^(1/4)*d)
 - (b^3*Sqrt[e]*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) - (2*b*Sqrt[e]*ArcTanh[(Sqrt[a]*S
qrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^2)^(1/
4)*d) - (2*b^2*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2
, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
```

```
 - (b^4*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
 d*x]]) - (2*b^2*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]
]) - (b^4*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^2*
d*Sqrt[Sin[c + d*x]]) - (b^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c
+ d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[Sin[c + d*x]]) + (b^2*(e*Sin[c + d*x])^(3/
2))/(a*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
```

```
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\sqrt{e\sin(c+dx)}}{(-b-a\cos(c+dx))^2}\,dx\\
&= \int \left(\frac{\sqrt{e\sin(c+dx)}}{a^2}+\frac{b^2\sqrt{e\sin(c+dx)}}{a^2(b+a\cos(c+dx))^2}-\frac{2b\sqrt{e\sin(c+dx)}}{a^2(b+a\cos(c+dx))}\right)dx\\
&= \frac{\int \sqrt{e\sin(c+dx)}\,dx}{a^2}-\frac{(2b)\int \frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{a^2}+\frac{b^2\int \frac{\sqrt{e\sin(c+dx)}}{(b+a\cos(c+dx))^2}\,dx}{a^2}\\
&= \frac{b^2(e\sin(c+dx))^{3/2}}{a\,(a^2-b^2)\,de(b+a\cos(c+dx))}+\frac{b^2\int \frac{\left(-b-\frac{1}{2}a\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{a^2\,(a^2-b^2)}\\
&\quad+\frac{(b^2e)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^3}-\frac{(b^2e)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^3}\\
&\quad+\frac{(2be)\operatorname{Subst}\left(\int \frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}\,dx,x,e\sin(c+dx)\right)}{ad}\\
&\quad+\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}\,dx}{a^2\sqrt{\sin(c+dx)}}\\
&= \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}}+\frac{b^2(e\sin(c+dx))^{3/2}}{a\,(a^2-b^2)\,de(b+a\cos(c+dx))}\\
&\quad-\frac{b^2\int \sqrt{e\sin(c+dx)}\,dx}{2a^2\,(a^2-b^2)}-\frac{b^3\int \frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{2a^2\,(a^2-b^2)}\\
&\quad+\frac{(4be)\operatorname{Subst}\left(\int \frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{ad}\\
&\quad+\frac{\left(b^2e\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^3\sqrt{e\sin(c+dx)}}\\
&\quad-\frac{\left(b^2e\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^3\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{2b^2 e \,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2b^2 e \,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2 d\sqrt{\sin(c+dx)}}+\frac{b^2(e\sin(c+dx))^{3/2}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}$$
$$+\frac{(b^4 e)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^3\left(a^2-b^2\right)}-\frac{(b^4 e)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^3\left(a^2-b^2\right)}$$
$$-\frac{(2be)\text{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^2 d}$$
$$+\frac{(2be)\text{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{a^2 d}$$
$$+\frac{(b^3 e)\,\text{Subst}\left(\int \frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}\,dx, x, e\sin(c+dx)\right)}{2a\left(a^2-b^2\right)d}$$
$$-\frac{\left(b^2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{2a^2\left(a^2-b^2\right)\sqrt{\sin(c+dx)}}$$

$$
\begin{aligned}
&= \frac{2b\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d} - \frac{2b\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d} \\
&\quad - \frac{2b^2e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b^2e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}} \\
&\quad - \frac{b^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\left(a^2-b^2\right)d\sqrt{\sin(c+dx)}} + \frac{b^2(e\sin(c+dx))^{3/2}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))} \\
&\quad + \frac{\left(b^3e\right)\text{Subst}\left(\int\frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a\left(a^2-b^2\right)d} \\
&\quad + \frac{\left(b^4e\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^3\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{\left(b^4e\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^3\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d} - \frac{2b\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d} \\
&\quad - \frac{2b^2e\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{b^4e\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^3(a^2-b^2)\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b^2e\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{b^4e\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^3(a^2-b^2)\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}} \\
&\quad - \frac{b^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2(a^2-b^2)d\sqrt{\sin(c+dx)}} + \frac{b^2(e\sin(c+dx))^{3/2}}{a(a^2-b^2)de(b+a\cos(c+dx))} \\
&\quad - \frac{(b^3e)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2a^2(a^2-b^2)d} \\
&\quad + \frac{(b^3e)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2a^2(a^2-b^2)d}
\end{aligned}
$$

$$= \frac{b^3\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d} + \frac{2b\sqrt{e}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d}$$
$$- \frac{b^3\sqrt{e}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d} - \frac{2b\sqrt{e}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d}$$
$$- \frac{2b^2e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{b^4e\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^3\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{2b^2e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{b^4e\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^3\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}}$$
$$- \frac{b^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\left(a^2-b^2\right)d\sqrt{\sin(c+dx)}} + \frac{b^2(e\sin(c+dx))^{3/2}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.87 (sec) , antiderivative size = 854, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\sqrt{e\sin(c+dx)}\left(\frac{\left(-2a^2+3b^2\right)\cos^2(c+dx)\left(3\sqrt{2}b\left(-a^2+b^2\right)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)\right.\right.}{}\right.}{}$$
$$+ \frac{b^2(b+a\cos(c+dx))\sec(c+dx)\sqrt{e\sin(c+dx)}\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}$$

[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x])^2,x]

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sqrt[e*Sin[c + d*x]]*(((-2*a^2 + 3*b
^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*S
qrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt
[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2
]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqr
t[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*S
in[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*
Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c + d*
x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2))
+ (4*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c +
 d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1
 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(S
qrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (
a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b^2)))*(b +
 a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2
])))/(2*a*(-a + b)*(a + b)*d*(a + b*Sec[c + d*x])^2*Sqrt[Sin[c + d*x]]) + (
b^2*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]]*Tan[c + d*x])/(a
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] (warning: unable to verify)

Time = 19.71 (sec) , antiderivative size = 1313, normalized size of antiderivative = 1.62

method	result	size
default	Expression too large to display	1313

```
[In] int((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e*a*b*(-1/2*b^2/a^2/(a^2-b^2)*(e*sin(d*x+c))^(3/2)/(-a^2*e^2*cos(d*x+c)
^2+b^2*e^2)-1/8*(4*a^2-3*b^2)/a^4/(a^2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/4)*(2*ar
ctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-ln(((e*sin(d*x+c))^(1/
2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/
4)))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e*(-1/a^2*(-sin(d*x+c)+1)^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(2*
EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((-sin(d*x+c)+1)^(1/2
),1/2*2^(1/2)))+2*b^4/a^2*(-1/2*a^2/e/b^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)+1/2/b^2/(a^2-b^2)*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^
2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1
```

```
)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a^2-b^2)/a^2*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b
^2)^(1/2)/a),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^
2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^
(1/2))+3/8/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*Ellipti
cPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)))-3/a^2*b^2*(
-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)
+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/a^2*(-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)
/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2
)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin{(c+dx)}}}{(a+b\sec{(c+dx)})^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*sec(d*x+c))**2,x)
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\sqrt{e\,\sin\left(c+d\,x\right)}}{{\left(b+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

```
[In] int((e*sin(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(1/2))/(b + a*cos(c + d*x))^2, x)
```

3.246 $\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 838

$$\begin{aligned}
\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx
&= -\frac{3b^3\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}}
-\frac{2b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}
-\frac{3b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}} \\
&\quad -\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}
+\frac{2\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}} \\
&\quad +\frac{b^2\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2(a^2-b^2)\,d\sqrt{e\sin(c+dx)}} \\
&\quad +\frac{2b^2\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad +\frac{3b^4\operatorname{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^2(a^2-b^2)\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad +\frac{2b^2\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad +\frac{3b^4\operatorname{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^2(a^2-b^2)\left(a^2-b^2+a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad +\frac{b^2\sqrt{e\sin(c+dx)}}{a(a^2-b^2)\,de(b+a\cos(c+dx))}
\end{aligned}$$

```
[Out] -3/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(3/
2)/(a^2-b^2)^(7/4)/d/e^(1/2)-2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b
^2)^(1/4)/e^(1/2))/a^(3/2)/(a^2-b^2)^(3/4)/d/e^(1/2)-3/2*b^3*arctanh(a^(1/2
)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(3/2)/(a^2-b^2)^(7/4)/d/e
^(1/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^
(3/2)/(a^2-b^2)^(3/4)/d/e^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c
)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(
d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*
d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a^2-b^2-a*(a^
2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(
a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(a^2-b^2-a*(a^2-b
^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b
^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*s
in(d*x+c))^(1/2)-3/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^
(1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(
d*x+c))^(1/2)+b^2*(e*sin(d*x+c))^(1/2)/a/(a^2-b^2)/d/e/(b+a*cos(d*x+c))
```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 838, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules

used = {3957, 2991, 2721, 2720, 2773, 2946, 2781, 2886, 2884, 335, 218, 214, 211}

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{3\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^2\left(a^2-b^2\right)\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{3\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a^2\left(a^2-b^2\right)\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{3\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}}-\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}}$$
$$+\frac{\sqrt{e\sin(c+dx)}b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}+\frac{\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^2\left(a^2-b^2\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^2\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a^2\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}$$
$$-\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}$$
$$+\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (-3*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])
/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(3/4)*d*Sqrt[
e]) - (3*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt
[e])])/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTanh[(Sqrt[a]*Sqrt
[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(3/4)*
d*Sqrt[e]) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*d
*Sqrt[e*Sin[c + d*x]]) + (b^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*EllipticPi[(2*a)/
(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*(a^2
 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (3*b^4*EllipticPi[(2*
a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^2
*(a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b
^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[
```

```
c + d*x]])/(a^2*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (
3*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(2*a^2*(a^2 - b^2)*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*S
in[c + d*x]]) + (b^2*Sqrt[e*Sin[c + d*x]])/(a*(a^2 - b^2)*d*e*(b + a*Cos[c
+ d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
```

```
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx\\
&= \int \left(\frac{1}{a^2\sqrt{e\sin(c+dx)}}+\frac{b^2}{a^2(-b-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\right.\\
&\qquad\left.+\frac{2b}{a^2(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\right)dx\\
&= \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{a^2}+\frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a^2}+\frac{b^2\int \frac{1}{(-b-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx}{a^2}\\
&= \frac{b^2\sqrt{e\sin(c+dx)}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}+\frac{b^2\int \frac{b-\frac{1}{2}a\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a^2\left(a^2-b^2\right)}\\
&\quad+\frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^2\sqrt{a^2-b^2}}+\frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^2\sqrt{a^2-b^2}}\\
&\quad+\frac{(2be)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(\left(-a^2+b^2\right)e^2+a^2x^2\right)}\,dx,x,e\sin(c+dx)\right)}{ad}\\
&\quad+\frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{a^2\sqrt{e\sin(c+dx)}}\\
&= \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}}+\frac{b^2\sqrt{e\sin(c+dx)}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}\\
&\quad+\frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{2a^2\left(a^2-b^2\right)}+\frac{\left(3b^3\right)\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2a^2\left(a^2-b^2\right)}\\
&\quad+\frac{(4be)\text{Subst}\left(\int \frac{1}{\left(-a^2+b^2\right)e^2+a^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{ad}\\
&\quad+\frac{\left(b^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a^2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}}\\
&\quad+\frac{\left(b^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a^2\sqrt{a^2-b^2}\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$= \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\sqrt{a^2-b^2}\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+\frac{b^2\sqrt{e\sin(c+dx)}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}+\frac{(3b^4)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^2\left(a^2-b^2\right)^{3/2}}$$
$$+\frac{(3b^4)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^2\left(a^2-b^2\right)^{3/2}}$$
$$-\frac{(2b)\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a\sqrt{a^2-b^2}d}$$
$$-\frac{(2b)\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a\sqrt{a^2-b^2}d}$$
$$+\frac{(3b^3e)\,\text{Subst}\left(\int\frac{1}{\sqrt{x}\left((-a^2+b^2)e^2+a^2x^2\right)}\,dx,x,e\sin(c+dx)\right)}{2a\left(a^2-b^2\right)d}$$
$$+\frac{\left(b^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{2a^2\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}$$

$$= -\frac{2b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{2b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}$$

$$+ \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}}$$

$$+ \frac{b^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2\right)d\sqrt{e\sin(c+dx)}}$$

$$+ \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$

$$+ \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\sqrt{a^2-b^2}\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$

$$+ \frac{b^2\sqrt{e\sin(c+dx)}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}$$

$$+ \frac{(3b^3e)\,\text{Subst}\left(\int\frac{1}{(-a^2+b^2)e^2+a^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{a\left(a^2-b^2\right)d}$$

$$+ \frac{\left(3b^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a^2\left(a^2-b^2\right)^{3/2}\sqrt{e\sin(c+dx)}}$$

$$+ \frac{\left(3b^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a^2\left(a^2-b^2\right)^{3/2}\sqrt{e\sin(c+dx)}}$$

$$= -\frac{2b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{2b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}$$
$$+ \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{b^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2\right)d\sqrt{e\sin(c+dx)}}$$
$$- \frac{3b^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^2\left(a^2-b^2\right)^{3/2}\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{3b^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^2\left(a^2-b^2\right)^{3/2}\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\sqrt{a^2-b^2}\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{b^2\sqrt{e\sin(c+dx)}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}$$
$$- \frac{(3b^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2a\left(a^2-b^2\right)^{3/2}d}$$
$$- \frac{(3b^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2a\left(a^2-b^2\right)^{3/2}d}$$

$$
\begin{aligned}
&= -\frac{3b^3\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}} - \frac{2b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} \\
&\quad - \frac{3b^3\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}} - \frac{2b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} \\
&\quad + \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{b^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2\right)d\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{3b^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^2\left(a^2-b^2\right)^{3/2}\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{3b^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a^2\left(a^2-b^2\right)^{3/2}\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a^2\sqrt{a^2-b^2}\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{b^2\sqrt{e\sin(c+dx)}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.65 (sec) , antiderivative size = 1246, normalized size of antiderivative = 1.49

$$
\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx
$$

$$
= \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\sqrt{\sin(c+dx)}\left(\frac{2\left(-2a^2+b^2\right)\cos^2(c+dx)\left(b+a\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{b\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c}}{\sqrt[4]{-a^2+}}\right.\right.}{}\right.}{}\right.}{}
$$

$$
+ \frac{b^2(b+a\cos(c+dx))\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}
$$

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]]*((2*(-2*a^2 + b^2
)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[
2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sq
rt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log
[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2
)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b
^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[
1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2
*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[
c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2))
)))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*a*b*Cos[c + d*x]*(b +
a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*S
qrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[
a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*S
qrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt[
a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sq
rt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*
x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/
4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1
[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c
 + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sqrt[
1 - Sin[c + d*x]^2])))/(2*a*(-a + b)*(a + b)*d*(a + b*Sec[c + d*x])^2*Sqrt[
e*Sin[c + d*x]]) + (b^2*(b + a*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(a*
(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]])
```

Maple [A] (warning: unable to verify)

Time = 23.71 (sec) , antiderivative size = 1239, normalized size of antiderivative = 1.48

method	result	size
default	Expression too large to display	1239

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4*a*b*e*(1/4*b^2/a^2/(a^2-b^2)*(e*sin(d*x+c))^(1/2)/(-a^2*e^2*cos(d*x+c)^2
+b^2*e^2)+1/16*(4*a^2-b^2)/a^2/(a^2-b^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^
2+b^2*e^2)*(ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x
+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(
a^2-b^2)/a^2)^(1/4))))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(-1/a^2*(-sin(d*x+
```

```
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/a^2*b^4*(-1/2*a^2
/e/b^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)-
1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*
2^(1/2))-1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/
a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/
8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((
-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/4/b^2/(a^2-b^2)
^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)
+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/8/(a^2-b^2)^(3/2)/a*(-sin(
d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+
(a^2-b^2)^(1/2)/a),1/2*2^(1/2)))-3/a^2*b^2*(-1/2/(a^2-b^2)^(1/2)/a*(-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a
^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1
+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a
),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sin{(c+dx)}}\,(a+b\sec{(c+dx)})^2}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*sin(c + d*x))*(a + b*sec(c + d*x))**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^2\sqrt{e\sin\left(dx+c\right)}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\sqrt{e\,\sin\left(c+d\,x\right)}\,\left(b+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/((e*sin(c + d*x))^(1/2)*(b + a*cos(c + d*x))^2), x)
```

3.247 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 1054

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx=\frac{5b^3\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\sqrt{a}\,(a^2-b^2)^{9/4}\,de^{3/2}}+\frac{2b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\,(a^2-b^2)^{5/4}\,de^{3/2}}-\frac{5b^3\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\sqrt{a}\,(a^2-b^2)^{9/4}\,de^{3/2}}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\,(a^2-b^2)^{5/4}\,de^{3/2}}-\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}}+\frac{b^2}{a\,(a^2-b^2)\,de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}+\frac{4b(a-b\cos(c+dx))}{a^2\,(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}+\frac{b^2(5ab-(3a^2+2b^2)\cos(c+dx))}{a^2\,(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}}-\frac{5b^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a\,(a^2-b^2)^2\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}-\frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\,(a^2-b^2)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}-\frac{5b^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2a\,(a^2-b^2)^2\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}-\frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\,(a^2-b^2)\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}-\frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2de^2\sqrt{\sin(c+dx)}}-\frac{4b^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\,(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}-\frac{b^2(3a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\,(a^2-b^2)^2\,de^2\sqrt{\sin(c+dx)}}$$

```
[Out] 5/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-b
^2)^(9/4)/d/e^(3/2)/a^(1/2)+2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^
2)^(1/4)/e^(1/2))/(a^2-b^2)^(5/4)/d/e^(3/2)/a^(1/2)-5/2*b^3*arctanh(a^(1/2)
*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(9/4)/d/e^(3/2)/a^
(1/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^
2-b^2)^(5/4)/d/e^(3/2)/a^(1/2)-2*cos(d*x+c)/a^2/d/e/(e*sin(d*x+c))^(1/2)+b^
```

```
2/a/(a^2-b^2)/d/e/(b+a*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+4*b*(a-b*cos(d*x+c)
)/a^2/(a^2-b^2)/d/e/(e*sin(d*x+c))^(1/2)+b^2*(5*a*b-(3*a^2+2*b^2)*cos(d*x+c
))/a^2/(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(1/2)+5/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x)
,2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/e/(a-(a^
2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^
2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)/d/e/(a-(a^2-b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+5/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2
)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/e/(a+(a^2-b^2)^(1/2))/(e*sin(d
*x+c))^(1/2)+2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2
*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))
*sin(d*x+c)^(1/2)/a/(a^2-b^2)/d/e/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+
2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(c
os(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/e^2/sin(d*x+c)
^(1/2)+4*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/(a^2-
b^2)/d/e^2/sin(d*x+c)^(1/2)+b^2*(3*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*(e*sin(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 3.29 (sec) , antiderivative size = 1054, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3957, 2991, 2716, 2721, 2719, 2773, 2945, 2946, 2780, 2886, 2884, 335, 304, 211,

214, 2775}

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx =$$

$$-\frac{5\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a\left(a^2-b^2\right)^2\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$

$$-\frac{5\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2a\left(a^2-b^2\right)^2\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$

$$+\frac{5\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2\sqrt{a}\left(a^2-b^2\right)^{9/4}de^{3/2}}-\frac{5\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2\sqrt{a}\left(a^2-b^2\right)^{9/4}de^{3/2}}$$

$$-\frac{\left(3a^2+2b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}b^2}{a^2\left(a^2-b^2\right)^2de^2\sqrt{\sin(c+dx)}}$$

$$-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}b^2}{a^2\left(a^2-b^2\right)de^2\sqrt{\sin(c+dx)}}$$

$$+\frac{\left(5ab-\left(3a^2+2b^2\right)\cos(c+dx)\right)b^2}{a^2\left(a^2-b^2\right)^2de\sqrt{e\sin(c+dx)}}$$

$$-\frac{2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$

$$-\frac{2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{a\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$

$$+\frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}+\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}}+\frac{4(a-b\cos(c+dx))b}{a^2\left(a^2-b^2\right)de\sqrt{e\sin(c+dx)}}$$

$$-\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2de^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}}$$

```
[In] Int[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (5*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/
(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) + (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c
 + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(5/4)*d*e^(3/2
)) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) - (2*b*ArcTanh[(Sqrt[a]*Sqrt[
e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(5/4)*d
```

```
*e^(3/2)) - (2*Cos[c + d*x])/(a^2*d*e*Sqrt[e*Sin[c + d*x]]) + b^2/(a*(a^2 -
 b^2)*d*e*(b + a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]) + (4*b*(a - b*Cos[c +
d*x]))/(a^2*(a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) + (b^2*(5*a*b - (3*a^2 +
2*b^2)*Cos[c + d*x]))/(a^2*(a^2 - b^2)^2*d*e*Sqrt[e*Sin[c + d*x]]) - (5*b^4
*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(2*a*(a^2 - b^2)^2*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]])
 - (2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d
*x]]) - (5*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(2*a*(a^2 - b^2)^2*(a + Sqrt[a^2 - b^2])*d*e*Sqrt[e*
Sin[c + d*x]]) - (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*e*Sq
rt[e*Sin[c + d*x]]) - (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(a^2*d*e^2*Sqrt[Sin[c + d*x]]) - (4*b^2*EllipticE[(c - Pi/2 + d*x)/2,
2]*Sqrt[e*Sin[c + d*x]])/(a^2*(a^2 - b^2)*d*e^2*Sqrt[Sin[c + d*x]]) - (b^2*
(3*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^2
*(a^2 - b^2)^2*d*e^2*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

IntegerQ[2*n]

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx \\
&= \int \left(\frac{1}{a^2(e\sin(c+dx))^{3/2}}+\frac{b^2}{a^2(-b-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\right. \\
&\qquad \left.+\frac{2b}{a^2(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\right)dx \\
&= \frac{\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{a^2}+\frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{a^2}+\frac{b^2\int \frac{1}{(-b-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx}{a^2} \\
&= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}}+\frac{b^2}{a\,(a^2-b^2)\,de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}} \\
&\quad +\frac{4b(a-b\cos(c+dx))}{a^2\,(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}+\frac{b^2\int \frac{b-\frac{3}{2}a\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{a^2\,(a^2-b^2)} \\
&\quad -\frac{\int \sqrt{e\sin(c+dx)}\,dx}{a^2e^2}+\frac{(4b)\int \frac{\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{1}{2}ab\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{a^2\,(a^2-b^2)\,e^2} \\
&= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}}+\frac{b^2}{a\,(a^2-b^2)\,de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}} \\
&\quad +\frac{4b(a-b\cos(c+dx))}{a^2\,(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}+\frac{b^2(5ab-(3a^2+2b^2)\cos(c+dx))}{a^2\,(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}} \\
&\quad +\frac{(2b^2)\int \frac{\left(\frac{1}{2}b(4a^2+b^2)+\frac{1}{4}a(3a^2+2b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{a^2\,(a^2-b^2)^2\,e^2}+\frac{(2b)\int \frac{\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{(a^2-b^2)\,e^2} \\
&\quad -\frac{(2b^2)\int \sqrt{e\sin(c+dx)}\,dx}{a^2\,(a^2-b^2)\,e^2}-\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}\,dx}{a^2e^2\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}$$
$$+ \frac{4b(a-b\cos(c+dx))}{a^2\left(a^2-b^2\right)de\sqrt{e\sin(c+dx)}} + \frac{b^2(5ab-(3a^2+2b^2)\cos(c+dx))}{a^2\left(a^2-b^2\right)^2de\sqrt{e\sin(c+dx)}}$$
$$- \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2de^2\sqrt{\sin(c+dx)}}$$
$$+ \frac{(5b^3)\int\frac{\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{2\left(a^2-b^2\right)^2e^2} - \frac{(b^2(3a^2+2b^2))\int\sqrt{e\sin(c+dx)}\,dx}{2a^2\left(a^2-b^2\right)^2e^2}$$
$$+ \frac{b^2\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a\left(a^2-b^2\right)e} - \frac{b^2\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a\left(a^2-b^2\right)e}$$
$$+ \frac{(2ab)\mathrm{Subst}\left(\int\frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}\,dx, x, e\sin(c+dx)\right)}{\left(a^2-b^2\right)de}$$
$$- \frac{\left(2b^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{a^2\left(a^2-b^2\right)e^2\sqrt{\sin(c+dx)}}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a(a^2-b^2)de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}$$
$$+ \frac{4b(a-b\cos(c+dx))}{a^2(a^2-b^2)de\sqrt{e\sin(c+dx)}} + \frac{b^2(5ab-(3a^2+2b^2)\cos(c+dx))}{a^2(a^2-b^2)^2de\sqrt{e\sin(c+dx)}}$$
$$- \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2de^2\sqrt{\sin(c+dx)}}$$
$$- \frac{4b^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2(a^2-b^2)de^2\sqrt{\sin(c+dx)}}$$
$$+ \frac{(5b^4)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a(a^2-b^2)^2e} - \frac{(5b^4)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a(a^2-b^2)^2e}$$
$$+ \frac{(5ab^3)\operatorname{Subst}\left(\int\frac{\sqrt{x}}{(-a^2+b^2)e^2+a^2x^2}\,dx, x, e\sin(c+dx)\right)}{2(a^2-b^2)^2de}$$
$$+ \frac{(4ab)\operatorname{Subst}\left(\int\frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)de}$$
$$+ \frac{\left(b^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{a(a^2-b^2)e\sqrt{e\sin(c+dx)}}$$
$$- \frac{\left(b^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{a(a^2-b^2)e\sqrt{e\sin(c+dx)}}$$
$$- \frac{\left(b^2(3a^2+2b^2)\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{2a^2(a^2-b^2)^2e^2\sqrt{\sin(c+dx)}}$$

$$
\begin{aligned}
&= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{4b(a-b\cos(c+dx))}{a^2\left(a^2-b^2\right)de\sqrt{e\sin(c+dx)}} + \frac{b^2(5ab-(3a^2+2b^2)\cos(c+dx))}{a^2\left(a^2-b^2\right)^2de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2de^2\sqrt{\sin(c+dx)}} \\
&\quad - \frac{4b^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\left(a^2-b^2\right)de^2\sqrt{\sin(c+dx)}} \\
&\quad - \frac{b^2(3a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\left(a^2-b^2\right)^2de^2\sqrt{\sin(c+dx)}} \\
&\quad + \frac{(5ab^3)\,\text{Subst}\left(\int\frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{\left(a^2-b^2\right)^2de} \\
&\quad - \frac{(2b)\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{\left(a^2-b^2\right)de} \\
&\quad + \frac{(2b)\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{\left(a^2-b^2\right)de} \\
&\quad + \frac{\left(5b^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4a\left(a^2-b^2\right)^2e\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{\left(5b^4\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4a\left(a^2-b^2\right)^2e\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b \arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4} de^{3/2}} - \frac{2b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4} de^{3/2}} \\
&\quad - \frac{2\cos(c+dx)}{a^2 de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right) de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{4b(a-b\cos(c+dx))}{a^2\left(a^2-b^2\right) de\sqrt{e\sin(c+dx)}} + \frac{b^2(5ab-(3a^2+2b^2)\cos(c+dx))}{a^2\left(a^2-b^2\right)^2 de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{5b^4 \text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2a\left(a^2-b^2\right)^2\left(a-\sqrt{a^2-b^2}\right) de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b^2 \text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right) de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{5b^4 \text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2a\left(a^2-b^2\right)^2\left(a+\sqrt{a^2-b^2}\right) de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b^2 \text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right) de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2de^2\sqrt{\sin(c+dx)}} \\
&\quad - \frac{4b^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\left(a^2-b^2\right) de^2\sqrt{\sin(c+dx)}} \\
&\quad - \frac{b^2(3a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\left(a^2-b^2\right)^2 de^2\sqrt{\sin(c+dx)}} \\
&\quad - \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2\left(a^2-b^2\right)^2 de} \\
&\quad + \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{2\left(a^2-b^2\right)^2 de}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{5b^3 \arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\sqrt{a}\left(a^2-b^2\right)^{9/4} de^{3/2}} + \frac{2b \arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4} de^{3/2}} \\
&\quad - \frac{5b^3 \operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\sqrt{a}\left(a^2-b^2\right)^{9/4} de^{3/2}} - \frac{2b \operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4} de^{3/2}} \\
&\quad - \frac{2\cos(c+dx)}{a^2 de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right) de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{4b(a-b\cos(c+dx))}{a^2\left(a^2-b^2\right) de\sqrt{e\sin(c+dx)}} + \frac{b^2(5ab-(3a^2+2b^2)\cos(c+dx))}{a^2\left(a^2-b^2\right)^2 de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{5b^4\, \text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2a\left(a^2-b^2\right)^2\left(a-\sqrt{a^2-b^2}\right) de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b^2\, \text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right) de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{5b^4\, \text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{2a\left(a^2-b^2\right)^2\left(a+\sqrt{a^2-b^2}\right) de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2b^2\, \text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right) de\sqrt{e\sin(c+dx)}} \\
&\quad - \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2 de^2\sqrt{\sin(c+dx)}} \\
&\quad - \frac{4b^2 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\left(a^2-b^2\right) de^2\sqrt{\sin(c+dx)}} \\
&\quad - \frac{b^2(3a^2+2b^2)\, E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2\left(a^2-b^2\right)^2 de^2\sqrt{\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.23 (sec) , antiderivative size = 772, normalized size of antiderivative = 0.73

$$
\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\, dx = \frac{(b+a\cos(c+dx))\left(-\frac{(a^2-b^2)\left(b+a\sqrt{\cos^2(c+dx)}\right)\sec^3(c+dx)\sin^{\frac{3}{2}}(c+dx}{\quad}\right.}{\quad}
$$

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(-(((a^2 - b^2)*(b + a*Sqrt[Cos[c + d*x]^2])*Sec[c +
d*x]^3*Sin[c + d*x]^(3/2)*((2*a^3 + 3*a*b^2)*Cos[c + d*x]*(3*Sqrt[2]*b*(-a^
2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b
^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)
^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^
2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3
/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c +
 d*x]^(3/2)) + (1 + I)*a*(6*a^2*b + 4*b^3)*Sqrt[Cos[c + d*x]^2]*(3*(a^2 - b
^2)^(3/4)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1
/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)]
 - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x
]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^
(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]) - (4 - 4*I)*Sqrt[a]*b*AppellF
1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c
 + d*x]^(3/2))))/(a^(3/2)*(a - b)^2*(a + b)^2)) + 24*(-2*(b + a*Cos[c + d*x
])*(-2*a*b + (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x] + a*b^2*Sin[c + d*x])*T
an[c + d*x]^2))/(24*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])
^(3/2))
```

Maple [A] (warning: unable to verify)

Time = 22.48 (sec) , antiderivative size = 1417, normalized size of antiderivative = 1.34

method	result	size
default	Expression too large to display	1417

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e*a*b*(-2/e^2/(a^2-b^2)^2/(e*sin(d*x+c))^(1/2)-2/e^2/(a-b)^2/(a+b)^2*(1
/4*(e*sin(d*x+c))^(3/2)*b^2/(-a^2*e^2*cos(d*x+c)^2+b^2*e^2)+1/4*(a^2+1/4*b^
2)/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*(2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b
^2)/a^2)^(1/4))-ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin
(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4))))))+(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/e*((a^2+b^2)/(a^2-b^2)^2*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)+2*b
^4/(a-b)/(a+b)*(-1/2*a^2/e/b^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)+1/2/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*E
llipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
```

```
in(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(
1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1
-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a
),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)
*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/
(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d
*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)))-b^2*(3*a^2-b^2)/(a-b)^
2/(a+b)^2*(-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((
-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/a^2*(-sin(d*x
+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*
x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^
2-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^2}{(e\,\sin(c+d\,x))^{3/2}\,(b+a\,\cos(c+d\,x))^2}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/((e*sin(c + d*x))^(3/2)*(b + a*cos(c + d*x))^2), x)
```

3.248 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 1089

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = -\frac{7\sqrt{a}b^3\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\left(a^2-b^2\right)^{11/4}de^{5/2}} - \frac{2\sqrt{a}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} - \frac{7\sqrt{a}b^3\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\left(a^2-b^2\right)^{11/4}de^{5/2}} - \frac{2\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} - \frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{4b(a-b\cos(c+dx))}{3a^2\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{b^2(7ab-(5a^2+2b^2)\cos(c+dx))}{3a^2\left(a^2-b^2\right)^2de(e\sin(c+dx))^{3/2}} + \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2de^2\sqrt{e\sin(c+dx)}} + \frac{4b^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2\left(a^2-b^2\right)de^2\sqrt{e\sin(c+dx)}} + \frac{b^2(5a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2\left(a^2-b^2\right)^2de^2\sqrt{e\sin(c+dx)}} + \frac{7b^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(a^2-b^2-a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}} + \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b^2\right)\left(a^2-b^2-a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}} + \frac{7b^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(a^2-b^2+a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}} + \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b^2\right)\left(a^2-b^2+a\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$

```
[Out] -2/3*cos(d*x+c)/a^2/d/e/(e*sin(d*x+c))^(3/2)+b^2/a/(a^2-b^2)/d/e/(b+a*cos(d
*x+c))/(e*sin(d*x+c))^(3/2)+4/3*b*(a-b*cos(d*x+c))/a^2/(a^2-b^2)/d/e/(e*sin
(d*x+c))^(3/2)+1/3*b^2*(7*a*b-(5*a^2+2*b^2)*cos(d*x+c))/a^2/(a^2-b^2)^2/d/e
/(e*sin(d*x+c))^(3/2)-7/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)
```

```
^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(11/4)/d/e^(5/2)-2*b*arctan(a^(1/2)*(e*si
n(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(7/4)/d/e^(5/2)-
7/2*b^3*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/
2)/(a^2-b^2)^(11/4)/d/e^(5/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2
-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(7/4)/d/e^(5/2)-2/3*(sin(1/2*c+1/4*P
i+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/
2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/e^2/(e*sin(d*x+c))^(1/2)-4/3*b^2*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/
2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/e^2/(e*sin(d*
x+c))^(1/2)-1/3*b^2*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c
)^(1/2)/a^2/(a^2-b^2)^2/d/e^2/(e*sin(d*x+c))^(1/2)-7/2*b^4*(sin(1/2*c+1/4*P
i+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1
/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2
/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*
d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2
-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-7/2*b^4*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(a^2-
b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2
*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b^2+a
*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 3.46 (sec) , antiderivative size = 1089, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3957, 2991, 2716, 2721, 2720, 2773, 2945, 2946, 2781, 2886, 2884, 335, 218, 214,

211, 2775}

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \frac{7\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2\left(a^2-b^2\right)^2\left(a^2-\sqrt{a^2-b^2}a-b^2\right)de^2\sqrt{e\sin(c+dx)}}$$

$$+\frac{7\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^4}{2\left(a^2-b^2\right)^2\left(a^2+\sqrt{a^2-b^2}a-b^2\right)de^2\sqrt{e\sin(c+dx)}}-\frac{7\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2\left(a^2-b^2\right)^{11/4}de^{5/2}}$$

$$-\frac{7\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2\left(a^2-b^2\right)^{11/4}de^{5/2}}+\frac{\left(5a^2+2b^2\right)\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{3a^2\left(a^2-b^2\right)^2de^2\sqrt{e\sin(c+dx)}}$$

$$+\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{3a^2\left(a^2-b^2\right)de^2\sqrt{e\sin(c+dx)}}$$

$$+\frac{2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{\left(a^2-b^2\right)\left(a^2-\sqrt{a^2-b^2}a-b^2\right)de^2\sqrt{e\sin(c+dx)}}$$

$$+\frac{2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}b^2}{\left(a^2-b^2\right)\left(a^2+\sqrt{a^2-b^2}a-b^2\right)de^2\sqrt{e\sin(c+dx)}}$$

$$+\frac{\left(7ab-\left(5a^2+2b^2\right)\cos(c+dx)\right)b^2}{3a^2\left(a^2-b^2\right)^2de(e\sin(c+dx))^{3/2}}$$

$$+\frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{\left(a^2-b^2\right)^{7/4}de^{5/2}}$$

$$-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{\left(a^2-b^2\right)^{7/4}de^{5/2}}+\frac{4(a-b\cos(c+dx))b}{3a^2\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}}$$

$$+\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{\sin(c+dx)}}{3a^2de^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}}$$

```
[In] Int[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-7*Sqrt[a]*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sq
rt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTan[(Sqrt[a]*Sq
rt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4)*d*e^(5
/2)) - (7*Sqrt[a]*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(
1/4)*Sqrt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTanh[(Sq
rt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4
)*d*e^(5/2)) - (2*Cos[c + d*x])/(3*a^2*d*e*(e*Sin[c + d*x])^(3/2)) + b^2/(a
*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)) + (4*b*(a - b
*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (b^2*(7*a*
b - (5*a^2 + 2*b^2)*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)^2*d*e*(e*Sin[c + d*x]
```

```
)^(3/2)) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*d
*e^2*Sqrt[e*Sin[c + d*x]]) + (4*b^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(3*a^2*(a^2 - b^2)*d*e^2*Sqrt[e*Sin[c + d*x]]) + (b^2*(5*a^2
+ 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*(a^2 -
 b^2)^2*d*e^2*Sqrt[e*Sin[c + d*x]]) + (7*b^4*EllipticPi[(2*a)/(a - Sqrt[a^2
 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^2*(a^2
- b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (2*b^2*EllipticPi[
(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a
^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (7*
b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(2*(a^2 - b^2)^2*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*S
in[c + d*x]]) + (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b^2 + a*Sqrt[a^2 - b^2])
*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$$

$$= \int \left(\frac{1}{a^2(e\sin(c+dx))^{5/2}} + \frac{b^2}{a^2(-b-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}} + \frac{2b}{a^2(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}} \right) dx$$

$$= \frac{\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{a^2} + \frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx}{a^2} + \frac{b^2\int \frac{1}{(-b-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx}{a^2}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\,(a^2-b^2)\,de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{4b(a-b\cos(c+dx))}{3a^2\,(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{b^2\int \frac{b-\frac{5}{2}a\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx}{a^2\,(a^2-b^2)} + \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^2e^2} + \frac{(4b)\int \frac{\frac{3a^2}{2}-\frac{b^2}{2}-\frac{1}{2}ab\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3a^2\,(a^2-b^2)\,e^2}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\,(a^2-b^2)\,de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{4b(a-b\cos(c+dx))}{3a^2\,(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{b^2(7ab-(5a^2+2b^2)\cos(c+dx))}{3a^2\,(a^2-b^2)^2\,de(e\sin(c+dx))^{3/2}} + \frac{(2b^2)\int \frac{\frac{1}{2}b(8a^2-b^2)-\frac{1}{4}a(5a^2+2b^2)\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3a^2\,(a^2-b^2)^2\,e^2} + \frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{(a^2-b^2)\,e^2} + \frac{(2b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^2\,(a^2-b^2)\,e^2} + \frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a^2e^2\sqrt{e\sin(c+dx)}}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\,(a^2-b^2)\,de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{4b(a-b\cos(c+dx))}{3a^2\,(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{b^2(7ab-(5a^2+2b^2)\cos(c+dx))}{3a^2\,(a^2-b^2)^2\,de(e\sin(c+dx))^{3/2}} + \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2de^2\sqrt{e\sin(c+dx)}} + \frac{(7b^3)\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2\,(a^2-b^2)^2\,e^2} + \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{(a^2-b^2)^{3/2}\,e^2} + \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{(a^2-b^2)^{3/2}\,e^2} + \frac{(b^2(5a^2+2b^2))\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{6a^2\,(a^2-b^2)^2\,e^2} + \frac{(2ab)\text{Subst}\left(\int \frac{1}{\sqrt{x}((-a^2+b^2)e^2+a^2x^2)}\,dx,x,e\sin(c+dx)\right)}{(a^2-b^2)\,de} + \frac{\left(2b^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a^2\,(a^2-b^2)\,e^2\sqrt{e\sin(c+dx)}}$$

$$
\begin{aligned}
&= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{4b(a-b\cos(c+dx))}{3a^2\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{b^2\left(7ab-\left(5a^2+2b^2\right)\cos(c+dx)\right)}{3a^2\left(a^2-b^2\right)^2 de(e\sin(c+dx))^{3/2}} \\
&\quad + \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2de^2\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{4b^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2\left(a^2-b^2\right)de^2\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(7b^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4\left(a^2-b^2\right)^{5/2}e^2} + \frac{\left(7b^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4\left(a^2-b^2\right)^{5/2}e^2} \\
&\quad + \frac{\left(7ab^3\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(\left(-a^2+b^2\right)e^2+a^2x^2\right)}\,dx, x, e\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2 de} \\
&\quad + \frac{(4ab)\text{Subst}\left(\int\frac{1}{\left(-a^2+b^2\right)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{\left(a^2-b^2\right)de} \\
&\quad + \frac{\left(b^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{\left(a^2-b^2\right)^{3/2}e^2\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(b^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{\left(a^2-b^2\right)^{3/2}e^2\sqrt{e\sin(c+dx)}} \\
&\quad + \frac{\left(b^2\left(5a^2+2b^2\right)\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{6a^2\left(a^2-b^2\right)^2 e^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\,(a^2-b^2)\,de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}$$
$$+ \frac{4b(a-b\cos(c+dx))}{3a^2\,(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{b^2(7ab-(5a^2+2b^2)\cos(c+dx))}{3a^2\,(a^2-b^2)^2\,de(e\sin(c+dx))^{3/2}}$$
$$+ \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2de^2\sqrt{e\sin(c+dx)}}$$
$$+ \frac{4b^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2\,(a^2-b^2)\,de^2\sqrt{e\sin(c+dx)}}$$
$$+ \frac{b^2(5a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2\,(a^2-b^2)^2\,de^2\sqrt{e\sin(c+dx)}}$$
$$- \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^{3/2}\left(a-\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$+ \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^{3/2}\left(a+\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$- \frac{(2ab)\text{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^{3/2}\,de^2}$$
$$- \frac{(2ab)\text{Subst}\left(\int \frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^{3/2}\,de^2}$$
$$+ \frac{(7ab^3)\,\text{Subst}\left(\int \frac{1}{(-a^2+b^2)e^2+a^2x^4}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{(a^2-b^2)^2\,de}$$
$$+ \frac{\left(7b^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{4\,(a^2-b^2)^{5/2}\,e^2\sqrt{e\sin(c+dx)}}$$
$$+ \frac{\left(7b^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{4\,(a^2-b^2)^{5/2}\,e^2\sqrt{e\sin(c+dx)}}$$

$$= -\frac{2\sqrt{a}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{7/4}de^{5/2}} - \frac{2\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{7/4}de^{5/2}}$$
$$- \frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a(a^2-b^2)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}$$
$$+ \frac{4b(a-b\cos(c+dx))}{3a^2(a^2-b^2)de(e\sin(c+dx))^{3/2}} + \frac{b^2(7ab-(5a^2+2b^2)\cos(c+dx))}{3a^2(a^2-b^2)^2de(e\sin(c+dx))^{3/2}}$$
$$+ \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2de^2\sqrt{e\sin(c+dx)}}$$
$$+ \frac{4b^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2(a^2-b^2)de^2\sqrt{e\sin(c+dx)}}$$
$$+ \frac{b^2(5a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2(a^2-b^2)^2de^2\sqrt{e\sin(c+dx)}}$$
$$- \frac{7b^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2(a^2-b^2)^{5/2}\left(a-\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$- \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^{3/2}\left(a-\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$+ \frac{7b^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2(a^2-b^2)^{5/2}\left(a+\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$+ \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^{3/2}\left(a+\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}$$
$$- \frac{(7ab^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e-ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2(a^2-b^2)^{5/2}de^2}$$
$$- \frac{(7ab^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{a^2-b^2}e+ax^2}\,dx,x,\sqrt{e\sin(c+dx)}\right)}{2(a^2-b^2)^{5/2}de^2}$$

$$
\begin{aligned}
&= -\frac{7\sqrt{a}b^3\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\left(a^2-b^2\right)^{11/4}de^{5/2}} - \frac{2\sqrt{a}b\arctan\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} \\
&- \frac{7\sqrt{a}b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\left(a^2-b^2\right)^{11/4}de^{5/2}} - \frac{2\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} \\
&- \frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}} \\
&+ \frac{4b(a-b\cos(c+dx))}{3a^2\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{b^2(7ab-(5a^2+2b^2)\cos(c+dx))}{3a^2\left(a^2-b^2\right)^2de(e\sin(c+dx))^{3/2}} \\
&+ \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2de^2\sqrt{e\sin(c+dx)}} \\
&+ \frac{4b^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2\left(a^2-b^2\right)de^2\sqrt{e\sin(c+dx)}} \\
&+ \frac{b^2(5a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3a^2\left(a^2-b^2\right)^2de^2\sqrt{e\sin(c+dx)}} \\
&- \frac{7b^4\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^{5/2}\left(a-\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}} \\
&- \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a-\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b^2\right)^{3/2}\left(a-\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}} \\
&+ \frac{7b^4\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^{5/2}\left(a+\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}} \\
&+ \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+\sqrt{a^2-b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b^2\right)^{3/2}\left(a+\sqrt{a^2-b^2}\right)de^2\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 12.52 (sec) , antiderivative size = 1320, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx =$$

$$-\frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\left(\frac{2\left(-2a^3-5ab^2\right)\cos^2(c+dx)\left(b+a\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{b\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin}}{\sqrt[4]{-a^2}}\right.\right.}{}\right.}{}\right.}{}$$

$$+\frac{(b+a\cos(c+dx))^2\left(\frac{ab^2}{(-a^2+b^2)^2(b+a\cos(c+dx))}-\frac{2\left(-2ab+a^2\cos(c+dx)+b^2\cos(c+dx)\right)\csc^2(c+dx)}{3(-a^2+b^2)^2}\right)\sin(c+dx)\tan^2(c+a}{d(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}$$

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -1/6*((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sin[c + d*x]^(5/2)*((2*(-2*a^3
- 5*a*b^2)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1
 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 +
(Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*
x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c
+ d*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(
a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2
)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]
^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2,
(a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c +
 d*x]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(10*a^2*b + 4
*b^3)*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*
(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*A
rcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqr
t[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*S
in[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt
[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]
*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4,
 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*
AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)
```

```
] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x
]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b
 + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/((a - b)^2*(a + b)^2*d*(a +
b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)) + ((b + a*Cos[c + d*x])^2*((a*b^2
)/((-a^2 + b^2)^2*(b + a*Cos[c + d*x])) - (2*(-2*a*b + a^2*Cos[c + d*x] + b
^2*Cos[c + d*x])*Csc[c + d*x]^2)/(3*(-a^2 + b^2)^2))*Sin[c + d*x]*Tan[c + d
*x]^2)/(d*(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2))
```

Maple [A] (warning: unable to verify)

Time = 31.29 (sec) , antiderivative size = 1331, normalized size of antiderivative = 1.22

method	result	size
default	Expression too large to display	1331

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4*e*a*b*(1/3/e^2/(a^2-b^2)^2/(e*sin(d*x+c))^(3/2)+1/e^2/(a-b)^2/(a+b)^2*(1
/4*(e*sin(d*x+c))^(1/2)*b^2/(-a^2*e^2*cos(d*x+c)^2+b^2*e^2)+1/16*(4*a^2+3*b
^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*(ln(((e*sin(d*x+c))^(1/2)+
(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4))
)+2*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4)))))+(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/e^2*(1/3*(a^2+b^2)/(a^2-b^2)^2/(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)/(cos(d*x+c)^2-1)*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2
*sin(d*x+c))+2*b^4/(a-b)/(a+b)*(-1/2*a^2/e/b^2/(a^2-b^2)*(cos(d*x+c)^2*e*si
n(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)^(3/2)*a*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2)
,1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)
^(1/2)/a),1/2*2^(1/2))+1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a
^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1
/2*2^(1/2))-5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a
)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)))-b^
2*(3*a^2-b^2)/(a-b)^2/(a+b)^2*(-1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/
(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)
/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1
```

```
/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)
)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(e\,\sin\left(c+d\,x\right)\right)^{5/2}\,\left(b+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/((e*sin(c + d*x))^(5/2)*(b + a*cos(c + d*x))^2), x)
```

3.249 $\int \sqrt{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \sqrt{a+b\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(1+\sec(e+fx))}{a+b\sec(e+fx)}}(a+b\sec(e+fx))}{\sqrt{a+b}f}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b)^(1/2)/(a+b*sec(f*x+e))^(1/2),a/(a+b),((a-b)/
(a+b))^(1/2))*(a+b*sec(f*x+e))*(-b*(1-sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2)*(
b*(1+sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2)/f/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3865}

$$\int \sqrt{a+b\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(\sec(e+fx)+1)}{a+b\sec(e+fx)}}(a+b\sec(e+fx))\operatorname{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{a-b}{a+b}\right)}{f\sqrt{a+b}}$$

```
[In] Int[Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*Cot[e + f*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[e
+ f*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[e + f*x]))/(a + b*Sec[e + f*x
]))]*Sqrt[(b*(1 + Sec[e + f*x]))/(a + b*Sec[e + f*x])]*(a + b*Sec[e + f*x])
)/(Sqrt[a + b]*f)
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

integral =
$$-\frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(1+\sec(e+fx))}{a+b\sec(e+fx)}}(a+b\sec(e+}{\sqrt{a+b}f}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.21

$$\int\sqrt{a+b\sec(e+fx)}\,dx$$
$$=\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((-a+b)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)+2a\right.}{f(b+a\cos(e+fx))}$$

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]],
 (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x]])/(f*(b + a*Cos[e + f*x]))
```

Maple [A] (verified)

Time = 6.61 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.46

method	result
default	$\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)a-\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)b-2a\,\text{EllipticPi}\left(\cot(fx+e)-\right.\right.}{f(b+a\cos(fx+e))}$

```
[In] int((a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a-
EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi(cot(f
*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*
```

```
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+
a*cos(f*x+e))
```

Fricas [F]

$$\int \sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)+a}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{a+b\sec\left(e+fx\right)}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)+a}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)+a}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(e+fx)}\,dx=\int \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(1/2),x)
[Out] int((a + b/cos(e + f*x))^(1/2), x)
```

3.250 $\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = \frac{\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f} - \frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

```
[Out] cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2
)/f-cot(f*x+e)*(a+b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3960, 3917}

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = \frac{\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[e + f*x]))/(a - b))])/f - (Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/f
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}+\frac{1}{2}b\int\frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx\\ &= \frac{\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}\\ &\quad-\frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int\csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = \frac{-\left((b+a\cos(e+fx))\csc(e+fx)\sqrt{\frac{1}{1+\sec(e+fx)}}\right)+b\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}{f\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-((b + a*Cos[e + f*x])*Csc[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)]) + b*Ell
ipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])
/((a + b)*(1 + Sec[e + f*x]))])/(f*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[a + b
*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 6.64 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.78

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)}\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,b\cos(fx+e)+\text{EllipticF}\left(\cot(fx\right.\right.}{f(b+a\cos(fx+e))}$

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))*(EllipticF(cot(f*x+e)-csc(f*x+
e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos
(f*x+e))/(cos(f*x+e)+1))^(1/2)*b*cos(f*x+e)+EllipticF(cot(f*x+e)-csc(f*x+e)
,((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f
*x+e))/(cos(f*x+e)+1))^(1/2)*b+a*cos(f*x+e)*cot(f*x+e)+b*cot(f*x+e))
```

Fricas [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)+a}\csc\left(fx+e\right)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

Sympy [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx=\int \sqrt{a+b\sec\left(e+fx\right)}\csc^2\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*csc(e + f*x)**2, x)
```

Maxima [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)+a}\csc\left(fx+e\right)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)+a}\csc\left(fx+e\right)^2\,dx$$

[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\sin\left(e+f\,x\right)^2}\,dx$$

[In] int((a + b/cos(e + f*x))^(1/2)/sin(e + f*x)^2,x)

[Out] int((a + b/cos(e + f*x))^(1/2)/sin(e + f*x)^2, x)

3.251 $\int (a+b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 309

$$\int (a+b\sec(e+fx))^{3/2}\,dx = -\frac{2(a-b)\sqrt{a+b}\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f} + \frac{2(2a-b)\sqrt{a+b}\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f} - \frac{2a\sqrt{a+b}\cot(e+fx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

```
[Out] -2*(a-b)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a
-b))^(1/2)/f+2*(2*a-b)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+
sec(f*x+e))/(a-b))^(1/2)/f-2*a*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)
/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+
b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3866, 4006, 3869, 3917, 4089}

$$\int (a+b\sec(e+fx))^{3/2}\,dx = \frac{2(2a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)\right.}{f}$$
$$-\frac{2(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}$$
$$-\frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

```
[In] Int[(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (2*(2*a - b)*Sqrt[a + b]*Cot[e +
f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
b))])/f - (2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a
 + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f
```

Rule 3866

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(3/2), x_Symbol] :> Int[(a^2 + b
*(2*a - b)*Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]], x] + Dist[b^2, Int[Csc[c
 + d*x]*((1 + Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]]), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
```

```
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\text{integral} = b^2\int \frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx + \int \frac{a^2+(2a-b)b\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f} + a^2\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx + ((2a-b)b)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f} + \frac{2(2a-b)\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f} - \frac{2a\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 684 vs. $2(309) = 618$.

Time = 6.73 (sec) , antiderivative size = 684, normalized size of antiderivative = 2.21

$$\int (a+b\sec(e+fx))^{3/2}\,dx = \frac{2b\cos(e+fx)(a+b\sec(e+fx))^{3/2}\sin(e+fx)}{f(b+a\cos(e+fx))}$$
$$+\frac{2(a+b\sec(e+fx))^{3/2}\left(ab\tan\left(\frac{1}{2}(e+fx)\right)+b^2\tan\left(\frac{1}{2}(e+fx)\right)-2ab\tan^3\left(\frac{1}{2}(e+fx)\right)+ab\tan^5\left(\frac{1}{2}(e+f\right.\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b*Cos[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(f*(b + a*Cos[e
+ f*x])) + (2*(a + b*Sec[e + f*x])^(3/2)*(a*b*Tan[(e + f*x)/2] + b^2*Tan[(e
 + f*x)/2] - 2*a*b*Tan[(e + f*x)/2]^3 + a*b*Tan[(e + f*x)/2]^5 - b^2*Tan[(e
 + f*x)/2]^5 - 2*a^2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a +
b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan
[(e + f*x)/2]^2)/(a + b)] - 2*a^2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]],
(a - b)/(a + b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a +
b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + b*(a + b)*Ellip
ticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2
]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e +
f*x)/2]^2)/(a + b)] + (a^2 - 2*a*b - b^2)*EllipticF[ArcSin[Tan[(e + f*x)/2]
], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*S
qrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]))/(f*(b
+ a*Cos[e + f*x])^(3/2)*Sec[e + f*x]^(3/2)*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-
1)]*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)^(3/2)*Sqrt[(a + b -
a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. $2(282) = 564$.

Time = 8.89 (sec) , antiderivative size = 1599, normalized size of antiderivative = 5.17

method	result	size
default	Expression too large to display	1599

```
[In] int((a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos
(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2*cos(f*
x+e)^2-2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a
```

```
*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b*co
s(f*x+e)^2-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b
+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2*
cos(f*x+e)^2+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*
b*cos(f*x+e)^2+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*
b^2*cos(f*x+e)^2-2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))
^(1/2))*a^2*cos(f*x+e)^2+2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1
/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*a^2*cos(f*x+e)-4*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^
(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*a*b*cos(f*x+e)-2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b)
)^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*b^2*cos(f*x+e)+2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a
-b)/(a+b))^(1/2))*a*b*cos(f*x+e)+2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),(
(a-b)/(a+b))^(1/2))*b^2*cos(f*x+e)-4*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e
),-1,((a-b)/(a+b))^(1/2))*a^2*cos(f*x+e)+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x
+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*a^2-2*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(
1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*a*b-(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*Ellipt
icF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*b^2+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x
+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b+(
1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x
+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2-2*(1/(a+b)*(
b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,(
(a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2+sin(f*x+e)*cos(f*
x+e)*a*b+sin(f*x+e)*b^2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))/(cos(f*x+e
)+1)
```

Fricas [F]

$$\int (a+b\sec(e+fx))^{3/2}\,dx=\int (b\sec(fx+e)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^(3/2), x)
```

Sympy [F]

$$\int (a+b\sec(e+fx))^{3/2}\,dx=\int (a+b\sec{(e+fx)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**(3/2),x)
[Out] Integral((a + b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\sec(e+fx))^{3/2}\,dx=\int (b\sec{(fx+e)}+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\sec(e+fx))^{3/2}\,dx=\int (b\sec{(fx+e)}+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))^{3/2}\,dx=\int \left(a+\frac{b}{\cos{(e+f\,x)}}\right)^{3/2}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(3/2),x)
[Out] int((a + b/cos(e + f*x))^(3/2), x)
```

3.252 $\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 228

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = -\frac{3(a-b)\sqrt{a+b}\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$
$$+\frac{3(a-b)\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$
$$-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}$$

```
[Out] -cot(f*x+e)*(a+b*sec(f*x+e))^(3/2)/f-3*(a-b)*cot(f*x+e)*EllipticE((a+b*sec(
f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e)
)/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f+3*(a-b)*cot(f*x+e)*Ellipti
cF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(
1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used

$= \{3960, 3914, 3917, 4089\}$

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \frac{3(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\right.}{f}$$
$$-\frac{3(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}$$
$$-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}$$

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (3*(a - b)*Sqrt[a + b]*Cot[e + f*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/f - (Cot[e + f*x]*(a + b*Sec[e + f*x])^(3/2))/f
```

Rule 3914

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
```

```
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f} + \frac{1}{2}(3b)\int \sec(e+fx)\sqrt{a+b\sec(e+fx)}\,dx \\
&= -\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f} + \frac{1}{2}(3(a-b)b)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx \\
&\quad + \frac{1}{2}(3b^2)\int \frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx \\
&= \\
&\quad -\frac{3(a-b)\sqrt{a+b}\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f} \\
&\quad +\frac{3(a-b)\sqrt{a+b}\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e-}{a-b}}}{f} \\
&\quad -\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.45 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.21

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \frac{\cos(e+fx)(a+b\sec(e+fx))^{3/2}((-b-a\cos(e+fx))\csc(e+fx)+3b\sin(e+fx))}{f(b+a\cos(e+fx))} + \frac{3b(a+b\sec(e+fx))^{3/2}\left(-\frac{(a+b)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-\operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}} -\right.}{f(b+a\cos(e+fx))^2\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sec^{\frac{3}{2}}(e+fx)\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec}}$$

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((-b - a*Cos[e + f*x])*Csc[e + f*x
] + 3*b*Sin[e + f*x]))/(f*(b + a*Cos[e + f*x])) + (3*b*(a + b*Sec[e + f*x])
^(3/2)*(-(((a + b)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*
(EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - EllipticF[ArcSin[Ta
n[(e + f*x)/2]], (a - b)/(a + b)]))/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])
- (b + a*Cos[e + f*x])*Tan[(e + f*x)/2]))/(f*(b + a*Cos[e + f*x])^2*Sqrt[Se
c[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(208) = 416.

Time = 9.18 (sec) , antiderivative size = 763, normalized size of antiderivative = 3.35

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)}\left(3\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,ab\cos(fx+e)+3\,\mathrm{EllipticF}\left(\cot(\right.\right.}{}$

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))*(3*EllipticF(cot(f*x+e)-csc(f*
x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b*cos(f*x+e)+3*EllipticF(cot(f*x+e)-csc(
f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2*cos(f*x+e)-3*(1/(a+b)*(b+a*cos(f*x+e
))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*
x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b*cos(f*x+e)-3*(1/(a+b)*(b+a*cos(f*x
+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(
f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^2*cos(f*x+e)+3*(1/(a+b)*(b+a*cos(f
*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^
(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b+3*(1/(a+b)*(b+a*cos(f*x+e))/(c
os(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2-3*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)
+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*a*b-3*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2
)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*b^2+a^2*cos(f*x+e)*cot(f*x+e)+3*a*b*cos(f*x+e)*cot(f*x+e)-a*b
*cot(f*x+e)+3*b^2*cot(f*x+e)-2*b^2*csc(f*x+e))
```

Fricas [F]

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx=\int (b\sec\left(fx+e\right)+a)^{\frac{3}{2}}\csc\left(fx+e\right)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e)^2*sec(f*x + e) + a*csc(f*x + e)^2)*sqrt(b*sec(f*x
+ e) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(3/2)/sin(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x))^(3/2)/sin(e + f*x)^2, x)
```

3.253 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{af}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))
/(a-b))^(1/2)/a/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3869}

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af}$$

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*f)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

integral =

$$-\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{af}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)-2\,\text{Ellipt}\right.}{f\sqrt{a+b\sec(e+fx)}}$$

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(EllipticF[ArcSin[Tan[(e + f*x)/2
]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(
a + b)])*Sec[e + f*x])/(f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

method	result
default	$\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)-2\,\text{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),-1,\sqrt{\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1}}}{f(b+a\cos(fx+e))}$

```
[In] int(1/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))-2*
EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*cos
```

```
(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f
*x+e))^(1/2)/(b+a*cos(f*x+e))
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\sec{(fx+e)}+a}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{1}{\sqrt{a+b\sec{(e+fx)}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral(1/sqrt(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\sec{(fx+e)}+a}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\sec{(fx+e)}+a}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(a + b/cos(e + f*x))^(1/2),x)
[Out] int(1/(a + b/cos(e + f*x))^(1/2), x)
```

3.254 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 255

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \frac{\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{\sqrt{a+b}f} - \frac{\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{\sqrt{a+b}f} - \frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)f\sqrt{a+b\sec(e+fx)}}$$

```
[Out] cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f/(a+b)^(1
/2)-cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f/(a+b
)^(1/2)-cot(f*x+e)/f/(a+b*sec(f*x+e))^(1/2)+b^2*tan(f*x+e)/(a^2-b^2)/f/(a+b
*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3960, 3918, 21, 3914, 3917, 4089}

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$
$$=\frac{b^2\tan(e+fx)}{f(a^2-b^2)\sqrt{a+b\sec(e+fx)}}$$
$$-\frac{\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f\sqrt{a+b}}$$
$$+\frac{\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f\sqrt{a+b}}$$
$$-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x
]))/(a - b))])/(Sqrt[a + b]*f) - (Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*
Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(Sqrt[a + b]*f) - Cot[e + f
*x]/(f*Sqrt[a + b*Sec[e + f*x]]) + (b^2*Tan[e + f*x])/((a^2 - b^2)*f*Sqrt[a
 + b*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3914

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
```

```
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}-\frac{1}{2}b\int\frac{\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx\\
&=-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}+\frac{b^2\tan(e+fx)}{(a^2-b^2)\,f\sqrt{a+b\sec(e+fx)}}+\frac{b\int\frac{\sec(e+fx)\left(-\frac{a}{2}-\frac{1}{2}b\sec(e+fx)\right)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a^2-b^2}\\
&=-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}+\frac{b^2\tan(e+fx)}{(a^2-b^2)\,f\sqrt{a+b\sec(e+fx)}}\\
&\quad-\frac{b\int\sec(e+fx)\sqrt{a+b\sec(e+fx)}\,dx}{2\,(a^2-b^2)}\\
&=-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}+\frac{b^2\tan(e+fx)}{(a^2-b^2)\,f\sqrt{a+b\sec(e+fx)}}\\
&\quad-\frac{b\int\frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{2(a+b)}-\frac{b^2\int\frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx}{2\,(a^2-b^2)}
\end{aligned}
$$

$$= \frac{\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{\sqrt{a+b}f}$$
$$-\frac{\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{\sqrt{a+b}f}$$
$$-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}+\frac{b^2\tan(e+fx)}{(a^2-b^2)\,f\sqrt{a+b\sec(e+fx)}}$$

Mathematica [A] (warning: unable to verify)

Time = 7.39 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int\frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$
$$=\frac{\sqrt{\sec(e+fx)}\left(\frac{(b+a\cos(e+fx))(-a+b\cos(e+fx))\csc(e+fx)}{(a^2-b^2)\sqrt{\sec(e+fx)}}+\frac{b\left(-\frac{(a+b)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-\text{Elliptic}\right.}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right.}{(-a^2+b^2)\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{}}\right.}{f\sqrt{a+b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[Sec[e + f*x]]*(((b + a*Cos[e + f*x])*(-a + b*Cos[e + f*x])*Csc[e + f*
x])/((a^2 - b^2)*Sqrt[Sec[e + f*x]]) + (b*(-(((a + b)*Sqrt[(b + a*Cos[e + f
*x])/((a + b)*(1 + Cos[e + f*x]))]*(EllipticE[ArcSin[Tan[(e + f*x)/2]], (a
- b)/(a + b)] - EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))/Sqrt
[Cos[e + f*x]/(1 + Cos[e + f*x])]) - (b + a*Cos[e + f*x])*Tan[(e + f*x)/2])
)/((-a^2 + b^2)*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*
x]])))/(f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. 2(233) = 466.

Time = 5.73 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.98

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)}\left(-\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,ab\cos(fx+e)-\text{EllipticF}\left(\cot\right.\right.}{}$

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/(a-b)/(a+b)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))*(-EllipticF(cot(f*
x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+
```

```
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b*cos(f*x+e)-EllipticF(cot(f*
x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2*cos(f*x+e)+(1/(a+b)*(b+a*co
s(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE
(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b*cos(f*x+e)+(1/(a+b)*(b+a*co
s(f*x+e))/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE
(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^2*cos(f*x+e)-(1/(a+b)*(b+a*co
s(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b
))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b-(1/(a+b)*(b+a*cos(f*x+e))/(
cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+
1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*a*b+(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*E
llipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*b^2+a^2*cos(f*x+e)*cot(f*x+e)-a*b*cos(f*x+e)*cot(f*x+e)+a*b*cot(
f*x+e)-b^2*cot(f*x+e))
```

Fricas [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\csc^2\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx$$

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)), x)
```

3.255 $\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 347

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{2\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}f}$$
$$-\frac{2\cot(e+fx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}f}$$
$$-\frac{2\sqrt{a+b}\cot(e+fx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a^2f}$$
$$+\frac{2b^2\tan(e+fx)}{a(a^2-b^2)f\sqrt{a+b\sec(e+fx)}}$$

```
[Out] 2*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a/f/(a+b
)^(1/2)-2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/
a/f/(a+b)^(1/2)-2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),
(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b
*(1+sec(f*x+e))/(a-b))^(1/2)/a^2/f+2*b^2*tan(f*x+e)/a/(a^2-b^2)/f/(a+b*sec(
f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3870, 4143, 4006, 3869, 3917, 4089}

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{a^2f}$$
$$+\frac{2b^2\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}$$
$$-\frac{2\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af\sqrt{a+b}}$$
$$+\frac{2\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{af\sqrt{a+b}}$$

```
[In] Int[(a + b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f
*x]))/(a - b))])/(a*Sqrt[a + b]*f) - (2*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*Sqrt[a + b]*f) - (
2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a^2*f) + (2*b^2*Tan[e + f*x])/(a*(
a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
```

```
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b^2\tan(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{2\int\frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\sec(e+fx)+\frac{1}{2}b^2\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\left(a^2-b^2\right)}\\
&= \frac{2b^2\tan(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}}\\
&\quad - \frac{2\int\frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int\frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\left(a^2-b^2\right)}
\end{aligned}$$

$$= \frac{2\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}f}$$

$$+ \frac{2b^2\tan(e+fx)}{a(a^2-b^2)f\sqrt{a+b\sec(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx}{a} - \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{a(a+b)}$$

$$= \frac{2\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}f}$$

$$- \frac{2\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}f}$$

$$- \frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+f}{a-b}}}{a^2f}$$

$$+ \frac{2b^2\tan(e+fx)}{a(a^2-b^2)f\sqrt{a+b\sec(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 972 vs. $2(347) = 694$.

Time = 6.60 (sec) , antiderivative size = 972, normalized size of antiderivative = 2.80

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}dx = \frac{(b+a\cos(e+fx))^2\sec^2(e+fx)\left(\frac{2b\sin(e+fx)}{a(-a^2+b^2)} + \frac{2b^2\sin(e+fx)}{a(a^2-b^2)(b+a\cos(e+fx))}\right)}{f(a+b\sec(e+fx))^{3/2}}$$

$$+ \frac{2(b+a\cos(e+fx))^{3/2}\sec^{\frac{3}{2}}(e+fx)\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+b\tan^2\left(\frac{1}{2}(e+fx)\right)}{1+\tan^2\left(\frac{1}{2}(e+fx)\right)}}\left(ab\tan\left(\frac{1}{2}(e+fx)\right)+b^2\tan\left(\frac{1}{2}(e+\right.\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] ((b + a*Cos[e + f*x])^2*Sec[e + f*x]^2*((2*b*Sin[e + f*x])/(a*(-a^2 + b^2))
 + (2*b^2*Sin[e + f*x])/(a*(a^2 - b^2)*(b + a*Cos[e + f*x]))))/(f*(a + b*Se
c[e + f*x])^(3/2)) + (2*(b + a*Cos[e + f*x])^(3/2)*Sec[e + f*x]^(3/2)*Sqrt[
(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]
^2)]*(a*b*Tan[(e + f*x)/2] + b^2*Tan[(e + f*x)/2] - 2*a*b*Tan[(e + f*x)/2]^
3 + a*b*Tan[(e + f*x)/2]^5 - b^2*Tan[(e + f*x)/2]^5 + 2*a^2*EllipticPi[-1,
ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqr
t[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - 2*b^2*El
lipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f
*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b
)] + 2*a^2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Tan[(e
```

```
 + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^
2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - 2*b^2*EllipticPi[-1, ArcSin[Tan[(e + f
*x)/2]], (a - b)/(a + b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*S
qrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + b*(a +
 b)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e +
f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*
Tan[(e + f*x)/2]^2)/(a + b)] - a*(a + b)*EllipticF[ArcSin[Tan[(e + f*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sq
rt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]))/(a*(a^2
 - b^2)*f*(a + b*Sec[e + f*x])^(3/2)*(-1 + Tan[(e + f*x)/2]^2)*Sqrt[(1 + Ta
n[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]*(a*(-1 + Tan[(e + f*x)/2]^2) -
b*(1 + Tan[(e + f*x)/2]^2)))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(318) = 636$.

Time = 6.48 (sec) , antiderivative size = 1799, normalized size of antiderivative = 5.18

method	result	size
default	Expression too large to display	1799

```
[In] int(1/(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/a/(a+b)/(a-b)*(-((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2
*csc(f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^
2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+
b))^(1/2))*a^2-((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x
+e)^2-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2*csc(
f*x+e)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc
(f*x+e)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(
1/2))*a*b+((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2
-a-b)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2*csc(f*x+e
)^2+1)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+
e)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))
*a*b+((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)
*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2*csc(f*x+e)^2+1
)^(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-
a-b)/(a+b))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^2+
2*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)*((
1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2*csc(f*x+e)^2+1)^(
1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b
)/(a+b))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^2
-2*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)*(
(1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(-(1-cos(f*x+e))^2*csc(f*x+e)^2+1)^
```

```
(1/2)*(-(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-
b)/(a+b))^(1/2)*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*b^
2-((1-cos(f*x+e))^4*a*csc(f*x+e)^4-(1-cos(f*x+e))^4*b*csc(f*x+e)^4-2*a*(1-c
os(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a*b*(1-cos(f*x+e))^3*csc(f*x+e)^3+((1-
cos(f*x+e))^4*a*csc(f*x+e)^4-(1-cos(f*x+e))^4*b*csc(f*x+e)^4-2*a*(1-cos(f*x
+e))^2*csc(f*x+e)^2+a+b)^(1/2)*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+((1-cos(f*
x+e))^4*a*csc(f*x+e)^4-(1-cos(f*x+e))^4*b*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2
*csc(f*x+e)^2+a+b)^(1/2)*a*b*(-cot(f*x+e)+csc(f*x+e))-((1-cos(f*x+e))^4*a*c
sc(f*x+e)^4-(1-cos(f*x+e))^4*b*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)
^2+a+b)^(1/2)*b^2*(-cot(f*x+e)+csc(f*x+e)))*((a*(1-cos(f*x+e))^2*csc(f*x+e)
^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^
(1/2)/((1-cos(f*x+e))^4*a*csc(f*x+e)^4-(1-cos(f*x+e))^4*b*csc(f*x+e)^4-2*a*
(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)/(a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b
*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)
```

Fricas [F]

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(f*x + e) + a)/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2), x)

Sympy [F]

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(a+b*sec(f*x+e))**(3/2),x)

[Out] Integral((a + b*sec(e + f*x))**(-3/2), x)

Maxima [F]

$$\int \frac{1}{(a + b\sec(e + fx))^{3/2}}\,dx = \int \frac{1}{(b\sec(fx + e) + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a + b\sec(e + fx))^{3/2}}\,dx = \int \frac{1}{(b\sec(fx + e) + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\sec(e + fx))^{3/2}}\,dx = \int \frac{1}{\left(a + \frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b/cos(e + f*x))^(3/2),x)
[Out] int(1/(a + b/cos(e + f*x))^(3/2), x)
```

3.256 $\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 318

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{4a\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)(a+b)^{3/2}f}$$
$$-\frac{(3a-b)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)(a+b)^{3/2}f}$$
$$-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}+\frac{b^2\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))^{3/2}}$$
$$+\frac{4ab^2\tan(e+fx)}{(a^2-b^2)^2\,f\sqrt{a+b\sec(e+fx)}}$$

```
[Out] -cot(f*x+e)/f/(a+b*sec(f*x+e))^(3/2)+4*a*cot(f*x+e)*EllipticE((a+b*sec(f*x+
e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(
-b*(1+sec(f*x+e))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/f-(3*a-b)*cot(f*x+e)*Ellip
ticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+
e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/f+b^2*ta
n(f*x+e)/(a^2-b^2)/f/(a+b*sec(f*x+e))^(3/2)+4*a*b^2*tan(f*x+e)/(a^2-b^2)^2/
f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3960, 3918, 4088, 4090, 3917, 4089}

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{4ab^2\tan(e+fx)}{f\left(a^2-b^2\right)^2\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}} - \frac{(3a-b)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f(a-b)(a+b)^{3/2}} + \frac{4a\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f(a-b)(a+b)^{3/2}} - \frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*a*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e +
 f*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*f) - ((3*a - b)*Cot[e + f*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/((a
 - b)*(a + b)^(3/2)*f) - Cot[e + f*x]/(f*(a + b*Sec[e + f*x])^(3/2)) + (b^2
*Tan[e + f*x])/((a^2 - b^2)*f*(a + b*Sec[e + f*x])^(3/2)) + (4*a*b^2*Tan[e
+ f*x])/((a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} - \frac{1}{2}(3b)\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx \\
&= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))^{3/2}} \\
&\quad + \frac{b\int \frac{\sec(e+fx)\left(-\frac{3a}{2}+\frac{1}{2}b\sec(e+fx)\right)}{(a+b\sec(e+fx))^{3/2}}\,dx}{a^2-b^2}
\end{aligned}
$$

$$= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))^{3/2}} + \frac{4ab^2\tan(e+fx)}{(a^2-b^2)^2 f\sqrt{a+b\sec(e+fx)}} - \frac{(2b)\int \frac{\sec(e+fx)\left(\frac{1}{4}(3a^2+b^2)+ab\sec(e+fx)\right)}{\sqrt{a+b\sec(e+fx)}}dx}{(a^2-b^2)^2}$$

$$= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))^{3/2}} + \frac{4ab^2\tan(e+fx)}{(a^2-b^2)^2 f\sqrt{a+b\sec(e+fx)}} - \frac{((3a-b)b)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{2(a-b)(a+b)^2} - \frac{(2ab^2)\int \frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}dx}{(a^2-b^2)^2}$$

$$= \frac{4a\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)(a+b)^{3/2}f} - \frac{(3a-b)\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)(a+b)^{3/2}f} - \frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))^{3/2}} + \frac{4ab^2\tan(e+fx)}{(a^2-b^2)^2 f\sqrt{a+b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 7.47 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.81

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{-((a-b)((3a-b)b+a(a-3b)\cos(e+fx))\csc(e+fx))+8ab(a+b)\cos^2\left(\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-((a - b)*((3*a - b)*b + a*(a - 3*b)*Cos[e + f*x])*Csc[e + f*x]) + 8*a*b*(
a + b)*Cos[(e + f*x)/2]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f
*x]))]*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sec[e + f*x]*Sq
rt[(1 + Sec[e + f*x])^(-1)] - 2*b*(3*a^2 + 4*a*b + b^2)*Cos[(e + f*x)/2]^2*
Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*EllipticF[ArcSin[Ta
n[(e + f*x)/2]], (a - b)/(a + b)]*Sec[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)
])/((a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(294) = 588$.

Time = 6.86 (sec) , antiderivative size = 977, normalized size of antiderivative = 3.07

method	result	size
default	Expression too large to display	977

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/(a-b)^2/(a+b)^2*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))*(4*EllipticE(c
ot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b*cos(f*x+e)+4*Ellipti
cE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2*cos(f*x+e)-3*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b)
)^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b*cos(f*x+e)-4
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2*cos(f*x+
e)-(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)
/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*b^3*cos(f*x+
e)+4*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-
b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b+4*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b
))^(1/2))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2-3*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2
))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a^2*b-4*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(
a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*a*b^2-(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a
*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*b^3+a^3*cos(f*x+e)*cot(f*x+e)-4*a^2*b*co
s(f*x+e)*cot(f*x+e)+3*a*b^2*cos(f*x+e)*cot(f*x+e)+3*a^2*b*cot(f*x+e)-4*a*b^
2*cot(f*x+e)+b^3*cot(f*x+e))
```

Fricas [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2/(b^2*sec(f*x + e)^2 + 2*a*
b*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e))**(3/2),x)
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^2}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^2}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^2\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(3/2)), x)
```

3.257 $\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 249

$$\begin{aligned}
&\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx\\
&=\frac{a^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}\\
&\quad+\frac{3a^2b\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}\\
&\quad+\frac{b^3\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}\\
&\quad+\frac{3ab^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{1+m}}{de(1+m)}
\end{aligned}$$

```
[Out] 3*a^2*b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(
1+m)/d/e/(1+m)+b^3*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*si
n(d*x+c))^(1+m)/d/e/(1+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/
2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+3*a*
b^2*hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*sin(
d*x+c))^(1+m)*(cos(d*x+c)^2)^(1/2)/d/e/(1+m)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3957, 2991, 2722, 2644, 371, 2657}

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx$$
$$=\frac{a^3\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}$$
$$+\frac{3a^2b(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$
$$+\frac{3ab^2\sqrt{\cos^2(c+dx)}\sec(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$
$$+\frac{b^3(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

```
[In] Int[(a + b*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b
*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(1 + m))/(d*e*(1 + m)) + (b^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2,
Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a*b^2*Sqrt[Cos
[c + d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*S
ec[c + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b-a\cos(c+dx))^3\sec^3(c+dx)(e\sin(c+dx))^m\,dx\\
&= -\int \left(-a^3(e\sin(c+dx))^m-3a^2b\sec(c+dx)(e\sin(c+dx))^m\right.\\
&\qquad\left.-3ab^2\sec^2(c+dx)(e\sin(c+dx))^m-b^3\sec^3(c+dx)(e\sin(c+dx))^m\right)\,dx\\
&= a^3\int (e\sin(c+dx))^m\,dx+\left(3a^2b\right)\int \sec(c+dx)(e\sin(c+dx))^m\,dx\\
&\quad+\left(3ab^2\right)\int \sec^2(c+dx)(e\sin(c+dx))^m\,dx+b^3\int \sec^3(c+dx)(e\sin(c+dx))^m\,dx\\
&= \frac{a^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}\\
&\quad+\frac{3ab^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{1+m}}{de(1+m)}\\
&\quad+\frac{(3a^2b)\,\text{Subst}\left(\int \frac{x^m}{1-\frac{x^2}{e^2}}\,dx,x,e\sin(c+dx)\right)}{de}+\frac{b^3\text{Subst}\left(\int \frac{x^m}{\left(1-\frac{x^2}{e^2}\right)^2}\,dx,x,e\sin(c+dx)\right)}{de}
\end{aligned}
$$

$$= \frac{a^3 \cos(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) (e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{3a^2 b \operatorname{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) (e\sin(c+dx))^{1+m}}{de(1+m)}$$
$$+ \frac{b^3 \operatorname{Hypergeometric2F1}\left(2, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) (e\sin(c+dx))^{1+m}}{de(1+m)}$$
$$+ \frac{3ab^2 \sqrt{\cos^2(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) \sec(c+dx)(e\sin(c+dx))^{1+m}}{de(1+m)}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.73

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx$$
$$= \frac{\left(a^3\sqrt{\cos^2(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) + b\left(3a^2\cos(c+dx) \operatorname{Hypergeometric2F\right.\right.}{}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] ((a^3*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin
[c + d*x]^2] + b*(3*a^2*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 + m
)/2, Sin[c + d*x]^2] + b*(3*a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (
1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + b*Cos[c + d*x]*Hypergeometric2F1[2,
(1 + m)/2, (3 + m)/2, Sin[c + d*x]^2])))*(e*Sin[c + d*x])^m*Tan[c + d*x])/(
d*(1 + m))
```

Maple [F]

$$\int (a+b\sec(dx+c))^3 (e\sin(dx+c))^m\,dx$$

```
[In] int((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

Fricas [F]

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^3(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)*(e*sin(d*x + c))^m, x)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx = \int (e\sin(c+dx))^m\,(a+b\sec(c+dx))^3\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**3*(e*sin(d*x+c))**m,x)
```

```
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x))**3, x)
```

Maxima [F]

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^3(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

Giac [F]

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^3(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx=\int (e\,\sin(c+d\,x))^m\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^3\,dx$$

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^3, x)
```

3.258 $\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 190

$$\begin{aligned}
&\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx\\
&=\frac{a^2\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin(c+dx)(e\sin(c+dx))^m}{d(1+m)\sqrt{\cos^2(c+dx)}}\\
&\quad+\frac{2ab\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}\\
&\quad+\frac{b^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^m\tan(c+dx)}{d(1+m)}
\end{aligned}$$

```
[Out] 2*a*b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+
m)/d/e/(1+m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+
c)^2)*sin(d*x+c)*(e*sin(d*x+c))^m/d/(1+m)/(cos(d*x+c)^2)^(1/2)+b^2*hypergeo
m([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^m*(cos(d*x+c)^2
)^(1/2)*tan(d*x+c)/d/(1+m)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used

$= \{3957, 2990, 2644, 371, 4483, 4486, 2722, 2657\}$

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx$$
$$= \frac{a^2\sin(c+dx)\cos(c+dx)(e\sin(c+dx))^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{2ab(e\sin(c+dx))^{m+1}\operatorname{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$
$$+ \frac{b^2\sqrt{\cos^2(c+dx)}\tan(c+dx)(e\sin(c+dx))^m \operatorname{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{d(m+1)}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(e*Sin[c + d*x])^m)/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*
a*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d
*x])^(1 + m))/(d*e*(1 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3
/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^m*Tan[c + d*x])/
(d*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 4483

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-b-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^m\,dx\\
&= (2ab)\int \sec(c+dx)(e\sin(c+dx))^m\,dx\\
&\quad+\int \left(b^2+a^2\cos^2(c+dx)\right)\sec^2(c+dx)(e\sin(c+dx))^m\,dx\\
&= \frac{(2ab)\text{Subst}\left(\int \frac{x^m}{1-\frac{x^2}{e^2}}\,dx,x,e\sin(c+dx)\right)}{de}+\left(\sin^{-m}(c+dx)(e\sin(c+dx))^m\right)\int \left(b^2\right.\\
&\quad\left.+a^2\cos^2(c+dx)\right)\sec^2(c+dx)\sin^m(c+dx)\,dx
\end{aligned}$$

$$= \frac{2ab\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} + \left(\sin^{-m}(c+dx)(e\sin(c+dx))^m\right)\int\left(a^2\sin^m(c+dx) + b^2\sec^2(c+dx)\sin^m(c+dx)\right)\,dx$$

$$= \frac{2ab\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} + \left(a^2\sin^{-m}(c+dx)(e\sin(c+dx))^m\right)\int\sin^m(c+dx)\,dx + \left(b^2\sin^{-m}(c+dx)(e\sin(c+dx))^m\right)\int\sec^2(c+dx)\sin^m(c+dx)\,dx$$

$$= \frac{a^2\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin(c+dx)(e\sin(c+dx))^m}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{2ab\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} + \frac{b^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^m\tan(c+dx)}{d(1+m)}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int(a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx$$
$$= \frac{(e\sin(c+dx))^m\left(2ab\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin(c+dx)+\sqrt{\cos^2(c+dx)}\left(a^2\,\text{Hyp}\right.\right.}{d(}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] ((e*Sin[c + d*x])^m*(2*a*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c
 + d*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*(a^2*Hypergeometric2F1[1/2,
(1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + b^2*Hypergeometric2F1[3/2, (1 + m)/
2, (3 + m)/2, Sin[c + d*x]^2])*Tan[c + d*x]))/(d*(1 + m))
```

Maple [F]

$$\int (a+b\sec(dx+c))^2 (e\sin(dx+c))^m \, dx$$

```
[In] int((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

Fricas [F]

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m \, dx = \int (b\sec(dx+c)+a)^2(e\sin(dx+c))^m \, dx$$

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*(e*sin(d*x + c))^m
, x)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m \, dx = \int (e\sin(c+dx))^m (a+b\sec(c+dx))^2 \, dx$$

```
[In] integrate((a+b*sec(d*x+c))**2*(e*sin(d*x+c))**m,x)
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x))**2, x)
```

Maxima [F]

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m \, dx = \int (b\sec(dx+c)+a)^2(e\sin(dx+c))^m \, dx$$

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

Giac [F]

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^2(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx = \int (e\,\sin(c+d\,x))^m\left(a+\frac{b}{\cos(c+d\,x)}\right)^2\,dx$$

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^2, x)
```

3.259 $\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = \frac{a\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}$$

```
[Out] b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d
/e/(1+m)+a*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*
(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2917, 2644, 371, 2722}

$$\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = \frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}} + \frac{b(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

```
[In] Int[(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m
))/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int (-b - a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^m\,dx \\
&= a\int (e\sin(c+dx))^m\,dx + b\int \sec(c+dx)(e\sin(c+dx))^m\,dx
\end{aligned}
$$

$$= \frac{a\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b\text{Subst}\left(\int\frac{x^m}{1-\frac{x^2}{e^2}}\,dx,x,e\sin(c+dx)\right)}{de}$$

$$= \frac{a\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int(a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = \frac{\left(a\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)+b\cos(c+dx)\,\text{Hypergeometric2F1}\,(1\right.}{d(1+m)}$$

```
[In] Integrate[(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
[Out] ((a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c
 + d*x]^2] + b*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[
c + d*x]^2])*(e*Sin[c + d*x])^m*Tan[c + d*x])/(d*(1 + m))
```

Maple [F]

$$\int(a+b\sec(dx+c))\,(e\sin(dx+c))^m\,dx$$

```
[In] int((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

Fricas [F]

$$\int(a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = \int(b\sec(dx+c)+a)(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [F]

$$\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = \int (e\sin{(c+dx)})^m\,(a+b\sec{(c+dx)})\;dx$$

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))**m,x)
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = \int (b\sec{(dx+c)}+a)(e\sin{(dx+c)})^m\;dx$$

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Giac [F]

$$\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = \int (b\sec{(dx+c)}+a)(e\sin{(dx+c)})^m\;dx$$

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = \int (e\,\sin{(c+d\,x)})^m\,\left(a+\frac{b}{\cos{(c+d\,x)}}\right)\;dx$$

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x)), x)
```

3.260 $\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 232

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = -\frac{be\,\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right)}{a^2d(1-m)} + \frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{ade(1+m)\sqrt{\cos^2(c+dx)}}$$

```
[Out] -b*e*AppellF1(1-m,1/2-1/2*m,1/2-1/2*m,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(b+
a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos(d
*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^2/d/(1-m)+cos(
d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^
(1+m)/a/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3957, 2946, 2722, 2782}

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = \frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{ade(m+1)\sqrt{\cos^2(c+dx)}} - \frac{be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{}{b+a}\right.}{a^2d(1-m)}$$

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
+ d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Co
s[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1
 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^2*d*(1 - m))) + (Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
+ m))/(a*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cos(c+dx)(e\sin(c+dx))^m}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{\int (e\sin(c+dx))^m\,dx}{a} + \frac{b\int \frac{(e\sin(c+dx))^m}{-b-a\cos(c+dx)}\,dx}{a}$$

$$= -\frac{be\,\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos}{b+a\cos}\right.}{a^2d(1-m)}$$

$$+\frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{ade(1+m)\sqrt{\cos^2(c+dx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 687 vs. 2(232) = 464.

Time = 4.96 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.96

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$$

$$= \frac{}{d(a+b\sec(c+dx))\left(\left(-b\,\text{AppellF1}\left(\frac{1+m}{2},m,1,\frac{3+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)+(a+b)\,\text{Hy}\right.\right.}$$

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*
Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 + m)/2, 1 + m,
 (3 + m)/2, -Tan[(c + d*x)/2]^2])*(e*Sin[c + d*x])^m*Tan[(c + d*x)/2])/(d*(
a + b*Sec[c + d*x])*((-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*
x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1
[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2 + 2*
m*Cot[c + d*x]*(-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^
2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 +
m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2] + 2*m*(-(b*A
ppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/
2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2 + ((1 + m)*Sec[(c + d*x)/2]^2*(
-((a + b)^2*(Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/
2]^2] - (Sec[(c + d*x)/2]^2)^(-1 - m))) + (2*b*((-a + b)*AppellF1[(3 + m)/2
, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b
)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d*x)/2]^2
, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)/(3 + m)))/(a +
 b)))
```

Maple [F]

$$\int \frac{(e\sin(dx+c))^m}{a+b\sec(dx+c)}dx$$

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x)
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x)
```

Fricas [F]

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^m}{b\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$$

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c)),x)
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^m}{b\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^m}{b\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^m}{b+a\,\cos(c+d\,x)}\,dx$$

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x)), x)
```

3.261 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 405

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx = -\frac{2be\,\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right)}{a^3d(1-m)} + \frac{b^2e\,\text{AppellF1}\left(2-m,\frac{1-m}{2},\frac{1-m}{2},3-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right)}{a^3d(2-m)(b+a\cos(c+dx))} + \frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{a^2de(1+m)\sqrt{\cos^2(c+dx)}}$$

```
[Out] -2*b*e*AppellF1(1-m,1/2-1/2*m,1/2-1/2*m,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(
b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos
(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^3/d/(1-m)+b^
2*e*AppellF1(2-m,1/2-1/2*m,1/2-1/2*m,3-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(b+a
*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos(d*
x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^3/d/(2-m)/(b+a*
cos(d*x+c))+cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)
*(e*sin(d*x+c))^(1+m)/a^2/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3957, 2991, 2722, 2782}

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\text{AppellF1}\left(2-m,\frac{1-m}{2},\frac{1-m}{2},3-m,-\frac{\quad}{b+a\,c}\right.}{a^3d(2-m)(a\cos(c+dx)+b)}$$
$$-\frac{2be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{\quad}{b-}\right.}{a^3d(1-m)}$$
$$+\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{a^2de(m+1)\sqrt{\cos^2(c+dx)}}$$

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
 + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*C
os[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((
1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^3*d*(1 - m)) + (b^2*e*AppellF1[2 -
m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b)/(b
 + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((1 -
m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[c +
d*x])^(-1 + m))/(a^3*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
+ m))/(a^2*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^m}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{(e\sin(c+dx))^m}{a^2} + \frac{b^2(e\sin(c+dx))^m}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^m}{a^2(b+a\cos(c+dx))}\right)dx$$

$$= \frac{\int (e\sin(c+dx))^m\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^m}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^m}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= -\frac{2be\,\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos\cdots}{b+a\cos\cdots}\right.}{a^3d(1-m)}$$

$$+\frac{b^2e\,\text{AppellF1}\left(2-m,\frac{1-m}{2},\frac{1-m}{2},3-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos\cdots}{b+a\cos\cdots}\right.}{a^3d(2-m)(b+a\cos(c+dx))}$$

$$+\frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{a^2de(1+m)\sqrt{\cos^2(c+dx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1433 vs. $2(405) = 810$.

Time = 11.66 (sec) , antiderivative size = 1433, normalized size of antiderivative = 3.54

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx =$$

$$-\frac{}{a^2d(a+b\sec(c+dx))^2\left(\operatorname{AppellF1}\left(\frac{1+m}{2},m,1,\frac{3+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)\sec^2\left(\frac{1}{2}(c+\right.\right.}$$

$$+\frac{}{a^2d(a+b\sec(c+dx))^2\left(\left((a+b)\operatorname{AppellF1}\left(\frac{1+m}{2},m,1,\frac{3+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)-\right.\right.}$$

$$-\frac{(b+a\cos(c+dx))^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3}{2},\cos^2(c+dx)\right)(e\sin(c+dx))^m\sin^2(c+dx)^{\frac{1}{2}(-1-m)}}{a^2d(a+b\sec(c+dx))^2}$$

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-4*b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)]*(b + a*Cos[c + d*x])*Sec[c + d*x]^2*(e*Sin[c + d
*x])^m*Tan[(c + d*x)/2])/(a^2*d*(a + b*Sec[c + d*x])^2*(AppellF1[(1 + m)/2,
 m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
]*Sec[(c + d*x)/2]^2 + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Cot[c + d*x]*Tan[(c + d*x)/
2] + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2 - (2*(1 + m)*((-a + b)*App
ellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) + (2*b^2*((a + b)*AppellF1[(1 + m)/
2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b
)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[c + d*x]^2*(e*Sin[c + d*x])^m*Tan[(c +
d*x)/2])/(a^2*d*(a + b*Sec[c + d*x])^2*(((a + b)*AppellF1[(1 + m)/2, m, 1,
(3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a
*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c
+ d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2 + 2*m*((a + b)*AppellF1[(1 + m)/2
, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b
)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)])*Cot[c + d*x]*Tan[(c + d*x)/2] + 2*m*((a + b)
*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c
+ d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c +
d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2 - (2*(
```

```
1 + m)*((-a^2 + b^2)*AppellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]
^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a*(a - b)*AppellF1[(3 + m)/2,
 m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
] + (a + b)*m*((a + b)*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(3 + m)/2, 1
 + m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)]))*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) - ((b + a*
Cos[c + d*x])^2*Hypergeometric2F1[1/2, (1 - m)/2, 3/2, Cos[c + d*x]^2]*(e*S
in[c + d*x])^m*(Sin[c + d*x]^2)^((-1 - m)/2)*Tan[c + d*x])/(a^2*d*(a + b*Se
c[c + d*x])^2)
```

Maple [F]

$$\int \frac{(e \sin (dx+c))^m}{(a+b \sec (dx+c))^2} dx$$

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x)
```

Fricas [F]

$$\int \frac{(e \sin(c+dx))^m}{(a+b \sec(c+dx))^2} \, dx = \int \frac{(e \sin (dx+c))^m}{(b \sec (dx+c)+a)^2} \, dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)
, x)
```

Sympy [F]

$$\int \frac{(e \sin(c+dx))^m}{(a+b \sec(c+dx))^2} \, dx = \int \frac{(e \sin (c+dx))^m}{(a+b \sec (c+dx))^2} \, dx$$

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**2, x)
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^m}{(b+a\,\cos(c+d\,x))^2}\,dx$$

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x))^2, x)
```

3.262 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 580

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx =$$

$$-\frac{3be\,\mathrm{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right)}{a^4d(1-m)}$$

$$-\frac{b^3e\,\mathrm{AppellF1}\left(3-m,\frac{1-m}{2},\frac{1-m}{2},4-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right)}{a^4d(3-m)(b+a\cos(c+dx))^2}$$

$$+\frac{3b^2e\,\mathrm{AppellF1}\left(2-m,\frac{1-m}{2},\frac{1-m}{2},3-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right)}{a^4d(2-m)(b+a\cos(c+dx))}$$

$$+\frac{\cos(c+dx)\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{a^3de(1+m)\sqrt{\cos^2(c+dx)}}$$

```
[Out] -3*b*e*AppellF1(1-m,1/2-1/2*m,1/2-1/2*m,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(
b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos
(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^4/d/(1-m)-b^
3*e*AppellF1(3-m,1/2-1/2*m,1/2-1/2*m,4-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(b+a
*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos(d*
x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^4/d/(3-m)/(b+a*
cos(d*x+c))^2+3*b^2*e*AppellF1(2-m,1/2-1/2*m,1/2-1/2*m,3-m,(-a+b)/(b+a*cos(
d*x+c)),(a+b)/(b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1
/2*m)*(a*(1+cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)
/a^4/d/(2-m)/(b+a*cos(d*x+c))+cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/
2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/a^3/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3957, 2991, 2722, 2782}

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx =$$

$$-\frac{b^3e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\operatorname{AppellF1}\left(3-m,\frac{1-m}{2},\frac{1-m}{2},4-m,-\frac{}{b+\ldots}\right.}{a^4d(3-m)(a\cos(c+dx)+b)^2}$$
$$+\frac{3b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\operatorname{AppellF1}\left(2-m,\frac{1-m}{2},\frac{1-m}{2},3-m,-\frac{}{b\ldots}\right.}{a^4d(2-m)(a\cos(c+dx)+b)}$$
$$-\frac{3be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\operatorname{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{}{b-\ldots}\right.}{a^4d(1-m)}$$
$$+\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{a^3de(m+1)\sqrt{\cos^2(c+dx)}}$$

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
 + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*C
os[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((
1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^4*d*(1 - m)) - (b^3*e*AppellF1[3 -
m, (1 - m)/2, (1 - m)/2, 4 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b)/(b
 + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((1 -
m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[c +
d*x])^(-1 + m))/(a^4*d*(3 - m)*(b + a*Cos[c + d*x])^2) + (3*b^2*e*AppellF1[
2 - m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b
)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((
1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[
c + d*x])^(-1 + m))/(a^4*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos[c + d*x]*Hy
pergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])
^(1 + m))/(a^3*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(-b-a\cos(c+dx))^3}\,dx \\
&= -\int \left(-\frac{(e\sin(c+dx))^m}{a^3}+\frac{b^3(e\sin(c+dx))^m}{a^3(b+a\cos(c+dx))^3}-\frac{3b^2(e\sin(c+dx))^m}{a^3(b+a\cos(c+dx))^2}\right. \\
&\qquad \left. +\frac{3b(e\sin(c+dx))^m}{a^3(b+a\cos(c+dx))}\right)dx \\
&= \frac{\int (e\sin(c+dx))^m\,dx}{a^3}-\frac{(3b)\int \frac{(e\sin(c+dx))^m}{b+a\cos(c+dx)}\,dx}{a^3} \\
&\quad +\frac{(3b^2)\int \frac{(e\sin(c+dx))^m}{(b+a\cos(c+dx))^2}\,dx}{a^3}-\frac{b^3\int \frac{(e\sin(c+dx))^m}{(b+a\cos(c+dx))^3}\,dx}{a^3}
\end{aligned}$$

$$= -\frac{3be\,\text{AppellF1}\left(1-m,\frac{1-m}{2},\frac{1-m}{2},2-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+c}{b+a\,c}\right.}{a^4d(1-m)}$$
$$-\frac{b^3e\,\text{AppellF1}\left(3-m,\frac{1-m}{2},\frac{1-m}{2},4-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+c}{b+a\,c}\right.}{a^4d(3-m)(b+a\cos(c+dx))^2}$$
$$+\frac{3b^2e\,\text{AppellF1}\left(2-m,\frac{1-m}{2},\frac{1-m}{2},3-m,-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+}{b+a}\right.}{a^4d(2-m)(b+a\cos(c+dx))}$$
$$+\frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{a^3de(1+m)\sqrt{\cos^2(c+dx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2700 vs. $2(580) = 1160$.

Time = 16.99 (sec) , antiderivative size = 2700, normalized size of antiderivative = 4.66

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-6*b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)]*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^3*(e*Sin[c +
 d*x])^m*Tan[(c + d*x)/2])/(a^3*d*(a + b*Sec[c + d*x])^3*(AppellF1[(1 + m)/
2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)]*Sec[(c + d*x)/2]^2 + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c +
 d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Cot[c + d*x]*Tan[(c + d*x
)/2] + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a -
b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2 - (2*(1 + m)*((-a + b)*A
ppellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -T
an[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) + (6*b^2*((a + b)*AppellF1[(1 + m
)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a
+ b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)])*(b + a*Cos[c + d*x])*Sec[c + d*x]^3*(e*Si
n[c + d*x])^m*Tan[(c + d*x)/2])/(a^3*d*(a + b*Sec[c + d*x])^3*(((a + b)*App
ellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2 + 2*m*((a
+ b)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan
[(c + d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(
```

```
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Cot[c + d*x]*Tan[(c +
 d*x)/2] + 2*m*((a + b)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2
, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Ta
n[(c + d*x)/2]^2 - (2*(1 + m)*((-a^2 + b^2)*AppellF1[(3 + m)/2, m, 2, (5 +
m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a*(a -
 b)*AppellF1[(3 + m)/2, m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[
(c + d*x)/2]^2)/(a + b)] + (a + b)*m*((a + b)*AppellF1[(3 + m)/2, 1 + m, 1,
 (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*
a*AppellF1[(3 + m)/2, 1 + m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)]))*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)/((a + b
)*(3 + m)))) - (2*b^3*((a + b)^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[
(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*(a + b)*AppellF
1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2
]^2)/(a + b)] + 4*a^2*AppellF1[(1 + m)/2, m, 3, (3 + m)/2, -Tan[(c + d*x)/2
]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[c + d*x]^3*(e*Sin[c + d*x])
^m*Tan[(c + d*x)/2])/(a^3*d*(a + b*Sec[c + d*x])^3*(((a + b)^2*AppellF1[(1
+ m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/
(a + b)] - 4*a*(a + b)*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/
2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a^2*AppellF1[(1 + m)/2, m,
3, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*S
ec[(c + d*x)/2]^2 + 2*m*((a + b)^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Ta
n[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*(a + b)*Appel
lF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)
/2]^2)/(a + b)] + 4*a^2*AppellF1[(1 + m)/2, m, 3, (3 + m)/2, -Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Cot[c + d*x]*Tan[(c + d*x)/2]
 + 2*m*((a + b)^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2,
 ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*(a + b)*AppellF1[(1 + m)/2, m,
 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] +
 4*a^2*AppellF1[(1 + m)/2, m, 3, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*T
an[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2 + (2*(1 + m)*((a + b)^2*((a
 - b)*AppellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)] - (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m
)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) - 4*a*(a +
 b)*(2*(a - b)*AppellF1[(3 + m)/2, m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((
a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*m*AppellF1[(3 + m)/2, 1 + m,
2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) +
 4*a^2*(3*(a - b)*AppellF1[(3 + m)/2, m, 4, (5 + m)/2, -Tan[(c + d*x)/2]^2,
 ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*m*AppellF1[(3 + m)/2, 1 +
m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]
))*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) - ((b + a*Cos
[c + d*x])^3*Hypergeometric2F1[1/2, (1 - m)/2, 3/2, Cos[c + d*x]^2]*Sec[c +
 d*x]*(e*Sin[c + d*x])^m*(Sin[c + d*x]^2)^((-1 - m)/2)*Tan[c + d*x])/(a^3*d
*(a + b*Sec[c + d*x])^3)
```

Maple [F]

$$\int \frac{(e \sin (dx+c))^m}{(a+b \sec (dx+c))^3} dx$$

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x)
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x)
```

Fricas [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3} dx = \int \frac{(e \sin (dx+c))^m}{(b \sec (dx+c)+a)^3} dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 +
3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3} dx = \int \frac{(e \sin (c+dx))^m}{(a+b \sec (c+dx))^3} dx$$

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**3,x)
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**3, x)
```

Maxima [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3} dx = \int \frac{(e \sin (dx+c))^m}{(b \sec (dx+c)+a)^3} dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos(c+d\,x)^3\,(e\,\sin(c+d\,x))^m}{(b+a\,\cos(c+d\,x))^3}\,dx$$

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^3,x)
[Out] int((cos(c + d*x)^3*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x))^3, x)
```

3.263 $\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \mathrm{Int}\left((a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

Mathematica [N/A]

Not integrable

Time = 39.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx=\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(e\sin\left(dx+c\right)\right)^{m}dx$$

```
[In] int((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx=\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\left(e\sin\left(dx+c\right)\right)^{m}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(e*sin(d*x+c))**m,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Giac [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Mupad [N/A]

Not integrable

Time = 18.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx=\int (e\,\sin(c+d\,x))^m\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(3/2),x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(3/2), x)
```

3.264 $\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \mathrm{Int}\left(\sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m,x\right)$$

```
[Out] Unintegrable((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$$

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

```
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\text{integral} = \int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$$

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.97 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (e\sin(dx+c))^m\sqrt{a+b\sec(dx+c)}dx$$

```
[In] int((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
[Out] int((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int \sqrt{b\sec(dx+c)+a}(e\sin(dx+c))^m\,dx$$

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [N/A]

Not integrable

Time = 5.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int (e\sin(c+dx))^m\sqrt{a+b\sec(c+dx)}\,dx$$

```
[In] integrate((e*sin(d*x+c))**m*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral((e*sin(c + d*x))**m*sqrt(a + b*sec(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int \sqrt{b\sec{(dx+c)}+a}(e\sin{(dx+c)})^m\,dx$$

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int \sqrt{b\sec{(dx+c)}+a}(e\sin{(dx+c)})^m\,dx$$

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Mupad [N/A]

Not integrable

Time = 15.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx = \int {\left(e\,\sin\left(c+d\,x\right)\right)}^m\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(1/2),x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(1/2), x)
```

3.265 $\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \mathrm{Int}\left(\frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

```
[In] Int[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\text{integral} = \int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 21.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

```
[In] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(e\sin(dx+c))^m}{\sqrt{a+b\sec(dx+c)}}dx$$

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{(e\sin(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral((e*sin(c + d*x))**m/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{(e\sin{(dx+c)})^m}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 78.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{(e\sin{(dx+c)})^m}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 18.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{(e\,\sin{(c+d\,x)})^m}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(1/2), x)
```

3.266 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 20.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(e\sin(dx+c))^m}{(a+b\sec(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 30.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin{(c+dx)})^m}{(a+b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin{(dx+c)})^m}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 38.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin{(dx+c)})^m}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 25.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\,\sin\left(c+d\,x\right))^m}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(3/2), x)
```

3.267 $\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx = \text{Int}((a+b\sec(c+dx))^n(e\sin(c+dx))^m,x)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx = \int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx=\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{n}\left(e\sin\left(dx+c\right)\right)^{m}dx$$

```
[In] int((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx=\int (b\sec\left(dx+c\right)+a)^n(e\sin\left(dx+c\right))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Sympy [N/A]

Not integrable

Time = 161.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx=\int (e\sin\left(c+dx\right))^m\left(a+b\sec\left(c+dx\right)\right)^n\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n*(e*sin(d*x+c))**m,x)
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x))**n, x)
```

Maxima [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx=\int (b\sec{(dx+c)}+a)^n(e\sin{(dx+c)})^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx=\int (b\sec{(dx+c)}+a)^n(e\sin{(dx+c)})^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Mupad [N/A]

Not integrable

Time = 15.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx=\int (e\,\sin{(c+d\,x)})^m\,\left(a+\frac{b}{\cos{(c+d\,x)}}\right)^n\,dx$$

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^n,x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^n, x)
```

3.268 $\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 150

$$\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx = \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2d(1+n)} - \frac{2b^3\,\text{Hypergeometric2F1}\left(4,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^4d(1+n)} + \frac{b^5\,\text{Hypergeometric2F1}\left(6,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^6d(1+n)}$$

```
[Out] b*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a^2/d/(
1+n)-2*b^3*hypergeom([4, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n
)/a^4/d/(1+n)+b^5*hypergeom([6, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c
))^(1+n)/a^6/d/(1+n)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{3959, 186, 67\}$

$$\int (a+b\sec(c+dx))^n\sin^5(c+dx)\,dx$$
$$= \frac{b^5(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(6,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{a^6d(n+1)} - \frac{2b^3(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(4,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{a^4d(n+1)} + \frac{b(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a^2*d*(1 + n)) - (2*b^3*Hypergeometric2F1[4, 1 + n, 2 + n
, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a^4*d*(1 + n)) + (
b^5*Hypergeometric2F1[6, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
 + d*x])^(1 + n))/(a^6*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 3959

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-f^(-1), Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p -
 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(-1+x)^2(1+x)^2(a-bx)^n}{x^6}\,dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{(a-bx)^n}{x^6} - \frac{2(a-bx)^n}{x^4} + \frac{(a-bx)^n}{x^2}\right) dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{x^6} dx, x, -\sec(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{x^2} dx, x, -\sec(c+dx)\right)}{d} + \frac{2\text{Subst}\left(\int \frac{(a-bx)^n}{x^4} dx, x, -\sec(c+dx)\right)}{d}$$

$$= \frac{b \text{ Hypergeometric2F1}\left(2, 1+n, 2+n, 1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2d(1+n)} - \frac{2b^3 \text{ Hypergeometric2F1}\left(4, 1+n, 2+n, 1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^4d(1+n)} + \frac{b^5 \text{ Hypergeometric2F1}\left(6, 1+n, 2+n, 1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^6d(1+n)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 562 vs. $2(150) = 300$.

Time = 6.79 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.75

$$\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx =$$

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)\left(192a^3(-1+n)(b+a\cos(c+dx))^2 - 240a^3(-1+n)(b+a\cos(c+dx))^2\right.}{}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] -1/120*(Cos[(c + d*x)/2]^6*Cos[c + d*x]*(192*a^3*(-1 + n)*(b + a*Cos[c + d*
x])^2 - 240*a^3*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2 - 24*a^2
*(2*a - b*(-4 + n))*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2 + 40
*a^2*(2*a - b*(-3 + n))*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^4
+ a*(1 - n)*(96*a^2 + 4*a*b*(6 - 4*n) - 4*b^2*(12 - 7*n + n^2))*(b + a*Cos[
c + d*x])^2*Sec[(c + d*x)/2]^4 - 10*a*((-1 + n)*(-14*a^2 + 2*a*b*(-1 + n) +
 b^2*(6 - 5*n + n^2))*(b + a*Cos[c + d*x])^2 + b*(24*a^3 + 12*a^2*b*(-1 + n
) - 4*a*b^2*(2 - 3*n + n^2) - b^3*(-6 + 11*n - 6*n^2 + n^3))*Hypergeometric
2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])*Sec[(c + d*x)/
2]^6 + ((-1 + n)*(-84*a^3 + 2*a^2*b*(18 - 7*n) + 4*a*b^2*(9 - 9*n + 2*n^2)
+ b^3*(-24 + 26*n - 9*n^2 + n^3))*(b + a*Cos[c + d*x])^2 + b*(120*a^4 + 120
*a^3*b*(-1 + n) - 10*a*b^3*(-6 + 11*n - 6*n^2 + n^3) - b^4*(24 - 50*n + 35*
n^2 - 10*n^3 + n^4))*Hypergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b
 + a*Cos[c + d*x])])*Sec[(c + d*x)/2]^6)*(a + b*Sec[c + d*x])^n)/(a^4*d*(-1
 + n)*(b + a*Cos[c + d*x]))
```

Maple [F]

$$\int (a + b\sec(dx + c))^n \sin(dx + c)^5 \, dx$$

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

Fricas [F]

$$\int (a + b\sec(c + dx))^n \sin^5(c + dx) \, dx = \int (b\sec(dx + c) + a)^n \sin(dx + c)^5 \, dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="fricas")
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*sec(d*x + c) + a)^n*sin
(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\sec(c + dx))^n \sin^5(c + dx) \, dx = \text{Timed out}$$

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int (a + b\sec(c + dx))^n \sin^5(c + dx) \, dx = \int (b\sec(dx + c) + a)^n \sin(dx + c)^5 \, dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

Giac [F]

$$\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)^5\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx = \int \sin(c+d\,x)^5 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^n, x)
```

3.269 $\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx = \frac{b(6a^2-b^2(2-3n+n^2))\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{6a^4d(1+n)} + \frac{\cos^3(c+dx)(a+b\sec(c+dx))^{1+n}(2a-b(2-n)\sec(c+dx))}{6a^2d}$$

```
[Out] 1/6*b*(6*a^2-b^2*(n^2-3*n+2))*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a
+b*sec(d*x+c))^(1+n)/a^4/d/(1+n)+1/6*cos(d*x+c)^3*(a+b*sec(d*x+c))^(1+n)*(2
*a-b*(2-n)*sec(d*x+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3959, 150, 67}

$$\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx = \frac{\cos^3(c+dx)(2a-b(2-n)\sec(c+dx))(a+b\sec(c+dx))^{n+1}}{6a^2d} + \frac{b(6a^2-b^2(n^2-3n+2))\,(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{6a^4d(n+1)}$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (b*(6*a^2 - b^2*(2 - 3*n + n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*
Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(6*a^4*d*(1 + n)) + (Cos[c +
 d*x]^3*(a + b*Sec[c + d*x])^(1 + n)*(2*a - b*(2 - n)*Sec[c + d*x]))/(6*a^2
*d)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 3959

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-f^(-1), Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p -
 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(-1+x)(1+x)(a-bx)^n}{x^4}\,dx, x, -\sec(c+dx)\right)}{d}\\
&= \frac{\cos^3(c+dx)(a+b\sec(c+dx))^{1+n}(2a-b(2-n)\sec(c+dx))}{6a^2d}\\
&\quad -\frac{\left(6-\frac{b^2(1-n)(2-n)}{a^2}\right)\text{Subst}\left(\int \frac{(a-bx)^n}{x^2}\,dx, x, -\sec(c+dx)\right)}{6d}\\
&= \frac{b(6a^2-b^2(2-3n+n^2))\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^1}{6a^4d(1+n)}\\
&\quad +\frac{\cos^3(c+dx)(a+b\sec(c+dx))^{1+n}(2a-b(2-n)\sec(c+dx))}{6a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.28

$$\int (a+b\sec(c+dx))^n\sin^3(c+dx)\,dx$$
$$=\frac{\cos(c+dx)\left(-\frac{2(2a-b(-2+n))(b+a\cos(c+dx))^2}{a}+8\cos^2\left(\frac{1}{2}(c+dx)\right)(b+a\cos(c+dx))^2-\frac{2b(-6a^2+b^2(2-3n+n^2))\,\text{Hy}}{}\right.}{12ad(b+a\cos(c+dx))}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (Cos[c + d*x]*((-2*(2*a - b*(-2 + n))*(b + a*Cos[c + d*x])^2)/a + 8*Cos[(c
+ d*x)/2]^2*(b + a*Cos[c + d*x])^2 - (2*b*(-6*a^2 + b^2*(2 - 3*n + n^2))*Hy
pergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])/(a
*(-1 + n)))*(a + b*Sec[c + d*x])^n)/(12*a*d*(b + a*Cos[c + d*x]))
```

Maple [F]

$$\int (a+b\sec(dx+c))^n\sin(dx+c)^3\,dx$$

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

Fricas [F]

$$\int (a+b\sec(c+dx))^n\sin^3(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\sin(dx+c)^3\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^n\sin^3(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + b\sec(c + dx))^n \sin^3(c + dx)\, dx = \int (b\sec(dx + c) + a)^n \sin(dx + c)^3\, dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a + b\sec(c + dx))^n \sin^3(c + dx)\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\sec(c + dx))^n \sin^3(c + dx)\, dx = \int \sin(c + d\,x)^3 \left(a + \frac{b}{\cos(c + d\,x)}\right)^n dx$$

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

3.270 $\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx = \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2d(1+n)}$$

```
[Out] b*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a^2/d/(
1+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3959, 67}

$$\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx = \frac{b(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3959

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-f^(-1), Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p -
 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{x^2}\,dx, x, -\sec(c+dx)\right)}{d}\\ &= \frac{b\,\text{Hypergeometric2F1}\left(2, 1+n, 2+n, 1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2d(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.50

$$\begin{aligned}&\int (a+b\sec(c+dx))^n\sin(c+dx)\,dx\\ &= \frac{b\cos(c+dx)\,\text{Hypergeometric2F1}\left(2, 1-n, 2-n, \frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)(a+b\sec(c+dx))^n}{d(-1+n)(b+a\cos(c+dx))}\end{aligned}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x],x]
[Out] (b*Cos[c + d*x]*Hypergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*
Cos[c + d*x])]*(a + b*Sec[c + d*x])^n)/(d*(-1 + n)*(b + a*Cos[c + d*x]))
```

Maple [F]

$$\int (a+b\sec(dx+c))^n\sin(dx+c)\,dx$$

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c),x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c),x)
```

Fricas [F]

$$\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c),x)
[Out] Integral((a + b*sec(c + d*x))**n*sin(c + d*x), x)
```

Maxima [F]

$$\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Giac [F]

$$\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int (a + b\sec(c + dx))^n \sin(c + dx)\, dx$$
$$= \frac{\cos\left(c + d\,x\right)\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^n\,{}_2\mathrm{F}_1\left(1 - n, -n;\ 2 - n;\ -\frac{a\,\cos(c+d\,x)}{b}\right)}{d\,\left(\frac{a\,\cos(c+d\,x)}{b} + 1\right)^n\,(n - 1)}$$

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x))^n,x)
[Out] (cos(c + d*x)*(a + b/cos(c + d*x))^n*hypergeom([1 - n, -n], 2 - n, -(a*cos(
c + d*x))/b))/(d*((a*cos(c + d*x))/b + 1)^n*(n - 1))
```

3.271 $\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)} - \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{2(a+b)d(1+n)}$$

```
[Out] 1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n)
/(a-b)/d/(1+n)-1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*se
c(d*x+c))^(1+n)/(a+b)/d/(1+n)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3959, 88, 70}

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)} - \frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}$$

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a + b
)*d*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 3959

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-f^(-1), Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p -
 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(-1+x)(1+x)}\,dx, x, -\sec(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{-1+x}\,dx, x, -\sec(c+dx)\right)}{2d}+\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{1+x}\,dx, x, -\sec(c+dx)\right)}{2d}\\
&= \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)}\\
&\quad-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{2(a+b)d(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx$$
$$= \frac{\left(\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{(a+b)\cos(c+dx)}{b+a\cos(c+dx)}\right)-2^n\,\text{Hypergeometric2F1}\left(-n,-n,1-n,\frac{(-a+b)\cos(c+dx}{2}\right.\right.}{2dn}$$

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((Hypergeometric2F1[1, -n, 1 - n, ((a + b)*Cos[c + d*x])/(b + a*Cos[c + d*x
])] - (2^n*Hypergeometric2F1[-n, -n, 1 - n, ((-a + b)*Cos[c + d*x]*Sec[(c +
 d*x)/2]^2)/(2*b)])/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/b)^n)*(a + b
*Sec[c + d*x])^n)/(2*d*n)
```

Maple [F]

$$\int \csc\left(dx+c\right)\left(a+b\sec\left(dx+c\right)\right)^{n}dx$$

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \int \left(b\sec\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Sympy [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^{n}\csc\left(c+dx\right)dx$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*csc(c + d*x), x)
```

Maxima [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\csc(dx+c)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Giac [F]

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\csc(dx+c)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)}\,dx$$

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x),x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x), x)
```

3.272 $\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 231

$$\begin{aligned}\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx &= \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)d(1+n)}\\ &\quad-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)d(1+n)}\\ &\quad+\frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)}\\ &\quad+\frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)^2d(1+n)}\end{aligned}$$

```
[Out] 1/4*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n)
/(a-b)/d/(1+n)-1/4*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*se
c(d*x+c))^(1+n)/(a+b)/d/(1+n)+1/4*b*hypergeom([2, 1+n],[2+n],(a+b*sec(d*x+c
))/(a-b))*(a+b*sec(d*x+c))^(1+n)/(a-b)^2/d/(1+n)+1/4*b*hypergeom([2, 1+n],[
2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*sec(d*x+c))^(1+n)/(a+b)^2/d/(1+n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3959, 186, 70, 726}

$$\begin{aligned}&\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx\\&=\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{4d(n+1)(a-b)}\\&-\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a+b}\right)}{4d(n+1)(a+b)}\\&+\frac{b(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{4d(n+1)(a-b)^2}\\&+\frac{b(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{a+b\sec(c+dx)}{a+b}\right)}{4d(n+1)(a+b)^2}\end{aligned}$$

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(4*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a + b
)*d*(1 + n)) + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(
a - b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a - b)^2*d*(1 + n)) + (b*Hypergeo
metric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x
])^(1 + n))/(4*(a + b)^2*d*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3959

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-f^(-1), Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p -
 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2(a-bx)^n}{(-1+x)^2(1+x)^2}\,dx, x, -\sec(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{(a-bx)^n}{4(-1+x)^2} + \frac{(a-bx)^n}{4(1+x)^2} + \frac{(a-bx)^n}{2(-1+x^2)}\right) dx, x, -\sec(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(-1+x)^2}\,dx, x, -\sec(c+dx)\right)}{4d} - \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(1+x)^2}\,dx, x, -\sec(c+dx)\right)}{4d} \\
&\quad - \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{-1+x^2}\,dx, x, -\sec(c+dx)\right)}{2d} \\
&= \frac{b\,\text{Hypergeometric2F1}\left(2, 1+n, 2+n, \frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} \\
&\quad + \frac{b\,\text{Hypergeometric2F1}\left(2, 1+n, 2+n, \frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)^2d(1+n)} \\
&\quad - \frac{\text{Subst}\left(\int \left(-\frac{(a-bx)^n}{2(1-x)} - \frac{(a-bx)^n}{2(1+x)}\right) dx, x, -\sec(c+dx)\right)}{2d} \\
&= \frac{b\,\text{Hypergeometric2F1}\left(2, 1+n, 2+n, \frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} \\
&\quad + \frac{b\,\text{Hypergeometric2F1}\left(2, 1+n, 2+n, \frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)^2d(1+n)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{1-x}\,dx, x, -\sec(c+dx)\right)}{4d} + \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{1+x}\,dx, x, -\sec(c+dx)\right)}{4d}
\end{aligned}
$$

$$= \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)d(1+n)} - \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)d(1+n)} + \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} + \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)^2d(1+n)}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1520 vs. 2(231) = 462.

Time = 16.83 (sec) , antiderivative size = 1520, normalized size of antiderivative = 6.58

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$= \frac{(a+b)(b+a\cos(c+dx))^{-n}\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(\frac{(-a+b)\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{b}\right)^n(\cos(c+dx)\,\text{se}}{}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((a + b)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(((-a + b)*Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)/b)^n*(a + b*Sec[c + d*x])^n*((1 - Tan[(c + d*x)/2]^2)^(-1))^n
*(1 - Tan[(c + d*x)/2]^4)^n*(b + (a - a*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2))^n*(-((Hypergeometric2F1[n, 1 + n, 2 + n, (a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(2*b)]*(((a - b)*(-1 + Tan[(c + d*x)/2]^2))/
b)^n*(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)^(1 + n))/((a - b
)*(1 + n)*(2 - 2*Tan[(c + d*x)/2]^2)^n)) + (Hypergeometric2F1[1, -n, 1 - n,
 ((a + b)*(-1 + Tan[(c + d*x)/2]^2))/(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 +
Tan[(c + d*x)/2]^2))]*(a + b + (-a + b)*Tan[(c + d*x)/2]^2)^n)/(n*(1 - Tan[
(c + d*x)/2]^2)^n) - (Cot[(c + d*x)/2]^2*(1 - Tan[(c + d*x)/2]^2)^(1 - n)*(
a + b + (-a + b)*Tan[(c + d*x)/2]^2)^(1 + n))/(a + b) - (2^n*Hypergeometric
2F1[-n, -n, 1 - n, ((a - b)*(-1 + Tan[(c + d*x)/2]^2))/(2*b)]*(a + b + (-a
+ b)*Tan[(c + d*x)/2]^2)^n)/(n*(1 - Tan[(c + d*x)/2]^2)^n*((a + b + (-a + b
)*Tan[(c + d*x)/2]^2)/b)^n) - ((Hypergeometric2F1[n, 1 + n, 2 + n, (a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(2*b)]*(((a - b)*(-1 + Tan[(c
 + d*x)/2]^2))/b)^n*(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)^(
1 + n))/((1 + n)*(2 - 2*Tan[(c + d*x)/2]^2)^n) + ((a - b)*Hypergeometric2F1
```

```
[1, -n, 1 - n, ((a + b)*(-1 + Tan[(c + d*x)/2]^2))/(a*(-1 + Tan[(c + d*x)/2
]^2) - b*(1 + Tan[(c + d*x)/2]^2))]*(a + b + (-a + b)*Tan[(c + d*x)/2]^2)^n
)/(n*(1 - Tan[(c + d*x)/2]^2)^n) - (2^n*(a - b)*Hypergeometric2F1[-n, -n, 1
 - n, ((a - b)*(-1 + Tan[(c + d*x)/2]^2))/(2*b)]*(a + b + (-a + b)*Tan[(c +
 d*x)/2]^2)^n)/(n*(1 - Tan[(c + d*x)/2]^2)^n*((a + b + (-a + b)*Tan[(c + d*
x)/2]^2)/b)^n) - (2*(a + b*n)*(a + b + (-a + b)*Tan[(c + d*x)/2]^2)^n*(-(2^
n*Hypergeometric2F1[-n, -n, 1 - n, ((a - b)*(-1 + Tan[(c + d*x)/2]^2))/(2*b
)]) + Hypergeometric2F1[1, -n, 1 - n, ((a + b)*(-1 + Tan[(c + d*x)/2]^2))/(
a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2))]*((a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/b)^n))/(n*(1 - Tan[(c + d*x)/2]^2)^n
*((a + b + (-a + b)*Tan[(c + d*x)/2]^2)/b)^n))/(a + b)))/(2*d*(b + a*Cos[c
+ d*x])^n*(Cos[c + d*x]*Sec[(c + d*x)/2]^4)^n*(Cos[(c + d*x)/2]^2*Sec[c + d
*x])^n*(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)^n*(a*(((-a + b
)*Cos[c + d*x]*Sec[(c + d*x)/2]^2)/b)^n + 4*b*(((-a + b)*Cos[c + d*x]*Sec[(
c + d*x)/2]^2)/b)^n + 3*a*(((a - b)*(-1 + Tan[(c + d*x)/2]^2))/b)^n + 4*a*C
os[c + d*x]*((((-a + b)*Cos[c + d*x]*Sec[(c + d*x)/2]^2)/b)^n - (((a - b)*(
-1 + Tan[(c + d*x)/2]^2))/b)^n) - a*Cos[2*(c + d*x)]*((((-a + b)*Cos[c + d*
x]*Sec[(c + d*x)/2]^2)/b)^n - (((a - b)*(-1 + Tan[(c + d*x)/2]^2))/b)^n)))
```

Maple [F]

$$\int \csc \left(dx+c\right)^{3} \left(a+b\sec \left(dx+c\right)\right)^{n}dx$$

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
[Out] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx=\int \left(b\sec \left(dx+c\right)+a\right)^{n}\csc \left(dx+c\right)^{3}\,dx$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\csc(dx+c)^3\,dx$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Giac [F]

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\csc(dx+c)^3\,dx$$

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^3}\,dx$$

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^3,x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^3, x)
```

3.273 $\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx = \text{Int}\left((a+b\sec(c+dx))^n \sin^4(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4, x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx=\int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (a+b\sec(dx+c))^n\sin(dx+c)^4\,dx$$

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\sin(dx+c)^4\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="fricas")
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*sec(d*x + c) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**4,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx=\int (b\sec{(dx+c)}+a)^n\sin{(dx+c)}^4\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx=\int (b\sec{(dx+c)}+a)^n\sin{(dx+c)}^4\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Mupad [N/A]

Not integrable

Time = 18.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx=\int \sin{(c+d\,x)}^4\,\left(a+\frac{b}{\cos{(c+d\,x)}}\right)^n\,dx$$

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

3.274 $\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx = \mathrm{Int}\big((a+b\sec(c+dx))^n \sin^2(c+dx),x\big)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2, x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx=\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (a+b\sec(dx+c))^n\sin(dx+c)^2\,dx$$

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\sin(dx+c)^2\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sec(d*x + c) + a)^n, x)
```

Sympy [N/A]

Not integrable

Time = 37.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx=\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**2,x)
[Out] Integral((a + b*sec(c + d*x))**n*sin(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx = \int (b\sec(dx+c)+a)^n\sin(dx+c)^2\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx = \int (b\sec(dx+c)+a)^n\sin(dx+c)^2\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 15.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx = \int \sin(c+d\,x)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

3.275 $\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} + \frac{\sqrt{2}bn\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},1-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{ta}}{(a+b)d\sqrt{1+\sec(c+dx)}}$$

```
[Out] -cot(d*x+c)*(a+b*sec(d*x+c))^n/d+b*n*AppellF1(1/2,1-n,1/2,3/2,b*(1-sec(d*x+
c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/(a+b)/d
/(((a+b*sec(d*x+c))/(a+b))^n)/(1+sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3960, 3919, 144, 143}

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx$$
$$= \frac{\sqrt{2}bn\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},1-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(}{a+}\right.}{d(a+b)\sqrt{\sec(c+dx)+1}}$$
$$- \frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d}$$

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] -((Cot[c + d*x]*(a + b*Sec[c + d*x])^n)/d) + (Sqrt[2]*b*n*AppellF1[1/2, 1/2
, 1 - n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*
```

```
Sec[c + d*x])^n*Tan[c + d*x])/((a + b)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec
[c + d*x])/(a + b))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rubi steps

$$\text{integral} = -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} + (bn)\int \sec(c+dx)(a+b\sec(c+dx))^{-1+n}\,dx$$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} - \frac{(bn\tan(c+dx))\text{Subst}\left(\int \frac{(a+bx)^{-1+n}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} - \frac{\left(bn(a+b\sec(c+dx))^n\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{-n}\tan(c+dx)\right)\text{Subst}\left(\int\frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{-1+n}}{\sqrt{1-x}\sqrt{1+x}}dx,x,\sec(c\right.}{(a+b)d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} + \frac{\sqrt{2}bn\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},1-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx}{a+b}\right.}{(a+b)d\sqrt{1+\sec(c+dx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3614 vs. $2(136) = 272$.

Time = 17.33 (sec) , antiderivative size = 3614, normalized size of antiderivative = 26.57

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx = \text{Result too large to show}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Csc[c + d*x]^2*Sec[c + d*x]^n*(a +
 b*Sec[c + d*x])^n*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n)/(((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*AppellF1[1/2
, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan
[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, (
(a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n
, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*
AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]
^2)/(a + b)])*Tan[(c + d*x)/2]^2)))/(2*d*(-1/4*((b + a*Cos[c + d*x])^n*Csc[
(c + d*x)/2]^2*Sec[c + d*x]^n*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/
2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^n)/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*A
ppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(
a + b)]*Tan[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/
2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]
 + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(
c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2))) - (a*n*(b + a*Cos[c + d*x])^
(-1 + n)*Cot[(c + d*x)/2]*Sec[c + d*x]^n*Sin[c + d*x]*(-((AppellF1[-1/2, n,
 -n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c
+ d*x]*Sec[(c + d*x)/2]^2)^n)/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b))^n) + (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a -
```

b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2
, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2
n((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[
(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)))/2 +
(n*(b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Sec[c + d*x]^(1 + n)*Sin[c + d*x
]*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)
/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n)/(((b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2)/
(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 +
 n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan
[(c + d*x)/2]^2)))/2 + ((b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Sec[c + d*x
]^n*(-(((Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(((a - b)*n*AppellF1[1/2, n, 1
- n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])/(a + b) - n*AppellF1[1/2, 1 + n, -n, 3/2, Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) -
(n*AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]
^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1 + n)*(-(Sec[(c + d*x)/2]
^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n + n*AppellF1[-1/2, n, -n,
1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x
]*Sec[(c + d*x)/2]^2)^n*(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))
^(-1 - n)*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)) + (3*(a + b)*AppellF1[
1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, T
an[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*Ap
pellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)
/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2) + (3*(a + b)*Tan[(c +
 d*x)/2]^2*(-1/3*((a - b)*n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2
, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
)/(a + b) + (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*T
an[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3))/(3*(a
+ b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2
]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]
^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n
, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c +
d*x)/2]^2) - (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2*(2*n*((-a + b)*AppellF1
[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +

```
b)] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 3*(a + b)
*(-1/3*((a - b)*n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)
+ (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3) + 2*n*Tan[(c + d*
x)/2]^2*((-a + b)*((3*(a - b)*(1 - n)*AppellF1[5/2, n, 2 - n, 7/2, Tan[(c +
 d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/(5*(a + b)) + (3*n*AppellF1[5/2, 1 + n, 1 - n, 7/2, Tan[(c + d*
x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/5) + (a + b)*((-3*(a - b)*n*AppellF1[5/2, 1 + n, 1 - n, 7/2, Tan[(
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2])/(5*(a + b)) + (3*(1 + n)*AppellF1[5/2, 2 + n, -n, 7/2, Tan[(
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2])/5))))/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^
2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1
 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a +
 b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x
)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)^2))/2))
```

Maple [F]

$$\int \csc \left(dx+c\right)^{2} \left(a+b\sec \left(dx+c\right)\right)^{n} dx$$

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
[Out] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int \left(b\sec\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right)^{2}\,dx$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Sympy [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx=\int(a+b\sec(c+dx))^n\csc^2(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**n,x)
[Out] Integral((a + b*sec(c + d*x))**n*csc(c + d*x)**2, x)
```

Maxima [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx=\int(b\sec(dx+c)+a)^n\csc(dx+c)^2\,dx$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Giac [F]

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx=\int(b\sec(dx+c)+a)^n\csc(dx+c)^2\,dx$$

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx=\int\frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^2}\,dx$$

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^2,x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^2, x)
```

3.276 $\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 424

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{3\,\mathrm{AppellF1}\left(-\frac{1}{2},\frac{5}{2},-n,\frac{1}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\cot(c+dx)\sqrt{1+\sec(c+dx)}(a+b\sec(c+}{2\sqrt{2}d}$$
$$-\frac{\mathrm{AppellF1}\left(-\frac{3}{2},\frac{5}{2},-n,-\frac{1}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\cot^3(c+dx)(1+\sec(c+dx))^{3/2}(a+b\sec(c}{6\sqrt{2}d}$$
$$+\frac{\mathrm{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\tan(c+dx}{\sqrt{2}d\sqrt{1+\sec(c+dx)}}$$
$$+\frac{\mathrm{AppellF1}\left(\frac{1}{2},\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\tan(c+dx}{2\sqrt{2}d\sqrt{1+\sec(c+dx)}}$$

```
[Out] -1/12*AppellF1(-3/2,-n,5/2,-1/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*
cot(d*x+c)^3*(1+sec(d*x+c))^(3/2)*(a+b*sec(d*x+c))^n/d/(((a+b*sec(d*x+c))/(
a+b))^n)*2^(1/2)-3/4*AppellF1(-1/2,-n,5/2,1/2,b*(1-sec(d*x+c))/(a+b),1/2-1/
2*sec(d*x+c))*cot(d*x+c)*(a+b*sec(d*x+c))^n*(1+sec(d*x+c))^(1/2)/d/(((a+b*s
ec(d*x+c))/(a+b))^n)*2^(1/2)+1/2*AppellF1(1/2,-n,3/2,3/2,b*(1-sec(d*x+c))/(
a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*tan(d*x+c)/d/(((a+b*sec(d*x+c))
/(a+b))^n)*2^(1/2)/(1+sec(d*x+c))^(1/2)+1/4*AppellF1(1/2,-n,5/2,3/2,b*(1-se
c(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*tan(d*x+c)/d/(((a+b*
sec(d*x+c))/(a+b))^n)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

Rubi [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]

[Out] Defer[Int][Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]

Rubi steps

$$\text{integral} = \int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5928 vs. $2(424) = 848$.

Time = 23.44 (sec) , antiderivative size = 5928, normalized size of antiderivative = 13.98

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \text{Result too large to show}$$

[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]

[Out] Result too large to show

Maple [F]

$$\int \csc(dx+c)^4 (a+b\sec(dx+c))^n\,dx$$

[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x)

[Out] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x)

Fricas [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \csc(dx+c)^4\,dx$$

[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="fricas")

[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)

Sympy [F(-1)]

Timed out.

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\csc(dx+c)^4\,dx$$

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Giac [F]

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\csc(dx+c)^4\,dx$$

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^4}\,dx$$

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^4,x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^4, x)
```

3.277 $\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \text{Int}\left((a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2), x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n\sin^{\frac{3}{2}}(c+dx)\,dx=\int (a+b\sec(c+dx))^n\sin^{\frac{3}{2}}(c+dx)\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(a+b\sec\left(dx+c\right)\right)^{n}\sin\left(dx+c\right)^{\frac{3}{2}}dx$$

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n\sin^{\frac{3}{2}}(c+dx)\,dx=\int \left(b\sec\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^n\sin^{\frac{3}{2}}(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 22.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 16.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \int \sin(c+d\,x)^{3/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(sin(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n, x)
```

3.278 $\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx = \text{Int}\Big((a+b\sec(c+dx))^n\sqrt{\sin(c+dx)},x\Big)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx = \int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]], x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(a+b\sec\left(dx+c\right)\right)^{n}\sqrt{\sin\left(dx+c\right)}dx$$

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int \left(b\sec\left(dx+c\right)+a\right)^{n}\sqrt{\sin\left(dx+c\right)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 16.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int \left(a+b\sec\left(c+dx\right)\right)^{n}\sqrt{\sin\left(c+dx\right)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))**n*sqrt(sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int (b\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int (b\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 15.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int \sqrt{\sin(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(sin(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n, x)
```

3.279 $\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]], x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec(dx+c))^n}{\sqrt{\sin(dx+c)}}dx$$

```
[In] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
[Out] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 3.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n/sin(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))**n/sqrt(sin(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 16.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sqrt{\sin(c+d\,x)}}\,dx$$

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(1/2),x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(1/2), x)
```

3.280 $\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)} dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}, x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)} dx = \int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)} dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)} dx$$

Mathematica [N/A]

Not integrable

Time = 10.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.79 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec(dx+c))^n}{\sin(dx+c)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
[Out] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral(-(b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c))/(cos(d*x + c)^2 - 1), x
)
```

Sympy [N/A]

Not integrable

Time = 111.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n/sin(d*x+c)**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**n/sin(c + d*x)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 10.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 16.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^{3/2}}\,dx$$

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(3/2),x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(3/2), x)
```

3.281 $\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 190

$$
\begin{aligned}
&\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \\
&-\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} \\
&+\frac{ae^2\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&+\frac{ae^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&+\frac{2ae^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d}
\end{aligned}
$$

```
[Out] -2/3*a*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/d-2/3*a*e^2*csc(d*x+c)*(e*csc(d*
x+c))^(1/2)/d+a*e^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)/d+a*e^2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(
1/2)/d-2/3*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d
*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3963, 3957, 2917, 2644, 331, 335, 218, 212, 209, 2716, 2720}

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \frac{ae^2\sqrt{\sin(c+dx)}\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d} + \frac{ae^2\sqrt{\sin(c+dx)}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{2ae^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{3d}$$

```
[In] Int[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (2*a*e^2*Csc[c + d*x]*
Sqrt[e*Csc[c + d*x]])/(3*d) + (a*e^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]])/d + (a*e^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e
*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a*e^2*Sqrt[e*Csc[c + d*x]]*Ellipt
icF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\left(\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx\right)$$

$$= \left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}\,dx + \left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{1}{3}\left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx + \frac{\left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{x^{5/2}(1-x^2)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{2ae^2\sqrt{e\csc(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d} + \frac{\left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{2ae^2\sqrt{e\csc(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d} + \frac{\left(2ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d}$$

$$
\begin{aligned}
&= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} \\
&\quad + \frac{2ae^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d} \\
&\quad + \frac{\left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d} \\
&\quad + \frac{\left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d} \\
&= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} \\
&\quad + \frac{ae^2\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{ae^2\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{2ae^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.71

$$
\int(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx =
$$

$$
-\frac{a(e\csc(c+dx))^{5/2}\left(6\arctan\left(\sqrt{\csc(c+dx)}\right)+4\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\csc(c+dx)}+3\log\left(1-\sqrt{\csc(c+dx)}\right.\right.}{6d\csc^{\frac{5}{2}}(c+d}
$$

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/6*(a*(e*Csc[c + d*x])^(5/2)*(6*ArcTan[Sqrt[Csc[c + d*x]]] + 4*Cot[(c + d
*x)/2]*Sqrt[Csc[c + d*x]] + 3*Log[1 - Sqrt[Csc[c + d*x]]] - 3*Log[1 + Sqrt[
Csc[c + d*x]]] + 4*Sqrt[Csc[c + d*x]]*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*S
qrt[Sin[c + d*x]]))/(d*Csc[c + d*x]^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.82

method	result
default	$\frac{a\sqrt{2}\left(\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}\right)^{\frac{5}{2}}(1-\cos(dx+c))^2\left(2i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{2}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx-}\right.}{6d\sqrt{(1-\cos(dx+c))^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx+c)}\,\sqrt{(1-\mathrm{co}}}$
parts	$\frac{a\sqrt{2}\left(\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}\right)^{\frac{5}{2}}(1-\cos(dx+c))^2\left(2i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{2}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx-}\right.}{6d\sqrt{(1-\cos(dx+c))^3\csc(dx+c)^3+\csc(dx+c)-\cot(dx+c)}\,\sqrt{(1-\mathrm{co}}}$

```
[In] int((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a/d*2^(1/2)*(e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)+sin(d*x+c)))
^(5/2)*(1-cos(d*x+c))^2*(2*I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*2^(1/2)*(
-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*Elli
pticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*(-cot(d*x+c)+csc(d*
x+c))+(1-cos(d*x+c))^4*csc(d*x+c)^4-1)/((1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d
*x+c)-cot(d*x+c))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)*c
sc(d*x+c))^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^2*csc(d*x+c)^2+1/3*a/d*e
^2*(e*csc(d*x+c))^(1/2)*(3*cos(d*x+c)*arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)*(cot(d*x+c)+csc(d*x+c)))-3*cos(d*x+c)*arctanh((sin(d*x+c)/(cos(d*x+c)
+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))-2*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
-3*arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))+3*ar
ctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c))))/(sin(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*csc(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 682, normalized size of antiderivative = 3.59

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \left[-\frac{6\,a\sqrt{-e}e^2\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)\sin\left(dx+c\right)-3\,a\sqrt{-e}e^2\,\mathrm{lo}}{}\right.$$

$$-\frac{6\,ae^{\frac{5}{2}}\arctan\left(\frac{\left(\cos(dx+c)^2+6\,\sin(dx+c)-2\right)\sqrt{e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)-e)}\right)\sin\left(dx+c\right)-3\,ae^{\frac{5}{2}}\log\left(\frac{e\cos(dx+c)^4-72\,e\cos(dx+c)^2+8\,\left(\cos(dx\right.}{}\right.}{}$$

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/24*(6*a*sqrt(-e)*e^2*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*
sqrt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*sin(d*x + c) - 3*a*sqrt
(-e)*e^2*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 -
9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e
/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x
+ c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))*sin(d
*x + c) + 8*I*a*sqrt(2*I*e)*e^2*sin(d*x + c)*weierstrassPInverse(4, 0, cos(
d*x + c) + I*sin(d*x + c)) - 8*I*a*sqrt(-2*I*e)*e^2*sin(d*x + c)*weierstras
sPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 16*(a*e^2*cos(d*x + c) + a
*e^2)*sqrt(e/sin(d*x + c)))/(d*sin(d*x + c)), -1/24*(6*a*e^(5/2)*arctan(1/4
*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(
d*x + c) - e))*sin(d*x + c) - 3*a*e^(5/2)*log((e*cos(d*x + c)^4 - 72*e*cos(
d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*
sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e
)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)
^2 - 2)*sin(d*x + c) + 8))*sin(d*x + c) + 8*I*a*sqrt(2*I*e)*e^2*sin(d*x + c
)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) - 8*I*a*sqrt(-2*
I*e)*e^2*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
c)) + 16*(a*e^2*cos(d*x + c) + a*e^2)*sqrt(e/sin(d*x + c)))/(d*sin(d*x + c)
)]
```

Sympy [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate((e*csc(d*x+c))**(5/2)*(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \int (e\csc(dx+c))^{\frac{5}{2}}\,(a\sec(dx+c)+a)\,dx$$

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \int (e\csc(dx+c))^{\frac{5}{2}}\,(a\sec(dx+c)+a)\,dx$$

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)\,\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}\,dx$$

```
[In] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2), x)
```

3.282 $\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = -\frac{2ae\sqrt{e\csc(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{ae\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} + \frac{ae\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} - \frac{2ae\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d}$$

```
[Out] -2*a*e*(e*csc(d*x+c))^(1/2)/d-2*a*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/d-a*e*a
rctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+a*e*arctanh
(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a*e*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4
*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3963, 3957, 2917, 2644, 331, 335, 304, 209, 212, 2716, 2719}

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = -\frac{ae\sqrt{\sin(c+dx)}\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d} + \frac{ae\sqrt{\sin(c+dx)}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\sqrt{\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{d}$$

```
[In] Int[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*e*Sqrt[e*Csc[c + d*x]])/d - (2*a*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])
/d - (a*e*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]
])/d + (a*e*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]])/d - (2*a*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{a+a\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx\\
&= -\left(\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{(-a-a\cos(c+dx))\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx\right)\\
&= \left(ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx\\
&\quad+\left(ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx\\
&= -\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\left(ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx\\
&\quad+\frac{\left(ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1}{x^{3/2}(1-x^2)}\,dx,x,\sin(c+dx)\right)}{d}\\
&= -\frac{2ae\sqrt{e\csc(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}\\
&\quad-\frac{2ae\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d}\\
&\quad+\frac{\left(ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int\frac{\sqrt{x}}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\
&= -\frac{2ae\sqrt{e\csc(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}\\
&\quad-\frac{2ae\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d}\\
&\quad+\frac{\left(2ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d}
\end{aligned}$$

$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{\left(ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d} - \frac{\left(ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d}$$

$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{ae\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} + \frac{ae\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} - \frac{2ae\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.86

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \frac{a(e\csc(c+dx))^{3/2}\left(2\arctan\left(\sqrt{\csc(c+dx)}\right) - 4(1+\cos(c+dx))\sqrt{\csc(c+dx)} - \log\left(\right.\right.}{2}$$

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(e*Csc[c + d*x])^(3/2)*(2*ArcTan[Sqrt[Csc[c + d*x]]] - 4*(1 + Cos[c + d*
x])*Sqrt[Csc[c + d*x]] - Log[1 - Sqrt[Csc[c + d*x]]] + Log[1 + Sqrt[Csc[c +
 d*x]]] + (2*Csc[c + d*x]^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d
*x]^2]*Sin[2*(c + d*x)])/Sqrt[-Cot[c + d*x]^2]))/(2*d*Csc[c + d*x]^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 639, normalized size of antiderivative = 3.78

method	result
default	$\frac{a\sqrt{2}\, e\sqrt{e\csc(dx+c)}\,\left(2\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\operatorname{EllipticE}\left(\sqrt{-}\right.\right.}{}$
parts	$\frac{a\sqrt{2}\, e\sqrt{e\csc(dx+c)}\,\left(2\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\operatorname{EllipticE}\left(\sqrt{-}\right.\right.}{}$

```
[In] int((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] a/d*2^(1/2)*e*(e*csc(d*x+c))^(1/2)*(2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*
(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*Ell
ipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-(-I*(I-
cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot
(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1
/2*2^(1/2))*cos(d*x+c)+2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*
x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I
-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(d*x+c)+csc(d*x+c)))^
(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/
2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-2^(1/2))-a/d
*(e*csc(d*x+c))^(1/2)*e/(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(2*(sin(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)+arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x
+c)+csc(d*x+c)))*cot(d*x+c)+arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(co
t(d*x+c)+csc(d*x+c)))*cot(d*x+c)-arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
*(cot(d*x+c)+csc(d*x+c)))*csc(d*x+c)-arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)*(cot(d*x+c)+csc(d*x+c)))*csc(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.60

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \left[-\frac{2\,a\sqrt{-e}e\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)-a\sqrt{-e}e\log\left(\frac{e\cos(dx+c)^4-}{}\right.}{}\right.$$

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a*sqrt(-e)*e*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqr
t(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e)) - a*sqrt(-e)*e*log((e*cos(
d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 + (
7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c)) + 28*
(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 8*a*sqrt(2*I*e)*e*weier
strassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 8*a*sqrt(-2*I*e)*e*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 16*(a*e*cos(d*x + c) + a*e)*sqrt(e/sin(d*x + c)
))/d, 1/8*(2*a*e^(3/2)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqr
t(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e)) + a*e^(3/2)*log((e*cos(d*x
+ c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*co
s(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*co
s(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2
 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 8*a*sqrt(2*I*e)*e*weierstras
sZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 8*a
*sqrt(-2*I*e)*e*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c
) - I*sin(d*x + c))) - 16*(a*e*cos(d*x + c) + a*e)*sqrt(e/sin(d*x + c)))/d]
```

Sympy [F]

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = a\left(\int (e\csc{(c+dx)})^{\frac{3}{2}}\,dx + \int (e\csc{(c+dx)})^{\frac{3}{2}}\sec{(c+dx)}\,dx\right)$$

```
[In] integrate((e*csc(d*x+c))**(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral((e*csc(c + d*x))**(3/2), x) + Integral((e*csc(c + d*x))**(3/2)*
sec(c + d*x), x))
```

Maxima [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \int (e\csc(dx+c))^{\frac{3}{2}}\,(a\sec(dx+c)+a)\,dx$$

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)\,\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}\,dx$$

```
[In] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2), x)
```

3.283 $\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\begin{aligned}\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx &= \frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}\\ &\quad+\frac{a\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}\\ &\quad+\frac{2a\sqrt{e\csc(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d}\end{aligned}$$

```
[Out] a*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+a*arctan
h(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d-2*a*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

$= \{3963, 3957, 2917, 2644, 335, 218, 212, 209, 2720\}$

$$\int \sqrt{e \csc(c+dx)}(a+a\sec(c+dx))\,dx = \frac{a\sqrt{\sin(c+dx)}\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d} + \frac{a\sqrt{\sin(c+dx)}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d} + \frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{d}$$

```
[In] Int[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d +
(a*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d +
 (2*a*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sqrt{\sin(c+dx)}}\,dx \\
&= -\left(\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{\sqrt{\sin(c+dx)}}\,dx\right) \\
&= \left(a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx \\
&\quad + \left(a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{\sin(c+dx)}}\,dx \\
&= \frac{2a\sqrt{e\csc(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{\left(a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)}\,dx, x, \sin(c+dx)\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2a\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{\left(2a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-x^4}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d} \\
&= \frac{2a\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{\left(a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d} \\
&\quad + \frac{\left(a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d} \\
&= \frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{a\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{2a\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$
\int\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx = -\frac{a\sqrt{e\csc(c+dx)}\left(2\arctan\left(\sqrt{\csc(c+dx)}\right)+\log\left(1-\sqrt{\csc(c+dx)}\right)-\log\left(1+\sqrt{\csc(c+dx)}\right)+4\sqrt{}\right.}{2d\sqrt{\csc(c+dx)}}
$$

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*(a*Sqrt[e*Csc[c + d*x]]*(2*ArcTan[Sqrt[Csc[c + d*x]]] + Log[1 - Sqrt[C
sc[c + d*x]]] - Log[1 + Sqrt[Csc[c + d*x]]] + 4*Sqrt[Csc[c + d*x]]*Elliptic
F[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]))/(d*Sqrt[Csc[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.67 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

method	result
default	$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)a\left(i\,\mathrm{EllipticPi}\left(\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticPi}\left(\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\right)(\cos(dx+c)}{d}$
parts	$\frac{ia(\cos(dx+c)+1)\sqrt{2}\,\sqrt{e\csc(dx+c)}\,\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\mathrm{Elli}}{d}$

```
[In] int((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2-1/2*I)*a/d*(I*EllipticPi((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2+1/2*
I,1/2*2^(1/2))-EllipticPi((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2-1/2*I,1/
2*2^(1/2)))*(cos(d*x+c)+1)*2^(1/2)*(e*csc(d*x+c))^(1/2)*(-I*(I-cot(d*x+c)+c
sc(d*x+c)))^(1/2)*(I*(-I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(
d*x+c)))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 538, normalized size of antiderivative = 4.45

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx$$
$$= \left[-\frac{2\,a\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)-a\sqrt{-e}\log\left(\frac{e\cos(dx+c)^4-72\,e\cos(dx+c)^2+8\,\left(\cos(dx+\right.}{}\right.}{}\right.$$
$$-\frac{2\,a\sqrt{e}\arctan\left(\frac{\left(\cos(dx+c)^2+6\,\sin(dx+c)-2\right)\sqrt{e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)-e)}\right)-a\sqrt{e}\log\left(\frac{e\cos(dx+c)^4-72\,e\cos(dx+c)^2+8\,\left(\cos(dx+c)^4-9\,\cos\right.}{\cos(dx+}\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(
-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e)) - a*sqrt(-e)*log((e*cos(d*x
+ c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 + (7*co
s(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c)) + 28*(e*c
```

```
os(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 8*I*a*sqrt(2*I*e)*weierstra
ssPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) - 8*I*a*sqrt(-2*I*e)*weiers
trassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))/d, -1/8*(2*a*sqrt(e)*ar
ctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))
/(e*sin(d*x + c) - e)) - a*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c
)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x
 + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d
*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)
*sin(d*x + c) + 8)) + 8*I*a*sqrt(2*I*e)*weierstrassPInverse(4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 8*I*a*sqrt(-2*I*e)*weierstrassPInverse(4, 0, cos(d*
x + c) - I*sin(d*x + c)))/d]
```

Sympy [F]

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{e\csc{(c+dx)}}\,dx + \int \sqrt{e\csc{(c+dx)}}\sec{(c+dx)}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))**(1/2),x)
[Out] a*(Integral(sqrt(e*csc(c + d*x)), x) + Integral(sqrt(e*csc(c + d*x))*sec(c
+ d*x), x))
```

Maxima [F]

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx = \int \sqrt{e\csc{(dx+c)}}(a\sec{(dx+c)}+a)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{e \csc(c+dx)}(a+a\sec(c+dx))\,dx = \int \sqrt{e \csc(dx+c)}(a\sec(dx+c)+a)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e \csc(c+dx)}(a+a\sec(c+dx))\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)\sqrt{\frac{e}{\sin(c+d\,x)}}\,dx$$

```
[In] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2), x)
```

3.284 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx = -\frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] -a*arctan(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a*arcta
nh(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-2*a*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4
*Pi+1/2*d*x),2^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3963, 3957, 2917, 2644, 335, 304, 209, 212, 2719}

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx = -\frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] -((a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]
)) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
```

```
*x]]) + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt
[Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (a+a\sec(c+dx))\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= -\frac{\int (-a-a\cos(c+dx))\sec(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= \frac{a\int \sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\int \sec(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= -\frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&\quad + \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.76 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx = \frac{a\Big(-4\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\csc^2(c+dx)\right)+\sqrt{-\cot^2(c+dx)}\sqrt{\csc(c+dx)}\Big(2\arctan}{2d\sqrt{-\cot^2(c+dx)}\sqrt{e\csc(c+dx}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] (a*(-4*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d*x]^2] + Sqr
t[-Cot[c + d*x]^2]*Sqrt[Csc[c + d*x]]*(2*ArcTan[Sqrt[Csc[c + d*x]]] - Log[1
 - Sqrt[Csc[c + d*x]]] + Log[1 + Sqrt[Csc[c + d*x]]])))/(2*d*Sqrt[-Cot[c +
d*x]^2]*Sqrt[e*Csc[c + d*x]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.39 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.46

method	result
parts	$-\frac{a\sqrt{2}\left(2\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{-i(i-\cot(dx+c)}\right.\right.}{}$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/d*2^(1/2)*(2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(
d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+
c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-(-I*(I-cot(d*x+c)+csc(d*x+c))
)^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(
1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)
+2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2
)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+c)
))^(1/2),1/2*2^(1/2))-(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c
)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-co
t(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(d*x+c)-2^(1/2))/(e*csc
(d*x+c))^(1/2)*csc(d*x+c)+a/d*(arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*
(cot(d*x+c)+csc(d*x+c)))+arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*
```

```
x+c)+csc(d*x+c))))/(cos(d*x+c)+1)/(e*csc(d*x+c))^(1/2)/(sin(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 554, normalized size of antiderivative = 4.54

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx$$
$$= \left[-\frac{2\,a\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)+a\sqrt{-e}\log\left(\frac{e\cos(dx+c)^4-72\,e\cos(dx+c)^2+8\,\left(\cos(dx\right.}{}\right.}{}\right.$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt(
-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e)) + a*sqrt(-e)*log((e*cos(d*x
+ c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 + (7*co
s(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c)) + 28*(e*c
os(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 8*a*sqrt(2*I*e)*weierstrass
Zeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 8*a*
sqrt(-2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) -
 I*sin(d*x + c))))/(d*e), 1/8*(2*a*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*s
in(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e)) + a*sqr
t(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*co
s(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(
d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^
4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 8*a*sqrt
(2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*si
n(d*x + c))) + 8*a*sqrt(-2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4
, 0, cos(d*x + c) - I*sin(d*x + c))))/(d*e)]
```

Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx = a\left(\int \frac{1}{\sqrt{e\csc\left(c+dx\right)}}\,dx + \int \frac{\sec\left(c+dx\right)}{\sqrt{e\csc\left(c+dx\right)}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(1/2),x)
[Out] a*(Integral(1/sqrt(e*csc(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*csc(c
 + d*x)), x))
```

Maxima [F]

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx = \int \frac{a\sec\left(dx+c\right)+a}{\sqrt{e\csc\left(dx+c\right)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*csc(d*x + c)), x)
```

Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx = \int \frac{a\sec\left(dx+c\right)+a}{\sqrt{e\csc\left(dx+c\right)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*csc(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx = \int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{\frac{e}{\sin(c+d\,x)}}}\,dx$$

```
[In] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(1/2), x)
```

3.285 $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx = -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] -2*a/d/e/(e*csc(d*x+c))^(1/2)-2/3*a*cos(d*x+c)/d/e/(e*csc(d*x+c))^(1/2)+a*a
rctan(sin(d*x+c)^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a*arctanh
(sin(d*x+c)^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-2/3*a*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1
/4*Pi+1/2*d*x),2^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules

used = {3963, 3957, 2917, 2644, 327, 335, 218, 212, 209, 2715, 2720}

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx = \frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}} + \frac{a\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}} - \frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] (-2*a)/(d*e*Sqrt[e*Csc[c + d*x]]) - (2*a*Cos[c + d*x])/(3*d*e*Sqrt[e*Csc[c
+ d*x]]) + (a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Si
n[c + d*x]]) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sq
rt[Sin[c + d*x]]) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d*e*Sqrt[e*Cs
c[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\int (a + a\sec(c+dx))\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int (-a - a\cos(c+dx))\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{a\int \sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\int \sec(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{x^{3/2}}{1-x^2}\,dx, x, \sin(c+dx)\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)}\,dx, x, \sin(c+dx)\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

Mathematica [A] (verified)

Time = 9.74 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.74

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx =$$

$$-\frac{a\Big(12+4\cos(c+dx)+6\arctan\left(\sqrt{\csc(c+dx)}\right)\sqrt{\csc(c+dx)}+3\sqrt{\csc(c+dx)}\log\left(1-\sqrt{\csc(c+dx)}\right)}{6de\sqrt{e\csc(c+dx)}}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] -1/6*(a*(12 + 4*Cos[c + d*x] + 6*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Csc[c + d*
x]] + 3*Sqrt[Csc[c + d*x]]*Log[1 - Sqrt[Csc[c + d*x]]] - 3*Sqrt[Csc[c + d*x
]]*Log[1 + Sqrt[Csc[c + d*x]]] + (4*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sq
rt[Sin[c + d*x]]))/(d*e*Sqrt[e*Csc[c + d*x]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.88 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.01

method	result
parts	$-\frac{a\sqrt{2}\left(i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\ \mathrm{EllipticF}\left(\sqrt{-i(i-\cot(dx+c}\right.\right.}{}$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/d*2^(1/2)*(I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-
csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(
d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+I*(-I*(I-cot(d*x+c)+csc(d
*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+
c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-cos(
d*x+c)*sin(d*x+c)*2^(1/2))/(e*csc(d*x+c))^(1/2)/(cos(d*x+c)-1)/e/(cos(d*x+c
)+1)*sin(d*x+c)+a/d*(arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)
+csc(d*x+c)))*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)-(sin(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)*arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+
c)+csc(d*x+c)))*cos(d*x+c)+arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(
d*x+c)+csc(d*x+c)))*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-(sin(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)*arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+cs
c(d*x+c)))+2*sin(d*x+c))/(e*csc(d*x+c))^(1/2)/(cos(d*x+c)-1)/e/(cos(d*x+c)+
1)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.34

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx = \left[-\frac{6\,a\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)+3\,a\sqrt{-e}\log\left(\frac{e\cos(dx\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{6\,a\sqrt{e}\arctan\left(\frac{\left(\cos(dx+c)^2+6\,\sin(dx+c)-2\right)\sqrt{e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)-e)}\right)-3\,a\sqrt{e}\log\left(\frac{e\cos(dx+c)^4-72\,e\cos(dx+c)^2+8\left(\cos(dx+c)^4-9\,\cos(d\cdots\right.}{\cos(dx+c\cdots}\right.}{\cdots}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(6*a*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqrt
(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e)) + 3*a*sqrt(-e)*log((e*cos(d
*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 + (7
*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c)) + 28*(
e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*cos(d*x + c) + 3*a
)*sqrt(e/sin(d*x + c))*sin(d*x + c) + 8*I*a*sqrt(2*I*e)*weierstrassPInverse
(4, 0, cos(d*x + c) + I*sin(d*x + c)) - 8*I*a*sqrt(-2*I*e)*weierstrassPInve
rse(4, 0, cos(d*x + c) - I*sin(d*x + c)))/(d*e^2), -1/24*(6*a*sqrt(e)*arcta
n(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e
*sin(d*x + c) - e)) - 3*a*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)
^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x
+ c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*
x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*
sin(d*x + c) + 8)) + 16*(a*cos(d*x + c) + 3*a)*sqrt(e/sin(d*x + c))*sin(d*x
 + c) + 8*I*a*sqrt(2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 8*I*a*sqrt(-2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin
(d*x + c)))/(d*e^2)]
```

Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx = a\left(\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec(c+dx)}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(3/2),x)
[Out] a*(Integral((e*csc(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*csc(c +
 d*x))**(3/2), x))
```

Maxima [F]

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx = \int \frac{a\sec(dx+c)+a}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx = \int \frac{a\sec(dx+c)+a}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx = \int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(3/2), x)
```

3.286 $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 197

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = &-\frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\ &+\frac{a\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\ &-\frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}-\frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}\end{aligned}$$

```
[Out] -2/3*a*sin(d*x+c)/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*a*cos(d*x+c)*sin(d*x+c)/d/
e^2/(e*csc(d*x+c))^(1/2)-a*arctan(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d*x+c))^(1
/2)/sin(d*x+c)^(1/2)+a*arctanh(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d*x+c))^(1/2)
/sin(d*x+c)^(1/2)-6/5*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d/e^2/(e*csc(d*x+c)
)^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules

used = {3963, 3957, 2917, 2644, 327, 335, 304, 209, 212, 2715, 2719}

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = -\frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}} + \frac{a\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{6aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)}{5de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] -((a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]])) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[S
in[c + d*x]]) + (6*a*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d*e^2*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]]) - (2*a*Sin[c + d*x])/(3*d*e^2*Sqrt[e*Csc[c +
d*x]]) - (2*a*Cos[c + d*x]*Sin[c + d*x])/(5*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\int (a + a\sec(c+dx))\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int (-a - a\cos(c+dx))\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{a\int \sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\int \sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{(3a)\int \sqrt{\sin(c+dx)}\,dx}{5e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{x^{5/2}}{1-x^2}\,dx, x, \sin(c+dx)\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \sin(c+dx)\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{(2a)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{a\arctan\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.84

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \frac{a\left(-72\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\csc^2(c+dx)\right)-2\sqrt{-\cot^2(c+d}\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] (a*(-72*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d*x]^2] - 2*
Sqrt[-Cot[c + d*x]^2]*(-30*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Csc[c + d*x]] +
15*Sqrt[Csc[c + d*x]]*(Log[1 - Sqrt[Csc[c + d*x]]] - Log[1 + Sqrt[Csc[c + d
*x]]]) + 20*Sin[c + d*x] + 6*Sin[2*(c + d*x)])))/(60*d*e^2*Sqrt[-Cot[c + d*
x]^2]*Sqrt[e*Csc[c + d*x]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 602, normalized size of antiderivative = 3.06

method	result
default	$\frac{a\sqrt{2}\left(-6\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{-i(i-\cot(dx+c)-\right.\right.}{}$
parts	$\frac{a\sqrt{2}\left(-6\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{-i(i-\cot(dx+c)-\right.\right.}{}$

```
[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a/d*2^(1/2)*(-6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-
csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(
d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+3*(-I*(I-cot(d*x+c)+csc(d
*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+
c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(
d*x+c)+cos(d*x+c)^3*2^(1/2)-6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+c
ot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((
-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+3*(-I*(I-cot(d*x+c)+csc(d*
x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c
)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-4*2^(
1/2)*cos(d*x+c)+3*2^(1/2))/(e*csc(d*x+c))^(1/2)/e^2*csc(d*x+c)-1/3*a/d*(3*a
rctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))*(sin(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)+3*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*arctanh((
```

```
sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))+2*cos(d*x+c)-2)
/(cos(d*x+c)-1)/e^2/(e*csc(d*x+c))^(1/2)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.32

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \left[-\frac{30\,a\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)+15\,a\sqrt{-e}\log\left(\cdots\right.}{\cdots}\right.$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(30*a*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sq
rt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e)) + 15*a*sqrt(-e)*log((e*co
s(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 +
 (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c)) + 2
8*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x
 + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 72*a*sqrt(2*I*e)*weie
rstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)))
 - 72*a*sqrt(-2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*
x + c) - I*sin(d*x + c))) - 16*(3*a*cos(d*x + c)^3 + 5*a*cos(d*x + c)^2 - 3
*a*cos(d*x + c) - 5*a)*sqrt(e/sin(d*x + c)))/(d*e^3), 1/120*(30*a*sqrt(e)*a
rctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c)
)/(e*sin(d*x + c) - e)) + 15*a*sqrt(e)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x
 + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin
(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*s
in(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2
- 2)*sin(d*x + c) + 8)) + 72*a*sqrt(2*I*e)*weierstrassZeta(4, 0, weierstras
sPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 72*a*sqrt(-2*I*e)*weierst
rassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) +
16*(3*a*cos(d*x + c)^3 + 5*a*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 5*a)*sqrt(
e/sin(d*x + c)))/(d*e^3)]
```

Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = a\left(\int \frac{1}{(e\csc(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{\sec(c+dx)}{(e\csc(c+dx))^{\frac{5}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(5/2),x)
[Out] a*(Integral((e*csc(c + d*x))**(-5/2), x) + Integral(sec(c + d*x)/(e*csc(c +
 d*x))**(5/2), x))
```

Maxima [F]

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{a\sec(dx+c)+a}{(e\csc(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{a\sec(dx+c)+a}{(e\csc(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(5/2), x)
```

3.287 $\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 270

$$\begin{aligned}\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx &= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}\\ &-\frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}\sec(c+dx)}{3d}\\ &+\frac{2a^2e^2\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}\\ &+\frac{2a^2e^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}\\ &+\frac{7a^2e^2\sqrt{e\csc(c+dx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d}\\ &+\frac{5a^2e^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{3d}\end{aligned}$$

```
[Out] -2/3*a^2*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/d-4/3*a^2*e^2*csc(d*x+c)*(e*cs
c(d*x+c))^(1/2)/d-2/3*a^2*e^2*csc(d*x+c)*sec(d*x+c)*(e*csc(d*x+c))^(1/2)/d+
2*a^2*e^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+
2*a^2*e^2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d
-7/3*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)/d+5/3*a^2*e^2*(e*csc(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3963, 3957, 2952, 2716, 2720, 2644, 331, 335, 218, 212, 209, 2650, 2651}

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \frac{2a^2e^2\sqrt{\sin(c+dx)}\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d} + \frac{2a^2e^2\sqrt{\sin(c+dx)}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{5a^2e^2\tan(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e^2\csc(c+dx)\sec(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{7a^2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{3d}$$

```
[In] Int[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (4*a^2*e^2*Csc[c + d
*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (2*a^2*e^2*Csc[c + d*x]*Sqrt[e*Csc[c + d*
x]]*Sec[c + d*x])/(3*d) + (2*a^2*e^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a^2*e^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sq
rt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (7*a^2*e^2*Sqrt[e*Csc[c + d*x]]*
EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d) + (5*a^2*e^2*Sqr
t[e*Csc[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{(a+a\sec(c+dx))^2}{\sin^{\frac{5}{2}}(c+dx)}\,dx\\
&= \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx\\
&= \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\left(\frac{a^2}{\sin^{\frac{5}{2}}(c+dx)}+\frac{2a^2\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}+\frac{a^2\sec^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\right)dx\\
&= \left(a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sin^{\frac{5}{2}}(c+dx)}\,dx\\
&\quad+\left(a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\sec^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx\\
&\quad+\left(2a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx
\end{aligned}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}\sec(c+dx)}{3d} + \frac{1}{3}\left(a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx + \frac{1}{3}\left(5a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\sec^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx + \frac{\left(2a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{x^{5/2}(1-x^2)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}\sec(c+dx)}{3d} + \frac{2a^2e^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d} + \frac{5a^2e^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{3d} + \frac{1}{6}\left(5a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx + \frac{\left(2a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}(1-x^2)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}\sec(c+dx)}{3d} + \frac{7a^2e^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d} + \frac{5a^2e^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{3d} + \frac{\left(4a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-x^4}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}\sec(c+dx)}{3d} + \frac{7a^2e^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d} + \frac{5a^2e^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{3d} + \frac{\left(2a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d} + \frac{\left(2a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}\sec(c+dx)}{3d} + \frac{2a^2e^2\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} + \frac{2a^2e^2\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} + \frac{7a^2e^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d} + \frac{5a^2e^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{3d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.72

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = -\frac{a^2e^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{e\csc(c+dx)}\left(-7+6\arctan\left(\sqrt{\csc(c+dx)}\right)\sqrt{\cos^2(c+dx)}\sqrt{\csc(c+dx)}-6\text{arcta}\right.}{}$$

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*(a^2*e^2*Cos[(c + d*x)/2]^4*Sqrt[e*Csc[c + d*x]]*(-7 + 6*ArcTan[Sqrt[C
sc[c + d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] - 6*ArcTanh[Sqrt[Csc[
c + d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] + 4*Csc[c + d*x]^2 + 4*S
```

```
qrt[Cos[c + d*x]^2]*Csc[c + d*x]^2 + 7*Sqrt[-Cot[c + d*x]^2]*Hypergeometric
2F1[1/4, 1/2, 5/4, Csc[c + d*x]^2])*Sec[ArcCsc[Csc[c + d*x]]/2]^4*Tan[c + d
*x])/d
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.70 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.25

method	result
default	$\frac{a^2 e^2 \sqrt{2}\left(6i\sin(dx+c)\cos(dx+c)\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{i(-\cot(dx+c)+\csc(dx+c))}\ \text{Elli}\right.}{}$
parts	Expression too large to display

```
[In] int((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2*e^2/d*2^(1/2)*(6*I*sin(d*x+c)*cos(d*x+c)*(-I*(I-cot(d*x+c)+csc(d*x+
c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c))
)^(1/2)*EllipticPi((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2+1/2*I,1/2*2^(1/
2))+6*I*sin(d*x+c)*cos(d*x+c)*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+c
ot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*EllipticPi(
(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I*sin(d*x+c)*
cos(d*x+c)*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c
)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+cs
c(d*x+c)))^(1/2),1/2*2^(1/2))+6*cos(d*x+c)*sin(d*x+c)*(-I*(I-cot(d*x+c)+csc
(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*
x+c)))^(1/2)*EllipticPi((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2+1/2*I,1/2*
2^(1/2))-6*cos(d*x+c)*sin(d*x+c)*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(
I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*Elliptic
Pi((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2-1/2*I,1/2*2^(1/2))-7*2^(1/2)*co
s(d*x+c)+3*2^(1/2))*(cos(d*x+c)+1)*(e*csc(d*x+c))^(1/2)*sec(d*x+c)*csc(d*x+
c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 806, normalized size of antiderivative = 2.99

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(6*a^2*sqrt(-e)*e^2*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2
)*sqrt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c)*sin(d*x
```

```
+ c) - 3*a^2*sqrt(-e)*e^2*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x
 + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin
(d*x + c) + 8)*sqrt(-e)*sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*
sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 - 2)*sin(d*x + c) + 8))*sin(d*x + c) + 14*I*a^2*sqrt(2*I*e)*e^2*cos(d*x +
c)*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) -
14*I*a^2*sqrt(-2*I*e)*e^2*cos(d*x + c)*sin(d*x + c)*weierstrassPInverse(4,
0, cos(d*x + c) - I*sin(d*x + c)) + 4*(7*a^2*e^2*cos(d*x + c)^2 + 4*a^2*e^2
*cos(d*x + c) - 3*a^2*e^2)*sqrt(e/sin(d*x + c)))/(d*cos(d*x + c)*sin(d*x +
c)), -1/12*(6*a^2*e^(5/2)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*
sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e))*cos(d*x + c)*sin(d*x + c
) - 3*a^2*e^(5/2)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2
+ 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x + c
) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x +
 c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin
(d*x + c) + 8))*sin(d*x + c) + 14*I*a^2*sqrt(2*I*e)*e^2*cos(d*x + c)*sin(d*
x + c)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) - 14*I*a^2*
sqrt(-2*I*e)*e^2*cos(d*x + c)*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*
x + c) - I*sin(d*x + c)) + 4*(7*a^2*e^2*cos(d*x + c)^2 + 4*a^2*e^2*cos(d*x
+ c) - 3*a^2*e^2)*sqrt(e/sin(d*x + c)))/(d*cos(d*x + c)*sin(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate((e*csc(d*x+c))**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \int (e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2\,dx$$

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^2\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}dx$$

```
[In] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2), x)
```

3.288 $\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 240

$$\begin{aligned}\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx &= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} \\ &- \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\sqrt{e\csc(c+dx)}\sec(c+dx)}{d} \\ &- \frac{2a^2e\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\ &+ \frac{2a^2e\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} \\ &- \frac{5a^2e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} \\ &+ \frac{3a^2e\sqrt{e\csc(c+dx)}\sin(c+dx)\tan(c+dx)}{d}\end{aligned}$$

```
[Out] -4*a^2*e*(e*csc(d*x+c))^(1/2)/d-2*a^2*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/d-2
*a^2*e*sec(d*x+c)*(e*csc(d*x+c))^(1/2)/d-2*a^2*e*arctan(sin(d*x+c)^(1/2))*(
e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a^2*e*arctanh(sin(d*x+c)^(1/2))*(e
*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+5*a^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+3*a^2*e*sin(d*x+c)*(e*csc(d*x+c)
)^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3963, 3957, 2952, 2716, 2719, 2644, 331, 335, 304, 209, 212, 2650, 2651}

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = -\frac{2a^2e\sqrt{\sin(c+dx)}\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d} + \frac{2a^2e\sqrt{\sin(c+dx)}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d} - \frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\sec(c+dx)\sqrt{e\csc(c+dx)}}{d} + \frac{3a^2e\sin(c+dx)\tan(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{5a^2e\sqrt{\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{d}$$

```
[In] Int[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*e*Sqrt[e*Csc[c + d*x]])/d - (2*a^2*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*
x]])/d - (2*a^2*e*Sqrt[e*Csc[c + d*x]]*Sec[c + d*x])/d - (2*a^2*e*ArcTan[Sq
rt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a^2*e*Arc
Tanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d - (5*a^
2*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/d + (3*a^2*e*Sqrt[e*Csc[c + d*x]]*Sin[c + d*x]*Tan[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sin^{\frac{3}{2}}(c+dx)}\,dx \\
&= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx \\
&= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2}{\sin^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}+\frac{a^2\sec^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\right)dx \\
&= \left(a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx \\
&\quad+\left(a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\sec^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx \\
&\quad+\left(2a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\sqrt{e\csc(c+dx)}\sec(c+dx)}{d} \\
&\quad - \left(a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx \\
&\quad + \left(3a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\sec^2(c+dx)\sqrt{\sin(c+dx)}\,dx \\
&\quad + \frac{\left(2a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}(1-x^2)}\,dx,x,\sin(c+dx)\right)}{d} \\
&= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} \\
&\quad - \frac{2a^2e\sqrt{e\csc(c+dx)}\sec(c+dx)}{d} \\
&\quad - \frac{2a^2e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{3a^2e\sqrt{e\csc(c+dx)}\sin(c+dx)\tan(c+dx)}{d} \\
&\quad - \frac{1}{2}\left(3a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx \\
&\quad + \frac{\left(2a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{\sqrt{x}}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d} \\
&= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} \\
&\quad - \frac{2a^2e\sqrt{e\csc(c+dx)}\sec(c+dx)}{d} \\
&\quad - \frac{5a^2e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} \\
&\quad + \frac{3a^2e\sqrt{e\csc(c+dx)}\sin(c+dx)\tan(c+dx)}{d} \\
&\quad + \frac{\left(4a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d}
\end{aligned}
$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\sqrt{e\csc(c+dx)}\sec(c+dx)}{d} - \frac{5a^2e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{3a^2e\sqrt{e\csc(c+dx)}\sin(c+dx)\tan(c+dx)}{d} + \frac{\left(2a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-x^2}dx, x, \sqrt{\sin(c+dx)}\right)}{d} - \frac{\left(2a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1+x^2}dx, x, \sqrt{\sin(c+dx)}\right)}{d}$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\sqrt{e\csc(c+dx)}\sec(c+dx)}{d} - \frac{2a^2e\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} + \frac{2a^2e\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} - \frac{5a^2e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{3a^2e\sqrt{e\csc(c+dx)}\sin(c+dx)\tan(c+dx)}{d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.91 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.81

$$\int (e\csc(c+dx))^{3/2}(a + a\sec(c+dx))^2\,dx = \frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{3/2}\left(3\arctan\left(\sqrt{\csc(c+dx)}\right)\sqrt{\cos^2(c+dx)} + 3\text{ar}\right.}{}$$

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*(e*Csc[c + d*x])^(3/2)*(3*ArcTan[Sqrt[Csc[c + d*x
]]]*Sqrt[Cos[c + d*x]^2] + 3*ArcTanh[Sqrt[Csc[c + d*x]]]*Sqrt[Cos[c + d*x]^
2] - 6*Sqrt[Csc[c + d*x]] - 6*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] + 5*S
```

```
qrt[-Cot[c + d*x]^2]*Sqrt[Csc[c + d*x]]*Hypergeometric2F1[3/4, 3/2, 7/4, Cs
c[c + d*x]^2])*Sec[c + d*x]*Sec[ArcCsc[Csc[c + d*x]]/2]^4)/(3*d*Csc[c + d*x
]^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.20 (sec) , antiderivative size = 1069, normalized size of antiderivative = 4.45

method	result	size
parts	Expression too large to display	1069
default	Expression too large to display	1231

```
[In] int((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*2^(1/2)*e*(e*csc(d*x+c))^(1/2)*(2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2
)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*E
llipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-(-I*(
I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(c
ot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)
,1/2*2^(1/2))*cos(d*x+c)+2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(
d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*
(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(d*x+c)+csc(d*x+c))
)^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(
1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-2^(1/2))+1
/2*a^2/d*2^(1/2)*e*(e*csc(d*x+c))^(1/2)*(6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(
1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2
)*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-3*
(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(
-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^
(1/2),1/2*2^(1/2))*cos(d*x+c)+6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I
+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE
((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-3*(-I*(I-cot(d*x+c)+csc(
d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x
+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-3*2
^(1/2)+2^(1/2)*sec(d*x+c))-2*a^2/d*(e*csc(d*x+c))^(1/2)*e/(sin(d*x+c)/(cos(
d*x+c)+1)^2)^(1/2)*(2*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+arctan((sin(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))*cot(d*x+c)+arctanh((sin(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))*cot(d*x+c)-arctan((
sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))*csc(d*x+c)-arct
anh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))*csc(d*x+c)
)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.04

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \left[-\frac{2\,a^2\sqrt{-e}e\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)\cos\left(dx+c\right)-a^2\sqrt{-e}e\,c\right.$$

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a^2*sqrt(-e)*e*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*s
qrt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c) - a^2*sqrt(
-e)*e*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(cos(d*x
 + c)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(
-e)*sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)
/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) +
 8)) + 10*a^2*sqrt(2*I*e)*e*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassP
Inverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 10*a^2*sqrt(-2*I*e)*e*cos(d
*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*si
n(d*x + c))) + 4*(5*a^2*e*cos(d*x + c)^2 + 4*a^2*e*cos(d*x + c) - a^2*e)*sq
rt(e/sin(d*x + c)))/(d*cos(d*x + c)), 1/4*(2*a^2*e^(3/2)*arctan(1/4*(cos(d*
x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c)
 - e))*cos(d*x + c) + a^2*e^(3/2)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e
*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2
- 8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2
- 2*e)*sin(d*x + c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x
 + c)^2 - 2)*sin(d*x + c) + 8)) - 10*a^2*sqrt(2*I*e)*e*cos(d*x + c)*weierst
rassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) -
10*a^2*sqrt(-2*I*e)*e*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 4*(5*a^2*e*cos(d*x + c)^2 + 4*a^2
*e*cos(d*x + c) - a^2*e)*sqrt(e/sin(d*x + c)))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate((e*csc(d*x+c))**(3/2)*(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \int (e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2\,dx$$

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^2\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}dx$$

```
[In] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2), x)
```

3.289 $\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 154

$$\begin{aligned}
&\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx\\
&=\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}\\
&\quad+\frac{2a^2\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}\\
&\quad+\frac{3a^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d}\\
&\quad+\frac{a^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{d}
\end{aligned}$$

```
[Out] 2*a^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a^
2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d-3*a^2*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(
1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+a^2*
(e*csc(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3963, 3957, 2952, 2720, 2644, 335, 218, 212, 209, 2651}

$$\begin{aligned}
&\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx\\
&=\frac{2a^2\sqrt{\sin(c+dx)}\arctan\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d}\\
&\quad+\frac{2a^2\sqrt{\sin(c+dx)}\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d}\\
&\quad+\frac{a^2\tan(c+dx)\sqrt{e\csc(c+dx)}}{d}\\
&\quad+\frac{3a^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{d}
\end{aligned}$$

```
[In] Int[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/
d + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*
x]])/d + (3*a^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/d + (a^2*Sqrt[e*Csc[c + d*x]]*Tan[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{(a+a\sec(c+dx))^2}{\sqrt{\sin(c+dx)}}\,dx\\
&= \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx\\
&= \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\left(\frac{a^2}{\sqrt{\sin(c+dx)}}+\frac{2a^2\sec(c+dx)}{\sqrt{\sin(c+dx)}}+\frac{a^2\sec^2(c+dx)}{\sqrt{\sin(c+dx)}}\right)dx\\
&= \left(a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx\\
&\quad+\left(a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\sec^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx\\
&\quad+\left(2a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\sec(c+dx)}{\sqrt{\sin(c+dx)}}\,dx\\
&= \frac{2a^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d}\\
&\quad+\frac{a^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{d}\\
&\quad+\frac{1}{2}\left(a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx\\
&\quad+\frac{\left(2a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}(1-x^2)}\,dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{3a^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d}\\
&\quad+\frac{a^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{d}\\
&\quad+\frac{\left(4a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-x^4}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d}\\
&= \frac{3a^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d}\\
&\quad+\frac{a^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{d}\\
&\quad+\frac{\left(2a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d}\\
&\quad+\frac{\left(2a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d}
\end{aligned}$$

$$= \frac{2a^2 \arctan\left(\sqrt{\sin(c+dx)}\right) \sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} + \frac{2a^2 \operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right) \sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} + \frac{3a^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d} + \frac{a^2\sqrt{e\csc(c+dx)}\tan(c+dx)}{d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.89 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.09

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx = -\frac{2a^2\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{e\csc(c+dx)}\left(-1+2\arctan\left(\sqrt{\csc(c+dx)}\right)\sqrt{\cos^2(c+dx)}\sqrt{\csc(c+dx)}-2a\right.}{}$$

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Cos[(c + d*x)/2]^5*Sqrt[e*Csc[c + d*x]]*(-1 + 2*ArcTan[Sqrt[Csc[c +
 d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] - 2*ArcTanh[Sqrt[Csc[c + d*
x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] + 3*Sqrt[-Cot[c + d*x]^2]*Hype
rgeometric2F1[1/4, 1/2, 5/4, Csc[c + d*x]^2])*Sec[c + d*x]*Sec[ArcCsc[Csc[c
 + d*x]]/2]^4*Sin[(c + d*x)/2])/d
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.56 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.09

method	result
parts	$\frac{ia^2(\cos(dx+c)+1)\sqrt{2}\sqrt{e\csc(dx+c)}\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\ \mathrm{E}}{d}$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*a^2/d*(cos(d*x+c)+1)*2^(1/2)*(e*csc(d*x+c))^(1/2)*(-I*(I-cot(d*x+c)+csc(d
*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+
```

```
c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+1/2*
a^2/d*2^(1/2)*(e*csc(d*x+c))^(1/2)*(I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*
(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*Ell
ipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+I*(-I*(
I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(c
ot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)
,1/2*2^(1/2))+2^(1/2)*tan(d*x+c))+2*a^2/d*sin(d*x+c)*(arctanh((sin(d*x+c)/(
cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c)))-arctan((sin(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(d*x+c))))*(e*csc(d*x+c))^(1/2)/(cos(d*x+c)+
1)/(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.31

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx$$

$$= \left[-\frac{2\,a^2\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)\cos(dx+c) - a^2\sqrt{-e}\cos(dx+c)\log\left(\frac{e\cos(d\ldots}{\ldots}\right.}{\ldots} \right.$$

$$-\frac{2\,a^2\sqrt{e}\arctan\left(\frac{\left(\cos(dx+c)^2+6\,\sin(dx+c)-2\right)\sqrt{e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)-e)}\right)\cos(dx+c) - a^2\sqrt{e}\cos(dx+c)\log\left(\frac{e\cos(dx+c)^4-72\ldots}{\ldots}\right.}{\ldots}$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a^2*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqr
t(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c) - a^2*sqrt(-e
)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c
)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*
sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(co
s(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))
 + 6*I*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c)
+ I*sin(d*x + c)) - 6*I*a^2*sqrt(-2*I*e)*cos(d*x + c)*weierstrassPInverse(4
, 0, cos(d*x + c) - I*sin(d*x + c)) - 4*a^2*sqrt(e/sin(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)), -1/4*(2*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*si
n(d*x + c) - 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e))*cos(d*x
+ c) - a^2*sqrt(e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2
 + 8*(cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x +
```

```
c) + 8)*sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x
+ c) + 72*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*si
n(d*x + c) + 8)) + 6*I*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassPInverse(4,
0, cos(d*x + c) + I*sin(d*x + c)) - 6*I*a^2*sqrt(-2*I*e)*cos(d*x + c)*weier
strassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 4*a^2*sqrt(e/sin(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F]

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sqrt{e\csc(c+dx)}\,dx + \int 2\sqrt{e\csc(c+dx)}\sec(c+dx)\,dx + \int \sqrt{e\csc(c+dx)}\sec^2(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*(e*csc(d*x+c))**(1/2),x)
[Out] a**2*(Integral(sqrt(e*csc(c + d*x)), x) + Integral(2*sqrt(e*csc(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*csc(c + d*x))*sec(c + d*x)**2, x))
```

Maxima [F]

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)^2\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)^2\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e \csc(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^2\sqrt{\frac{e}{\sin(c+d\,x)}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2), x)
```

3.290 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = -\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a^2\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}}$$

```
[Out] -2*a^2*arctan(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+2*a
^2*arctanh(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-a^2*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1
/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a^2*t
an(d*x+c)/d/(e*csc(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules

used = {3963, 3957, 2952, 2719, 2644, 335, 304, 209, 212, 2651}

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = -\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}} + \frac{2a^2\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} + \frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] (-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*
x]]) + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin
[c + d*x]]) + (a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(d*Sqrt[e*Csc[c + d*x]
]*Sqrt[Sin[c + d*x]]) + (a^2*Tan[c + d*x])/(d*Sqrt[e*Csc[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (a+a\sec(c+dx))^2\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= \frac{\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= \frac{\int \left(a^2\sqrt{\sin(c+dx)}+2a^2\sec(c+dx)\sqrt{\sin(c+dx)}+a^2\sec^2(c+dx)\sqrt{\sin(c+dx)}\right)\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}
\end{aligned}$$

$$= \frac{a^2 \int \sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2 \int \sec^2(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a^2)\int \sec(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{2a^2 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} - \frac{a^2 \int \sqrt{\sin(c+dx)}\,dx}{2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{a^2 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} + \frac{(4a^2)\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{a^2 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2 E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 18.33 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.88

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = -\frac{\left(1+\cos\left(2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\right)^2\cos(c+dx)\left(-1+\csc^2(c+dx)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(-\arctan\left(\sqrt{\right.\right.}{2d\left(1+\cos\left(2\left(\frac{c}{2}+\frac{1}{2}(-\right.\right.\right.}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] -1/2*((1 + Cos[2*(c/2 + (d*x)/2)])^2*Cos[c + d*x]*(-1 + Csc[c + d*x]^2)*Sec
[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(-ArcTan[Sqrt[Csc[c + d*x]]] - Arc
```

```
Tanh[Sqrt[Csc[c + d*x]]] - (2*Sqrt[Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]^2]*H
ypergeometric2F1[3/4, 3/2, 7/4, Csc[c + d*x]^2])/(3*Sqrt[1 - Sin[c + d*x]^2
]) + (Sqrt[Csc[c + d*x]]*Hypergeometric2F1[-1/4, 3/2, 3/4, Csc[c + d*x]^2]*
Sqrt[1 - Sin[c + d*x]^2])/Sqrt[1 - Csc[c + d*x]^2]))/(d*(1 + Cos[2*(c/2 + (
-c + ArcCsc[Csc[c + d*x]])/2)])^2*Csc[c + d*x]^(3/2)*Sqrt[e*Csc[c + d*x]]*S
qrt[1 - Sin[c + d*x]^2])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.83 (sec) , antiderivative size = 996, normalized size of antiderivative = 6.51

method	result	size
parts	Expression too large to display	996
default	Expression too large to display	1290

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d*2^(1/2)*(2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-cs
c(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*
x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-(-I*(I-cot(d*x+c)+csc(d*x+c
)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))
^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+
c)+2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1
/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+
c)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x
+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-
cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(d*x+c)-2^(1/2))/(e*c
sc(d*x+c))^(1/2)*csc(d*x+c)-1/2*a^2/d*2^(1/2)/(e*csc(d*x+c))^(1/2)*(-2*(-I*
(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(
cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2
),1/2*2^(1/2))*cot(d*x+c)+(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d
*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(
I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cot(d*x+c)-2*(-I*(I-cot(d*x+c)
+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-cs
c(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2)
)*csc(d*x+c)+(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x
+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+
csc(d*x+c)))^(1/2),1/2*2^(1/2))*csc(d*x+c)+2^(1/2)*csc(d*x+c)-2^(1/2)*sec(d
*x+c)*csc(d*x+c))+2*a^2/d*(arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot
(d*x+c)+csc(d*x+c)))+arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)
+csc(d*x+c))))/(cos(d*x+c)+1)/(e*csc(d*x+c))^(1/2)/(sin(d*x+c)/(cos(d*x+c)+
1)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.56

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx$$

$$= \left[-\frac{2\,a^2\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)\cos(dx+c)+a^2\sqrt{-e}\cos(dx+c)\log\left(\frac{e\cos(d}{}\right.}{}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a^2*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sqr
t(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c) + a^2*sqrt(-e
)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c
)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-e)*
sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(co
s(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))
 - 2*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse
(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 2*a^2*sqrt(-2*I*e)*cos(d*x + c)*we
ierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)
)) + 4*(a^2*cos(d*x + c)^2 - a^2)*sqrt(e/sin(d*x + c)))/(d*e*cos(d*x + c)),
 1/4*(2*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sqrt(e
)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e))*cos(d*x + c) + a^2*sqrt(e)*cos
(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x + c)^4 -
 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(e)*sqrt(e
/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(cos(d*x
+ c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 2*a
^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
 cos(d*x + c) + I*sin(d*x + c))) + 2*a^2*sqrt(-2*I*e)*cos(d*x + c)*weierstr
assZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 4
*(a^2*cos(d*x + c)^2 - a^2)*sqrt(e/sin(d*x + c)))/(d*e*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{e\csc(c+dx)}}\,dx + \int \frac{2\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(1/2),x)
[Out] a**2*(Integral(1/sqrt(e*csc(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
csc(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*csc(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\csc(dx+c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*csc(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\csc(dx+c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*csc(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{e}{\sin(c+d\,x)}}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(1/2), x)
```

3.291 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 222

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} + \frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{a^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] -4*a^2/d/e/(e*csc(d*x+c))^(1/2)-2/3*a^2*cos(d*x+c)/d/e/(e*csc(d*x+c))^(1/2)
+a^2*sec(d*x+c)/d/e/(e*csc(d*x+c))^(1/2)+2*a^2*arctan(sin(d*x+c)^(1/2))/d/e
/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+2*a^2*arctanh(sin(d*x+c)^(1/2))/d/e/
(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+1/3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules

used = {3963, 3957, 2952, 2715, 2720, 2644, 327, 335, 218, 212, 209, 2646}

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}} + \frac{2a^2\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}} - \frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} - \frac{a^2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] (-4*a^2)/(d*e*Sqrt[e*Csc[c + d*x]]) - (2*a^2*Cos[c + d*x])/(3*d*e*Sqrt[e*Cs
c[c + d*x]]) + (a^2*Sec[c + d*x])/(d*e*Sqrt[e*Csc[c + d*x]]) + (2*a^2*ArcTa
n[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (2*a
^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]
]) - (a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d*e*Sqrt[e*Csc[c + d*x]]*Sqr
t[Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (a+a\sec(c+dx))^2\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \left(a^2\sin^{\frac{3}{2}}(c+dx)+2a^2\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)+a^2\sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\right)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{a^2\int \sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a^2\int \sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&\quad+\frac{(2a^2)\int \sec(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}}+\frac{a^2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&\quad-\frac{a^2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^{3/2}}{1-x^2}\,dx,x,\sin(c+dx)\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}}-\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}}\\
&\quad-\frac{a^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)}\,dx,x,\sin(c+dx)\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}}-\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}}\\
&\quad-\frac{a^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} \\
&\quad - \frac{a^2\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a^2)\operatorname{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&\quad + \frac{(2a^2)\operatorname{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} \\
&\quad + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} + \frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&\quad + \frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{a^2\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.71 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.74

$$
\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{e\csc(c+dx)}\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(3-6\sqrt{\cos^2(c-
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*Sqrt[e*Csc[c + d*x]]*Sec[ArcCsc[Csc[c + d*x]]/2]^
4*(3 - 6*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[-1/4, 1, 3/4, Csc[c + d*x]^
2] + 3*Sqrt[-Cot[c + d*x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Csc[c + d*x]^
2] + Sqrt[-Cot[c + d*x]^2]*Hypergeometric2F1[-3/4, 3/2, 1/4, Csc[c + d*x]^2
]*Sin[c + d*x]^2)*Tan[c + d*x])/(3*d*e^2)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.70 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.65

method	result	size
parts	Expression too large to display	810
default	Expression too large to display	1083

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^2/d*2^(1/2)*(I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c
)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-co
t(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+I*(-I*(I-cot(d*x+c)+csc
(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*
x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-co
s(d*x+c)*sin(d*x+c)*2^(1/2))/(e*csc(d*x+c))^(1/2)/(cos(d*x+c)-1)/e/(cos(d*x
+c)+1)*sin(d*x+c)+1/2*a^2/d*2^(1/2)*(I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)
*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*El
lipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+I*(-
I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I
*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1
/2),1/2*2^(1/2))*cos(d*x+c)-2^(1/2)*sin(d*x+c))/(e*csc(d*x+c))^(1/2)/e/(cos
(d*x+c)^2-1)*tan(d*x+c)+2*a^2/d*(arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
*(cot(d*x+c)+csc(d*x+c)))*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)-(s
in(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/
2)*(cot(d*x+c)+csc(d*x+c)))*cos(d*x+c)+arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*(cot(d*x+c)+csc(d*x+c)))*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-(sin(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(c
ot(d*x+c)+csc(d*x+c)))+2*sin(d*x+c))/(e*csc(d*x+c))^(1/2)/(cos(d*x+c)-1)/e/
(cos(d*x+c)+1)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.26

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \left[-\frac{6\,a^2\sqrt{-e}\arctan\left(-\frac{\left(\cos(dx+c)^2-6\,\sin(dx+c)-2\right)\sqrt{-e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)+e)}\right)\cos\left(dx+c\right)+3\,a\right.$$

$$-\frac{6\,a^2\sqrt{e}\arctan\left(\frac{\left(\cos(dx+c)^2+6\,\sin(dx+c)-2\right)\sqrt{e}\sqrt{\frac{e}{\sin(dx+c)}}}{4\,(e\sin(dx+c)-e)}\right)\cos\left(dx+c\right)-3\,a^2\sqrt{e}\cos\left(dx+c\right)\log\left(\frac{e\cos(dx+c)^4-72\,e}{}\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*a^2*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*sq
rt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c) + 3*a^2*sqrt
(-e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 - 8*(cos(d*x
+ c)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(-
```

```
e)*sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/
(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) +
8)) - 2*I*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassPInverse(4, 0, cos(d*x +
c) + I*sin(d*x + c)) + 2*I*a^2*sqrt(-2*I*e)*cos(d*x + c)*weierstrassPInvers
e(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 4*(2*a^2*cos(d*x + c)^2 + 12*a^2*c
os(d*x + c) - 3*a^2)*sqrt(e/sin(d*x + c))*sin(d*x + c))/(d*e^2*cos(d*x + c)
), -1/12*(6*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c) - 2)*sq
rt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e))*cos(d*x + c) - 3*a^2*sqrt(
e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*x +
c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt(e)*
sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e)/(co
s(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))
 - 2*I*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 2*I*a^2*sqrt(-2*I*e)*cos(d*x + c)*weierstrassPInverse(4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 4*(2*a^2*cos(d*x + c)^2 + 12*a^2*cos(
d*x + c) - 3*a^2)*sqrt(e/sin(d*x + c))*sin(d*x + c))/(d*e^2*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = a^2\left(\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{2\sec(c+dx)}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec^2(c+dx)}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*csc(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*cs
c(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*csc(c + d*x))**(3/2),
x))
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(3/2), x)
```

3.292 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 236

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx &= -\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\ &+\frac{2a^2\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}-\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\ &-\frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}-\frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}+\frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}}\end{aligned}$$

```
[Out] -4/3*a^2*sin(d*x+c)/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*a^2*cos(d*x+c)*sin(d*x+c
)/d/e^2/(e*csc(d*x+c))^(1/2)-2*a^2*arctan(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d*
x+c))^(1/2)/sin(d*x+c)^(1/2)+2*a^2*arctanh(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d
*x+c))^(1/2)/sin(d*x+c)^(1/2)+9/5*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d/e^2
/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a^2*tan(d*x+c)/d/e^2/(e*csc(d*x+c))^
(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules

used = {3963, 3957, 2952, 2715, 2719, 2644, 327, 335, 304, 209, 212, 2646}

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = -\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}} + \frac{2a^2\text{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} + \frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} - \frac{9a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c
+ d*x]]) + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*
Sqrt[Sin[c + d*x]]) - (9*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d*e^2*Sqr
t[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) - (4*a^2*Sin[c + d*x])/(3*d*e^2*Sqrt[
e*Csc[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[c + d*x])/(5*d*e^2*Sqrt[e*Csc[c
+ d*x]]) + (a^2*Tan[c + d*x])/(d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (a+a\sec(c+dx))^2\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \left(a^2\sin^{\frac{5}{2}}(c+dx)+2a^2\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)+a^2\sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\right)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{a^2\int \sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a^2\int \sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&\quad+\frac{(2a^2)\int \sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}+\frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}}+\frac{(3a^2)\int \sqrt{\sin(c+dx)}\,dx}{5e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&\quad-\frac{(3a^2)\int \sqrt{\sin(c+dx)}\,dx}{2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^{5/2}}{1-x^2}\,dx,x,\sin(c+dx)\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}-\frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}\\
&\quad-\frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}+\frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}}\\
&\quad+\frac{(2a^2)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx,x,\sin(c+dx)\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}-\frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}\\
&\quad-\frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}+\frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}}\\
&\quad+\frac{(4a^2)\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}} + \frac{(2a^2)\operatorname{Subst}\left(\int\frac{1}{1-x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{(2a^2)\operatorname{Subst}\left(\int\frac{1}{1+x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a^2\arctan\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a^2\operatorname{arctanh}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.03 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(-10\sqrt{\cos^2(c+dx)}\,\text{Hypergeom}\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*Sec[ArcCsc[Csc[c + d*x]]/2]^4*(-10*Sqrt[Cos[c + d
*x]^2]*Hypergeometric2F1[-3/4, 1, 1/4, Csc[c + d*x]^2] + 3*Sqrt[-Cot[c + d*
x]^2]*(-10*Hypergeometric2F1[-1/4, 3/2, 3/4, Csc[c + d*x]^2] + Hypergeometr
ic2F1[-5/4, 3/2, -1/4, Csc[c + d*x]^2]*Sin[c + d*x]^2))*Tan[c + d*x])/(15*d
*e^2*Sqrt[e*Csc[c + d*x]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.64 (sec) , antiderivative size = 1069, normalized size of antiderivative = 4.53

method	result	size
parts	Expression too large to display	1069
default	Expression too large to display	1333

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^2/d*2^(1/2)*(-6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c
)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-co
t(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+3*(-I*(I-cot(d*x+c)+csc
(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*
x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*co
s(d*x+c)+cos(d*x+c)^3*2^(1/2)-6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I
+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE
((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+3*(-I*(I-cot(d*x+c)+csc(
d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x
+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-4*2
^(1/2)*cos(d*x+c)+3*2^(1/2))/(e*csc(d*x+c))^(1/2)/e^2*csc(d*x+c)+1/2*a^2/d*
2^(1/2)/(e*csc(d*x+c))^(1/2)/e^2*(6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-
I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*Ellip
ticE((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cot(d*x+c)-3*(-I*(I-
cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot
(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1
/2*2^(1/2))*cot(d*x+c)+6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*
x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I
-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*csc(d*x+c)-3*(-I*(I-cot(d*x+c)+
csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc
(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))
*csc(d*x+c)+2*2^(1/2)*cot(d*x+c)-3*2^(1/2)*csc(d*x+c)+2^(1/2)*sec(d*x+c)*cs
c(d*x+c))-2/3*a^2/d*(3*arctan((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+
c)+csc(d*x+c)))*(sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+3*(sin(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)*arctanh((sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cot(d*x+c)+csc(
d*x+c)))+2*cos(d*x+c)-2)/(cos(d*x+c)-1)/e^2/(e*csc(d*x+c))^(1/2)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 776, normalized size of antiderivative = 3.29

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(30*a^2*sqrt(-e)*arctan(-1/4*(cos(d*x + c)^2 - 6*sin(d*x + c) - 2)*s
qrt(-e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) + e))*cos(d*x + c) + 15*a^2*sq
rt(-e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(cos(d*
x + c)^4 - 9*cos(d*x + c)^2 + (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*sqrt
(-e)*sqrt(e/sin(d*x + c)) + 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 72*e
)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c)
+ 8)) + 54*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPI
nverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 54*a^2*sqrt(-2*I*e)*cos(d*x
+ c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d
*x + c))) - 4*(6*a^2*cos(d*x + c)^4 + 20*a^2*cos(d*x + c)^3 - 21*a^2*cos(d*
x + c)^2 - 20*a^2*cos(d*x + c) + 15*a^2)*sqrt(e/sin(d*x + c)))/(d*e^3*cos(d
*x + c)), 1/60*(30*a^2*sqrt(e)*arctan(1/4*(cos(d*x + c)^2 + 6*sin(d*x + c)
- 2)*sqrt(e)*sqrt(e/sin(d*x + c))/(e*sin(d*x + c) - e))*cos(d*x + c) + 15*a
^2*sqrt(e)*cos(d*x + c)*log((e*cos(d*x + c)^4 - 72*e*cos(d*x + c)^2 + 8*(co
s(d*x + c)^4 - 9*cos(d*x + c)^2 - (7*cos(d*x + c)^2 - 8)*sin(d*x + c) + 8)*
sqrt(e)*sqrt(e/sin(d*x + c)) - 28*(e*cos(d*x + c)^2 - 2*e)*sin(d*x + c) + 7
2*e)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x +
c) + 8)) - 54*a^2*sqrt(2*I*e)*cos(d*x + c)*weierstrassZeta(4, 0, weierstras
sPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 54*a^2*sqrt(-2*I*e)*cos(d
*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*si
n(d*x + c))) + 4*(6*a^2*cos(d*x + c)^4 + 20*a^2*cos(d*x + c)^3 - 21*a^2*cos
(d*x + c)^2 - 20*a^2*cos(d*x + c) + 15*a^2)*sqrt(e/sin(d*x + c)))/(d*e^3*co
s(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(5/2), x)
```

3.293 $\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad} + \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad} - \frac{2e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7ad} + \frac{4e^2\sqrt{e\csc(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21ad}$$

```
[Out] -4/21*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d+2/7*e^2*cot(d*x+c)*csc(d*x+c)
^2*(e*csc(d*x+c))^(1/2)/a/d-2/7*e^2*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a/d-4
/21*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellip
ticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/
2)/a/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3963, 3957, 2918, 2644, 30, 2647, 2716, 2720}

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = -\frac{2e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7ad} + \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad} - \frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad} + \frac{4e^2\sqrt{\sin(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{21ad}$$

```
[In] Int[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(21*a*d) + (2*e^2*Cot[c + d*x]*C
sc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(7*a*d) - (2*e^2*Csc[c + d*x]^3*Sqrt[e*
Csc[c + d*x]])/(7*a*d) + (4*e^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{(a+a\sec(c+dx))\sin^{\frac{5}{2}}(c+dx)}\,dx\\
&= -\left(\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sin^{\frac{5}{2}}(c+dx)}\,dx\right)\\
&= \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a}\\
&\quad-\frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a}\\
&= \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad}\\
&\quad+\frac{\left(2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{7a}\\
&\quad+\frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{x^{9/2}}\,dx,x,\sin(c+dx)\right)}{ad}\\
&= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad}+\frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad}\\
&\quad-\frac{2e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7ad}+\frac{\left(2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21a}\\
&= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad}+\frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad}\\
&\quad-\frac{2e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7ad}\\
&\quad+\frac{4e^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{5/2}\sec^4\left(\frac{1}{2}(c+dx)\right)\left((2+\cos(c+dx)-2\cos(2(c+dx))-\cos(3(c+dx))\right.}{168a}$$

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/168*(Csc[(c + d*x)/2]^2*(e*Csc[c + d*x])^(5/2)*Sec[(c + d*x)/2]^4*((2 +
Cos[c + d*x] - 2*Cos[2*(c + d*x)] - Cos[3*(c + d*x)])*EllipticF[(-2*c + Pi
- 2*d*x)/4, 2] + 2*(4 + 2*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[Sin[c + d*x
]])*Sin[c + d*x]^(5/2))/(a*d)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.68 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.32

method	result
default	$\frac{\sqrt{2}\left(\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}\right)^{\frac{5}{2}}(1-\cos(dx+c))^2\left(8i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{2}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\right.}{84ad\left((1-\cos(dx+c))^2\csc(dx+c)^2+\right.}$

```
[In] int((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/84/a/d*2^(1/2)*(e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)+sin(d*x+c))
)^(5/2)*(1-cos(d*x+c))^2*(8*I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*2^(1/2)*
(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*Ell
ipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*(-cot(d*x+c)+csc(d
*x+c))-3*(1-cos(d*x+c))^6*csc(d*x+c)^6-5*(1-cos(d*x+c))^4*csc(d*x+c)^4-9*(1
-cos(d*x+c))^2*csc(d*x+c)^2-7)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^2/((1-cos(
d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))
^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))^(1/2)*csc(d*x+c)^2
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.05

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx =$$

$$-\frac{2\left(\left(i\,e^2\cos\left(dx+c\right)+i\,e^2\right)\sqrt{2i\,e}\sin\left(dx+c\right)\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(-i\,e\right.\right.}{}$$

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/21*((I*e^2*cos(d*x + c) + I*e^2)*sqrt(2*I*e)*sin(d*x + c)*weierstrassPIn
verse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (-I*e^2*cos(d*x + c) - I*e^2)*
sqrt(-2*I*e)*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*
x + c)) + (2*e^2*cos(d*x + c)^2 + 2*e^2*cos(d*x + c) + 3*e^2)*sqrt(e/sin(d*
x + c)))/((a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\left(e\csc\left(dx+c\right)\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e \csc(c+dx))^{5/2}}{a+a \sec(c+dx)}\,dx = \int \frac{(e \csc(dx+c))^{\frac{5}{2}}}{a \sec(dx+c)+a}\,dx$$

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e \csc(c+dx))^{5/2}}{a+a \sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

```
[In] int((e/sin(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e/sin(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

3.294 $\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - \frac{2e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - \frac{4e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{5ad}$$

```
[Out] -4/5*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d+2/5*e*cot(d*x+c)*csc(d*x+c)*(e*c
sc(d*x+c))^(1/2)/a/d-2/5*e*csc(d*x+c)^2*(e*csc(d*x+c))^(1/2)/a/d+4/5*e*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3963, 3957, 2918, 2644, 30, 2647, 2716, 2719}

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\frac{2e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - \frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - \frac{4e\sqrt{\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{5ad}$$

```
[In] Int[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])/(5*a*d) + (2*e*Cot[c + d*x]*Csc[c
+ d*x]*Sqrt[e*Csc[c + d*x]])/(5*a*d) - (2*e*Csc[c + d*x]^2*Sqrt[e*Csc[c + d
*x]])/(5*a*d) - (4*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sin^{\frac{3}{2}}(c+dx)}\,dx\\
&= -\left(\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sin^{\frac{3}{2}}(c+dx)}\,dx\right)\\
&= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a}\\
&\quad -\frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a}\\
&= \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad}\\
&\quad +\frac{\left(2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{5a}\\
&\quad +\frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{x^{7/2}}\,dx,x,\sin(c+dx)\right)}{ad}\\
&= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad}+\frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad}\\
&\quad -\frac{2e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{\left(2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{5a}\\
&= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad}+\frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad}\\
&\quad -\frac{2e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{4e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{5ad}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.81 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.59

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{3/2}\left(\frac{8\sqrt{2}e^{i(c-dx)}\sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}}\left(3-3e^{2i(c+dx)}+e^{2idx}\left(1+e^{2ic}\right)\right.}{d(1+e^{2ic}}\right.}{15a(1+}$$

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(e*Csc[c + d*x])^(3/2)*((8*Sqrt[2]*E^(I*(c - d*x))*Sqrt
[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((2*I)*(c + d*x))
 + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*x))]*Hypergeome
tric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*(1 + E^((2*I)
*c))*Csc[c + d*x]^(3/2)) - (6*(4*Cos[d*x]*Sec[c] + Sec[(c + d*x)/2]^2)*Tan[
c + d*x])/d))/(15*a*(1 + Sec[c + d*x]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.11 (sec) , antiderivative size = 642, normalized size of antiderivative = 4.43

method	result
default	$\frac{\sqrt{2}\left(4\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\operatorname{EllipticE}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot(dx\right.}{}$

```
[In] int((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/a/d*2^(1/2)*(4*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc
(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))
*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)^2-2*(-I*(I+cot(d*x+c)-csc(
d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x
+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos
(d*x+c)^2+8*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)
))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*(I*(-I
+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)-4*(-I*(I+cot(d*x+c)-csc(d*x+c)))^
(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/
2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+4
*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*El
lipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*(I*(-I+cot(d*x+c)
-csc(d*x+c)))^(1/2)-2*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-
csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((I*(-I+co
```

```
t(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))-2*2^(1/2)*cos(d*x+c)-3*2^(1/2))*e*
(e*csc(d*x+c))^(1/2)/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\frac{2\left((e\cos(dx+c)+e)\sqrt{2i\,e}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))+\right.}{}$$

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/5*((e*cos(d*x + c) + e)*sqrt(2*I*e)*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + (e*cos(d*x + c) + e)*sqrt(-2*
I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d
*x + c))) + (2*e*cos(d*x + c)^2 + 2*e*cos(d*x + c) + e)*sqrt(e/sin(d*x + c)
))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\csc(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate((e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral((e*csc(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\csc(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

```
[In] int((e/sin(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e/sin(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.295 $\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad} - \frac{2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3ad} + \frac{4\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3ad}$$

```
[Out] 2/3*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d-2/3*csc(d*x+c)*(e*csc(d*x+c))^(1/2)
/a/d-4/3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1
/2)/a/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3963, 3957, 2918, 2644, 30, 2647, 2720}

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = -\frac{2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3ad} + \frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad} + \frac{4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{3ad}$$

```
[In] Int[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*a*d) - (2*Csc[c + d*x]*Sqrt[e*Csc[
c + d*x]])/(3*a*d) + (4*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{(a+a\sec(c+dx))\sqrt{\sin(c+dx)}}\,dx\\
&= -\left(\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sqrt{\sin(c+dx)}}\,dx\right)\\
&= \frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a}\\
&\quad-\frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a}\\
&= \frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad}+\frac{\left(2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a}\\
&\quad+\frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1}{x^{5/2}}\,dx,x,\sin(c+dx)\right)}{ad}\\
&= \frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad}-\frac{2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3ad}\\
&\quad+\frac{4\sqrt{e\csc(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

$$
\int\frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx
=\frac{2(e\csc(c+dx))^{3/2}\left(-1+\cos(c+dx)-2\operatorname{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3ade}
$$

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x]),x]
[Out] (2*(e*Csc[c + d*x])^(3/2)*(-1 + Cos[c + d*x] - 2*EllipticF[(-2*c + Pi - 2*d
*x)/4, 2]*Sin[c + d*x]^(3/2)))/(3*a*d*e)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.08 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.71

method	result
default	$\frac{\sqrt{2}\sqrt{\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}}\,(1-\cos(dx+c))\left(2i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{2}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\right.}{3ad\sqrt{(1-\cos(dx+c))\left((1-\cos(dx+c))^2\csc(dx+c)^2+1\right)}}$

[In] int((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/3/a/d*2^(1/2)*(e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)+sin(d*x+c)))^(1/2)*(1-cos(d*x+c))*(2*I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*2^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-(1-cos(d*x+c))^3*csc(d*x+c)^3-csc(d*x+c)+cot(d*x+c))/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))^(1/2)*csc(d*x+c)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = -\frac{2\left(\sqrt{2i\,e}(i\,\cos(dx+c)+i)\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{-2i\,e}(-i\,\cos(d\right.}{3\,(ad\cos(dx+c)}$$

[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -2/3*(sqrt(2*I*e)*(I*cos(d*x + c) + I)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(-2*I*e)*(-I*cos(d*x + c) - I)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(e/sin(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)

Sympy [F]

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{e\csc(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate((e*csc(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
[Out] Integral(sqrt(e*csc(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\csc(dx+c)}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\csc(dx+c)}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\sqrt{\frac{e}{\sin(c+d\,x)}}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

```
[In] int((e/sin(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e/sin(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.296 $\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 99

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}} - \frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}} + \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{ad\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] 2*cot(d*x+c)/a/d/(e*csc(d*x+c))^(1/2)-2*csc(d*x+c)/a/d/(e*csc(d*x+c))^(1/2)
-4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(
cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2
)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3963, 3957, 2918, 2644, 30, 2647, 2719}

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = -\frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}} + \frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}} + \frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (2*Cot[c + d*x])/(a*d*Sqrt[e*Csc[c + d*x]]) - (2*Csc[c + d*x])/(a*d*Sqrt[e*
Csc[c + d*x]]) + (4*EllipticE[(c - Pi/2 + d*x)/2, 2])/(a*d*Sqrt[e*Csc[c + d
*x]]*Sqrt[Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{\sin(c+dx)}}{a+a\sec(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int \frac{\cos(c+dx)\sqrt{\sin(c+dx)}}{-a-a\cos(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}} + \frac{2\int \sqrt{\sin(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \sin(c+dx)\right)}{ad\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}} - \frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}} + \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{ad\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx$$
$$= \frac{6(2i+\cot(c+dx)-\csc(c+dx)) - 4\sqrt{1-e^{2i(c+dx)}}(i+\cot(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{2i(c+dx}\right.}{3ad\sqrt{e\csc(c+dx)}}$$

```
[In] Integrate[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])),x]
[Out] (6*(2*I + Cot[c + d*x] - Csc[c + d*x]) - 4*Sqrt[1 - E^((2*I)*(c + d*x))]*(I
 + Cot[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])/(3*
a*d*Sqrt[e*Csc[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.46 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.36

method	result
default	$-\frac{\sqrt{2}\left(4\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{-i(i-\cot(dx+c\right.\right.}{}$

```
[In] int(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/d*2^(1/2)*(4*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-cs
c(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*
x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-2*(-I*(I-cot(d*x+c)+csc(d*x
+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)
))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*
x+c)+4*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^
(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc(d*
x+c)))^(1/2),1/2*2^(1/2))-2*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot
(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I
*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(d*x+c)-2^(1/2))/
(e*csc(d*x+c))^(1/2)*csc(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{2\left(\sqrt{\frac{e}{\sin(dx+c)}}(\cos(dx+c)-1)+\sqrt{2i\,e}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(\ldots$$
$$\overline{\qquad\qquad ade}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(e/sin(d*x + c))*(cos(d*x + c) - 1) + sqrt(2*I*e)*weierstrassZeta(4,
 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(-2*I*e
)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x
+ c))))/(a*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\sqrt{e\csc(c+dx)}\sec(c+dx)+\sqrt{e\csc(c+dx)}}\,dx}{a}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*csc(c + d*x))*sec(c + d*x) + sqrt(e*csc(c + d*x))), x)/a
```

Maxima [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{\sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{\sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+d\,x)}{a\,\sqrt{\frac{e}{\sin(c+d\,x)}}\,(\cos(c+d\,x)+1)}\,dx$$

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2)),x)
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.297 $\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{2}{ade\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3ade\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] 2/a/d/e/(e*csc(d*x+c))^(1/2)-2/3*cos(d*x+c)/a/d/e/(e*csc(d*x+c))^(1/2)+4/3*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3963, 3957, 2918, 2644, 30, 2649, 2720}

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{2}{ade\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3ade\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] 2/(a*d*e*Sqrt[e*Csc[c + d*x]]) - (2*Cos[c + d*x])/(3*a*d*e*Sqrt[e*Csc[c + d
*x]]) - (4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*a*d*e*Sqrt[e*Csc[c + d*x]]*
Sqrt[Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sin^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{3}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}} - \frac{2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \sin(c+dx)\right)}{ade\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{2}{ade\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3ade\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.66

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{4\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right) - 2(-3+\cos(c+dx))\sqrt{\sin(}}{3ad(e\csc(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (4*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 2*(-3 + Cos[c + d*x])*Sqrt[Sin[c +
 d*x]])/(3*a*d*(e*Csc[c + d*x])^(3/2)*Sin[c + d*x]^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.59

method	result
default	$\frac{\sqrt{2}\left(2i\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticF}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot(dx+}\right.}{}$

```
[In] int(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a/d*2^(1/2)*(2*I*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-c
sc(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2
```

```
))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)+2*I*(-I*(I+cot(d*x+c)-cs
c(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*
x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)+c
os(d*x+c)*sin(d*x+c)*2^(1/2)-3*2^(1/2)*sin(d*x+c))/(e*csc(d*x+c))^(1/2)/e/(
cos(d*x+c)-1)/(cos(d*x+c)+1)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.84

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = -\frac{2\left(\sqrt{\frac{e}{\sin(dx+c)}}(\cos(dx+c)-3)\sin(dx+c)-i\sqrt{2ie}\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c$$

$$3\,ade^2$$

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -2/3*(sqrt(e/sin(d*x + c))*(cos(d*x + c) - 3)*sin(d*x + c) - I*sqrt(2*I*e)*
weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(-2*I*e)*w
eierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))/(a*d*e^2)
```

Sympy [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\csc(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

```
[In] integrate(1/(e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
[Out] Integral(1/((e*csc(c + d*x))**(3/2)*sec(c + d*x) + (e*csc(c + d*x))**(3/2))
, x)/a
```

Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+dx)}{a\left(\frac{e}{\sin(c+dx)}\right)^{3/2}(\cos(c+dx)+1)}\,dx$$

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2)),x)
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.298 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = -\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5ade^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\sin(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}}$$

```
[Out] 2/3*sin(d*x+c)/a/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*cos(d*x+c)*sin(d*x+c)/a/d/e
^2/(e*csc(d*x+c))^(1/2)+4/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/e^2/(e*csc(
d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3963, 3957, 2918, 2644, 30, 2649, 2719}

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}} - \frac{2\sin(c+dx)\cos(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5ade^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-4*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*a*d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[
Sin[c + d*x]]) + (2*Sin[c + d*x])/(3*a*d*e^2*Sqrt[e*Csc[c + d*x]]) - (2*Cos
[c + d*x]*Sin[c + d*x])/(5*a*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sin^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{5}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \cos(c+dx)\sqrt{\sin(c+dx)}\,dx}{ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \cos^2(c+dx)\sqrt{\sin(c+dx)}\,dx}{ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2\cos(c+dx)\sin(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}} - \frac{2\int \sqrt{\sin(c+dx)}\,dx}{5ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\text{Subst}\left(\int \sqrt{x}\,dx, x, \sin(c+dx)\right)}{ade^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5ade^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\sin(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{8\sqrt{1-e^{2i(c+dx)}}(i+\cot(c+dx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e\right.}{30ade^2\sqrt{e\csc(c-}}$$

```
[In] Integrate[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
[Out] (8*Sqrt[1 - E^((2*I)*(c + d*x))]*(I + Cot[c + d*x])*Hypergeometric2F1[1/2,
3/4, 7/4, E^((2*I)*(c + d*x))] + 20*Sin[c + d*x] - 6*(4*I + Sin[2*(c + d*x)
]))/(30*a*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.59 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.85

method	result
default	$\frac{\sqrt{2}\left(12\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\text{EllipticE}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot(dx\right.}{}$

```
[In] int(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/a/d*2^(1/2)*(12*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-c
sc(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2
))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)-6*(-I*(I+cot(d*x+c)-csc(
d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x
+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos
(d*x+c)+12*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c))
)^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*(I*(-I+
cot(d*x+c)-csc(d*x+c)))^(1/2)-6*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(c
ot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF
((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))+3*cos(d*x+c)^3*2^(1/2)-5
*2^(1/2)*cos(d*x+c)^2+3*2^(1/2)*cos(d*x+c)-2^(1/2))/(e*csc(d*x+c))^(1/2)/e^
2*csc(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{2\left(\left(3\cos\left(dx+c\right)^{3}-5\cos\left(dx+c\right)^{2}-3\cos\left(dx+c\right)+5\right)\sqrt{\frac{e}{\sin\cdots}}\cdots\right.}{\cdots}$$

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - 3*cos(d*x + c) + 5)*sqrt(e/sin
(d*x + c)) - 3*sqrt(2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(-2*I*e)*weierstrassZeta(4, 0, weie
rstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{5}{2}}\,(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{5}{2}}\,(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+d\,x)}{a\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}\,(\cos(c+d\,x)+1)}\,dx$$

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2)),x)
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.299 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 149

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx=-\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}}-\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{21ade^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}}$$

```
[Out] -2/21*cos(d*x+c)/a/d/e^3/(e*csc(d*x+c))^(1/2)+2/7*cos(d*x+c)^3/a/d/e^3/(e*c
sc(d*x+c))^(1/2)+2/5*sin(d*x+c)^2/a/d/e^3/(e*csc(d*x+c))^(1/2)+4/21*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+
1/4*Pi+1/2*d*x),2^(1/2))/a/d/e^3/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3963, 3957, 2918, 2644, 30, 2648, 2649, 2720}

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx=\frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}}+\frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}}-\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21ade^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-2*Cos[c + d*x])/(21*a*d*e^3*Sqrt[e*Csc[c + d*x]]) + (2*Cos[c + d*x]^3)/(7
*a*d*e^3*Sqrt[e*Csc[c + d*x]]) - (4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*a
```

```
*d*e^3*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (2*Sin[c + d*x]^2)/(5*a*d
*e^3*Sqrt[e*Csc[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sin^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{7}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \cos(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \cos^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{7ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\text{Subst}\left(\int x^{3/2}\,dx, x, \sin(c+dx)\right)}{ade^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}}\\
&\quad + \frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}} - \frac{2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}}\\
&\quad - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{21ade^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.61

$$
\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \frac{\sqrt{e\csc(c+dx)}\left(80\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sqrt{\sin(c+d
$$

```
[In] Integrate[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
[Out] (Sqrt[e*Csc[c + d*x]]*(80*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c +
d*x]] + 126*Sin[c + d*x] + 10*Sin[2*(c + d*x)] - 42*Sin[3*(c + d*x)] + 15*S
in[4*(c + d*x)]))/(420*a*d*e^4)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.01 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.12

method	result
default	$\frac{\sqrt{2}\left(-10i\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\ \mathrm{EllipticF}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot}\right.}{}$

[In] int(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/105/a/d*2^(1/2)*(-10*I*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+15*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-10*I*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))-21*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-5*cos(d*x+c)*sin(d*x+c)*2^(1/2)+21*2^(1/2)*sin(d*x+c))/e^3/(e*csc(d*x+c))^(1/2)/(cos(d*x+c)-1)^2/(cos(d*x+c)+1)^2*sin(d*x+c)^3

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \frac{2\left(\left(15\cos\left(dx+c\right)^{3}-21\cos\left(dx+c\right)^{2}-5\cos\left(dx+c\right)+21\right)\right.}{}$$

[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] 2/105*((15*cos(d*x + c)^3 - 21*cos(d*x + c)^2 - 5*cos(d*x + c) + 21)*sqrt(e/sin(d*x + c))*sin(d*x + c) + 5*I*sqrt(2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(-2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))/(a*d*e^4)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*csc(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+d\,x)}{a\left(\frac{e}{\sin(c+d\,x)}\right)^{7/2}(\cos(c+d\,x)+1)}\,dx$$

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(7/2)),x)
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(7/2)*(cos(c + d*x) + 1)), x)
```

3.300 $\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 268

$$\begin{aligned}\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx &= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d} \\ &+ \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} \\ &- \frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} - \frac{4e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} \\ &- \frac{2e^2\cot(c+dx)\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} + \frac{4e^2\csc^5(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} \\ &+ \frac{4e^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231a^2d}\end{aligned}$$

```
[Out] -4/231*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d+16/77*e^2*cot(d*x+c)*csc(d
*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d-2/11*e^2*cot(d*x+c)^3*csc(d*x+c)^2*(e*cs
c(d*x+c))^(1/2)/a^2/d-4/7*e^2*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a^2/d-2/11*
e^2*cot(d*x+c)*csc(d*x+c)^4*(e*csc(d*x+c))^(1/2)/a^2/d+4/11*e^2*csc(d*x+c)^
5*(e*csc(d*x+c))^(1/2)/a^2/d-4/231*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*c
sc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3963, 3957, 2954, 2952, 2647, 2716, 2720, 2644, 14}

$$\begin{aligned}
\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx &= \frac{4e^2\csc^5(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} \\
&- \frac{4e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} \\
&- \frac{2e^2\cot(c+dx)\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} \\
&+ \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} - \frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d} \\
&+ \frac{4e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{231a^2d}
\end{aligned}$$

```
[In] Int[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(231*a^2*d) + (16*e^2*Cot[c + d*
x]*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(77*a^2*d) - (2*e^2*Cot[c + d*x]^3*
Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(11*a^2*d) - (4*e^2*Csc[c + d*x]^3*Sqr
t[e*Csc[c + d*x]])/(7*a^2*d) - (2*e^2*Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[e*Cs
c[c + d*x]])/(11*a^2*d) + (4*e^2*Csc[c + d*x]^5*Sqrt[e*Csc[c + d*x]])/(11*a
^2*d) + (4*e^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(231*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
```

```
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{(a+a\sec(c+dx))^2\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{13}{2}}(c+dx)}\,dx}{a^4}$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}-\frac{2a^2\cos^3(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}+\frac{a^2\cos^4(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}\right)dx}{a^4}$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}\,dx}{a^2} + \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^4(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}\,dx}{a^2} - \frac{\left(2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^3(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} - \frac{2e^2\cot(c+dx)\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} - \frac{\left(2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{11a^2} - \frac{\left(6e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{11a^2} - \frac{\left(2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int \frac{1-x^2}{x^{13/2}}\,dx, x, \sin(c+dx)\right)}{a^2d}$$

$$= \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} - \frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} - \frac{2e^2\cot(c+dx)\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} - \frac{\left(10e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{77a^2} + \frac{\left(12e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{77a^2} - \frac{\left(2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int \left(\frac{1}{x^{13/2}}-\frac{1}{x^{9/2}}\right)dx, x, \sin(c+dx)\right)}{a^2d}$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d} + \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} - \frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} - \frac{4e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{2e^2\cot(c+dx)\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} + \frac{4e^2\csc^5(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} - \frac{\left(10e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{231a^2} + \frac{\left(4e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{77a^2}$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d} + \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} - \frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} - \frac{4e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{2e^2\cot(c+dx)\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} + \frac{4e^2\csc^5(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} + \frac{4e^2\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231a^2d}$$

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.43

$$\int\frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = -\frac{e^3\csc^2\left(\frac{1}{2}(c+dx)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(52+97\cos(c+dx)+4\cos(2(c+dx))+\cos(3(c+dx))+\csc^4\left(\frac{1}{2}(c\right.\right.}{3696a^2d\sqrt{e\csc(c+dx)}}$$

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3696*(e^3*Csc[(c + d*x)/2]^2*Sec[(c + d*x)/2]^6*(52 + 97*Cos[c + d*x] +
4*Cos[2*(c + d*x)] + Cos[3*(c + d*x)] + Csc[(c + d*x)/2]^4*EllipticF[(-2*c
+ Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(11/2)))/(a^2*d*Sqrt[e*Csc[c + d*x]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.06 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{2}\left(\frac{e\left(\left(1-\cos(dx+c)\right)^{2}\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}\right)^{\frac{5}{2}}\left(1-\cos(dx+c)\right)^{2}\left(21\left(1-\cos(dx+c)\right)^{8}\csc(dx+c)^{8}+16i\sqrt{-i\left(i-\cot(dx+c)+\csc(dx+c)\right)}\right.}{1848a^{2}d\left(\left(1-\cos(dx\right.\right.}$

[In] int((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/1848/a^2/d*2^(1/2)*(e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)+sin(d*x
+c)))^(5/2)*(1-cos(d*x+c))^2*(21*(1-cos(d*x+c))^8*csc(d*x+c)^8+16*I*(-I*(I-
cot(d*x+c)+csc(d*x+c)))^(1/2)*2^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*
(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))
^(1/2),1/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+c))-6*(1-cos(d*x+c))^6*csc(d*x+c)^
6-136*(1-cos(d*x+c))^4*csc(d*x+c)^4-186*(1-cos(d*x+c))^2*csc(d*x+c)^2-77)/(
(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^2/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*
x+c)^2+1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d
*x+c))^(1/2)*csc(d*x+c)^2

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.82

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\left(\left(i\,e^2\cos\left(dx+c\right)^2+2i\,e^2\cos\left(dx+c\right)+i\,e^2\right)\sqrt{2i\,e}\sin\left(dx+c\right)\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\right.}{}$$

[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] -2/231*((I*e^2*cos(d*x + c)^2 + 2*I*e^2*cos(d*x + c) + I*e^2)*sqrt(2*I*e)*s
in(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (-I*
e^2*cos(d*x + c)^2 - 2*I*e^2*cos(d*x + c) - I*e^2)*sqrt(-2*I*e)*sin(d*x + c
)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + (2*e^2*cos(d*x
 + c)^3 + 4*e^2*cos(d*x + c)^2 + 47*e^2*cos(d*x + c) + 24*e^2)*sqrt(e/sin(d
*x + c)))/((a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x +
c))

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\csc(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}}{a^2\left(\cos(c+d\,x)+1\right)^2}\,dx$$

```
[In] int((e/sin(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e/sin(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.301 $\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 250

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d} + \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d} - \frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} - \frac{4e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5a^2d} - \frac{2e\cot(c+dx)\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} + \frac{4e\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} - \frac{4e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{15a^2d}$$

```
[Out] -4/15*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d+16/45*e*cot(d*x+c)*csc(d*x+c)
*(e*csc(d*x+c))^(1/2)/a^2/d-2/9*e*cot(d*x+c)^3*csc(d*x+c)*(e*csc(d*x+c))^(1
/2)/a^2/d-4/5*e*csc(d*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d-2/9*e*cot(d*x+c)*cs
c(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a^2/d+4/9*e*csc(d*x+c)^4*(e*csc(d*x+c))^(1/
2)/a^2/d+4/15*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*
x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*
x+c)^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3963, 3957, 2954, 2952, 2647, 2716, 2719, 2644, 14}

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{4e\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} - \frac{4e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5a^2d} - \frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d} - \frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} - \frac{2e\cot(c+dx)\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} + \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d} - \frac{4e\sqrt{\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{15a^2d}$$

```
[In] Int[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])/(15*a^2*d) + (16*e*Cot[c + d*x]*Cs
c[c + d*x]*Sqrt[e*Csc[c + d*x]])/(45*a^2*d) - (2*e*Cot[c + d*x]^3*Csc[c + d
*x]*Sqrt[e*Csc[c + d*x]])/(9*a^2*d) - (4*e*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*
x]])/(5*a^2*d) - (2*e*Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[e*Csc[c + d*x]])/(9*
a^2*d) + (4*e*Csc[c + d*x]^4*Sqrt[e*Csc[c + d*x]])/(9*a^2*d) - (4*e*Sqrt[e*
Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(15*a^2*
d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
```

```
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{(a+a\sec(c+dx))^2\sin^{\frac{3}{2}}(c+dx)}\,dx\\
&= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sin^{\frac{3}{2}}(c+dx)}\,dx\\
&= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{11}{2}}(c+dx)}\,dx}{a^4}\\
&= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}-\frac{2a^2\cos^3(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}+\frac{a^2\cos^4(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}\right)dx}{a^4}\\
&= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^2(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}\,dx}{a^2}\\
&\quad+\frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^4(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}\,dx}{a^2}\\
&\quad-\frac{\left(2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^3(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}\,dx}{a^2}\\
&= -\frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}\\
&\quad-\frac{2e\cot(c+dx)\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}\\
&\quad-\frac{\left(2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{9a^2}\\
&\quad-\frac{\left(2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{3a^2}\\
&\quad-\frac{\left(2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1-x^2}{x^{11/2}}\,dx,x,\sin(c+dx)\right)}{a^2d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d} \\
&- \frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} \\
&- \frac{2e\cot(c+dx)\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} \\
&- \frac{\left(2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{15a^2} \\
&+ \frac{\left(4e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{15a^2} \\
&- \frac{\left(2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\left(\frac{1}{x^{11/2}}-\frac{1}{x^{7/2}}\right)\,dx,x,\sin(c+dx)\right)}{a^2d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d} + \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d} \\
&- \frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} - \frac{4e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5a^2d} \\
&- \frac{2e\cot(c+dx)\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} + \frac{4e\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} \\
&+ \frac{\left(2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{15a^2} \\
&- \frac{\left(4e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\sqrt{\sin(c+dx)}\,dx}{15a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d} + \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d} \\
&- \frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} - \frac{4e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5a^2d} \\
&- \frac{2e\cot(c+dx)\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} + \frac{4e\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} \\
&- \frac{4e\sqrt{e\csc(c+dx)}E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{15a^2d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.36 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.99

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{3/2}\sec(c+dx)\left(\frac{16\sqrt{2}e^{i(c-dx)}\sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}}\left(3-3e^{2i(c-}\right.}{}\right.}{}$$

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(e*Csc[c + d*x])^(3/2)*Sec[c + d*x]*((16*Sqrt[2]*E^(I*(
c - d*x))*Sqrt[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((2
*I)*(c + d*x)) + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d
*(1 + E^((2*I)*c))*Csc[c + d*x]^(3/2)) - (2*(24*Cos[d*x]*Sec[c] + (8 + 13*C
os[c + d*x])*Sec[(c + d*x)/2]^4)*Tan[c + d*x])/d))/(45*a^2*(1 + Sec[c + d*x
])^2)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.25 (sec) , antiderivative size = 859, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	859

```
[In] int((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/45/a^2/d*2^(1/2)*(12*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+
c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-c
ot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)^3-6*(-I*(I-cot(d*x+c)+
csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc
(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))
*cos(d*x+c)^3+36*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc
(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x
+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-18*(-I*(I-cot(d*x+c)+csc(d
*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+
c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(
d*x+c)^2+36*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+
c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+c
sc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)-18*(-I*(I-cot(d*x+c)+csc(d*x+c)))
^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1
```

```
/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+
12*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2
)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((-I*(I-cot(d*x+c)+csc(d*x+c)
))^(1/2),1/2*2^(1/2))-6*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(-I*(I+cot(d*x
+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((-I*(I-
cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))-6*2^(1/2)*cos(d*x+c)^2-25*2^(1/2
)*cos(d*x+c)-14*2^(1/2))*e*(e*csc(d*x+c))^(1/2)/(cos(d*x+c)+1)^2
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.73

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{2\left(3\left(e\cos\left(dx+c\right)^2+2\,e\cos\left(dx+c\right)+e\right)\sqrt{2i\,e}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)\right.}{}$$

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/45*(3*(e*cos(d*x + c)^2 + 2*e*cos(d*x + c) + e)*sqrt(2*I*e)*weierstrassZ
eta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(e*
cos(d*x + c)^2 + 2*e*cos(d*x + c) + e)*sqrt(-2*I*e)*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + (6*e*cos(d*x + c
)^3 + 12*e*cos(d*x + c)^2 + 19*e*cos(d*x + c) + 8*e)*sqrt(e/sin(d*x + c)))/
(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{(e\csc{(c+dx)})^{\frac{3}{2}}}{\sec^2{(c+dx)}+2\sec{(c+dx)}+1}\,dx}{a^2}$$

```
[In] integrate((e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*csc(c + d*x))**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
/a**2
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e \csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(e \csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\csc(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e \csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int((e/sin(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e/sin(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.302 $\int \frac{\sqrt{e \csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\begin{aligned}\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx &= \frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d} - \frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} \\ &\quad - \frac{4\csc(c+dx)\sqrt{e\csc(c+dx)}}{3a^2d} \\ &\quad - \frac{2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} \\ &\quad + \frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} \\ &\quad + \frac{20\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d}\end{aligned}$$

```
[Out] 16/21*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d-2/7*cot(d*x+c)^3*(e*csc(d*x+c))
^(1/2)/a^2/d-4/3*csc(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d-2/7*cot(d*x+c)*csc(d
*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d+4/7*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a^
2/d-20/21*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(
1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3963, 3957, 2954, 2952, 2647, 2716, 2720, 2644, 14}

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{4\csc(c+dx)\sqrt{e\csc(c+dx)}}{3a^2d} - \frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} + \frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d} + \frac{20\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{21a^2d}$$

```
[In] Int[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (16*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(21*a^2*d) - (2*Cot[c + d*x]^3*Sqrt[
e*Csc[c + d*x]])/(7*a^2*d) - (4*Csc[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*a^2*d
) - (2*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(7*a^2*d) + (4*Csc
[c + d*x]^3*Sqrt[e*Csc[c + d*x]])/(7*a^2*d) + (20*Sqrt[e*Csc[c + d*x]]*Elli
pticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{(a+a\sec(c+dx))^2\sqrt{\sin(c+dx)}}\,dx\\ &= \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sqrt{\sin(c+dx)}}\,dx\end{aligned}$$

$$=\frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a^4}$$

$$=\frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}-\frac{2a^2\cos^3(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}+\frac{a^2\cos^4(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\right)\,dx}{a^4}$$

$$=\frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a^2}+\frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^4(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a^2}-\frac{\left(2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^3(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a^2}$$

$$=-\frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{\left(2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{7a^2}-\frac{\left(6\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{7a^2}-\frac{\left(2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1-x^2}{x^{9/2}}\,dx,x,\sin(c+dx)\right)}{a^2d}$$

$$=\frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d}-\frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{\left(2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{21a^2}+\frac{\left(4\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\sin(c+dx)}}\,dx}{7a^2}-\frac{\left(2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\left(\frac{1}{x^{9/2}}-\frac{1}{x^{5/2}}\right)\,dx,x,\sin(c+dx)\right)}{a^2d}$$

$$
\begin{aligned}
&= \frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d} - \frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} \\
&\quad - \frac{4\csc(c+dx)\sqrt{e\csc(c+dx)}}{3a^2d} - \frac{2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} \\
&\quad + \frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} \\
&\quad + \frac{20\sqrt{e\csc(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.41

$$
\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx =
-\frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}\left(2(8+11\cos(c+dx))\sin^4\left(\frac{1}{2}(c+dx)\right)+5\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right.\right.}{21a^2d}
$$

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*Csc[c + d*x]^3*Sqrt[e*Csc[c + d*x]]*(2*(8 + 11*Cos[c + d*x])*Sin[(c + d
*x)/2]^4 + 5*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(21*a
^2*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.06 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.54

method	result
default	$\frac{\sqrt{2}\sqrt{\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)+\sin(dx+c)\right)}{1-\cos(dx+c)}}\,(1-\cos(dx+c))\left(20i\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{2}\,\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{\right.}{42a^2d\sqrt{(1-\cos(dx+c))\left((1-\cos(dx+\right.}}$

```
[In] int((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/42/a^2/d*2^(1/2)*(e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)+sin(d*x+c
)))^(1/2)*(1-cos(d*x+c))*(20*I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*2^(1/2)
*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*El
lipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))+3*(1-cos(d*x+c))^
5*csc(d*x+c)^5-16*(1-cos(d*x+c))^3*csc(d*x+c)^3-19*csc(d*x+c)+19*cot(d*x+c)
)/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)*csc(d*x+c))^(1/2)/((1-c
os(d*x+c))^3*csc(d*x+c)^3+csc(d*x+c)-cot(d*x+c))^(1/2)*csc(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{e}{\sin(dx+c)}}(11\cos(dx+c)+8)\sin(dx+c)+5\left(i\cos(dx+c)^2+2i\cos(dx+c)+i\right)\sqrt{2ie}\,\mathrm{weierstr}\right.}{}$$

[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] -2/21*(sqrt(e/sin(d*x + c))*(11*cos(d*x + c) + 8)*sin(d*x + c) + 5*(I*cos(d*x + c)^2 + 2*I*cos(d*x + c) + I)*sqrt(2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*cos(d*x + c)^2 - 2*I*cos(d*x + c) - I)*sqrt(-2*I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F]

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{e\csc(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

[In] integrate((e*csc(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)

[Out] Integral(sqrt(e*csc(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

Giac [F]

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\csc\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\sqrt{\frac{e}{\sin(c+d\,x)}}}{a^2\,\left(\cos\left(c+d\,x\right)+1\right)^2}\,dx$$

```
[In] int((e/sin(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e/sin(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.303 $\int \frac{1}{\sqrt{e \csc(c+dx)}(a+a\sec(c+dx))^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 199

$$
\begin{aligned}
&\int \frac{1}{\sqrt{e \csc(c+dx)}(a+a\sec(c+dx))^2} \, dx \\
&= \frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}} \\
&\quad - \frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} + \frac{4\csc^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} + \frac{28E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5a^2d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}
\end{aligned}
$$

```
[Out] 16/5*cot(d*x+c)/a^2/d/(e*csc(d*x+c))^(1/2)-2/5*cot(d*x+c)^3/a^2/d/(e*csc(d*
x+c))^(1/2)-4*csc(d*x+c)/a^2/d/(e*csc(d*x+c))^(1/2)-2/5*cot(d*x+c)*csc(d*x+
c)^2/a^2/d/(e*csc(d*x+c))^(1/2)+4/5*csc(d*x+c)^3/a^2/d/(e*csc(d*x+c))^(1/2)
-28/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

$= \{3963, 3957, 2954, 2952, 2647, 2716, 2719, 2644, 14\}$

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{4\csc^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} + \frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} + \frac{28E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5a^2d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (16*Cot[c + d*x])/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) - (2*Cot[c + d*x]^3)/(5*a^
2*d*Sqrt[e*Csc[c + d*x]]) - (4*Csc[c + d*x])/(a^2*d*Sqrt[e*Csc[c + d*x]]) -
 (2*Cot[c + d*x]*Csc[c + d*x]^2)/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) + (4*Csc[c
+ d*x]^3)/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) + (28*EllipticE[(c - Pi/2 + d*x)/2
, 2])/(5*a^2*d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)\sqrt{\sin(c+dx)}}{(-a-a\cos(c+dx))^2}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a^4\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{7}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\right)dx}{a^4\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2\int \frac{\cos^3(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\int \frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{5a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{6\int \frac{\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{5a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1-x^2}{x^{7/2}}\,dx, x, \sin(c+dx)\right)}{a^2d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} + \frac{2\int \sqrt{\sin(c+dx)}\,dx}{5a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{12\int \sqrt{\sin(c+dx)}\,dx}{5a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2\text{Subst}\left(\int \left(\frac{1}{x^{7/2}} - \frac{1}{x^{3/2}}\right)dx, x, \sin(c+dx)\right)}{a^2d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} + \frac{4\csc^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} + \frac{28E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5a^2d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\csc(c+dx)}\sec^2(c+dx)\left(-\frac{28\sqrt{2}e^{i(c-dx)}\sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}}\left(3-3e^{2i(c+dx)}+e^{2idx}\left(1+e^{2ic}\right)\sqrt{1-e^{2i(c+dx)}}\right.}{1+e^{2ic}}\right.}{15a^2d\sqrt{}}$$

```
[In] Integrate[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sqrt[Csc[c + d*x]]*Sec[c + d*x]^2*((-28*Sqrt[2]*E^(I*
(c - d*x))*Sqrt[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((
2*I)*(c + d*x)) + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))]))/(1 + E^((2*I)*
c)) - 3*Sqrt[Csc[c + d*x]]*((-23 + 5*Cos[2*c])*Cos[d*x]*Sec[c] - 2*(-10 + S
ec[(c + d*x)/2]^2 + 5*Sin[c]*Sin[d*x]))))/(15*a^2*d*Sqrt[e*Csc[c + d*x]]*(1
 + Sec[c + d*x])^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.70 (sec) , antiderivative size = 659, normalized size of antiderivative = 3.31

method	result
default	$-\frac{\sqrt{2}\left(28\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\,\mathrm{EllipticE}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot}\right.}{}$

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a^2/d*2^(1/2)*(28*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)
-csc(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1
/2))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)^2-14*(-I*(I+cot(d*x+c)
-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-cs
c(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2)
)*cos(d*x+c)^2+56*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(
d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*
(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)-28*(-I*(I+cot(d*x+c)-csc(d*
x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x+c
)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d
*x+c)+28*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^
```

```
(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*(I*(-I+co
t(d*x+c)-csc(d*x+c)))^(1/2)-14*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(co
t(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF(
(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))+5*2^(1/2)*cos(d*x+c)^2+2^
(1/2)*cos(d*x+c)-6*2^(1/2))/(cos(d*x+c)+1)/(e*csc(d*x+c))^(1/2)*csc(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{2\left(7\sqrt{2i\,e}(\cos(dx+c)+1)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))+\right.}{}$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(7*sqrt(2*I*e)*(cos(d*x + c) + 1)*weierstrassZeta(4, 0, weierstrassPInv
erse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 7*sqrt(-2*I*e)*(cos(d*x + c) +
 1)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*
x + c))) + (9*cos(d*x + c)^2 - cos(d*x + c) - 8)*sqrt(e/sin(d*x + c)))/(a^2
*d*e*cos(d*x + c) + a^2*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sqrt{e\csc(c+dx)}\sec^2(c+dx)+2\sqrt{e\csc(c+dx)}\sec(c+dx)+\sqrt{e\csc(c+dx)}}\,dx}{a^2}$$

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*csc(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*csc(c + d*x))*s
ec(c + d*x) + sqrt(e*csc(c + d*x))), x)/a**2
```

Maxima [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\csc\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\csc\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{a^2\,\sqrt{\frac{e}{\sin(c+d\,x)}}\,\left(\cos\left(c+d\,x\right)+1\right)^2}\,dx$$

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2)),x)
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.304 $\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 213

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{4}{a^2de\sqrt{e\csc(c+dx)}} - \frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} + \frac{4\csc^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{4\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{a^2de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] 4/a^2/d/e/(e*csc(d*x+c))^(1/2)-4/3*cos(d*x+c)/a^2/d/e/(e*csc(d*x+c))^(1/2)-
2/3*cos(d*x+c)*cot(d*x+c)^2/a^2/d/e/(e*csc(d*x+c))^(1/2)-2/3*cot(d*x+c)*csc
(d*x+c)/a^2/d/e/(e*csc(d*x+c))^(1/2)+4/3*csc(d*x+c)^2/a^2/d/e/(e*csc(d*x+c)
)^(1/2)+4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/e/(e*csc(d*x+c))^(1/2)/sin(
d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3963, 3957, 2954, 2952, 2647, 2720, 2644, 14, 2649}

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{4\csc^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} + \frac{4}{a^2de\sqrt{e\csc(c+dx)}} - \frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{4\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^2de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] 4/(a^2*d*e*Sqrt[e*Csc[c + d*x]]) - (4*Cos[c + d*x])/(3*a^2*d*e*Sqrt[e*Csc[c
 + d*x]]) - (2*Cos[c + d*x]*Cot[c + d*x]^2)/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]
) - (2*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]) + (4*Csc
[c + d*x]^2)/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]) - (4*EllipticF[(c - Pi/2 + d*
x)/2, 2])/(a^2*d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
```

```
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\sin^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{3}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^4e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}-\frac{2a^2\cos^3(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}+\frac{a^2\cos^4(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\right)dx}{a^4e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&\quad -\frac{2\int \frac{\cos^3(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}
\end{aligned}$$

$$= -\frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1-x^2}{x^{5/2}}\,dx, x, \sin(c+dx)\right)}{a^2de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3a^2de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{4\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2\text{Subst}\left(\int \left(\frac{1}{x^{5/2}}-\frac{1}{\sqrt{x}}\right)\,dx, x, \sin(c+dx)\right)}{a^2de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{4}{a^2de\sqrt{e\csc(c+dx)}} - \frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} + \frac{4\csc^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{a^2de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.47

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(12(1+\cos(c+dx))\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-\right.\right.}{6a^2d(e\csc(c+}$$

```
[In] Integrate[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Sec[(c + d*x)/2]^2*(12*(1 + Cos[c + d*x])*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2] + (15 + 10*Cos[c + d*x] - Cos[2*(c + d*x)])*Sqrt[Sin[c + d*x]]))/(6*a^2
*d*(e*Csc[c + d*x])^(3/2)*Sin[c + d*x]^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.99 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.86

method	result
default	$\frac{\sqrt{2}\left(6i\sqrt{i(-i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{i(-\cot(dx+c)+\csc(dx+c))}\,\operatorname{EllipticF}\left(\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))},\frac{\sqrt{2}}{2}\right)\sqrt{-i(i-\cot(dx+\right.}{}$

```
[In] int(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^2/d*2^(1/2)*(6*I*(I*(-I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)
+csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1
/2))*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*cos(d*x+c)^2+12*I*(I*(-I-cot(d*x+
c)+csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*EllipticF((-I*(I-c
ot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1
/2)*cos(d*x+c)+6*I*(I*(-I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc
(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))
*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)+cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-5*cos
(d*x+c)*sin(d*x+c)*2^(1/2)-8*2^(1/2)*sin(d*x+c))/(cos(d*x+c)-1)/(cos(d*x+c)
+1)^2/e/(e*csc(d*x+c))^(1/2)*sin(d*x+c)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.63

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = -\frac{2\left(\left(\cos\left(dx+c\right)^2-5\cos\left(dx+c\right)-8\right)\sqrt{\frac{e}{\sin(dx+c)}}\sin\left(dx+c\right)+3\sqrt{2i\,e}(-i\cos\left(dx+c\right)-i)\mathrm{weierstrassPIn}\right.}{3\,(a^2}$$

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*((cos(d*x + c)^2 - 5*cos(d*x + c) - 8)*sqrt(e/sin(d*x + c))*sin(d*x +
c) + 3*sqrt(2*I*e)*(-I*cos(d*x + c) - I)*weierstrassPInverse(4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 3*sqrt(-2*I*e)*(I*cos(d*x + c) + I)*weierstrassPIn
verse(4, 0, cos(d*x + c) - I*sin(d*x + c)))/(a^2*d*e^2*cos(d*x + c) + a^2*d
*e^2)
```

Sympy [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}\sec^2(c+dx)+2(e\csc(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\csc(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

```
[In] integrate(1/(e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
[Out] Integral(1/((e*csc(c + d*x))**(3/2)*sec(c + d*x)**2 + 2*(e*csc(c + d*x))**(
3/2)*sec(c + d*x) + (e*csc(c + d*x))**(3/2)), x)/a**2
```

Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2}{a^2\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2)),x)
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

3.305 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 215

$$\begin{aligned}\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx =&-\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}\\&+\frac{4\csc(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{44E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5a^2de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\&+\frac{4\sin(c+dx)}{3a^2de^2\sqrt{e\csc(c+dx)}}-\frac{12\cos(c+dx)\sin(c+dx)}{5a^2de^2\sqrt{e\csc(c+dx)}}\end{aligned}$$

```
[Out] -2*cot(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)-2*cos(d*x+c)^2*cot(d*x+c)/a^2/
d/e^2/(e*csc(d*x+c))^(1/2)+4*csc(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)+4/3*
sin(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)-12/5*cos(d*x+c)*sin(d*x+c)/a^2/d/
e^2/(e*csc(d*x+c))^(1/2)+44/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/e^2/(e*
csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

= {3963, 3957, 2954, 2952, 2647, 2719, 2644, 14, 2649}

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \frac{4\csc(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} + \frac{4\sin(c+dx)}{3a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{12\sin(c+dx)\cos(c+dx)}{5a^2de^2\sqrt{e\csc(c+dx)}} - \frac{44E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5a^2de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-2*Cot[c + d*x])/(a^2*d*e^2*Sqrt[e*Csc[c + d*x]]) - (2*Cos[c + d*x]^2*Cot[
c + d*x])/(a^2*d*e^2*Sqrt[e*Csc[c + d*x]]) + (4*Csc[c + d*x])/(a^2*d*e^2*Sq
rt[e*Csc[c + d*x]]) - (44*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*a^2*d*e^2*Sq
rt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (4*Sin[c + d*x])/(3*a^2*d*e^2*Sqrt
[e*Csc[c + d*x]]) - (12*Cos[c + d*x]*Sin[c + d*x])/(5*a^2*d*e^2*Sqrt[e*Csc[
c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\sin^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{5}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{a^4e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}
\end{aligned}$$

$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{3}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\right)dx}{a^4e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx}{a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx}{a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2\int \frac{\cos^3(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx}{a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\int\sqrt{\sin(c+dx)}dx}{a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{6\int\cos^2(c+dx)\sqrt{\sin(c+dx)}dx}{a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1-x^2}{x^{3/2}}dx, x, \sin(c+dx)\right)}{a^2de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{a^2de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{12\cos(c+dx)\sin(c+dx)}{5a^2de^2\sqrt{e\csc(c+dx)}} - \frac{12\int\sqrt{\sin(c+dx)}dx}{5a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2\text{Subst}\left(\int\left(\frac{1}{x^{3/2}}-\sqrt{x}\right)dx, x, \sin(c+dx)\right)}{a^2de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} + \frac{4\csc(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{44E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5a^2de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{4\sin(c+dx)}{3a^2de^2\sqrt{e\csc(c+dx)}} - \frac{12\cos(c+dx)\sin(c+dx)}{5a^2de^2\sqrt{e\csc(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.82 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}dx = \frac{-123\cot(c+dx) + 88\sqrt{1-e^{2i(c+dx)}}(i+\cot(c+dx))\,\text{Hyperg}}{}$$

```
[In] Integrate[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

[Out] (-123*Cot[c + d*x] + 88*Sqrt[1 - E^((2*I)*(c + d*x))]*(I + Cot[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))] + Csc[c + d*x]*(140 - 20*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)] - (264*I)*Sin[c + d*x]))/(30*a^2*d*e^2*Sqrt[e*Csc[c + d*x]])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.86 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.15

method	result
default	$\frac{\sqrt{2}\left(132\sqrt{-i(i+\cot(dx+c)-\csc(dx+c))}\,\sqrt{-i(\cot(dx+c)-\csc(dx+c))}\ \mathrm{EllipticE}\left(\sqrt{i(-i+\cot(dx+c)-\csc(dx+c))}, \frac{\sqrt{2}}{2}\right)\sqrt{i(-i+\cot(dx\right.}{}$

[In] int(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/15/a^2/d*2^(1/2)*(132*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*cos(d*x+c)-66*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+3*cos(d*x+c)^3*2^(1/2)+132*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticE((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)-66*(-I*(I+cot(d*x+c)-csc(d*x+c)))^(1/2)*(-I*(cot(d*x+c)-csc(d*x+c)))^(1/2)*(I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2)*EllipticF((I*(-I+cot(d*x+c)-csc(d*x+c)))^(1/2),1/2*2^(1/2))-10*2^(1/2)*cos(d*x+c)^2+33*2^(1/2)*cos(d*x+c)-26*2^(1/2))/(e*csc(d*x+c))^(1/2)/e^2*csc(d*x+c)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.52

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \frac{2\left(\left(3\,\cos\left(dx+c\right)^3-10\,\cos\left(dx+c\right)^2-33\,\cos\left(dx+c\right)+40\right)\right.}{}$$

[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] 2/15*((3*cos(d*x + c)^3 - 10*cos(d*x + c)^2 - 33*cos(d*x + c) + 40)*sqrt(e/sin(d*x + c)) - 33*sqrt(2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 33*sqrt(-2*I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*e^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\left(e\csc\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\left(e\csc\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{5/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2)),x)
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

3.306 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = -\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}} + \frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}} + \frac{52\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{21a^2de^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{4\sin^2(c+dx)}{5a^2de^3\sqrt{e\csc(c+dx)}}$$

```
[Out] -4/a^2/d/e^3/(e*csc(d*x+c))^(1/2)+26/21*cos(d*x+c)/a^2/d/e^3/(e*csc(d*x+c))
^(1/2)+2/7*cos(d*x+c)^3/a^2/d/e^3/(e*csc(d*x+c))^(1/2)+4/5*sin(d*x+c)^2/a^2
/d/e^3/(e*csc(d*x+c))^(1/2)-52/21*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/e^3
/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3963, 3957, 2954, 2952, 2649, 2720, 2644, 14}

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = -\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}} + \frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}} + \frac{4\sin^2(c+dx)}{5a^2de^3\sqrt{e\csc(c+dx)}} + \frac{52\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21a^2de^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[In] Int[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] -4/(a^2*d*e^3*Sqrt[e*Csc[c + d*x]]) + (26*Cos[c + d*x])/(21*a^2*d*e^3*Sqrt[
e*Csc[c + d*x]]) + (2*Cos[c + d*x]^3)/(7*a^2*d*e^3*Sqrt[e*Csc[c + d*x]]) +
(52*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*a^2*d*e^3*Sqrt[e*Csc[c + d*x]]*Sq
rt[Sin[c + d*x]]) + (4*Sin[c + d*x]^2)/(5*a^2*d*e^3*Sqrt[e*Csc[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sin^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{7}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sqrt{\sin(c+dx)}}\,dx}{a^4e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}-\frac{2a^2\cos^3(c+dx)}{\sqrt{\sin(c+dx)}}+\frac{a^2\cos^4(c+dx)}{\sqrt{\sin(c+dx)}}\right)dx}{a^4e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{a^2e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{\int \frac{\cos^4(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{a^2e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&\quad-\frac{2\int \frac{\cos^3(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{a^2e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{2\cos(c+dx)}{3a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a^2e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&\quad+\frac{6\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{7a^2e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}-\frac{2\text{Subst}\left(\int \frac{1-x^2}{\sqrt{x}}\,dx,x,\sin(c+dx)\right)}{a^2de^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}}+\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3a^2de^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&\quad+\frac{4\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{7a^2e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}-\frac{2\text{Subst}\left(\int \left(\frac{1}{\sqrt{x}}-x^{3/2}\right)dx,x,\sin(c+dx)\right)}{a^2de^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$= -\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}} + \frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}} + \frac{52\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{21a^2de^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{4\sin^2(c+dx)}{5a^2de^3\sqrt{e\csc(c+dx)}}$$

Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.55

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \frac{\sqrt{e\csc(c+dx)}\Big(-520\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+(-7$$

```
[In] Integrate[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Sqrt[e*Csc[c + d*x]]*(-520*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (-756 + 3
05*Cos[c + d*x] - 84*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqrt[Sin[c + d
*x]])*Sqrt[Sin[c + d*x]])/(210*a^2*d*e^4)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.86 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.84

method	result
default	$\frac{\sqrt{2}\left(130i\sqrt{i(-i-\cot(dx+c)+\csc(dx+c))}\,\sqrt{i(-\cot(dx+c)+\csc(dx+c))}\,\sqrt{-i(i-\cot(dx+c)+\csc(dx+c))}\ \text{EllipticF}\left(\sqrt{-i(i-\cot(dx+\right.\right.}{}$

```
[In] int(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105/a^2/d*2^(1/2)*(130*I*(-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-I-cot(
d*x+c)+csc(d*x+c)))^(1/2)*(I*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*EllipticF((-I*
(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^(1/2))*cos(d*x+c)+130*I*(-I*(I-cot(d
*x+c)+csc(d*x+c)))^(1/2)*(I*(-I-cot(d*x+c)+csc(d*x+c)))^(1/2)*(I*(-cot(d*x+
c)+csc(d*x+c)))^(1/2)*EllipticF((-I*(I-cot(d*x+c)+csc(d*x+c)))^(1/2),1/2*2^
(1/2))+15*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-42*cos(d*x+c)^2*2^(1/2)*sin(d*x+c
)+65*cos(d*x+c)*sin(d*x+c)*2^(1/2)-168*2^(1/2)*sin(d*x+c))/e^3/(e*csc(d*x+c
))^(1/2)/(cos(d*x+c)-1)^2/(cos(d*x+c)+1)^2*sin(d*x+c)^3
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.65

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \frac{2\left(\left(15\,\cos\left(dx+c\right)^3-42\,\cos\left(dx+c\right)^2+65\,\cos\left(dx+c\right)-16\right.\right.}{}$$

[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] 2/105*((15*cos(d*x + c)^3 - 42*cos(d*x + c)^2 + 65*cos(d*x + c) - 168)*sqrt
(e/sin(d*x + c))*sin(d*x + c) - 65*I*sqrt(2*I*e)*weierstrassPInverse(4, 0,
cos(d*x + c) + I*sin(d*x + c)) + 65*I*sqrt(-2*I*e)*weierstrassPInverse(4, 0
, cos(d*x + c) - I*sin(d*x + c)))/(a^2*d*e^4)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

[In] integrate(1/(e*csc(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\left(e\csc\left(dx+c\right)\right)^{\frac{7}{2}}\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)

Giac [F]

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\left(e\csc\left(dx+c\right)\right)^{\frac{7}{2}}\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{7/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(7/2)),x)
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(7/2)*(cos(c + d*x) + 1)^2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                  end if
              else #result contains complex but optimal is not
                  if debug then
                          print("result contains complex but optimal is not");
                  fi;
                  return "C","Result contains complex when optimal does not.";
              fi;
          else # result do not contain complex
               # this assumes optimal do not as well. No check is needed here.
              if debug then
                     print("result do not contain complex, this assumes optimal do not as well")
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  if debug then
                      print("leaf_count_result<=2*leaf_count_optimal");
                  fi;
                  return "A"," ";
              else
                  if debug then
                      print("leaf_count_result>2*leaf_count_optimal");
                  fi;
                  return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                         convert(ExpnType_result,string)," vs. order ",
                         convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```